FUNDAMENTALS OF STRUCTURAL ANALYSIS

3rd Edition

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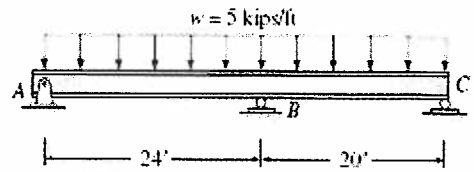
SOLUTIONS MANUAL

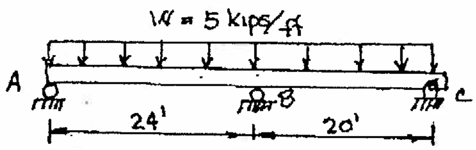
CHAPTER 15:

APPROXIMATE ANALYSIS OF INDETERMINATE STRUCTURES

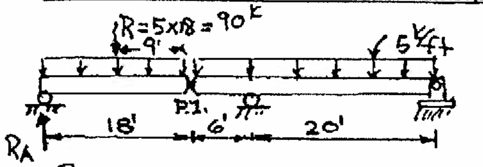
P15.1 Use an approximate analysis (assume the location of a point of inflection) to estimate the moment in the beam at support *B* (Figure P15.1). Draw the shear and moment curves for the beam. Check results by moment distribution or use the computer program. *EI* is constant.



P15.1

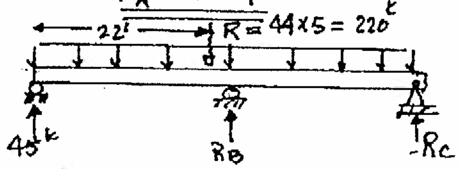


ASSUME P.I. 0.25L To left of support B



$\Sigma M_{P.I.} = R_A 18 - 90^k \times 9' = 0$

$R_A = 45$ kips



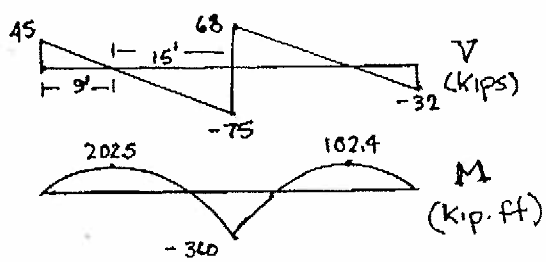
$\Sigma M_C = 0 = 45^k(44) + R_B 20 - 220(22)$

$R_B = 143^K$

$\uparrow \Sigma F_y = 0 = -220 + 45 + 143 + R_C$

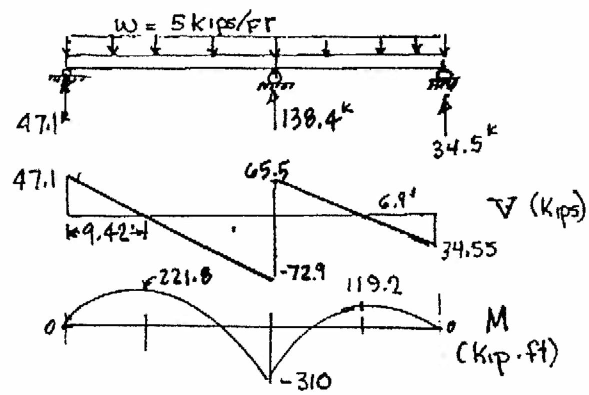
$R_C = 32$ Kips

RESULTS OF APPROX. ANALYSIS

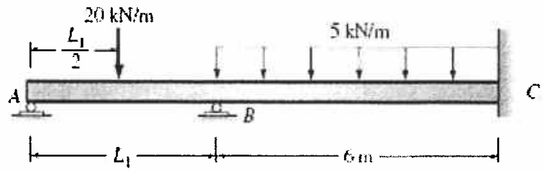


MOMENT DISTR. RESULTS

	$\frac{5}{11}$	$\frac{6}{11}$	
-240	+240	-166.7	+166.7
+240 →	+120	-83.33	-166.7
	-50	-60.00	
	+310	-310	0



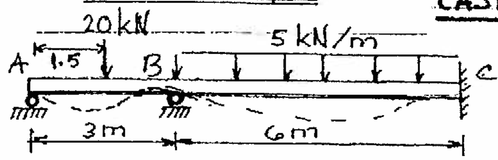
P15.2 Guess the location of the points of inflection in each span in Figure P15.2. Compute the values of moment at supports B and C, and draw the shear and moment curves. EI is constant.
Case 1: $L_1 = 3$ m
Case 2: $L_1 = 12$ m
Check your results by using moment distribution.



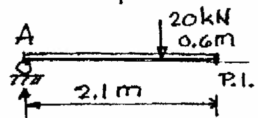
P15.2

CASE 1

APPROX. ANALYSIS



Assume P.I. at .3L to left of SUPPORT B. where .3L = 0.9m



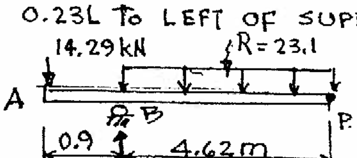
$R_A = 5.71$

$\Sigma M_{P.I.} = 0$

$0 = R_A 2.1 - 20 \times 0.6$

$R_A = 5.71$ kN

IN-SPAN BC Assume P.I located 0.23L To LEFT OF SUPPORT C; i.e. 1.38m



R_B

$\Sigma M_{P.I} = 0$

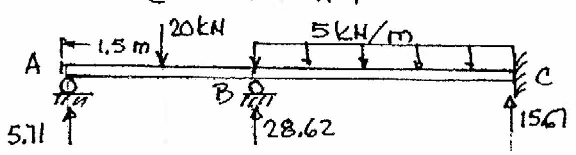
$0 = 14.29(5.52) + R_B 4.62 - 23.1 \times 2.31$

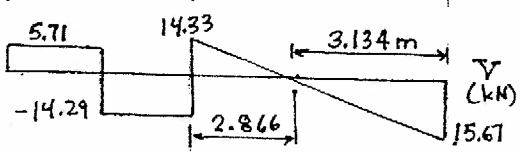
$R_B = 28.62$ kN

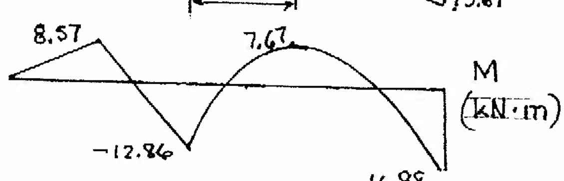
COMPUTE R_C; $+\uparrow \Sigma F_y = 0$. ENTIRE STRUC.

$5.71 - 20 + 28.62 - 30 + R_C = 0$

$R_C = 15.67$ kN $\uparrow$







EXACT ANALYSIS

ANALYSIS BY MOMENT DISTRIBUTION

FEM'S

$\frac{PL}{8} = \frac{20(3)}{8} = \pm 7.5$ kN·m

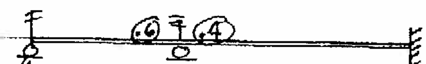
$\frac{WL^2}{12} = \frac{5(6)^2}{12} = \pm 15$ kN·m

D.F.

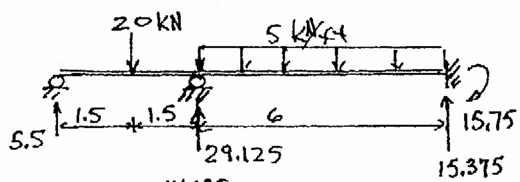
$K_{AB} = \frac{3}{4}\frac{I}{3} = I/4 = .25I$ 0.6

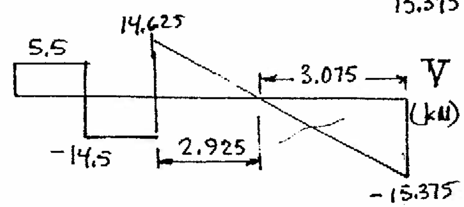
$K_{BC} = I/6 = 0.167I$ 0.4

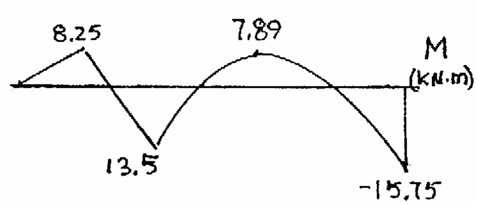
$\Sigma K_s = 0.417I$



A	BA	BC	C
-7.5	+7.5	-15	+15
+7.5	+3.75		
	2.25	+1.5	+.75
0	+13.5	-13.5	15.75





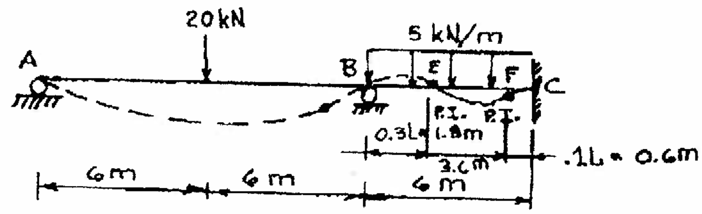


CONCLUSION: EXACT AND APPROX. ANALYSIS COMPARE CLOSELY.

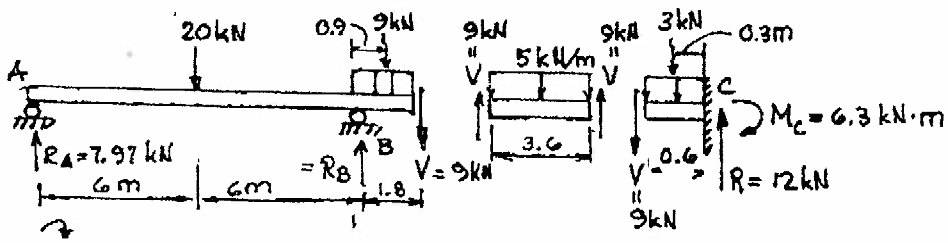
P15.2 Continued

P15.2 Continued

CASE 2 $L_1 = 12$ m

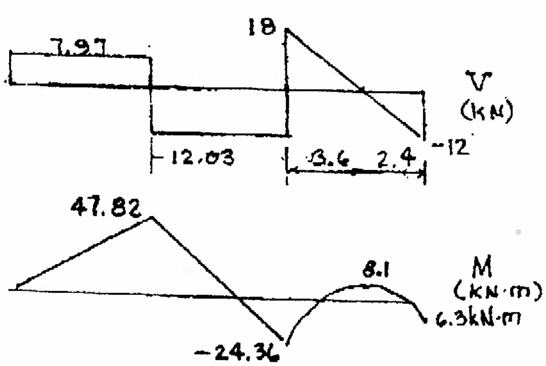


IN THIS CASE, BECAUSE OF THE LONGER LENGTH OF SPAN AB, A MUCH LARGER COUNTERCLOCKWISE MOMENT IS APPLIED MEMBER AB TO THE LEFT END OF MEMBER BC. THIS MOMENT ROTATES THE B-END OF MEMBER BC IN THE COUNTERCLOCKWISE DIRECTIONS CAUSING THE POINTS OF INFLECTIONS TO SHIFT TO THE RIGHT. WE WILL ASSUME THE P.I. ADJACENT TO B AT 0.3L AND THE ONE ON THE RIGHT AT 0.1L AS SHOWN ABOVE

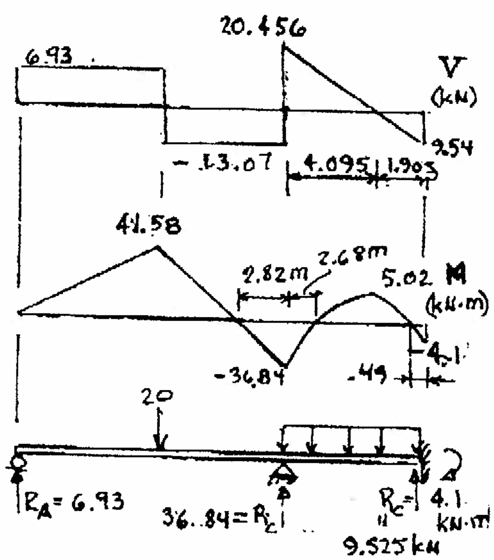


$\Sigma M_A = 0$: $20 \times 6 + 9 \times .9 + 9 \times 13.8 - R_B 12$

$R_B = 30.03$ kN



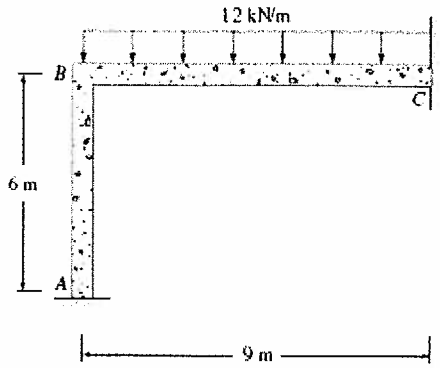
APPROXIMATE ANALYSIS



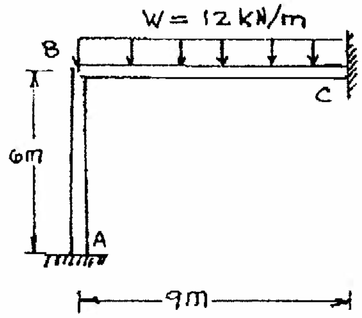
EXACT ANALYSIS BY MOMENT DIST.

ALTHOUGH DIFFERENCES EXIST BETWEEN THE MAGNITUDE OF FORCES IN THE TWO SOLUTIONS, THE ORDER OF MAGNITUDE IN CORRESPONDING SPANS IS SIMILAR AND ONE CAN HAVE CONFIDENCE THE EXACT ANALYSIS IS CORRECT.

P15.3 Assume values for member end moments and compute all reactions in Figure P15.3 based on your assumption. Given: EI is constant. If $I_{BC} = 8I_{AB}$, how would you adjust your assumptions of member end moments?



