FUNDAMENTALS OF STRUCTURAL ANALYSIS

3rd Edition

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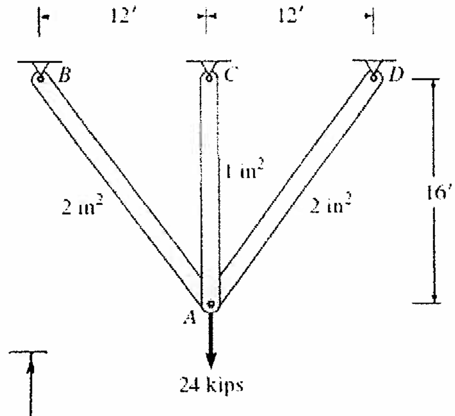
SOLUTIONS MANUAL

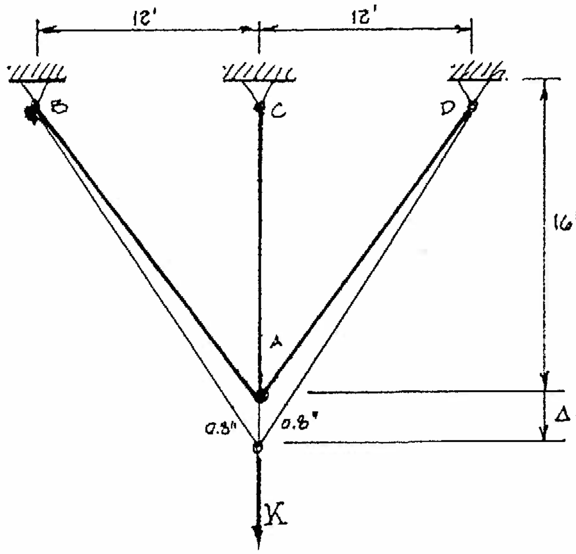
CHAPTER 16:

INTRODUCTION TO GENERAL STIFFNESS METHOD

P16.1 The structure in Figure P16.1 is composed of three pin-connected bars. The bar areas are shown in the figure. Given: E = 30,000 kips/in^2.
(a) Compute the stiffness coefficient K associated with a 1-in vertical displacement of joint A. (b) Determine the vertical displacement at A produced by a vertical load of 24 kips directed downward. (c) Determine the axial forces in all bars.

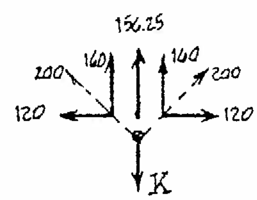


P16.1



a) $F_{AC} = \dfrac{AE\Delta}{L} = \dfrac{1(30{,}000)\,1''}{16(12)} = \underline{156.25^K}$

$F_{AB} = F_{AD} = \dfrac{2(30{,}000)\,0.8''}{20\,(12)} = \underline{200^K}$



JOINT EQUILIBRIUM

$\Sigma F_Y = 0; \quad 160(2) + 156.25 - K = 0 \qquad \underline{K = 476.25\ ^K/_{''}}$

b) DISPLACEMENT DUE TO 24^K

$F = \Delta K$

$24^K = \Delta\,(476.25)$

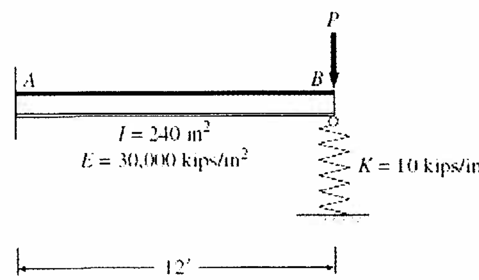
$\underline{\Delta = 0.0504''}$

c.) BAR FORCES DUE TO 24^k @ A

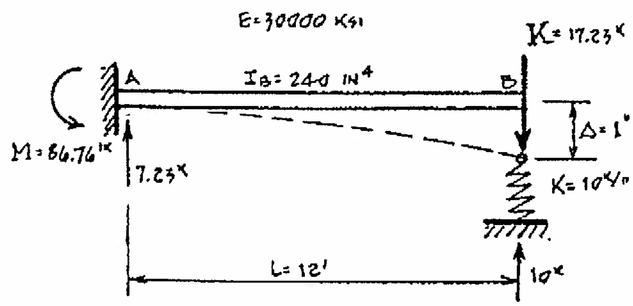
$F_{AC} = 0.0504''\dfrac{(156.25)}{1''} = \underline{7.88^K}$

$F_{AB} = F_{AD} = 0.0504''\dfrac{(200)}{1''} = \underline{10.08^K}$

P16.2 The cantilever beam in Figure P16.2 is supported on a spring at joint B. The spring stiffness is 10 kips/in. (a) Compute the stiffness coefficient associated with a 1-in vertical displacement at joint B. (b) Compute the vertical deflection of the spring produced by a vertical load of 15 kips acting downward at B. (c) Determine all support reactions produced by the 15-kip load.



