

***FUNDAMENTALS OF STRUCTURAL ANALYSIS***

3<sup>rd</sup> Edition

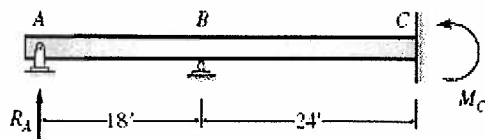
Kenneth M. Leet, Chia-Ming Uang, and Anne M. Gilbert

**SOLUTIONS MANUAL**

**CHAPTER 14:**

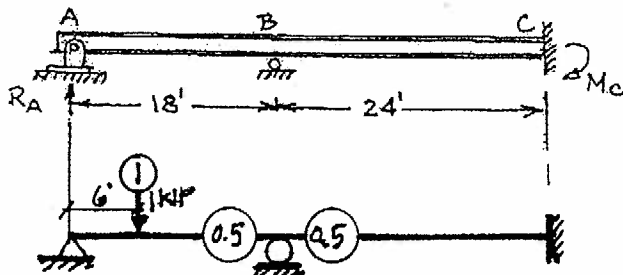
**INDETERMINATE STRUCTURES:**  
**INFLUENCE LINES**

**P14.1** Construct the influence lines for the vertical reaction at support *A* and the moment at support *C*. Evaluate the ordinates at 6 ft intervals of the influence line. *EI* is constant.



P14.1

LOAD @ ①:

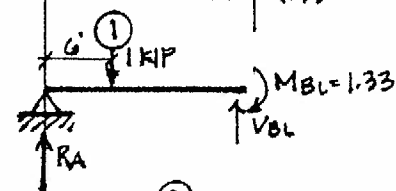


FEM	-2.67	+1.33	0	0
DEM	+2.67	COM → +1.33		
DEM		-1.33	-1.33	COM → -0.67
FINAL	0	+1.33	-1.33	-0.67 = Mc

FBD:  $\sum M_B = 0$

$$R_A(18') - 1k(12') + 1.33 = 0$$

$$R_A = +0.593$$



MOMENT DISTRIBUTION  
DISTRIBUTION FACTORS:

$$K_{AB} = \left(\frac{I}{18}\right)^3 = \frac{I}{24}$$

$$K_{BC} = \frac{I}{24}$$

$$DF_{BA} = 0.5$$

$$DF_{BC} = 0.5$$

LOAD @ ①:

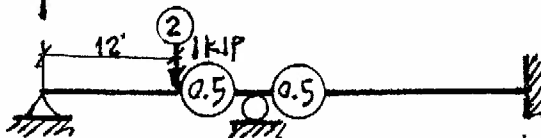
$$FEM_{AB} = -\frac{Pa^2b^2}{L^2} = -\frac{1(6)(12)^2}{(18)^2}$$

$$= -2.67$$

$$FEM_{BA} = \frac{Pa^2b}{L^2}$$

$$= +1.33$$

LOAD @ ②:

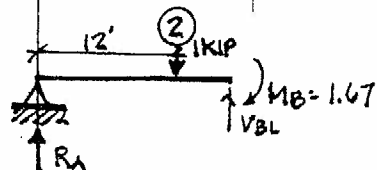


FEM	-1.33	+2.67	0	0
DEM	+1.33	COM → +0.67		
DEM		-1.67	-1.67	COM → -0.833
FINAL	0	+1.67	-1.67	-0.833 Mc

FBD:  $\sum M_B = 0$

$$R_A(18') - 1k(6') + 1.67 = 0$$

$$R_A = +0.241$$



LOAD @ ② MAGNITUDES

LOAD @ ③

$$FEM_{BC} = -\frac{PL}{8} = -3$$

$$FEM_{CB} = \frac{PL}{8} = +3$$

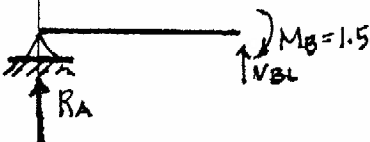
LOAD @ ③:

FEM			-3	+3
DEM		+1.5	+1.5	+0.75
	0	+1.5	-1.5	+3.75 = Mc

FBD:  $\sum M = 0$

$$R_A(18') + 1.5 = 0$$

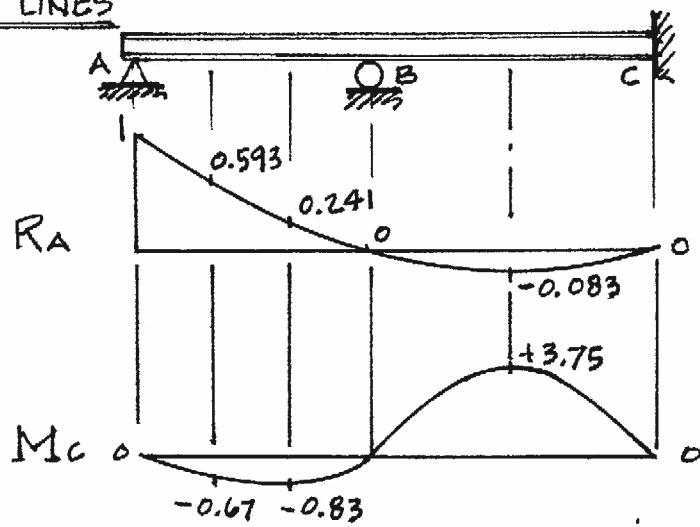
$$R_A = -0.083$$



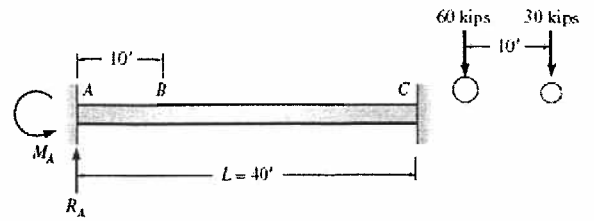
P14.1 Continued

P14.1 Continued

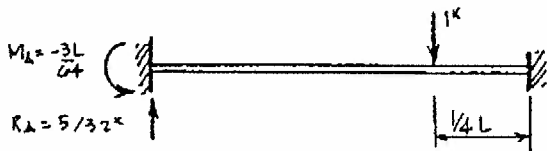
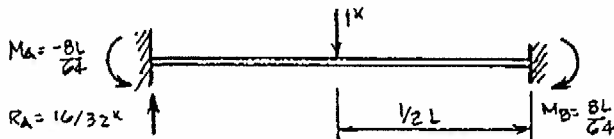
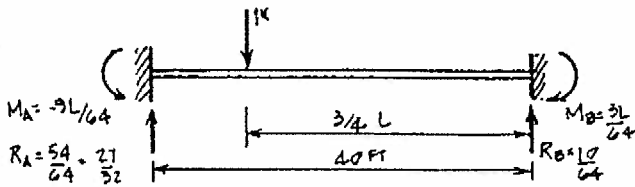
INFLUENCE LINES



**P14.2** (a) Using moment distribution, construct the influence lines for the moment and the vertical reaction  $R_A$  at support A for the beam in Figure P14.2. Evaluate the influence line ordinates at the quarter points of the span. (b) Using the influence lines for reactions, construct the influence line for moment at point B. Compute the maximum value of  $R_A$  produced by the set of wheel loads.