P15.3



If BOTH ENDS OF GIRDER BC ARE FIXED: $FEM = \frac{WL^2}{12} = \frac{12(9)^2}{12}$

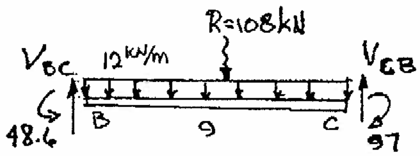
$\underline{FEM = 81 \text{ kN·m}}$

SEE FIG 15.6: $M_{BC} = 0.6\,FEM = \underline{-48.6 \text{ kN·m}}$

$M_{BA} = -M_{BC} = \underline{+48.6 \text{ kN·m}}$

$M_{AB} = \frac{1}{2} M_{BA} = \underline{+24.6 \text{ kN·m}}$

Assume $M_{CB} = 1.2\,FEM = 1.2 \times 81 = 97 \text{ kN·m}$



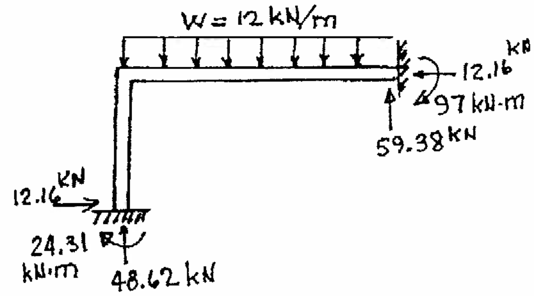
V_{CB}: $\Sigma M_B = 0$

$108 \times \frac{9}{2} + 97 - 48.6 - V_{CB}\,9 = 0$

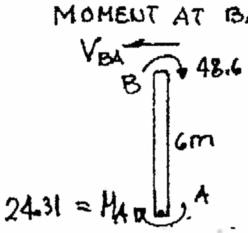
$V_{CB} = \underline{59.38 \text{ kips}}$

$+\uparrow \Sigma F_y = 0 = V_{BC} + 59.38 - 108$

$V_{BC} = \underline{48.62 \text{ kN}}$



MOMENT AT BASE OF COLUMN $= \frac{1}{2}(48.6)$



$\Sigma M_A = 0 = 48.6 + 24.31 - V_B\,6$

$\underline{V_{BA} = 12.16 \text{ kN}}$

$R_{AX} = V_{BA} = 12.16 \text{ kN}$

$\uparrow R_{AY} = V_{BC} = 48.62 \text{ kN}.$

P15.3 Continued

P15.3 Continued

For $I_{BC} = 8I_{AB}$, then $\dfrac{I_C/L_C}{I_G/L_G} = \dfrac{1/6}{8/9} = 0.1875$

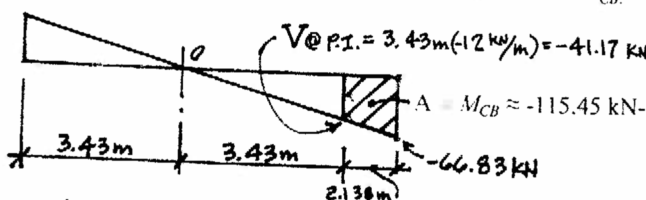
Read in **Fig. 15.6, Case A,** $M_{BC} \approx 0.15$ FEM $= 0.15(-81\text{kN-m}) = -12.2$ kN-m

and $M_{AB} = ½M_{BA}$, where $M_{BA} = -M_{BC}$
$M_{AB} = ½(12.2\text{kN-m}) = 6.1$ kN-m

The column ($EI_C/L_C = 0.167$) is flexible compared with the girder ($EI_G/L_G = 0.89$), resulting in $M_{BC} = -12.2$kN-m, which is about 10% of $wL^2/8$. The column's flexibility causes the moment M_{CB} at the fixed support C to approach $-wL^2/8$, and causes the distance of girder's left $P.I.$ to approach joint B and right $P.I.$ to approach $0.25L$ from the fixed support at C. Using Fig. 15.1d, estimate the location of the girder's right $P.I.$ to be about 95% of $0.25L$, we then can estimate M_{CB}.

Approximate location of $P.I.$ from the fixed support at $C \approx (0.95)0.25(9m) = 2.138m$
And estimate the shear diagram based on the approximated $P.I.$, we then estimate M_{CB}.

Approximate V Diagram for estimating M_{CB}



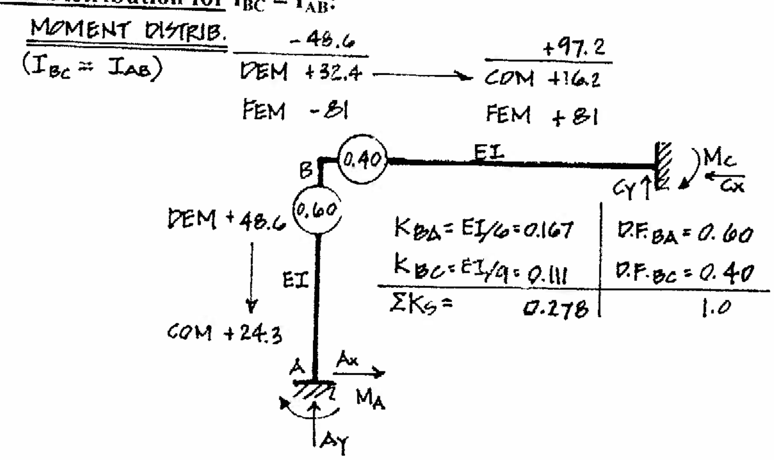
Evaluate member BC using statics, results for the approximate analyses for $I_{BC} = 8I_{AB}$ are:

$M_{AB} = 6.1$ kN-m
$M_{CB} = -115.45$ kN-m
Joint B, $M_B = -12.2$kN-m

$A_X = 3.1$ kN, $C_X = -3.1$ kN
$A_Y = 42.5$ kN
$C_Y = 65.5$ kN

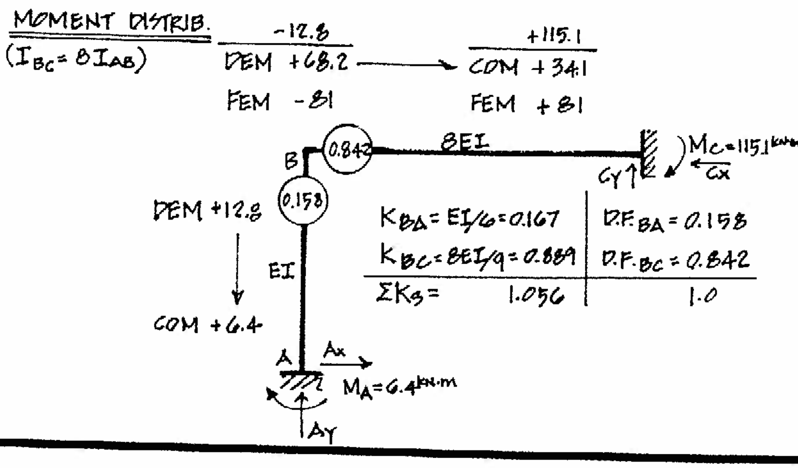
Check Approximate Analyses using Moment Distribution.

Moment Distribution for $I_{BC} = I_{AB}$:



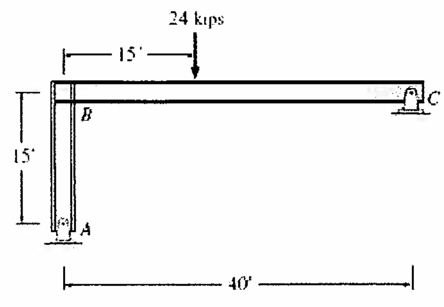
REMAINING REACTIONS BY STATICS
MA = +24.3 kN·m
Ax = +12.2 kN →
AY = +48.6 kN ↑
Mc = +97.2 kN·m
Cx = -12.2 kN ←
Cy = +59.4 kN ↑

Moment Distribution for $I_{BC} = 8I_{AB}$:

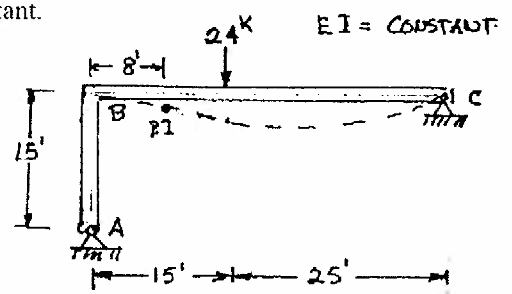


REMAINING REACTIONS BY STATICS
MA = +6.4 kN·m
Ax = +3.2 kN →
AY = +42.63 kN ↑
Mc = +115.1 kN·m
Cx = -3.2 kN ←
Cy = +65.4 kN ↑

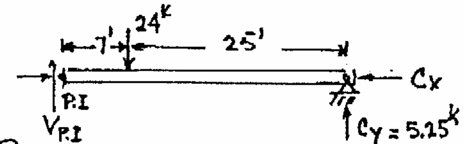
P15.4 Assuming the location of the point of inflection in the girder in Figure P15.4, estimate the moment at *B*. Then compute the reactions at *A* and *C*. Given: *EI* is constant.



P15.4



ASSUME THE POINT of inflection is $.2L = 8'$ FROM JOINT B.

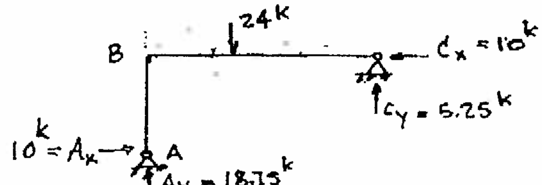


$\curvearrowright \Sigma M_{P.I.} = 0 = 24^K \times 7 - C_Y(32)$

$C_Y = 5.25^K$

$\uparrow^+ \Sigma F_Y = 0 = V_{P.I} + 5.25 - 24$

$V_{P.I.} = 18.75^K$



$\curvearrowleft^+ \Sigma M_A = 0 = 24 \times 15 - 5.25 \times 40 - C_x 15$

$C_x = 10\text{ k}$

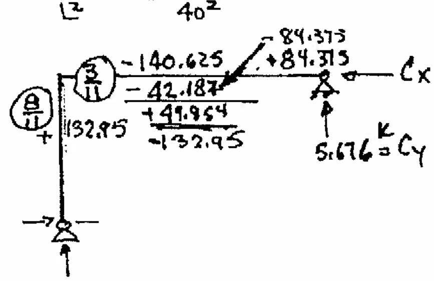
$\uparrow^+ \Sigma F_Y = 0 = A_Y - 24 + 5.25$

$A_Y = 18.75^K$

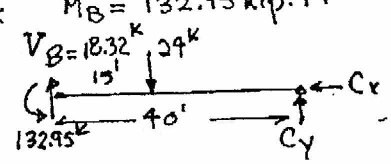
$M_B = 5.25(40) - 24 \times 15 = 150 \text{ kip}\cdot\text{ft}$

EXACT VALUE BY MOMENT DISTRIBUTION

$FEM_{BC} = \dfrac{Pb^2a}{L^2} = \dfrac{24(25)^2 15}{40^2} = -140.625^{\,k\cdot ft} \qquad FEM_{CB} = \dfrac{Pba^2}{L^2} = \dfrac{24(25)15^2}{40^2} = 84.375^{\,k\cdot ft}$



$M_B = 132.95 \text{ kip}\cdot\text{ft}$



$\curvearrowright^+ \Sigma M_B = 0 = 24 \times 15 - C_Y 40 - 132.95$

$C_Y = 5.676 \text{ Kips}$

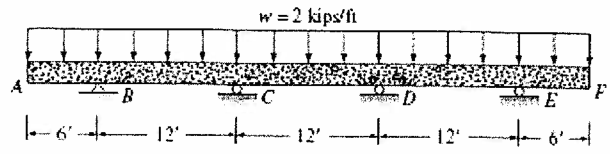
LOCATION P.I. $\quad 132.95 - 18.32 x = 0$

$x = 7.26'$ from B

$\curvearrowright^+ \Sigma M_A = 0 = 24 \times 15 - C_x 15 - 5.676 \times 40$

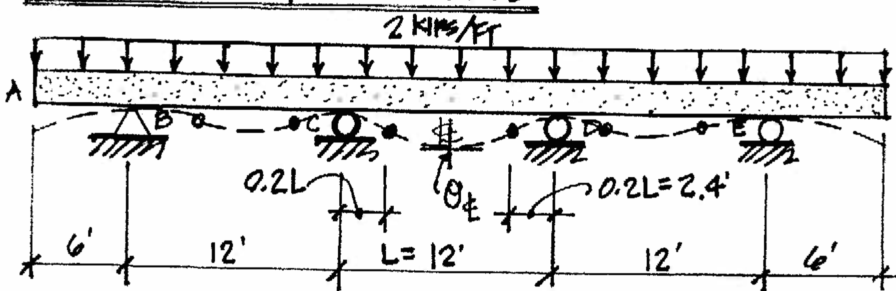
$C_x = 8.864$

P15.5 Estimate the moment in the beam in Figure P15.5 at support *C* and the maximum positive moment in span *CD* by guessing the location of one of the points of inflection in span *CD*.



P15.5

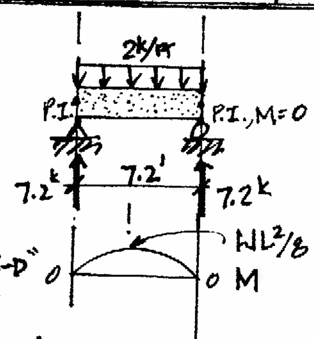
APPROXIMATE DEFLECTED SHAPE



SYMMETRIC STRUCTURE W/SYMMETRIC LOADS, $\theta_{℄} = 0$

ASSUME P.I. OCCUR 0.2L FROM SUPPORTS C & D:

FREEBODY MIDSPAN PORTION BETW. HINGES

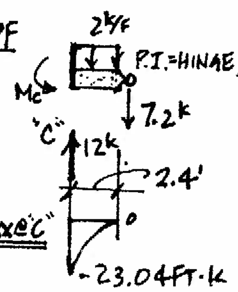


M^{+}_{MAX} "C-D"

$$wL^2/8 = \frac{2^{k}/ft(7.2')^2}{8} = +12.96\ ft\cdot k$$

(IS EQUIVALENT TO $wL^2/22.2$)

FREEBODY LEFT OF HINGE TO "C"



M^{-}_{MAX} @ "C"

$$-M_C = \frac{2^{k}(2.4')^2}{2} + 7.2^{k}(2.4')$$

$$M_C = -23.04\ ft\cdot k$$

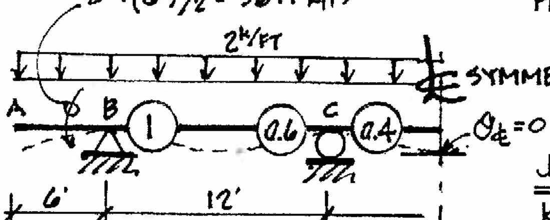
(EQUIV. TO $wL^2/12.5$)

P15.5 Continued

P15.5 Continued

Check Approximate Analysis using Moment Distribution:

$2^{k/ft}(6')^2/2 = 36$ FT·KIPS



$FEM_{BC} = -FEM_{CB} = WL^2/12 = 24$ FT·K

JOINT C

$K_{CB} = 3/4(EI/12') = 0.0625$ | D.F. $_{CB}$ = 0.60

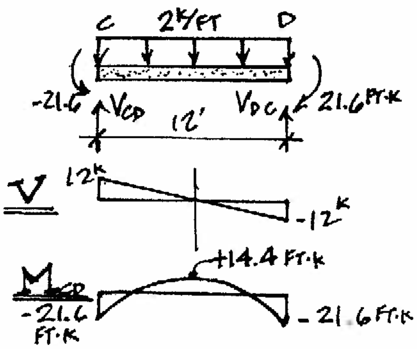
$K_{CD} = 1/2(EI/12') = 0.0417$ | D.F. $_{CD}$ = 0.40

0.1042 | 1.0

FEM	36	-24	+24	-24
DEM		-12		
COM			-6	
DEM			+3.6	+2.4
FINAL	36	-36	+21.6	-21.6

$\therefore M_C = -21.6$ FT·KIPS

SPAN "CD":



$\Sigma M_C = 0$,

$-21.6 + 21.6 + 2^{k/'}(12)^2/2 - V_{CD}(12') = 0$

$V_{DC} = 12^k \uparrow$

$V_{CD} = 12^k \uparrow$

$+M_{MAX_{CD}} = +14.4$ FT·KIPS

Compare Approximate Analysis with Moment Distribution:

+max M @ midspan on *CD* = +14.4 ft-kips is approximately 10% higher than the approximate analysis (12.96 ft-kips), and is equivalent to $wL^2/20$.

M @ support *C* = -21.6 ft-kips is approximately 6.7% lower than the approximate analysis (23.04 ft-kips), and is equivalent to $wL^2/13.3$.

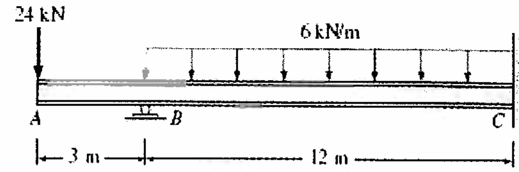
Actual location of P.I. from supports *C* and *D*:

14.4 ft-kips = $2^{k/ft}(L')^2/8$

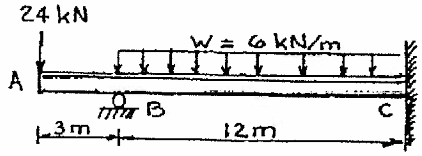
L' = 7.59 ft = 0.632 L

Thus, Points of Inflection are 0.184L ≈ 2.21ft. from supports *C* and *D*.