P16.2



$\Delta_B = PL^3/3EI$

a.) FORCE P_B FOR 1" OF DEFLECTION @ B

$$P_B = 3EI\Delta_B / L^3 = \frac{3(30{,}000)(240)\,1"}{(12(12))^3} = 7.23\ ^{K}/_{''}$$

FORCE P_S REQUIRED TO COMPRESS SPRING 1" $P_S = 10^{K}/_{''}$

STIFFNESS COEFFICIENT K

$K = 7.23 + 10 = 17.23^K$

b.) VERTICAL DEFLECTION @ B DUE TO $P = 15^K$

$P = \Delta K$

$15 = \Delta\,(17.23)$

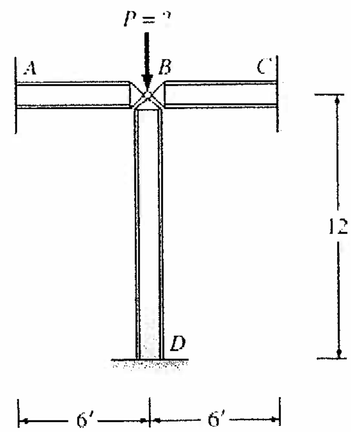
$\Delta = 0.87"$

c.) $R_A = 0.87(7.23) = 6.29^K$

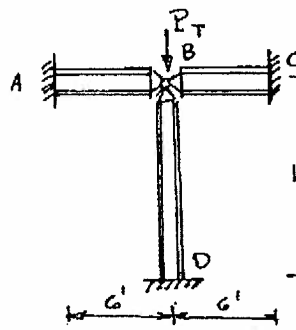
$M_A = 0.87\,(86.76) = 75.48$ FT·K

$R_B = 0.87\,(10) = 8.7^K$

P16.3 The structural system in Figure P16.3 is composed of steel members–two cantilever beams and a column–connected through a pin joint at B. Given: E = 29,000 kips/in^2, $I_{AB} = I_{BC} = 600$ in^4, and $A_{BD} = 3.6$ in^2. (a) Compute the stiffness coefficient K associated with a 1-in vertical displacement at joint B. (b) Determine the magnitude of the force P if it produces a vertical deflection of 1/8 in at joint B.



P16.3



a.) BEAM $\Delta = \dfrac{PL^3}{3EI}$ SET $\Delta = 1''$

$$P_{BM} = \frac{1'' \times 29{,}000\,(3)600}{(6\times12)^3}$$

$$P_{BM} = \underline{\underline{139.85 \text{ kips}}}$$

COLUMN

$$P_{COL} = \frac{\Delta AE}{L} = \frac{1''(3.6)\,29{,}000}{12\times12}$$

$$P_{COL} = \underline{\underline{725 \text{ kips}}}$$

$$K_{11} = P_{COL} + 2\,P_{BM}$$

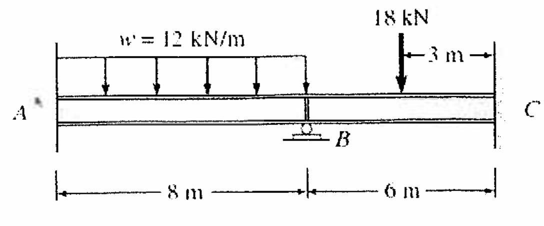
$$= 725 + 2(139.85)$$

$$K_{11} = \underline{1004.7 \text{ kips/in}}$$

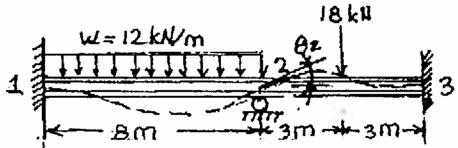
b.) FOR $\Delta = 1/8''$

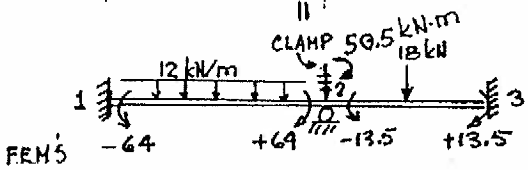
$$P_T = \tfrac{1}{8} K_{11} = \frac{1004.7}{8} = \underline{\underline{125.59 \text{ kips}}}$$