P14.2



$$M_A = FEM_{AB} = -Pab^2/L^2 = -9L/64$$

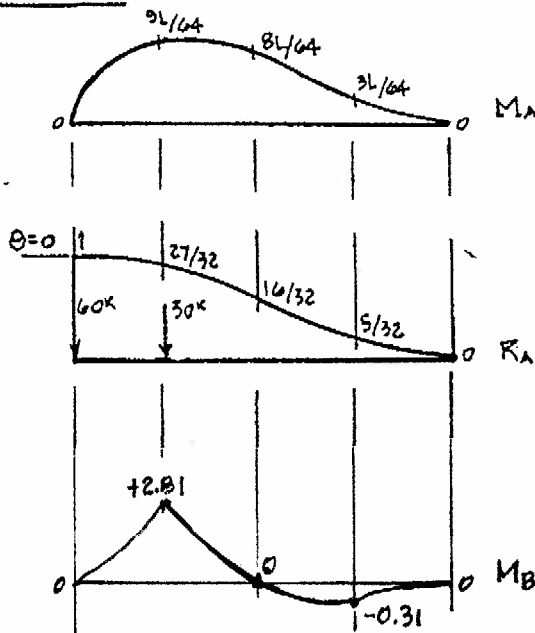
$$M_B = FEM_{BA} = Pba^2/L^2 = 3L/64$$

$$\sum M_B = 0$$

$$R_A L - 1(3/4 L) - 9L/64 + 3L/64 = 0$$

$$R_A = 54/64$$

INFLUENCE LINES



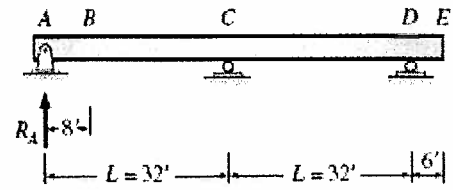
$$\text{Max } R_A = 60^k(1) + 30^k(27/32) = \underline{85.31^k}$$

$$\text{LOAD @ } 1/4 L: \frac{27}{32}(10) - \frac{9(40)}{64} = 2.81 \text{ FT}\cdot\text{K}$$

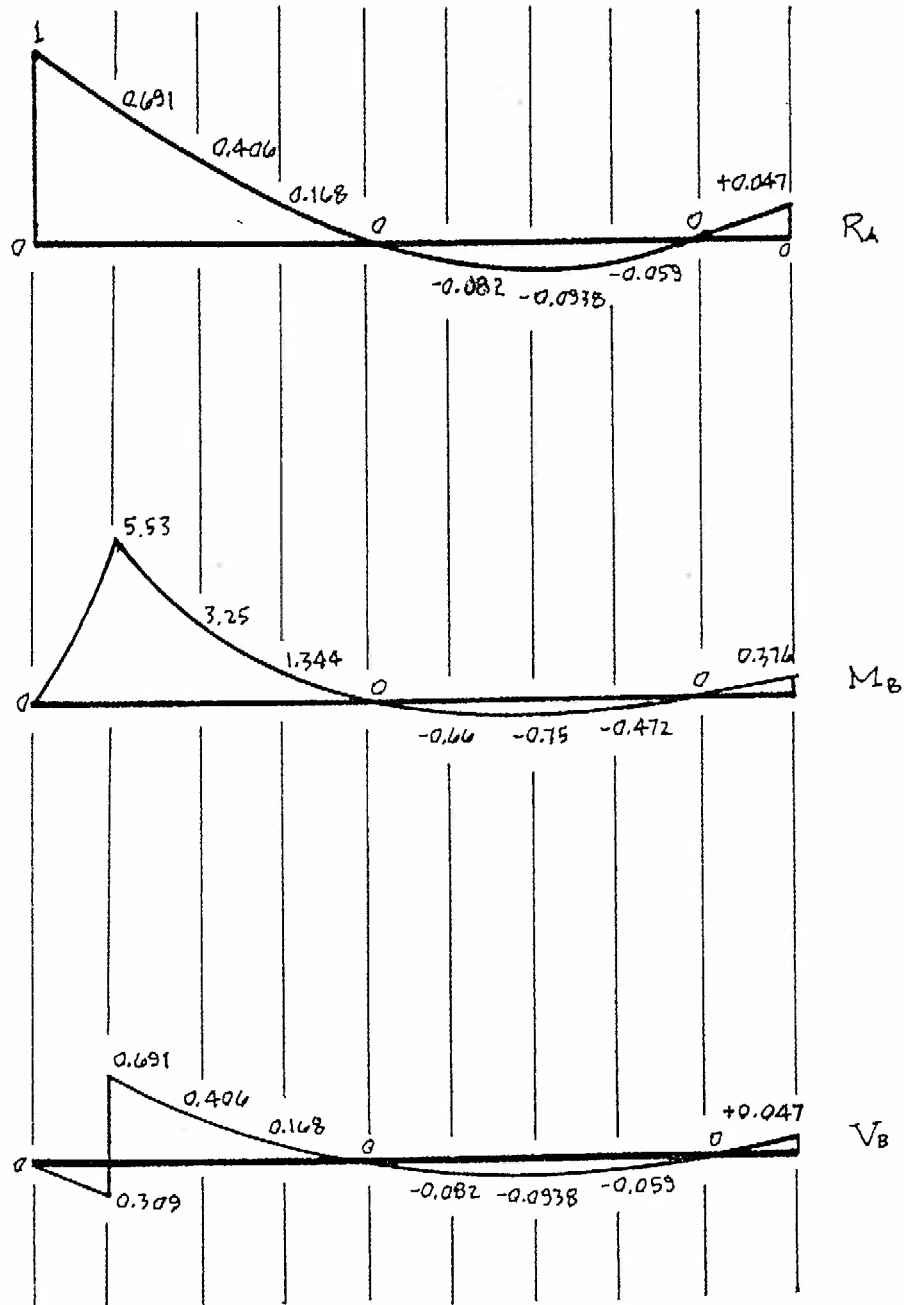
$$\text{LOAD @ } 1/2 L: \frac{16}{32}(10) - \frac{8(40)}{64} = 0$$