P15.6 Estimate the moment at support C in Figure P15.6. Based on your estimate, compute the reactions at B and C.



P15.6

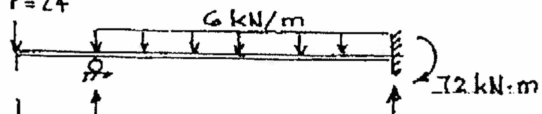


$FEM_{BC} = \frac{wL^2}{12} = \frac{6(12)^2}{12} = 72$ kN·m

M_{BA} IN CANTILEVER $= 3 \times 24 = 72$ kN·m

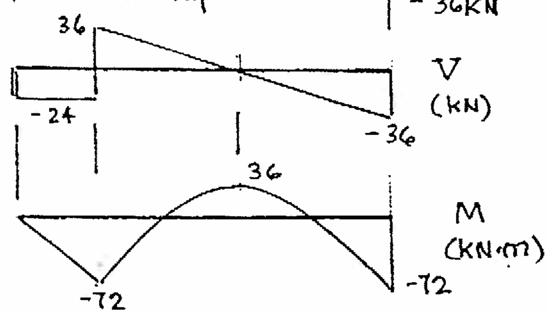
SINCE MOMENTS ON JOINT B ARE EQUAL AND OPPOSITE, JOINT DOES NOT ROTATE ∴ SEGMENT BC ACTS LIKE A FIXED-ENDED BEAM.

ASSUME M_{CB} = FEM = 72 kN·m

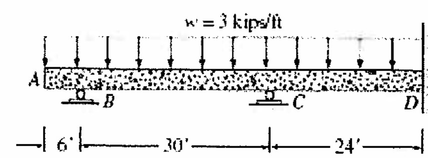


$R_A = P + \frac{wL}{2}$
$= 24 + \frac{6(12)}{2}$
$= 60$ kips

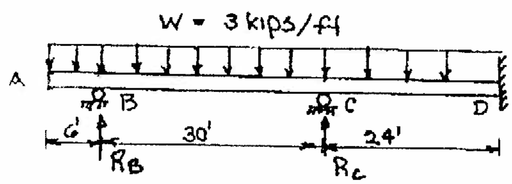
$R = \frac{wL}{2}$
$= \frac{6(12)}{2}$
$= 36$ kN



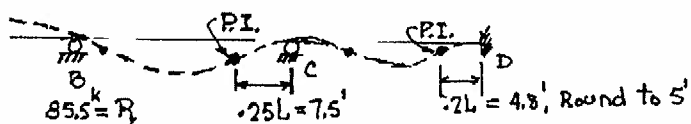
P15.7 The beam is indeterminate to the second degree. Assume the location of the minimum number of points Of inflection required to analyze the beam. Compute all reactions and draw the shear and moment diagrams. Check your results, using moment distribution.

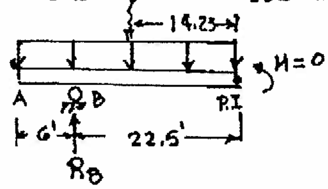


P15.7



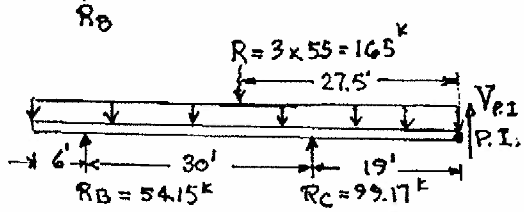
SINCE STRUCTURE INDET TO 2°,
WE MUST GUESS LOCATION OF TWO P.I.'s :





$\Sigma M_{P.I.} = 0 = R_B\, 22.5 - 85.5(14.25)$

$\underline{R_B = 54.15^k}$



$\Sigma M_{P.I} = 0$

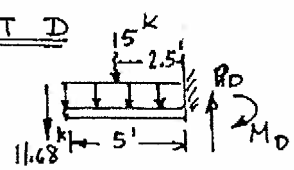
$0 = 54.15(49) - 165(27.5) + R_C 19$

$\underline{R_C = 99.17^k}\uparrow$

$+\uparrow \Sigma F_y = 0 = 54.15 + 99.17 - 165 + V_{P.I}$

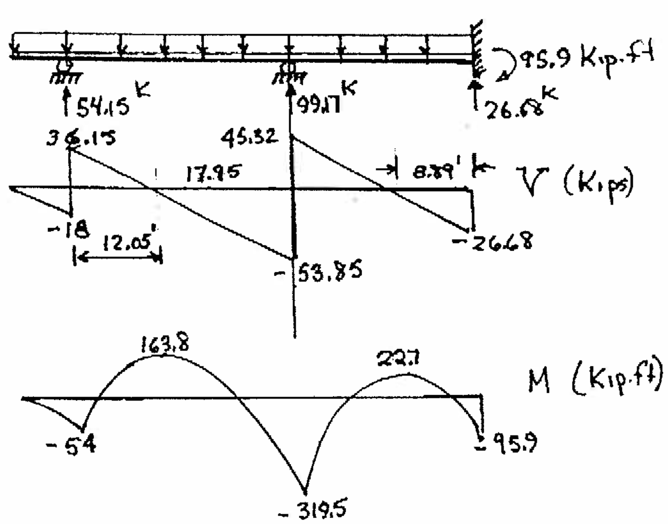
$\underline{V_{P.I} = 11.68^k}\uparrow$

V & M AT D



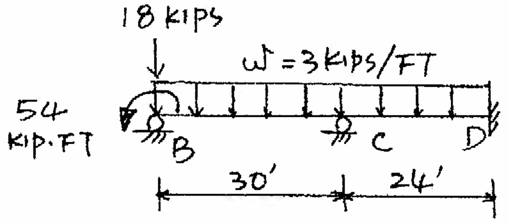
$R_D = 15^k + 11.68 = \underline{26.68^k}\uparrow$

$M_D = 11.68 \times 5 + 15 \times 2.5 = \underline{95.9\text{ kip}\cdot\text{ft}}$



P15.7 Continued

SOLVE BY MOMENT DISTRIBUTION.



$K_{BC} = \frac{3}{4}\left(\frac{EI}{30}\right)$

$K_{CD} = \frac{EI}{24}$

$DF_{CB} = \frac{3}{8}$

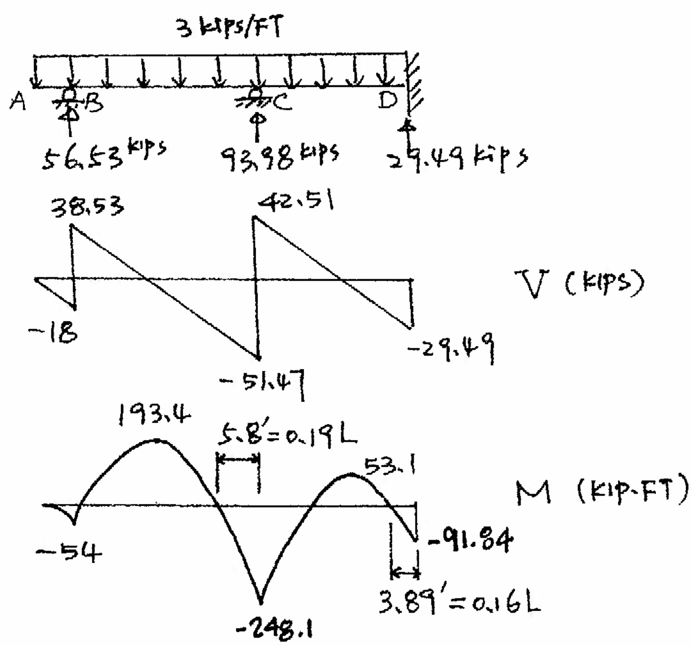
$DF_{CD} = \frac{5}{8}$

$FEM_{BC} = -FEM_{CB} = \frac{-3(30)^2}{12} = -225$ KIP·FT

$FEM_{CD} = -FEM_{DC} = \frac{-3(24)^2}{12}$

$= -144$ KIP·FT

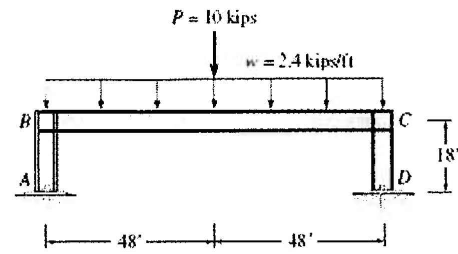
	1	3/8	5/8	0
54	-225	+225	-144	+144
	+171 →	85.5		
		-62.4	-104.1 →	-52.1
+54	-54	+248.1	-248.1	+91.9



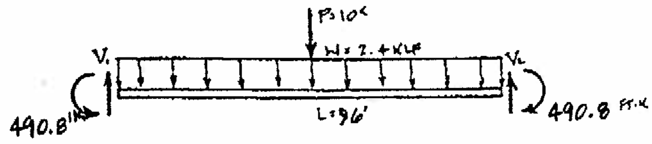
COMMENTS:

1. THE SHEAR DIAGRAM IS CLOSE TO THE EXACT VALUES.
2. THE MOMENT DIAGRAM DEVIATES MORE FROM THE EXACT SOLUTION. THE P.I. IN SPAN CD IS 0.16L FROM D RATHER THAN 0.2L. THE P.I. IN SPAN BC IS 0.19L TO THE LEFT OF C RATHER THAN 0.25L ASSUMED.

P15.8 The frame in Figure P15.8 is to be constructed with a deep girder to limit deflections. However, to satisfy architectural requirements, the depth of the columns will be as small as possible. Assuming that the moments at the ends of the girder are 25 percent of the fixed-ended moments, compute the reactions and draw the moment curve for the girder.



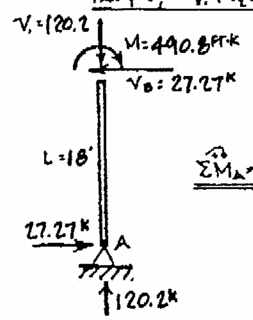
P15.8



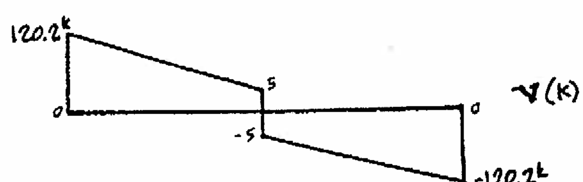
$$FEM = \frac{WL^2}{12} + \frac{PL}{8} = \frac{2.4(96)^2}{12} + \frac{10(96)}{8} = 1963^{FT\cdot K}$$

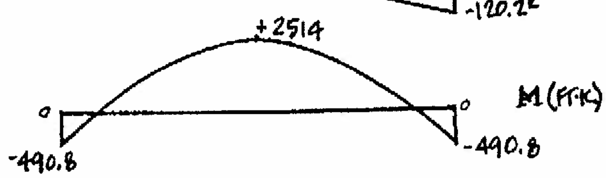
GIRDER END MOMENTS: $0.25\,(FEM) = 490.8^{FT\cdot K}$

$\uparrow \Sigma F_Y = 0$; $V_1 + V_2 - 10 - 2.4(96) = 0$; BY SYMMETRY $V_1 = V_2 = 120.2^k$

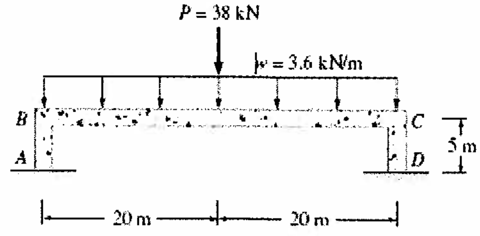


$\Sigma M_A = 0$ $\quad 18V_B - 490.8 = 0$ $\quad V_B = 27.27^k$

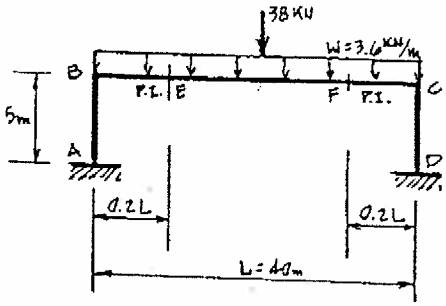




P15.9 The cross sections of the columns and girder of the frame in Figure P15.9 are identical. Carry out an approximate analysis of the frame by estimating the location of the points of inflection in the girder. The analysis is to include evaluating the support reactions and drawing the moment curves for column AB and girder BC.



P15.9

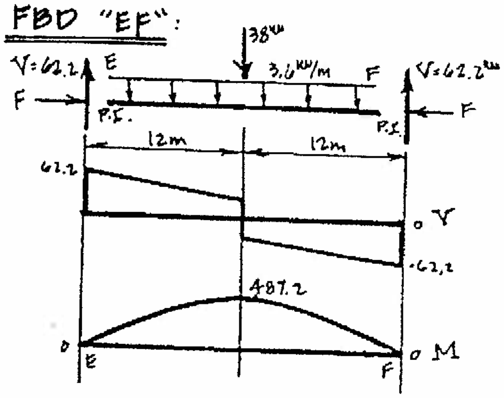


APPROXIMATE ANALYSIS OF FRAME

SINCE COLUMNS ARE STIFF RELATIVE TO THE GIRDER, ASSUME P.I. IN GIRDER ARE LOCATED 0.2 L FROM ENDS

$0.2L = 0.2(40\text{ m}) = 8\text{ m}$

FBD "EF":

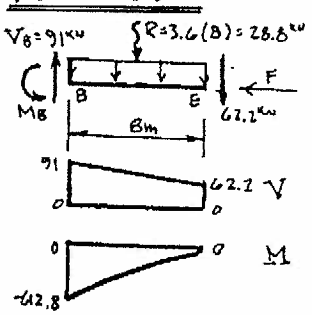


FBD OF GIRDER BETWEEN P.I.'S E-F

BY SYMMETRY V = 1/2 LOAD

$V = 38/2 + 3.6(12\text{ m}) = 62.2^{\text{kN}}$

FBD "BE":

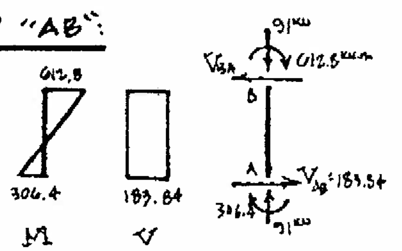


FBD OF GIRDER BETWEEN B AND P.I.