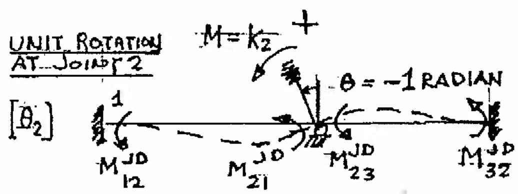
P16.4 Analyze the beam in Figure P16.4 by the stiffness method described in Section 16.3. After member end moments are determined, compute all reactions and draw the moment diagrams. *EI* is constant.



P16.4







θ_2 = ACTUAL JOINT ROTATION AT JOINT 2

RESTRAINDED STRUCTURE: CLAMP JOINT 2

$FEM_{12} = \frac{WL^2}{12} = \frac{12 \times 8^2}{12} = \pm 64$ kN·m

$FEM_{23} = \pm PL/8 = 18 \times 6/8 = \pm 13.5$ kN·m

MOMENT IN CLAMP $= 64 - 13.5 = 50.5$ kN·m

MOMENTS DUE TO UNIT ROTATION AT 2

$M_{12}^{JD} = \frac{2EI}{L}(-1) = -\frac{2}{8}EI = -EI/4$

$M_{21}^{JD} = \frac{2EI}{L}(2(-1)) = -\frac{4}{8}EI = -EI/2$

$M_{23}^{JD} = \frac{2EI}{L}(2(-1)) = -\frac{4}{6}EI = -\frac{2}{3}EI$

$M_{32}^{JD} = \frac{2EI}{L}(-1) = -\frac{2EI}{6} = -\frac{EI}{3}$

$K_2 = M_{21}^{JD} + M_{23}^{JD} = -\frac{EI}{2} + \left(-\frac{2EI}{3}\right) = -\frac{7}{6}EI$

EQULIBRIUM AT JOINT 2

$\Sigma M_2 = 0$

$50.5 + K_2\theta_2 = 0$ (1)

SUBSTI: $K_2 = -7/6\, EI$ (2)

$50.5 + (-7/6\, EI)\theta_2 = 0$

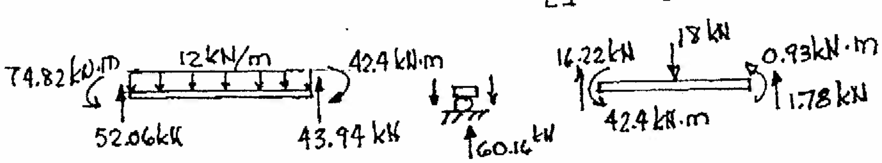
$\theta_2 = \frac{43.286}{EI}$

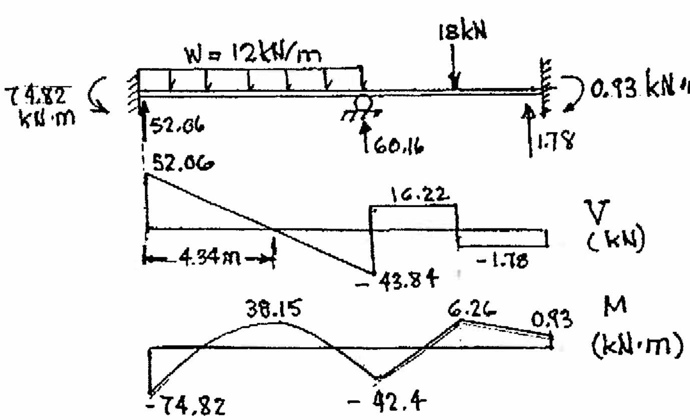
$M_{12} = -64 + \theta_2 M_{12}^{JD} = -64 + \frac{43.286}{EI}\left(-\frac{EI}{4}\right) = -74.82$ kN·m