$$\text{LOAD @ } 3/4 L: \frac{5}{32}(10) - \frac{3(40)}{64} = -0.31 \text{ FT}\cdot\text{K}$$

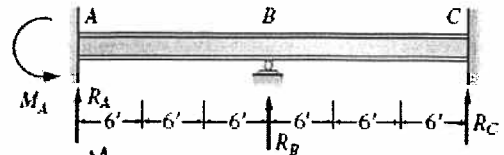
P14.3 Using moment distribution, construct the influence lines for the reaction at  $A$  and the shear and moment at section  $B$  (Figure P14.3). Evaluate influence line ordinates at 8-ft intervals in span  $AC$  and  $CD$  and at  $E$ .  $EI$  is constant.



P14.3

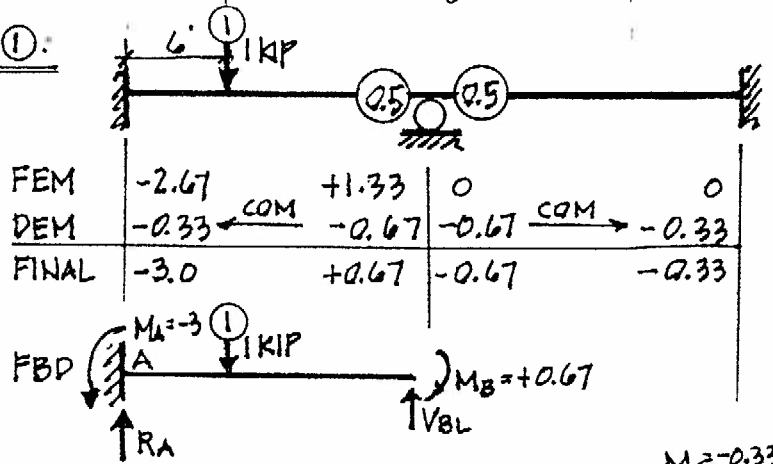


P14.4 Construct the influence lines for  $R_A$ ,  $R_B$ ,  $R_C$ , and the moments at supports A and B. Evaluate the ordinates at 6 ft intervals.  $EI$  is constant.

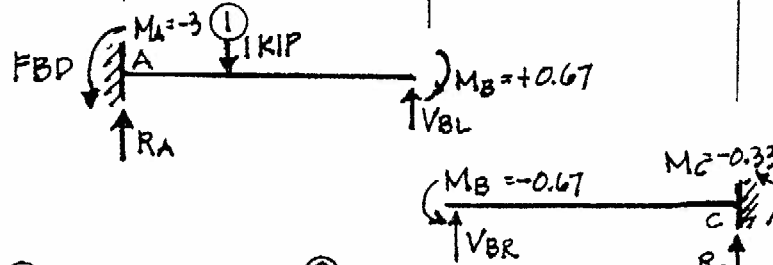


P14.4

LOAD @ ①:



FEM	-2.67	+1.33	0	0
DEM	-0.33	-0.67	-0.67	-0.33
FINAL	-3.0	+0.67	-0.67	-0.33



MOMENT DISTRIBUTION

LOAD @ ①:

$$FEM_{AB} = -\frac{Pab^2}{L^2} = -2.67$$

$$FEM_{BA} = +\frac{Pa^2b}{L^2} = +1.33$$

FBD: LEFT OF B:  $\sum M_B = 0$

$$R_A(18') - 1(12') - 3 + 0.67 = 0$$

$$R_A = 0.796 \uparrow$$

$\sum F_y = 0$ ;

$$V_{BL} + 0.796 - 1 = 0$$

$$V_{BL} = 0.204 \uparrow$$

FBD: RIGHT OF B:  $\sum M_B = 0$

$$-0.67 - 0.33 - R_C(18) = 0$$

$$R_C = -0.056$$

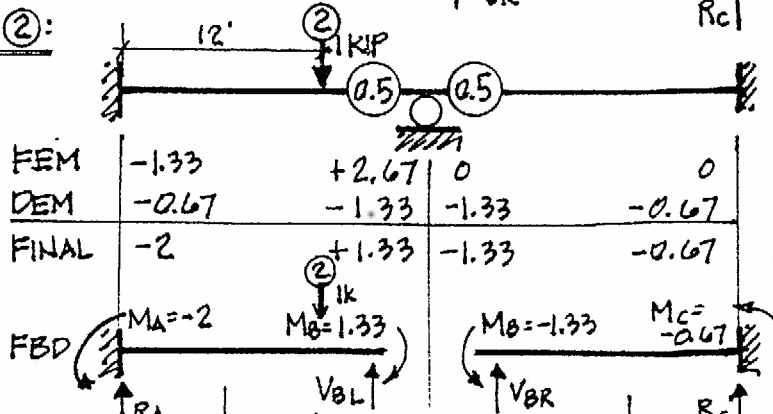
$\sum F_y = 0$

$$V_{BR} - 0.056 = 0$$

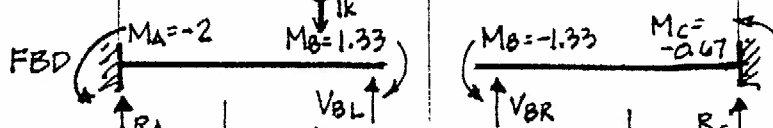
JT. B EQUILIBRIUM:

$$R_B = V_{BL} + V_{BR} = 0.26 \uparrow$$

LOAD @ ②:



FEM	-1.33	+2.67	0	0
DEM	-0.67	-1.33	-1.33	-0.67
FINAL	-2	+1.33	-1.33	-0.67



LOAD @ ②:

FBD: LEFT OF B:  $\sum M_B = 0$

$$R_A(18) - 2 - 1(6) + 1.33 = 0$$

$$R_A = 0.37 \uparrow$$

$\sum F_y = 0$ ;

$$V_{BL} + 0.37 - 1 = 0$$

$$V_{BL} = 0.63 \uparrow$$

FBD: RIGHT OF B:  $\sum M_B = 0$

$$-R_C(18) - 1.33 - 0.67 = 0$$

$$R_C = 0.111 \downarrow$$

$\sum F_y = 0$ ;