ΣM_B $\quad M_B = 62.2(8) + 28.8(4) = 612.8\text{ kN·m}$

$\uparrow\Sigma F_y$ $\quad V_B = 28.8 + 62.2 = 91\text{ kN}$

FBD "AB":



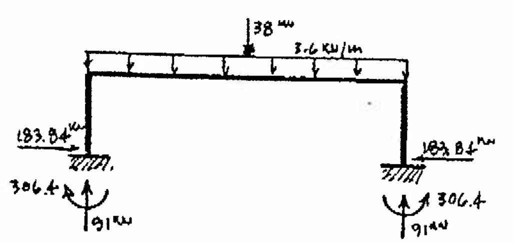
FBD OF COLUMN

BASED ON C.O. FACTOR OF 1/2, ASSUME $M_A = \frac{1}{2}M_B = 306.4$ kN·m

SHEAR IN COLUMNS

$\Sigma M_A = 0;\quad 612.8 + 306.4 - V_{BA}(5) = 0$

$V_{BA} = V_{AB} = 183.84$ kN



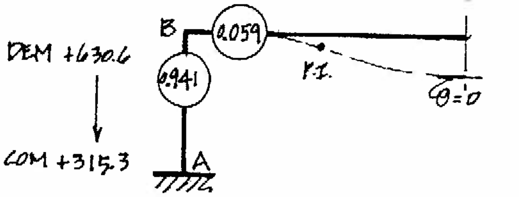
REACTIONS

P15.9 Continued

P15.9 Continued : **Check Approximate Analysis using Moment Distribution:**

FINAL −630.6
DEM +39.41
FEM −670

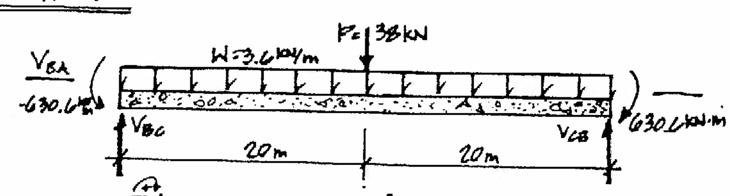
$K_{BA} = EI/5m = 0.20$
$K_{BC} = \frac{1}{2}(EI/40) = 0.0125$
$\Sigma K_i = 0.2125$



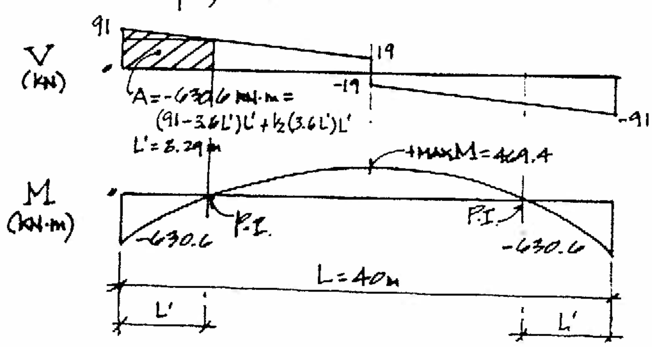
D.F. BA = 0.941
D.F. BC = 0.059

$FEM_{BC} = \frac{WL^2}{12} + \frac{PL}{8} = -670\ kN\cdot m$

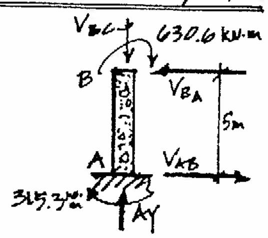
FBD GIRDER "BC":



$\Sigma M_B = 0;\ 38(20) + 3.6\frac{(40)^2}{2} - V_{CB}(40)$
$-630.6 + 630.6 = 0$
$V_{CB} = 91\ kN\uparrow$
$+\uparrow\Sigma F_Y = 0;\ V_{BC} = 91\ kN\uparrow$



FBD COLUMNS "AB", "CD":



$\Sigma M_A = 0;\ 630.6 + 315.3 - V_{BA}(5m) = 0$
$V_{BA} = 189.2\ kN\leftarrow$
$V_{AB} = 189.2\ kN \rightarrow = A_X$
$+\uparrow\Sigma F_Y = 0;\ A_Y = V_{BC} = 91\ kN\uparrow$

Final Reactions:

M_A = 315.3 kN-m ↻	M_D = 315.3 kN-m ↺
A_Y = 91kN↑	C_Y = 91kN↑
A_X = 189.2 kN→	C_X = - 189.2kN←

Compare Approximate Analysis with Moment Distribution:

+max M @ midspan *BC* = +469.4 kN-m is about 3.8% lower than the approximate analysis (487.2 kN-m).

M_A* and *-M_D = 315.3 kN-m is about 2.8% higher than the approximate analysis (306.4 kN-m), and similarly ***M_B* and *M_C*** = -630.6 kN-m is about 2.8% higher than the approximate analysis (612.8 kN-m).

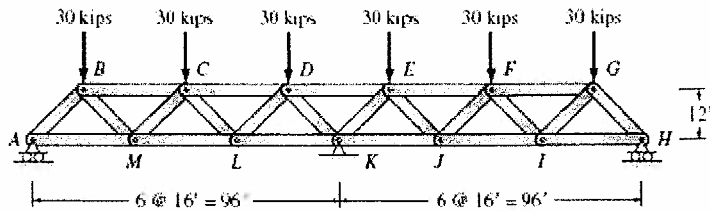
Location of P.I. from supports *B* and *C*:

630.6 kN-m = [91kN − (3.6$^{kN/m}$)L'](L') + ½ (3.6$^{kN/m}$)(L')^2

Thus, Points of Inflection are L' = 8.29 m = 0.207L from supports *B* and *C*.

This is within 3.5% of the approximate analysis at 0.2L

P15.10 Carry out an approximate analysis of the truss in Figure P15.10 by treating it as a continuous beam of constant cross section. As part of the analysis, evaluate the forces in members *DE* and *EF* and compute the reactions at *A* and *K*.

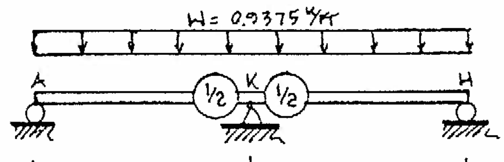


P15.10

REPLACE TRUSS BY AN EQUIVALENT UNIFORMILY LOADED BEAM.

EQN. 1.3: $W = \frac{\Sigma P_H}{L} = \frac{30^K(3)}{96'} = 0.9375^K/FT$

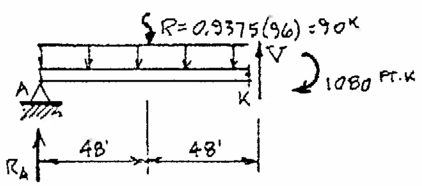
(EQUIV. UNIFORM LOAD = $0.9375^K/FT$)



ANALYZE BY MOMENT DISTRIBUTION

FEM = $WL^2/12 = 0.9375\,(96)^2/12 = 720$ FT·K

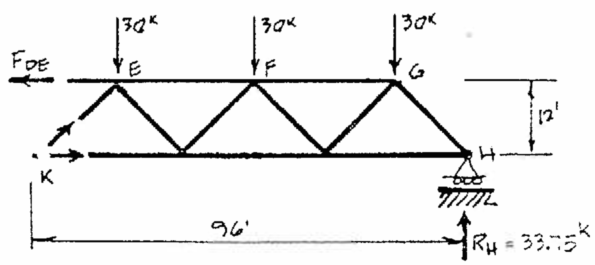
-720	+720	-720	+720
+720 →	+360	-360	← -720
0	1080	-1080	0



$\Sigma M_K = 0;$ $R_A(96) - 90(48) + 1080$ $R_A = 33.75^K$

$\uparrow \Sigma F_Y = 0;$ $V = 90 - 33.75$ $V = 56.25^K$

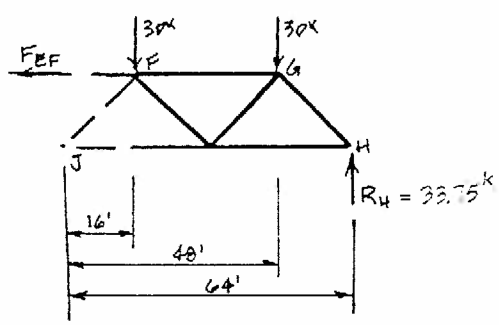
$R_K = 2V$ $R_K = 112.50^K$



COMPUTE F_{DE}

$\Sigma M_K = 0;$ $30(16) + 30(18) + 30(80) - 33.75(96) - F_{DE}(12)$

$F_{DE} = 90^K$ TENSION

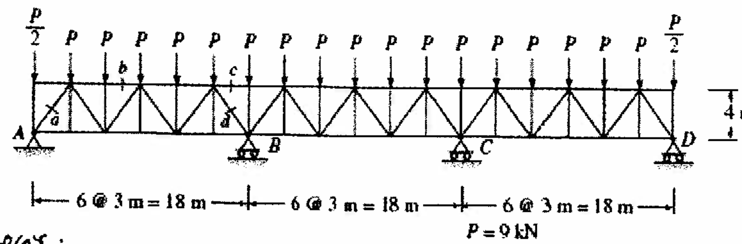


COMPUTE F_{EF}

$\Sigma M_J = 0;$ $30(16) + 30(48) - 33.75(64) - F_{EF}(12) = 0$

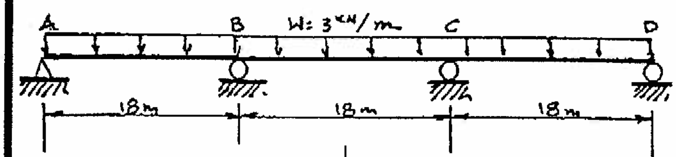
$F_{EF} = -20^K$ COMPRESSION

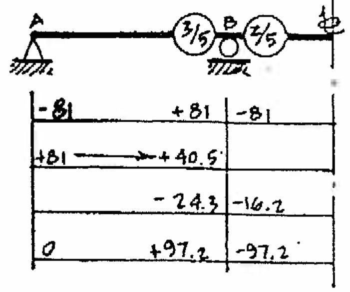
P15.11. Use an approximate analysis of the continuous truss in Figure P15.11 to determine the reactions at *A* and *B*. Also evaluate the forces in bars *a*, *b*, *c*, and *d*. Given: $P = 9$ kN.



P15.11

ANALYSIS BY BEAM ANALOGY;
EQUIVALENT UNIFORM LOAD $= W = \frac{18P}{54} = 3^{kN}/m$

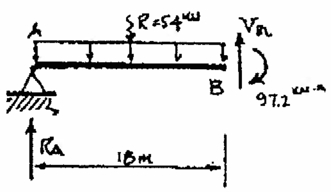




$FEM = WL^2/12 = \frac{3(18)^2}{12} = 81^{kN\cdot m}$

$K_{AB} = \frac{3}{4}\left[\frac{I}{18}\right] = \frac{I}{24}$ D.F. = 0.6

$K_{BC} = \frac{1}{2}\left[\frac{I}{18}\right] = \frac{I}{36}$ D.F. = 0.4

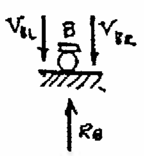


$\overset{\curvearrowright}{\Sigma M_B} = 0;\ R_A(18) - 54(9) + 97.2 = 0$

$R_A = 21.6^{kN}$

$+\!\uparrow \Sigma F_y = 0;\ V_{BL} + 21.6 - 54 = 0$

$V_{BL} = 32.4^{kN}$

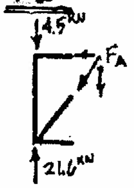


$V_{BL} = 32.4^{kN}$

$V_{BR} = WL/2 = 27^{kN}$

$R_B = V_{BL} + V_{BR} = 59.4^{kN}$

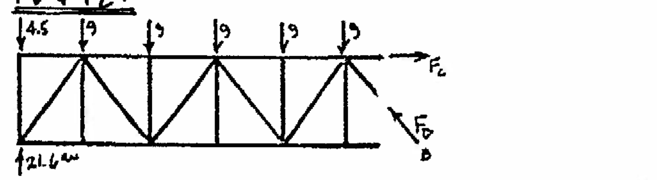
Fa:



$\Sigma F_y = 0;\ F_{Ay} = 21.6 - 4.5 = 17.1^{kN}$

$F_A = \frac{5}{4} F_{Ay} = 21.38^{kN}$ COMPRESSION

F_D & F_C:



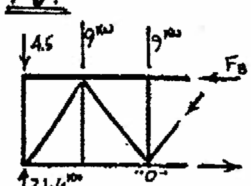
$+\!\uparrow \Sigma F_y = 0;\ 21.6 - 4.5 - 9(5) + F_{Dy} = 0;\ F_{Dy} = 27.9^{kN};\ F_D = \frac{5}{4} F_{Dy} = 34.88^{kN}$

$\Sigma M_B = 0;\ (21.6 - 4.5)18 - 9(15) - 9(12) - 9(9) - 9(6) - 9(3) + F_C(4) = 0$ $F_C = 24.3^{kN}$ TENSION

(F_C ALT. BY BEAM ANALOGY: $F_C = M_B/h$

$F_C = 97.2/4\text{m} = 24.3^{kN}$.)

Fb:

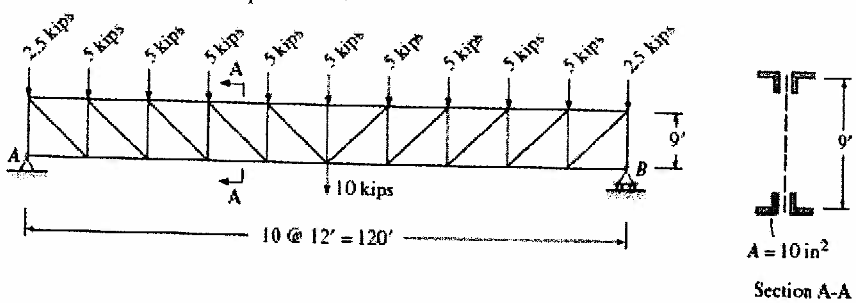


$\overset{\curvearrowright}{\Sigma M_b} = 0$

$(21.6 - 4.5)6 - 9(3) - F_b(4) = 0$

$F_b = 18.9^{kN}$ COMPRESSION

P15.12. Estimate the deflection at midspan of the truss in Figure P15.12, treating it as a beam of constant cross section. The area of both the top and bottom chords is 10 in^2. E = 29,000 kips/in^2. The distance between the centroids of the top and bottom chords equals 9 ft.



P15.12

TREAT TRUSS AS A uniformly loaded beam

$$\text{LOAD/ft. } W = \frac{\Sigma P}{L} = \frac{5^k \times 9 + 2.5^k \times 2}{120\text{ ft}} = 0.417 \text{ kips/ft}$$

I of truss considering flanges ONLY.

$$I = Ad^2 \times 2 \quad \text{where } A = 10\text{ in}^2,\ d = \frac{9'}{2} = 4.5 = 54''$$

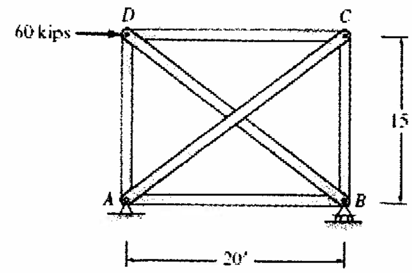
$$I = 10 \times (54)^2 \times 2 = 58{,}320 \text{ in}^4$$

$$\Delta_{\mathfrak{L}} = \frac{5WL^4}{384EI} = \frac{5(0.417)(120)^4 1728}{384 \times 29{,}000 \times 58320} = 1.15 \text{ in}$$

DOUBLE Δ to ACCOUNT FOR CONTRIBUTION OF WEB MEMBERS

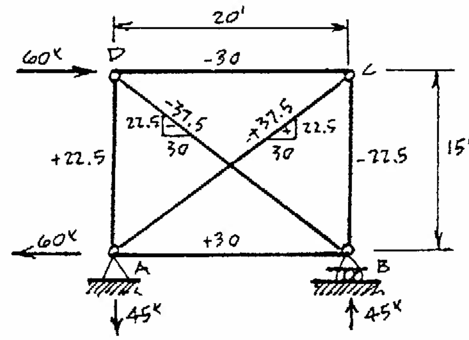
ESTIMATED DEFL.: $\Delta = 2 \times 1.15 = \underline{\underline{2.3 \text{ in}}}$

P15.13. Determine the approximate values of force in each member of the truss in Figure P15.13. Assume that the diagonals can carry either tension or compression.