$M_{21} = 64 + \theta_2 M_{21}^{JD} = 64 + \frac{43.286}{EI}\left(-\frac{EI}{2}\right) = 42.4$ kN·m

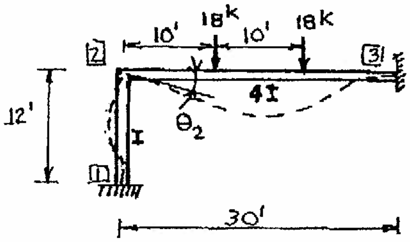
$M_{23} = -13.5 + \theta_2 M_{23}^{JD} = -13.5 + \frac{43.286}{EI}\left(-\frac{2EI}{3}\right) = -42.4$ kN·m

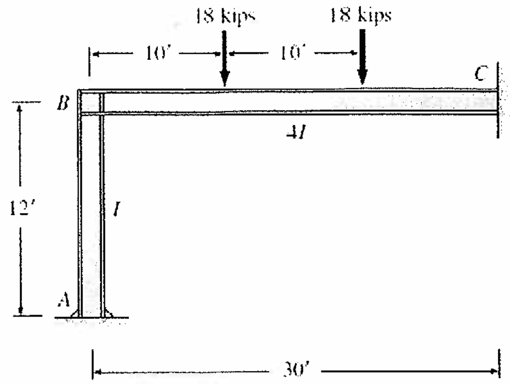
$M_{32} = 13.5 + \theta_2 M_{32}^{JD} = 13.5 + \frac{43.286}{EI}\left(-\frac{EI}{3}\right) = -0.93$ kN·m





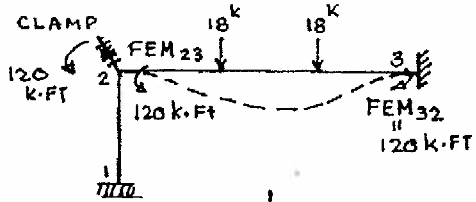
P16.5 Analyze the steel rigid frame in Figure P16.5 by the stiffness method of Section 16.3. After member end moments are evaluated, compute all reactions and the moment diagram for beam *BC*. Supports at *A* and *C* are detailed to produce fixed ends.





P16.5

θ_2 IS THE UNKNOWN DISPL.



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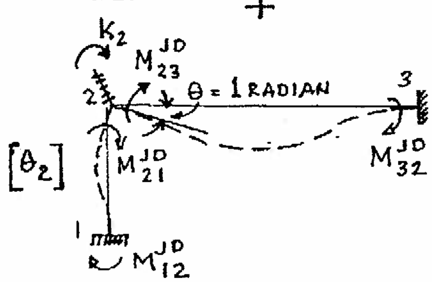


RESTRAINED STRUCTURE; CLAMP AT JOINT 2.

$FEM_{23} = -\frac{2PL}{9} = -\frac{2(18)30}{9} = -120 \underline{\text{kip·FT}}$ (PAGE 448 FIG 12.5)

$FEM_{32} = \frac{2PL}{9} = \underline{120\,\text{kip·FT}}$

ROTATION OF θ_2 AT JOINT 2. i.e.,
UNIT ROTATION INTRODUCED AT JOINT 2, (to be MULTIPLIED BY θ_2)

$$(1)\begin{cases} M_{12}^{JD} = \frac{2EI}{L}(1) = \frac{2EI}{12} = \frac{EI}{6} = \\ M_{21}^{JD} = \frac{2EI}{L}(2(1)) = \frac{4EI}{12} = \frac{EI}{3} \\ M_{23}^{JD} = \frac{2E(4I)}{30}(2\times 1) = \frac{8EI}{15} \\ M_{32}^{JD} = \frac{2E(4I)(1)}{30} = \frac{4EI}{15} \end{cases}$$

$\underline{K_2} = M_{21}^{JD} + M_{23}^{JD} = \frac{EI}{3} + \frac{8}{15}EI = \underline{\frac{13EI}{15}}$

JOINT EQUIL EQ

$\Sigma M_2 = 0$

$-120 + K_2\theta_2 = 0$

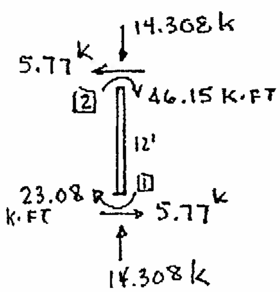
$\theta_2 = \frac{120}{K_2} = \frac{120}{13EI/15} = \underline{\frac{138.46}{EI}}$ (2)