$$V_{BR} - R_C = 0$$

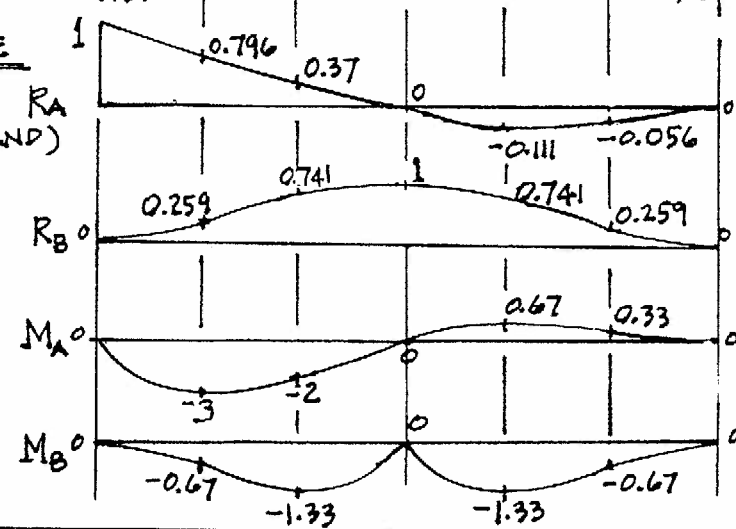
$$V_{BR} = 0.111 \uparrow$$

JT. B EQUILIBRIUM

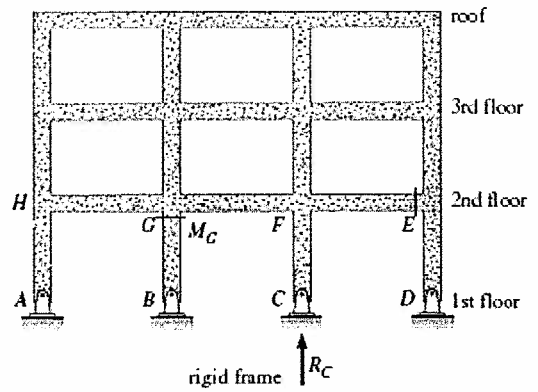
$$R_B = V_{BL} + V_{BR} = 0.741 \uparrow$$

INFLUENCE LINES

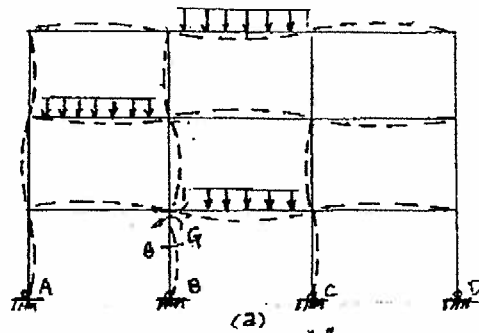
( $R_C$  OFF. HAND)



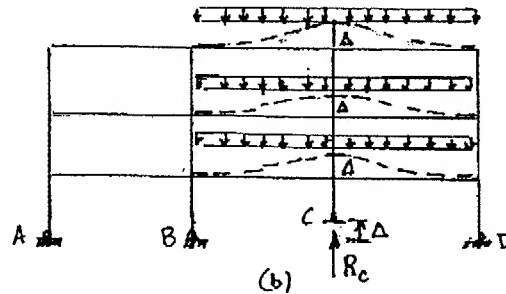
**P14.5** (a) Draw the qualitative influence lines for (1) the moment at a section located at the top of the first-floor column  $BG$  and (2) the vertical reaction at support  $C$ . Columns are equally spaced. (b) Indicate the spans on which a uniformly distributed load should be placed to maximize the moment on a section at the top of column  $BG$ . (c) Draw a qualitative influence line for negative moment on a vertical section through the floor beam at  $E$ .



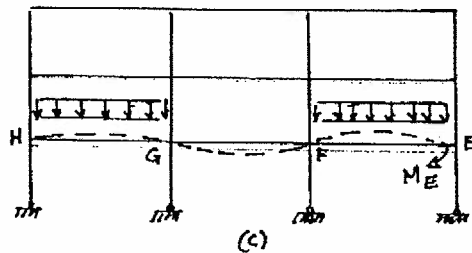
P14.5



NOTE: FOR MAX. MOMENT AT "G", LOAD ALTERNATE FLOORS ON EITHER SIDE OF COLUMN.

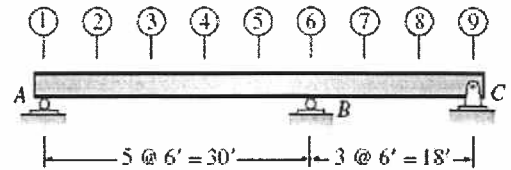


AXIAL LOAD AT SUPPORT "C"

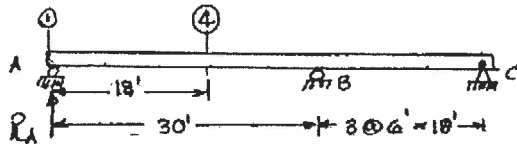


LOADING TO MAXIMIZE NEG. MOMENT AT E

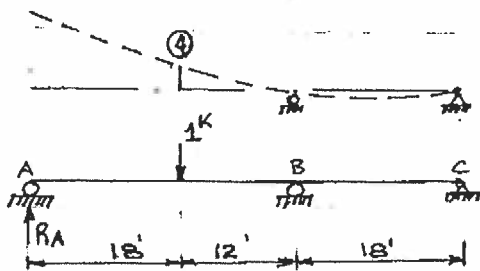
P14.6 (a) Draw a qualitative influence line for the reaction at support A for the beam in Figure P14.6. Using *moment distribution*, calculate the ordinate of the influence line at section 4. (b) Draw the qualitative influence line for the moment at B. Using the *conjugate beam or moment distribution method*, calculate the ordinate of the influence line at Section 8.  $EI$  is constant.