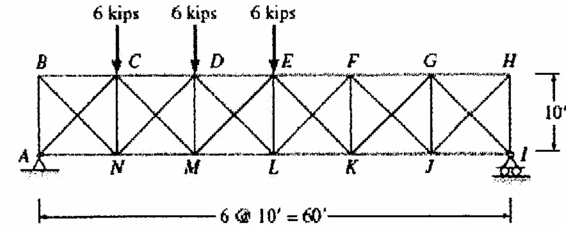


P15.13

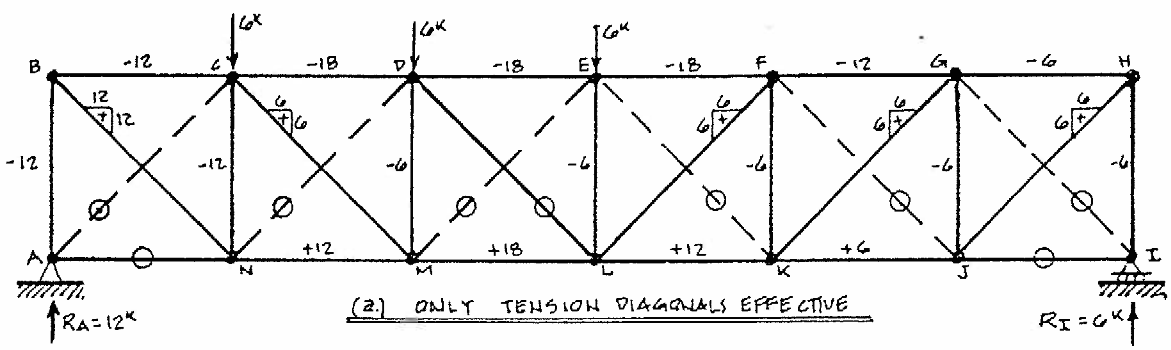
ASSUME SHEAR OF 60^K DIVIDES EQUALLY BETWEEN DIAGONALS



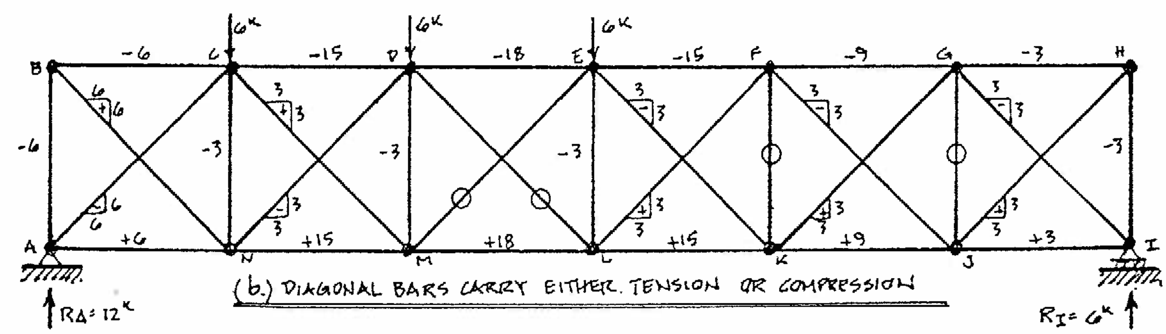
P15.14. Determine the approximate values of bar force in the members of the truss in Figure P15.14 for the following two cases.
(*a*) Diagonal bars are slender and can carry only tension.
(*b*) Diagonal bars do not buckle and may carry either tension or compression.



P15.14

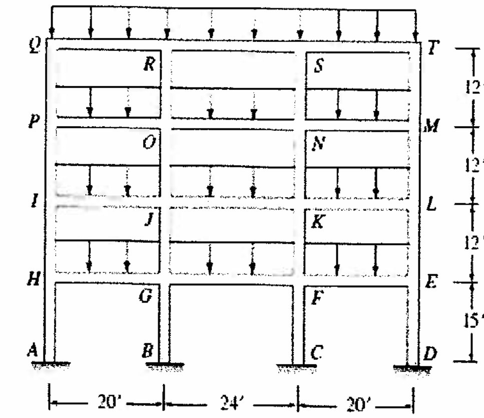


(a.) ONLY TENSION DIAGONALS EFFECTIVE



(b.) DIAGONAL BARS CARRY EITHER TENSION OR COMPRESSION

P15.15. (*a*) All beams of the frame in Figure P15.15 have the same cross section and carry a uniformly distributed gravity load of 3.6 kips/ft. Estimate the approximate value of axial load and the moment at the top of columns *AH* and *BG*. Also estimate the shear and moment at each end of beams *IJ* and *JK*. (*b*) Assuming that all columns are 12 in square (I = 1728 in^4) and the moment of inertia of all girders equals 12,000 in^4, carry out an approximate analysis of the second floor by analyzing the second-floor beams and the attached columns (above and below) as a rigid frame.



P15.15

(a.) ESTIMATE AXIAL LOAD AND MOMENT IN COLUMN AH.

AXIAL LOAD: ASSUME 0.45 WL TRANSFERRED TO EXTERIOR COLUMN FROM EACH OF THE 4 FLOOR BEAMS.

$$P_{AH} = 4(0.45\,WL) = 4(0.45(3.6\times 20)) = 129.6^K$$

MOMENT: MOMENT TO TOP OF COLUMN CREATED PRIMARILY BY LOAD ON GIRDER HG. ASSUME MOMENT AT END OF GIRDER HG ≈ 0.5 FEM OF WHICH 45% GOES TO BOTTOM COLUMN AH.

$$M_{HA} = 0.45\left(0.50\,\frac{WL^2}{12}\right) = 0.45\left(0.50\left(\frac{3.6(20)^2}{12}\right)\right) = 27 \text{ FT}\cdot\text{K}$$

ESTIMATE AXIAL LOAD AND MOMENT IN COLUMN BG.

AXIAL LOAD: ASSUME LOAD OF 0.55 WL FROM EXTERIOR GIRDERS AND 0.50 WL FROM INTERIOR GIRDERS.

$$P_{BG} = (0.55\,WL + 0.50\,WL')(4) = [0.55(3.6\times 20) + 0.50(3.6\times 24)]4 = 331.2^K$$

MOMENT: SINCE THE SPANS OF THE GIRDERS ARE NEARLY THE SAME ON EACH SIDE THE GIRDERS APPLY MOMENTS THAT ARE APPROXIMATELY EQUAL AND OPPOSITE TO EACH OTHER; THEREFORE, THE MOMENT CREATED AT THE TOP OF THE COLUMN IS VERY SMALL, SAY $M_{GB} = 0$.

BEAM IJ

$M_{IJ} \cong 0.50 \text{ FEM} = 0.50\,\frac{WL^2}{12} = 60^{\text{FT}\cdot\text{K}}$ $\qquad V_{IJ} \cong 0.45(20'\times 3.6) = 32^K$

$M_{JI} \cong WL^2/9 = 3.6(20)^2/9 = 160 \text{ FT}\cdot\text{K}$ $\qquad V_{JI} \cong 0.55(20'\times 3.6) = 40^K$

BEAM JK

SINCE INTERIOR BEAM AND SPANS ARE ABOUT THE SAME THEN:

$$M_{JK} = M_{KJ} = \frac{WL^2}{12} = \frac{3.6(24)^2}{12} = 172.8 \text{ FT}\cdot\text{K}$$

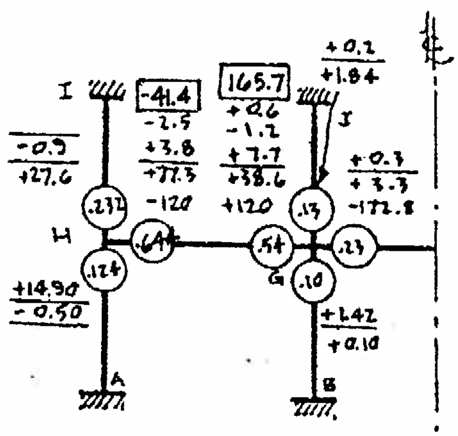
$$V \text{ @ EACH END} = 0.50\,WL = 0.50(3.6)24 = 43.2^K$$

P15.15 Continued

P15.15 Continued

(b.) APPROXIMATE ANALYSIS FOR IMPROVED VALUES OF MOMENT.

ANALYSIS OF 2ND FLOOR AND ATTACHED COLUMNS. ANALYSIS OF 1/2 OF STRUCTURE DUE TO SYMMETRY.



$$FEM_{HG} = \frac{WL^2}{12} = \frac{3.6(20)^2}{12} = 120 \text{ ft·k}$$

$$FEM_{GF} = \frac{WL^2}{12} = \frac{3.6(24)^2}{12} = 172.8 \text{ ft·k}$$

JOINT H

$K_{IH} = \frac{1728(1.5)}{12} = 216$ $D.F._{IH} = 0.232$

$K_{HA} = 1728/15 = 115.2$ $D.F._{HA} = 0.124$

$K_{HG} = 12000/20 = 600$ $D.F._{HG} = 0.644$

$\Sigma K_i = 931.2$

JOINT G

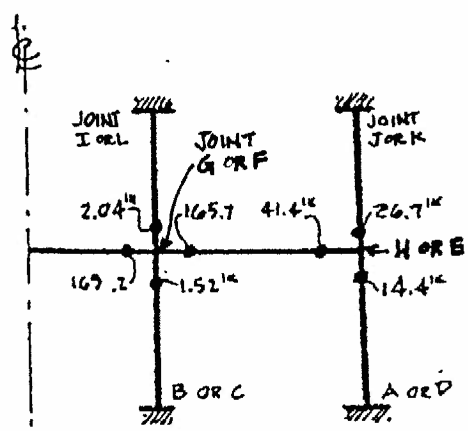
$K_{JG} = 1728/12 = 144$ $D.F._{JG} = 0.13$

$K_{GB} = 1728/15 = 115.2$ $D.F._{GB} = 0.10$

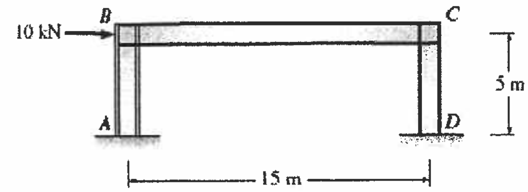
$K_{GH} = 12000/20 = 600$ $D.F._{GH} = 0.54$

$K_{GF} = \frac{1}{2}\frac{12000}{24} = 250$ $D.F._{GF} = 0.23$

$\Sigma K_i = 1109.2$



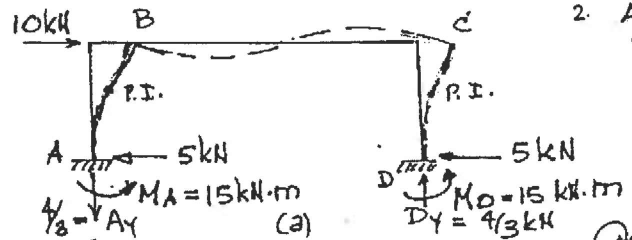
P15.16. (*a*) Use an approximate analysis to compute the reactions and draw the moment curves for column *AB* and girder *BC* in Figure P15.16. (*b*) Repeat the computations if the base of the columns connects to hinged supports at *A* and *D*. *EI* is constant for all members.



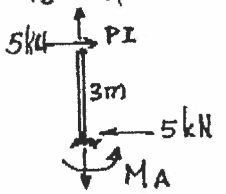
P15.16

(a.) FIXED SUPPORTS

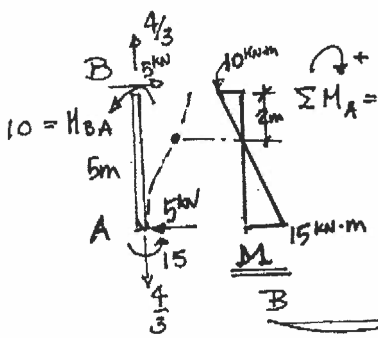
1. ASSUME SHEAR IN COLUMNS ARE EQUAL.
2. ASSUME P.I. IN COLUMN LOCATED $0.6h = 3m$ above BASE.



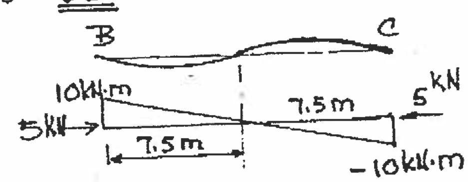
$$\Sigma M_A = 0 = 10 \times 5 - 15 - 15 - D_Y 15$$
$$D_Y = \frac{20}{15} = 4/3 \text{ kN}$$



$$\Sigma M_{P.I.} = 0 = 5 \times 3 - M_A$$
$$\underline{M_A = 15 \text{ kN·m}}$$

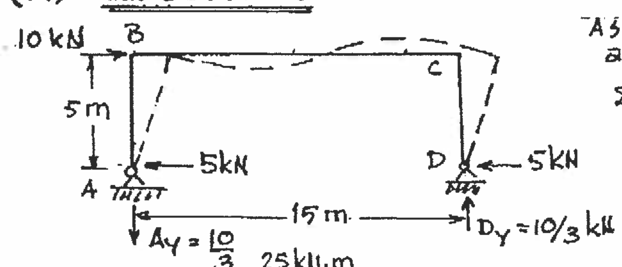


$$\Sigma M_A = 0 = 5 \times 5 - M_{BA} - 15$$
$$M_{BA} = 10 \text{ kN·m}$$



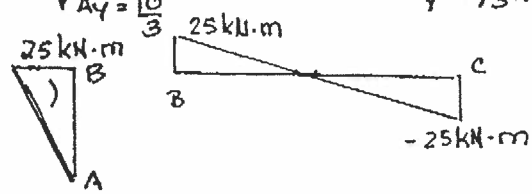
MOMENT DIAGRAM FOR GIRDER

(b.) PIN SUPPORTS

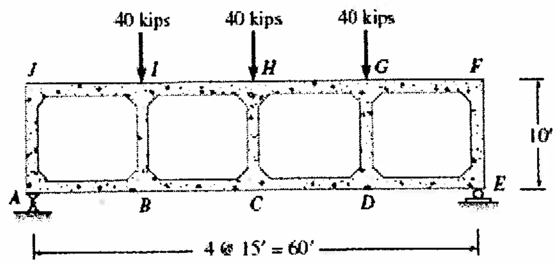


Assume SHEARS IN COLUMNS are equal.

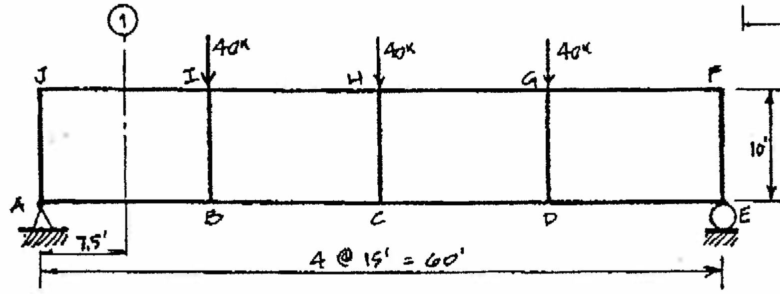
$$\Sigma M_A = 0 = 10 \times 5 - D_Y 15$$
$$D_Y = 50/15 = 10/3 \text{ kN} \uparrow$$
$$A_Y = -D_Y = 10/3 \text{ kN} \downarrow$$



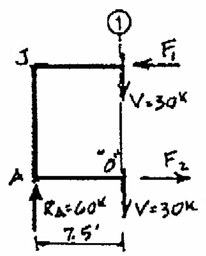
P15.17. Using an approximate analysis of the Vierendeel truss in Figure P15.17, determine the moments and axial forces acting on free bodies of members *AB*, *BC*, *IB*, and *HC*.