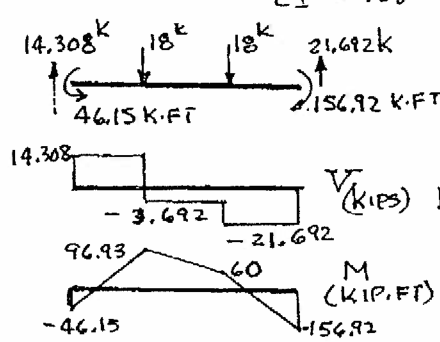
$M_{12} = 0 + \theta_2 M_{12}^{JD} = \frac{138.46}{EI}\left(\frac{EI}{6}\right) = 23.08\ \text{kip·FT}$

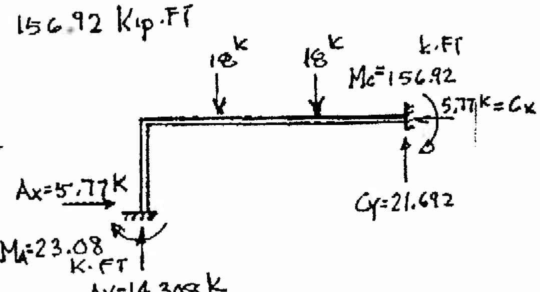
$M_{21} = 0 + \theta_2 M_{21}^{JD} = \frac{138.46}{EI}\left(\frac{EI}{3}\right) = 46.15\ \text{KIP·FT}$

$M_{23} = -120 + \theta_2 M_{23}^{JD} = -120 + \frac{138.46}{EI}\left(\frac{8EI}{15}\right) = -46.15\ \text{kip·FT}$

$M_{32} = 120 + \theta_2 M_{32}^{JD} = 120 + \frac{138.46}{EI}\left(\frac{4EI}{15}\right) = 156.92\ \text{Kip·FT}$

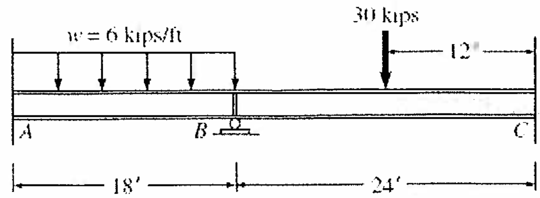




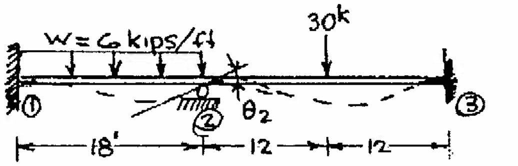


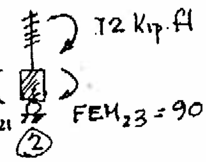
REACTIONS

P16.6 Analyze the beam in Figure P16.6 by the general stiffness method. Compute all reactions and draw the shear and moment diagrams. Given: EI is constant.



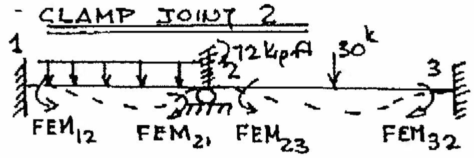
P16.6





slope θ_2 is unknown displacement

CLAMP JOINT 2



CLAMP JOINT ② APPLY LOADS.

$FEM_{12} = -\frac{wl^2}{12} = -\frac{6(18)^2}{12} = -162$ kip·ft

$FEM_{21} = 162$ kip·ft

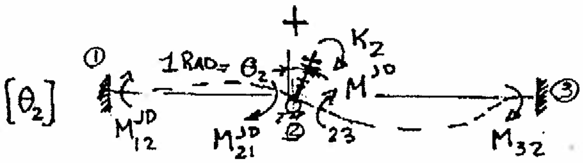
$FEM_{23} = -\frac{PL}{8} = -\frac{30(24)}{8} = -90$ kip·ft