P14.6



(a) INF. LINE FOR  $R_A$



use Moment Distrib.

$$K_{AB} = \frac{I}{30} \times \frac{3}{4} = \frac{I}{40} = \frac{3I}{120}$$

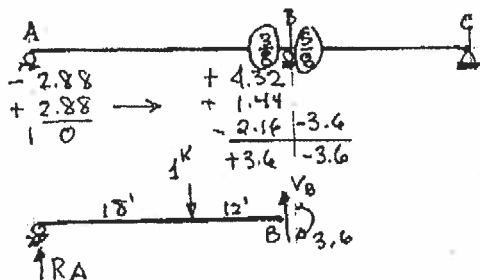
$$K_{BC} = \frac{I}{18} \times \frac{3}{4} = \frac{I}{24} = \frac{5I}{120}$$

$$\Sigma K_b = \frac{8I}{120}$$

D.F.
$\frac{3}{8}$
$\frac{5}{8}$

$$FEM_{AB} = -\frac{Pab^2}{L^2} = -\frac{1(18)(12)^2}{30^2} = -2.88$$

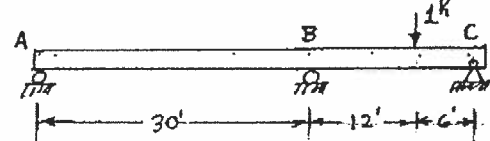
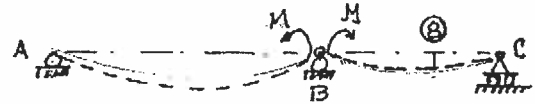
$$FEM_{BA} = \frac{Pba^2}{L^2} = \frac{1(12)(18)^2}{30^2} = 4.32 \text{ kip-ft}$$



$$\sum M_B = 0 = R_A 30 - 1 \times 12 + 3.6$$

$$R_A = 0.28 \text{ kips}$$

(b) QUALITATIVE INFLU. LINE FOR  $M_B$



$$FEM_B = \frac{Pab^2}{L^2} = -\frac{1(12)(6)^2}{18^2} = -1.333 \text{ k-ft}$$

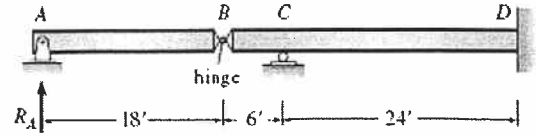
$$FEM_{CB} = \frac{Pba^2}{L^2} = \frac{1(6)(12)^2}{324} = 2.666 \text{ k-ft}$$

	$\frac{3}{8}$	$\frac{5}{8}$	
	-1.333	+2.666	
	-1.333	-2.666	
	+1	+1.666	
	+1	-1	

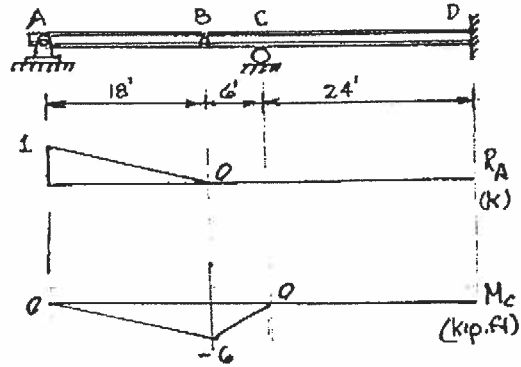
ORDINATE OF I.L. AT 8 = -1 kip-ft



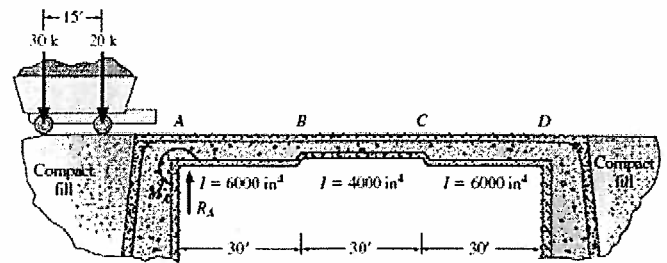
**P14.7** Construct the influence lines for  $R_A$  and  $M_C$  in Figure P14.7, using the Müller-Breslau method. Evaluate the ordinates at points A, B, C, and D.  $EI$  is constant.