P15.17



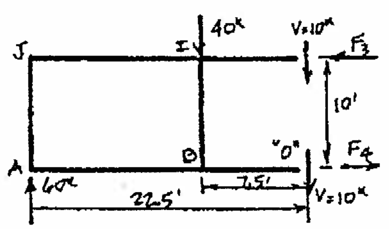
ASSUME (1) P.I. @ MIDSPAN AND (2) SHEARS DIVIDE EQUALLY



$\Sigma M_o = 0;\quad 60(7.5) - F_1(10) = 0 \qquad F_1 = 45^K\ C$

$F_2 = F_1 \qquad F_2 = 45^K\ T$

FBD MEMBER AB

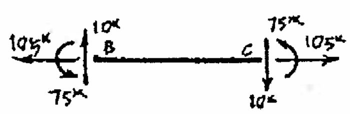


$\Sigma M_o = 0;$

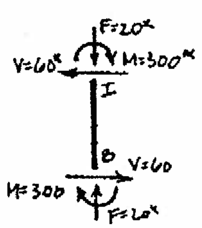
$60(22.5) - 40(7.5) - F_3(10) = 0$

$F_3 = 105^K\ C$

$F_4 = F_3 = 105^K\ T$



FBD MEMBER BC

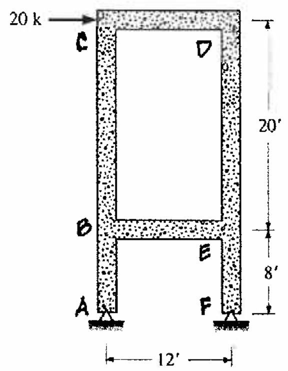


FBD MEMBER BI

FBD MEMBER HC

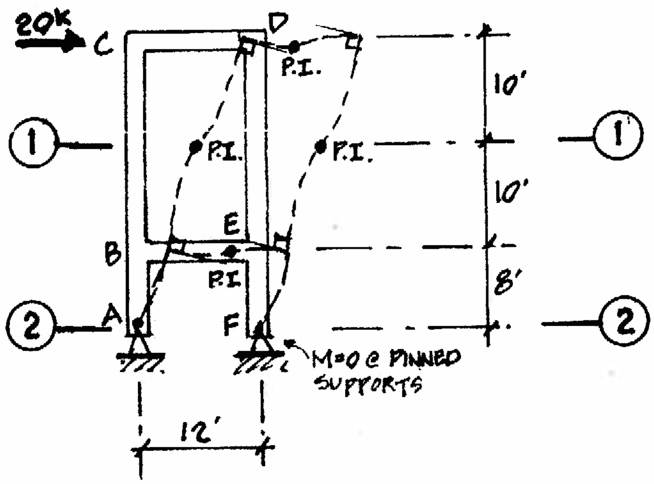
NOTE: NO MOMENT OR SHEAR ONLY AXIAL FORCE

P15.18. Determine the moments and axial forces in members of the frame in Figure P15.18, using the portal method. Compare the results with those produced by the cantilever method.



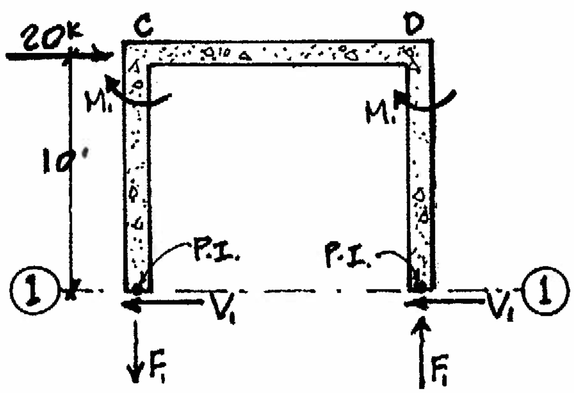
P15.18

FIGURE BELOW USED FOR BOTH PORTAL & CANTILEVER METHODS. ASSUMPTIONS INCLUDE: P.I. OCCUR @ MID-HEIGHT OF CONTINUOUS COLS. & P.I. @ MIDSPAN OF GIRDERS



(a) PORTAL METHOD:

SECTION CUT @ (1)



SECTION CUT @ (1) THRU P.I. OF COLUMNS BC & ED, FREEBODY ABOVE SECTION

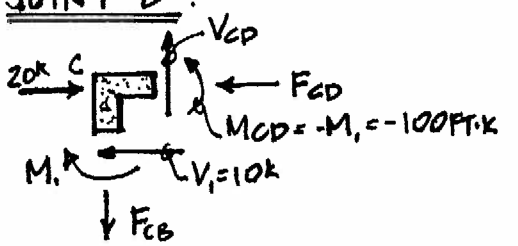
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 20^k - 2V_1 = 0$

$V_1 = 10^k \leftarrow$

MOMENT @ TOP OF UPPER COL.:

$M_1 = 10^k(10') = 100\text{ FT}\cdot\text{K}$ @ C & D.

JOINT "C":



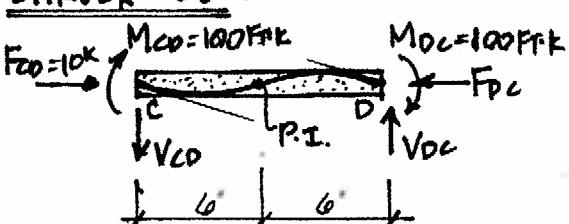
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 20^k - V_1 - F_{CD} = 0$

$F_{CD} = 10^k$

$\Sigma M_C = 0;\quad M_1 - M_{CD} = 0$

$M_{CD} = -100\text{ FT}\cdot\text{K}$

GIRDER "CD":



$\Sigma M_{P.I.} = 0;$ (UTILIZE LEFT 1/2 OF GIRDER "CD")

$M_{CD} - V_{CD}(6') = 0;\quad V_{CD} = 100^{FT\cdot K}/6' = 16.67^k \downarrow$

$+\uparrow\Sigma F_Y = 0;$ (UTILIZE ENTIRE GIRDER "CD")

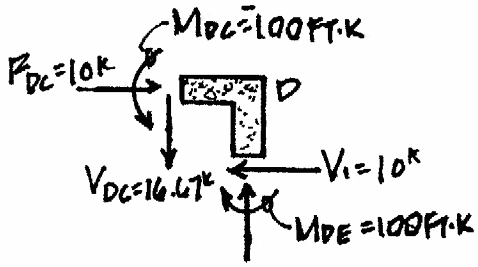
$-16.67 + V_{DC} = 0;\quad V_{DC} = 16.67^k \uparrow$

INSERT VALUE FOR V_{CD} IN FREEBODY OF JOINT "C":

$+\uparrow\Sigma F_Y = 0;\quad V_{CD} - F_{CB} = 0;\quad F_{CB} = 16.67^k$ TENSION

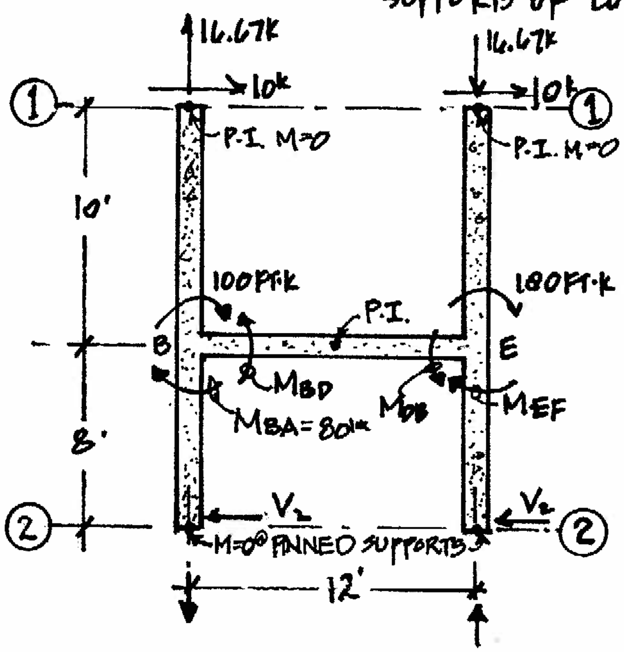
P15.18 CONTINUED

JOINT "D":



SECTION CUT ②

FREEBODY BETW. P.I. OF UPPER COLUMNS PINNED SUPPORTS OF LOWER COLUMNS WHERE M=0.



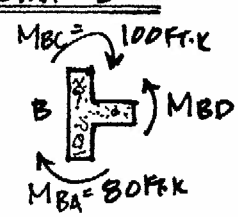
$\overset{+}{\rightarrow}\Sigma F_x=0;$

$2(10^k)-2V_2=0$ $\quad V_2=10^k\leftarrow$

$M_{BA}=M_{EF}=M_2$

$M_2=V_2(8')=80$ FT·K

JOINT "B":

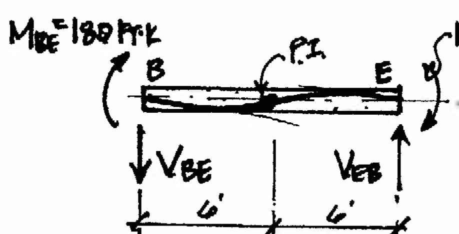


$\Sigma M_B=0;$

100 FT·K $+80-M_{BD}=0$

$M_{BD}=-180$ FT·K ↺

GIRDER "BE"



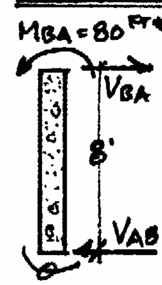
$M_{EB}=M_{BE}=180$ FT·K

$\Sigma M_{P.I.}=0;$ (LEFT ½ OF GIRDER)

$(180^{'k})-V_{BE}(6')=0$

$V_{BE}=30^k\downarrow \cdot V_{EB}=30^k\uparrow$

COLUMN "AB":



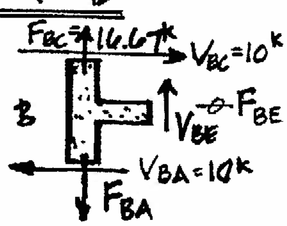
$\Sigma M_A=0;$

$-80^{'k}+V_{BA}(8')=0$

$V_{BA}=10^k\rightarrow$

$V_{AB}=10^k\leftarrow$

JOINT "B"



$+\uparrow\Sigma F_y=0;$

$16.67^k+30^k-F_{BA}=0$

$F_{BA}=46.67^k\downarrow$ TENSION

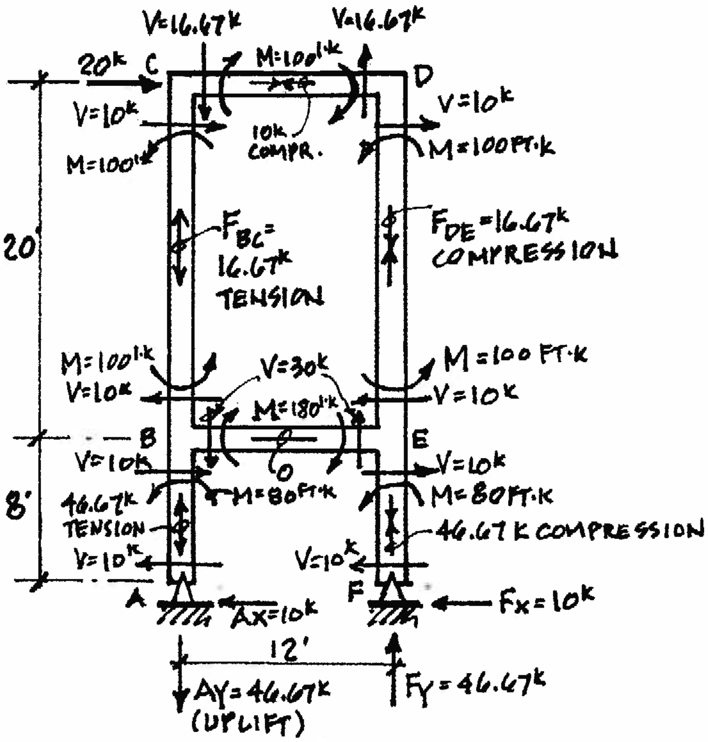
$\overset{+}{\rightarrow}\Sigma F_x=0;$ $\quad F_{BE}+V_{BC}-V_{BA}=0$

$F_{BE}=0$

P15.18 CONTINUED

P15.18 CONTINUED (PORTAL & CANTILEVER METHODS)

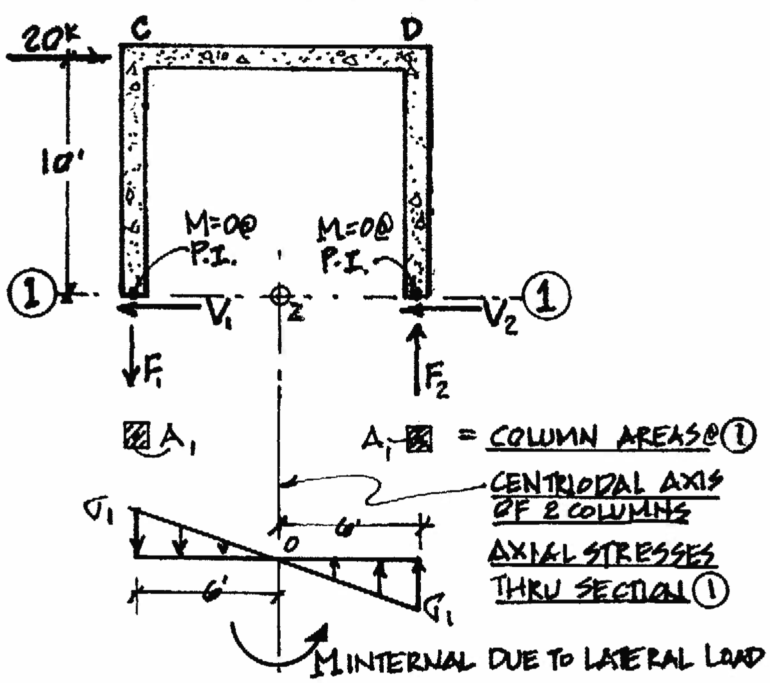
SUMMARY OF PORTAL METHOD:



MEMBER END FORCES & REACTIONS

(b.) CANTILEVER METHOD

SECTION CUT @ (1) FREEBODY ABOVE (1)



EXTERNAL MOMENT:

$\overset{+}{\Sigma} M_Z = 0;$

$M_{Z\,EXT.} = 20^k(10') = 200\,\text{FT}\cdot\text{k}$

INTERNAL MOMENT:

$\overset{+}{\Sigma} M_Z = 0$

$M_{Z\,INT.} = F_1(6') + F_2(6') = 12A\sigma_1$

WHERE $F_1 = A\sigma_1 = F_2$

$M_{EXT.} = M_{INT.}$

$200\,\text{FT}\cdot\text{K} = 12A\sigma_1$

$\sigma_1 A_1 = 16.67^k$

THUS $F_1 = -F_2 = \underline{16.67^k}$

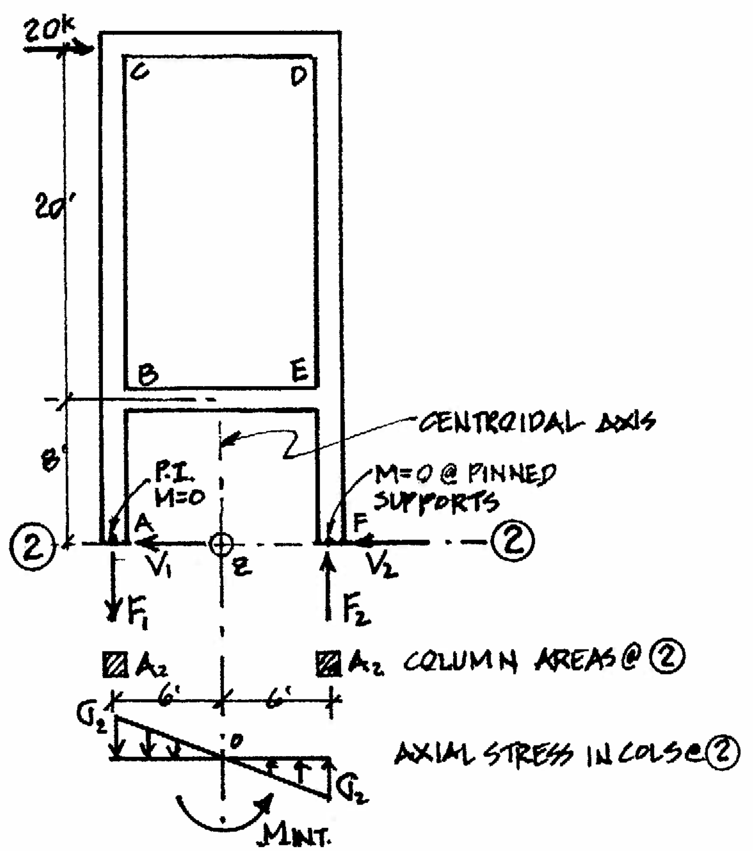
$F_1 = F_{CB} = 16.67^k$ TENSION

$F_2 = F_{DE} = 16.67^k$ COMPR.