$FEM_{32} = PL/8 = 90$ kip·ft

CLAMP FORCE $= FEM_{21} + FEM_{23} = 72$ Kip·ft

+

$[\theta_2]$



INTRODUCE $\theta = 1$ RADIAN AT JOINT 2

$M_{12}^{JD} = \frac{2EI}{18}(1) = EI/9$

$M_{21}^{JD} = \frac{2EI}{18}(2) = 2EI/9$

$M_{23}^{JD} = \frac{2EI}{24}(2) = EI/6$

$M_{32}^{JD} = \frac{2EI}{24}(1) = EI/12$

$K_2 = M_{21}^{JD} + M_{23}^{JD} = \frac{2EI}{9} + \frac{EI}{6} = \frac{7EI}{18}$

EQUIL. AT JOINT ②

$\Sigma M_2 = 0$

$72 + \theta_2 K_2 = 0$

$72 + \theta_2 \frac{7EI}{18} = 0$

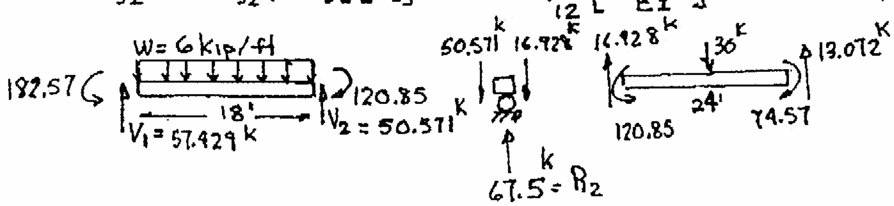
$\theta_2 = -\frac{72 \times 18}{7EI} = -\frac{185.14}{EI}$

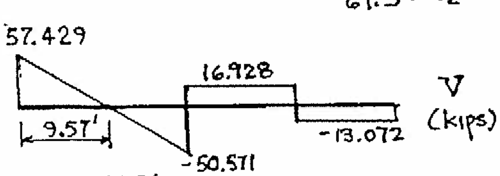
$M_{12} = FEM_{12} + M_{12}^{JD}[\theta_2] = -162 + \frac{EI}{9}\left[-\frac{185.14}{EI}\right] = -182.57$ kip·ft

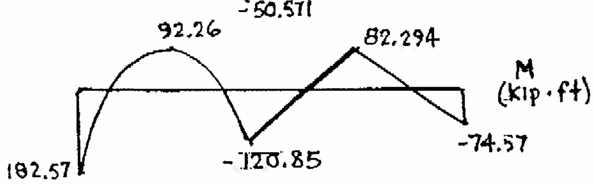
$M_{21} = FEM_{21} + M_{21}^{JD}[\theta_2] = +162 + \frac{2EI}{9}\left[-\frac{185.14}{EI}\right] = 120.85$ kip·ft

$M_{23} = FEM_{23} + M_{23}^{JD}[\theta_2] = -90 + \frac{EI}{6}\left[-\frac{185.14}{EI}\right] = -120.85$ kip·ft

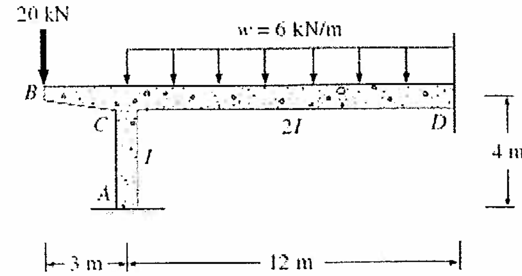
$M_{32} = FEM_{32} + M_{32}^{JD}[\theta_2] = 90 + \frac{EI}{12}\left[-\frac{185.14}{EI}\right] = 74.57$ Kip·ft



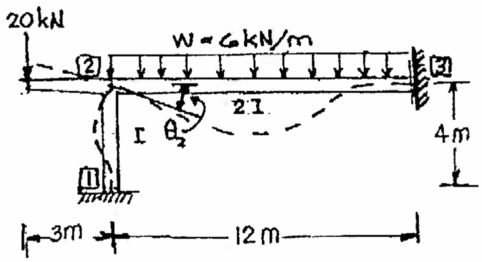




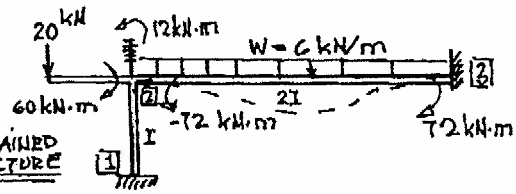
P16.7 Analyze the reinforced concrete frame in Figure P16.7 by the general stiffness method. Determine all reactions. E is constant.