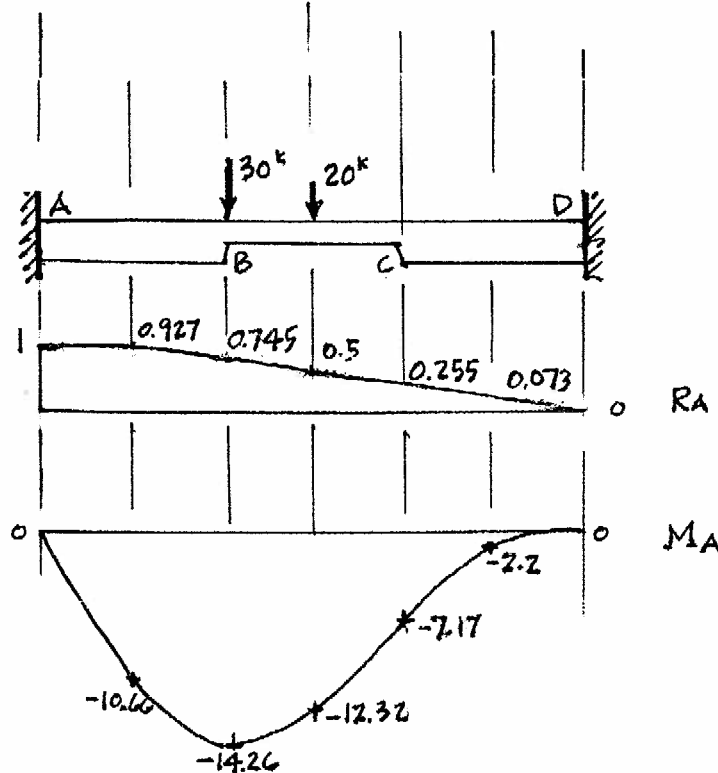
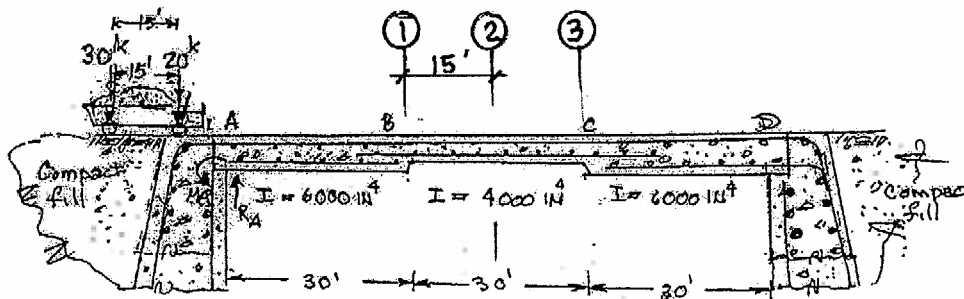
P14.7



**P14.8** Computer analysis of beam of varying depth.  
 The reinforced concrete bridge girder, attached to the massive end wall as shown in Figure P14.8, may be treated as a fixed-ended beam of varying depth.  
 (a) Construct the influence lines for the reactions  $R_A$  and  $M_A$  at support A. Evaluate the ordinates at 15-ft intervals. (b) Evaluate the moment  $M_A$  and the Vertical reaction  $R_A$  at end A produced by the loaded orecarrier when its 30-kip rear wheel is positioned at point B.  $E = 3000 \text{ kips/in}^2$ .



P14.8



$$R_A = 30^k(0.745) + 20^k(0.5) = \underline{\underline{32.35^k}}$$

$$M_A = 30^k(-14.26) + 20^k(-12.32) = \underline{\underline{674.2 \text{ Ft. k}}}$$