P15.18 CONTINUED

F15.18 CONTINUED (CANTILEVER METHOD)

SECTION CUT ② FREEBODY ABOVE ②



$M_{EXT.} = 20^k(28') = 560\,FT\cdot K$

$M_{INT.} = F_1(6') + F_2(6') = 12A_2\sigma_2$

<u>$M_{EXT.} = M_{INT.}$</u>

$560\,FT\cdot K = 12A_2\sigma_2$

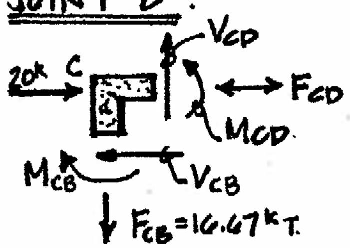
$A_2\sigma_2 = 560/12 = 46.67^k$

$F_1 = -F_2 = 46.67^k$

<u>$F_1 = F_{AB} = 46.67^k$ TENSION</u>

<u>$F_2 = F_{ED} = 46.67^k$ COMPRESSION</u>

JOINT "C":

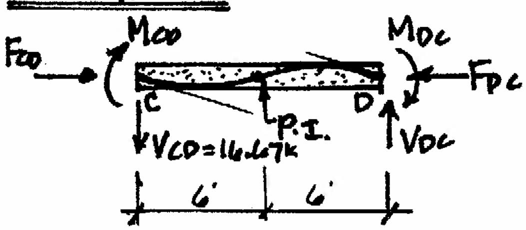


$+\uparrow \Sigma F_y = 0; \quad -F_{CB} + V_{CD} = 0$

WHERE $F_{CB} = 16.67^k$ FROM F.B ①

<u>$V_{CD} = 16.67^k\uparrow$</u>

GIRDER "CD":



<u>$\Sigma M_{P.I.} = 0$; (LEFT ½ OF GIRDER "CD"):</u>

$M_{CD} - \overset{16.67^k}{V_{CD}}(6') = 0; \qquad \underline{M_{CD} = 100^{1k}}$

<u>ENTIRE GIRDER:</u>

$+\uparrow \Sigma F_y = 0; \quad -16.67^k + V_{DC} = 0; \qquad \underline{V_{DC} = 16.67^k\uparrow}$

$\Sigma M_D = 0; \quad 100^{1k} - 16.67(12') + M_{DC} = 0; \qquad \underline{M_{DC} = 100^{1k}}$

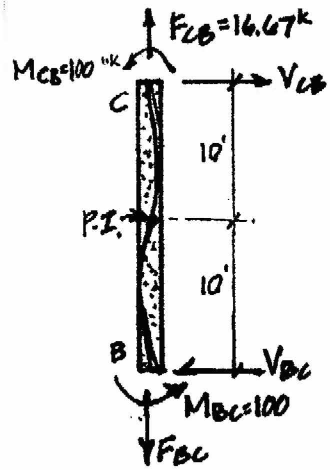
<u>USING JOINT "C": $\Sigma M_C = 0$;</u>

$-M_{CD} + M_{CB} = 0 \qquad \therefore \underline{M_{CB} = 100\,FT\cdot K}$

<u>F15.18 CONTINUED</u>

P15.18 CONTINUED (CANTILEVER METHOD):

COLUMN "BC":



$\overset{+}{\curvearrowleft}\Sigma M_{P.I.}=0;$ (UTILIZE TOP 1/2 OF COLUMN "BC"):
$(-100\text{ FT·K}) + V_{CB}(10')=0;$ $\underline{V_{CB}=10^k \rightarrow}$

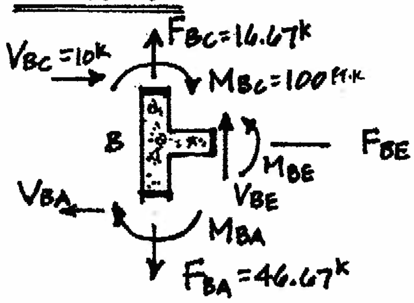
ENTIRE COLUMN "BC":
$\overset{+}{\curvearrowleft}\Sigma M_B=0;$ $-100\text{ FT·K}+10^k(20')-M_{BC}=0;$ $\underline{M_{BC}=100'^k}$

$\overset{+}{\rightarrow}\Sigma F_x=0;$ $\underline{V_{BC}=10^k\leftarrow}$

$+\uparrow\Sigma F_y=0;$ $\underline{F_{BC}=16.67^k \text{ TENSION}}$

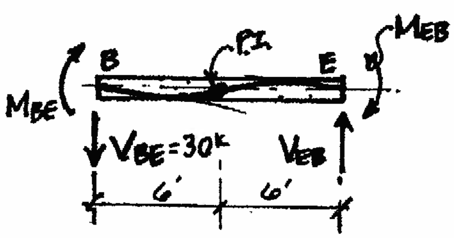
USING JOINT "C" & APPLY $V_{CB}=10^k\leftarrow$
$\overset{+}{\rightarrow}\Sigma F_x=0;$ $F_{CD}-V_{CB}=0$ $\underline{F_{CD}=10^k \text{ COMPR.}}$

JOINT "B":



$+\uparrow\Sigma F_y=0;$ $F_{BC}-F_{BA}+V_{BE}=0$
$V_{BE}=16.67^k-46.67^k=\underline{30^k\uparrow}$

GIRDER "BE"



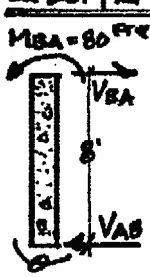
$\overset{+}{\curvearrowleft}\Sigma M_{P.I.}=0;$ (LEFT 1/2 "BE")
$M_{BE}-30^k(6')=0;$ $\underline{M_{BE}=180'^k}$

ENTIRE GIRDER "BE":
$\overset{+}{\curvearrowleft}\Sigma M_E=0;$ $180\text{ FT·K}+M_{EB}-30^k(12')=0;$ $\underline{M_{EB}=180'^k}$

$+\uparrow\Sigma F_y=0;$ $-30^k+V_{EB}=0;$ $\underline{V_{EB}=30^k\uparrow}$

USING JOINT "B", APPLY $M_{BE}=180\text{ FT·K}$
$\overset{+}{\curvearrowleft}\Sigma M_B=0;$ $100\text{ FT·K}-180\text{ FT·K}+M_{BA}=0$
$\underline{M_{BA}=80\text{ FT·K}}$

COLUMN "AB":

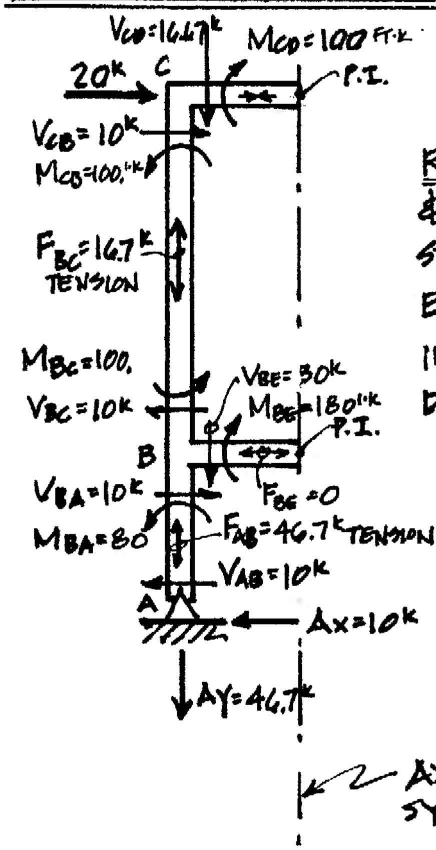


$\overset{+}{\curvearrowleft}\Sigma M_A=0;$
$-80'^k+V_{BA}(8')=0$
$\underline{V_{BA}=10^k\rightarrow}$
$\therefore \underline{V_{AB}=10^k\leftarrow}$

USING JOINT "B" AGAIN & APPLY $V_{BA}=10^k\leftarrow$
$\overset{+}{\rightarrow}\Sigma F_x=0;$ $V_{BC}-V_{BA}+F_{BE}=0$
$10^k-10^k+F_{BE}=0$
$\underline{F_{BE}=0}$

P15.18 CONTINUED (CANTILEVER METHOD)

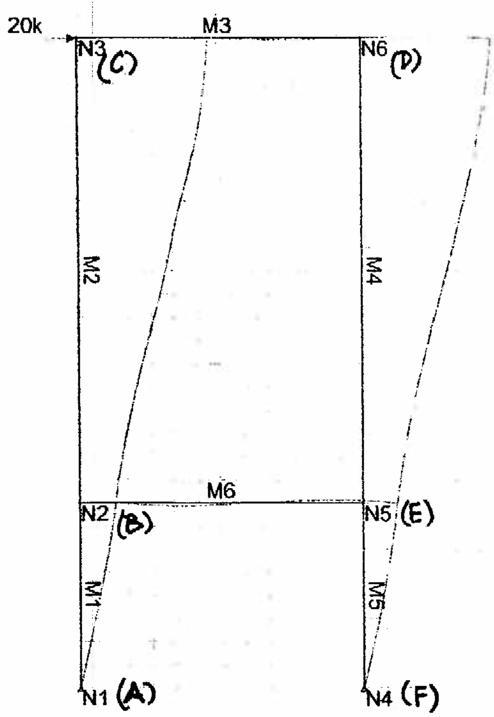
SUMMARY OF CANTILEVER METHOD



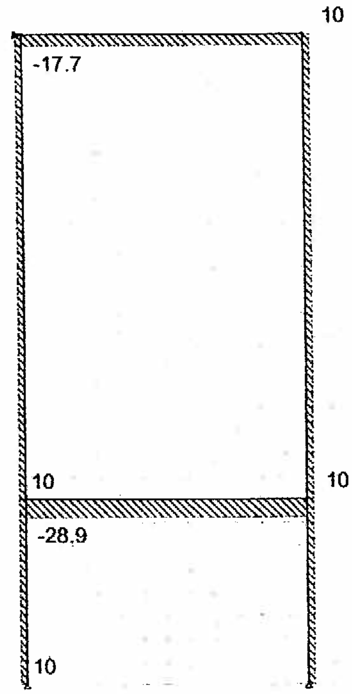
RIGHT SIDE OF AXIS: SHEAR & MOMENT ARE EQUAL TO LEFT OF STRUCTURE. AXIAL FORCES ARE EQUAL IN MAGNITUDE, BUT OPPOSITE IN DIRECTION. THUS, COLUMNS DE & EF ARE IN COMPRESSION.

(C.) THE RESULTS FROM CANTILEVER METHOD ARE THE SAME AS THE PORTAL METHOD FOR THIS PROBLEM.

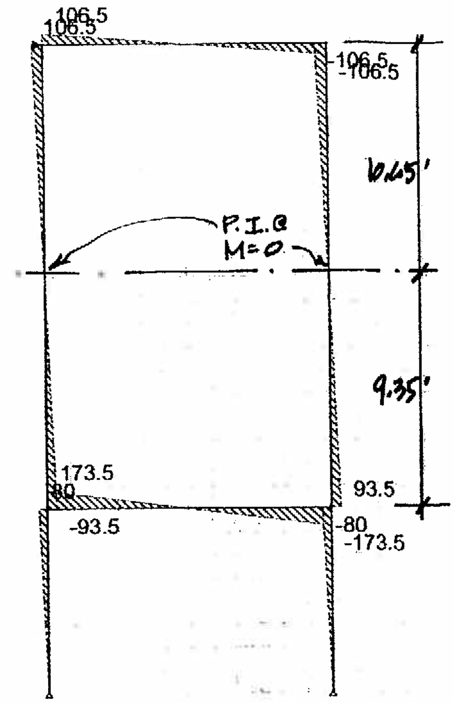
P15.18 CONTINUED : RISA RESULTS



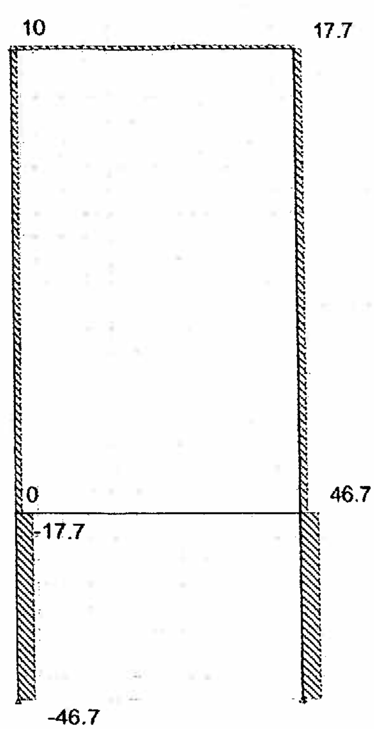
UNBRACED FRAME WITH RIGID JTS, & PINNED SUPPORTS. DEFLECTED SHAPE.



SHEAR DIAGRAM (KIPS)



MOMENT DIAGRAM (FT·KIPS)



AXIAL FORCE DIAGRAM (KIPS)

Member Section Forces

Member Label	Section	Axial (k)	Shear (k)	Moment (k-ft)
M1	1	-46.667	9.997	0
	2	-46.667	9.997	19.994
	3	-46.667	9.997	39.988
	4	-46.667	9.997	59.982
	5	-46.667	9.997	79.976
M2	1	-17.749	10.003	-93.539
	2	-17.749	10.003	-43.524
	3	-17.749	10.003	6.491
	4	-17.749	10.003	56.506
	5	-17.749	10.003	106.521
M3	1	9.997	-17.749	106.521
	2	9.997	-17.749	53.273
	3	9.997	-17.749	.025
	4	9.997	-17.749	-53.222
	5	9.997	-17.749	-106.47
M4	1	17.749	9.997	-106.47
	2	17.749	9.997	-56.485
	3	17.749	9.997	-6.5
	4	17.749	9.997	43.485
	5	17.749	9.997	93.47
M5	1	46.667	10.003	-80.024
	2	46.667	10.003	-60.018
	3	46.667	10.003	-40.012
	4	46.667	10.003	-20.006
	5	46.667	10.003	0
M6	1	.006	-28.917	173.515
	2	.006	-28.917	86.763
	3	.006	-28.917	.011
	4	.006	-28.917	-86.742
	5	.006	-28.917	-173.494

Reactions

Joint Label	X Force (k)	Y Force (k)	Moment (k-ft)
N1	-9.997	-46.667	0
N4	-10.003	46.667	0
Totals:	-20	0	

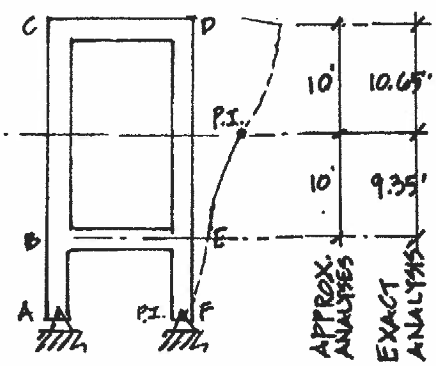
P15.18 CONTINUED

P15.18 Continued

The Portal Method and Cantilever Method yield essentially the same results for this particular problem consisting of a single-bay, unbraced, rigid frame with pinned supports. A comparison of the member end force magnitudes for the approximate Portal and Cantilever Methods versus the RISA Program results are provided in the Table below.

SUMMARY TABLE for problem P15.18

MEMBER	METHOD	AXIAL	SHEAR	MOMENT (TOP OR LEFT)	MOMENT (BOTTOM OR RIGHT)
COLUMNS "BC" & "DE"	PORTAL & CANTILEVER	16.67^k	10^k	100 FT·k	100 FT·k
(M2 & M4)	RISA	17.75^k	$\sim 10^k$	106.5 FT·k	93.5 FT·k
BEAM "CD"	PORTAL & CANTILEVER	10^k	16.67^k	~100 FT·k	100 FT·k
(M3)	RISA	$\sim 10^k$	17.75^k	106.5 FT·k	106.5 FT·k
BEAM "BE"	PORTAL & CANTILEVER	0	30^k	180 FT·k	180 FT·k
(M6)	RISA	~0	28.92^k	173.5 FT·k	173.5 FT·k
COLUMNS "AB" & "EF"	PORTAL & CANTILEVER	46.67^k	10^k	80 FT·k	0
(M1 & M5)	RISA	46.67^k	$\sim 10^k$	80 FT·k	0



The primary difference between the approximate methods (Portal and Cantilever Methods) versus RISA Program is due to variation in locations of the points of inflection (PI) of the upper columns "BC" and "DE".

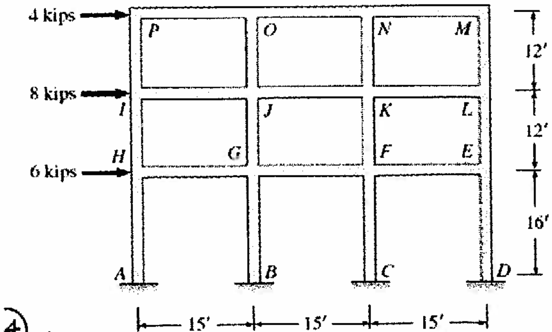
- Compared with the exact analysis, the approximate methods result in about 6% lower values in axial forces and top end moments in columns "BC" and "DE"; and 6% lower values in shear for the upper girder "CD".
  - This is due to the assumed location of the PI at mid-height of the upper columns BC and DE in the approximate method, which generates a shorter moment arm and thus a smaller moment at the top of the columns, i.e. V_{CB} x10 f t.= 100ft-kips, compared with the exact analysis, which generates a moment of V_{CB} x10.65 ft.= 106.5ft-kips. The variation in shear values in the adjoining girders are due to equilibrating the column end moment magnitudes. Exact analyses account for the variation in stiffness of all frame members, and influence the locations of PIs. Stiffness' in this problem vary by the member lengths only.

- Approximate methods result in about 7% higher values in moment at the bottom of the upper columns "BC" and "DE"; as well as, 3.7% higher values in shear and moment in the lower girder "BE";
  - As discussed above, this is due to the difference in location of the P.I. in columns "BC" and "DE" in the approximate vs. exact analyses.

The results of the approximate methods of this problem, a single bay, rigid frame, could provide reasonable information for estimating initial member sizes; or would be useful for verifying relative accuracy of an exact analysis.

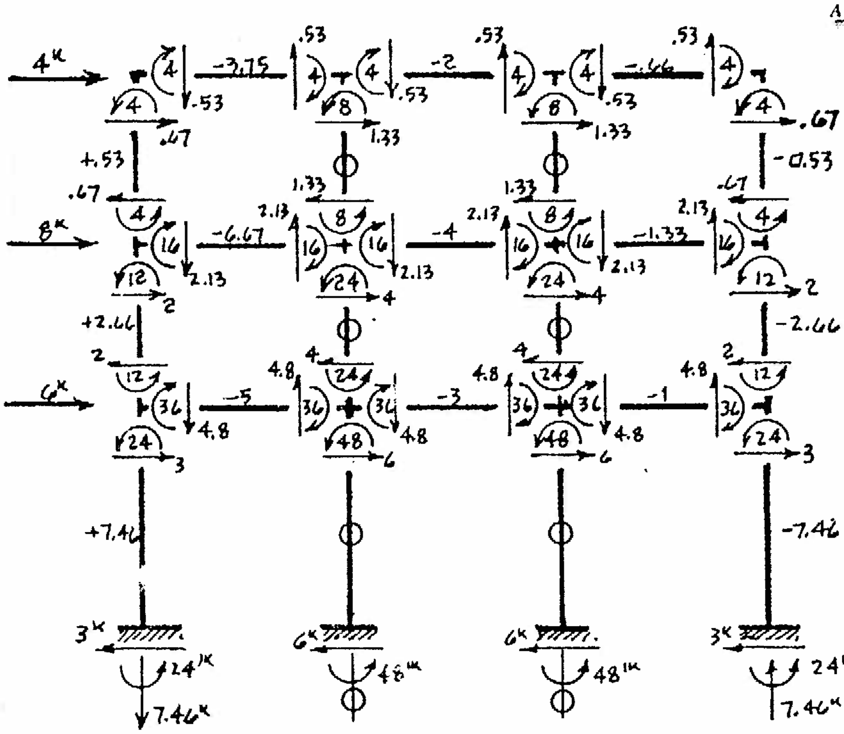
For highly indeterminate structures, such as multi-bay and multi-story rigid frames, the approximate methods may yield more significant differences in the results (10-20%) compared with an exact analysis utilizing computer software.

P15.19. Determine the moments and axial forces in the members of the frame in Figure P15.19, using the portal method. Compare the results with those produced by the cantilever method. Assume the area of the interior columns is twice the area of the exterior columns.



P15.19

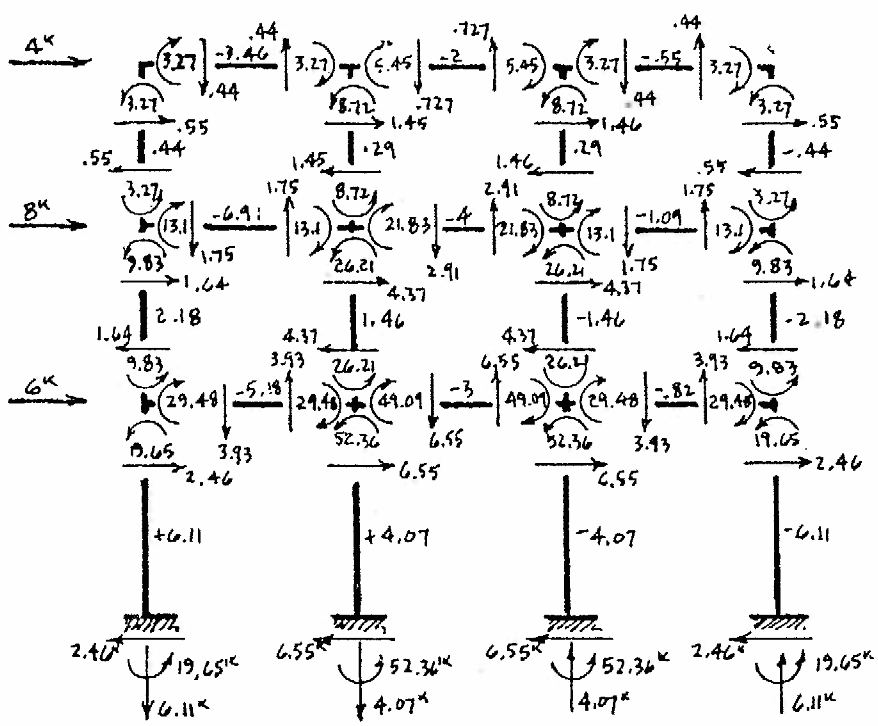
a.) ANALYSIS BY PORTAL METHOD



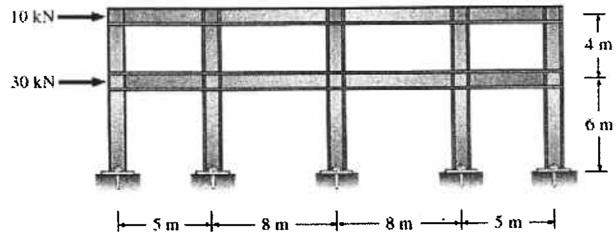
LEGEND:

1. FORCES ARE IN KIPS, FEET.
2. ARROWS INDICATE FORCE DIRECTION ACTING ON MEMBER ENDS.
3. MEMBER AXIAL FORCES DENOTED BY MAGNITUDE SHOWN @ MIDSPAN.

b.) ANALYSIS BY CANTILEVER METHOD



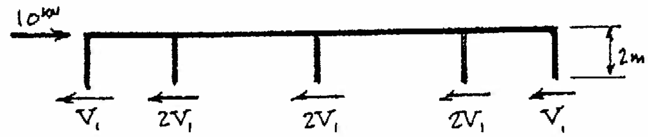
P15.20. Analyze the two-story frame in Figure P15.20 by the portal method. Repeat the analysis using the cantilever method. Assume the area of the interior columns is twice the area of the exterior columns. Assume the baseplates connecting all columns to the foundations can be treated as a pin support.



P15.20

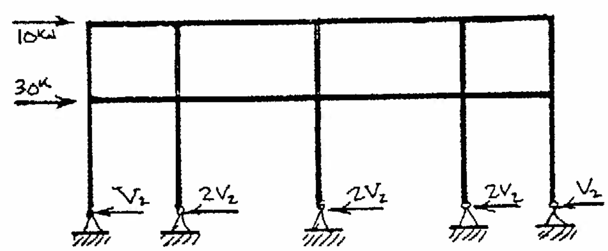
ANALYSIS BY PORTAL METHOD

(a.) CUT HORIZONTAL SECTION THROUGH P.I. IN SECOND STORY

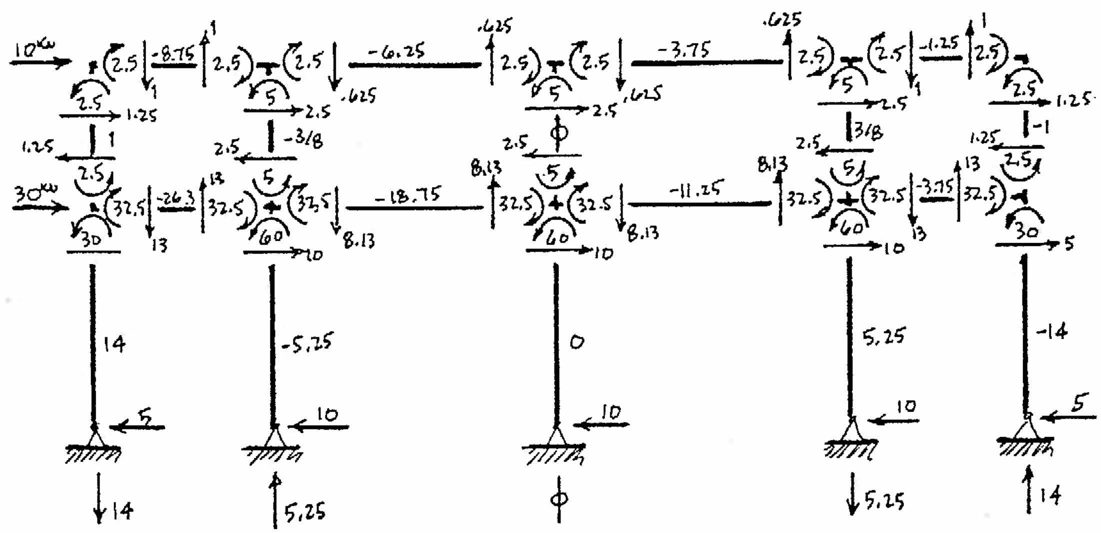


$\Sigma \vec{F}_x = 0;$ $8V_1 = 10$ SO $V_1 = 5/4^{kN}$

(b.) SHEAR @ BASE OF COLUMNS



$\Sigma \vec{F}_x = 0;$ $8V_2 = 40$ SO $V_2 = 5^{kN}$



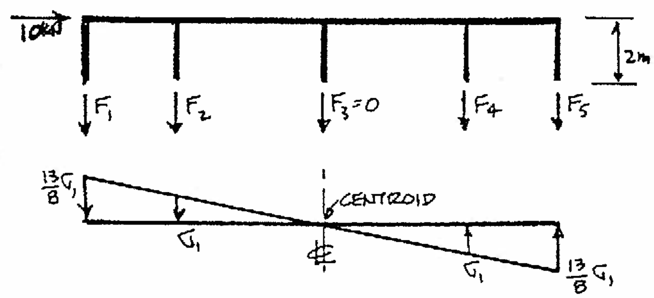
PORTAL METHOD RESULTS (UNITS IN KN, m)

P15.20 Continued

P15.20 Continued

(b) ANALYSIS BY CANTILEVER METHOD

ASSUME AREA OF INTERIOR COLUMNS IS TWICE THAT OF THE EXTERIOR COLUMNS



(a.) $F_1 = F_5 = \frac{13}{8}\sigma_1 A = 0.44\text{ KN}$ WHERE $\sigma_1 A$ IS COMPUTED IN (b)

$F_2 = F_4 = \sigma_1 2A = 0.54\text{ KN}$

(b.) $\Sigma M_{CENTROID}: 10(2) = 8F_2 + 8F_4 + 13F_1 + 13F_5$

$20 = (\sigma_1 2A\ 8)2 + \frac{13}{8}(\sigma_1 A\ 13)2$

$\sigma_1 A = 20/74.25$

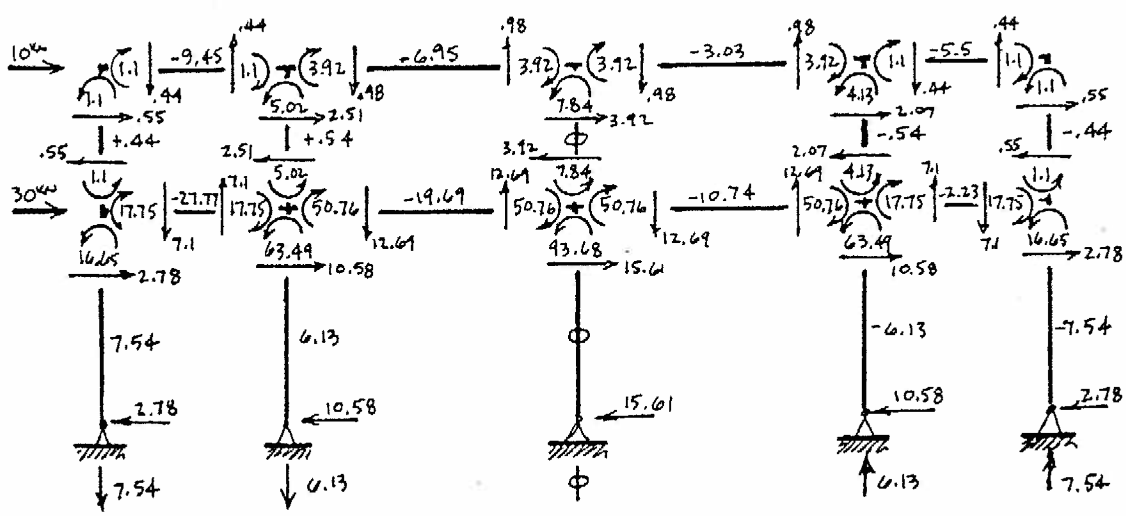
(c) $\Sigma M_{BASE}: 10(10) + 30(6) = (\sigma_2 2A\ 8)2 + \frac{13}{8}\sigma_1 A\ 13(2)$

$\sigma_2 A = 280/74.25$

$F_1 = F_5 = \frac{13}{8}(280/74.25) = 6.13^{KN}$

$F_2 = F_4 = 2\sigma_2 A = 7.54^{KN}$

NOTE: COMPUTATIONS CONTAIN A ROUND OFF ERROR WHICH RESULTS IN A HORIZONTAL SHEAR AT THE BASE 5% GREATER THAN THE CORRECT VALUE



CANTILEVER METHOD RESULTS (UNITS IN KN, m)