P16.7



θ_2 is the unknown displ.



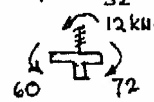
RESTRAINED STRUCTURE

+

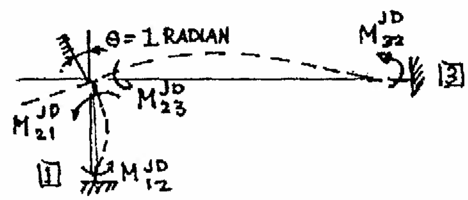
CLAMP JOINT 2

$FEM_{23} = -\frac{wL^2}{12} = \frac{6(12)^2}{12} = -72\text{ kN·m}$

$FEM_{32} = +72\text{ kN·m}$



JOINT 2



$[\theta_2]$

MOMENTS PRODUCED BY A 1 RADIAN ROTATION OF JOINT 2, MULTIPLIED BY θ_2

$M_{12}^{JD} = \frac{2EI}{4}(-1) = -\frac{EI}{2}$

$M_{21}^{JD} = \frac{2EI}{4}(2(-1)) = -EI$

$M_{23}^{JD} = \frac{2E(2I)}{12}(2[-1]) = -\frac{2}{3}EI$

$M_{32}^{JD} = \frac{2E(2I)(-1)}{12} = -\frac{EI}{3}$

$K_2 = M_{21}^{JD} + M_{23}^{JD} = -EI - \frac{2}{3}EI$

$K_2 = -\frac{5}{3}EI$

JOINT EQUIL EQ: $\Sigma M_2 = 0$

$-12\text{ kN} + K_2\theta = 0$

$-12\text{ kN} + -\frac{5}{3}EI\theta = 0$

$\theta = -\frac{36}{5EI}$

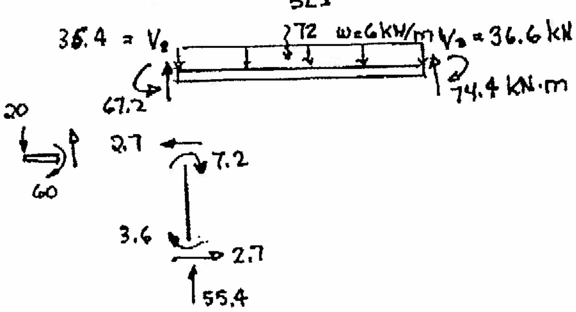
MEMBER END MOMENTS

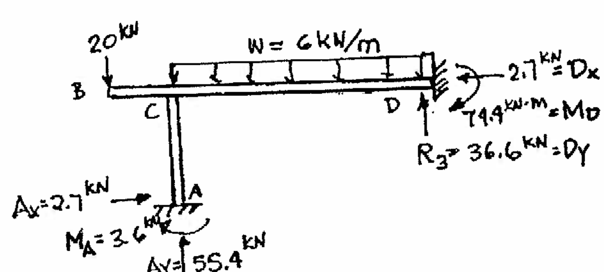
$M_{12} = 0 + \theta_2 M_{12}^{JD} = \frac{-36}{5EI}\left(-\frac{EI}{2}\right) = +\frac{18}{5} = 3.6\text{ kN·m}$

$M_{21} = 0 + \theta_2 M_{21}^{JD} = \frac{-36}{5EI}(-EI) = 7.2\text{ kN·m}$

$M_{23} = -72 + \theta_2 M_{23}^{JD} = -72 + \left(-\frac{36}{6EI}\right)\left(-\frac{2}{3}EI\right) = -67.2\text{ kN·m}$

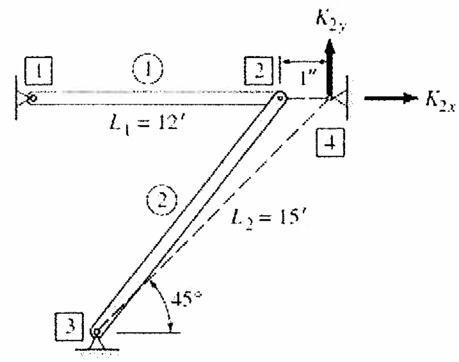
$M_{32} = 72 + -\frac{36}{5EI}\left(-\frac{EI}{3}\right) = 74.4\text{ kN·m}$



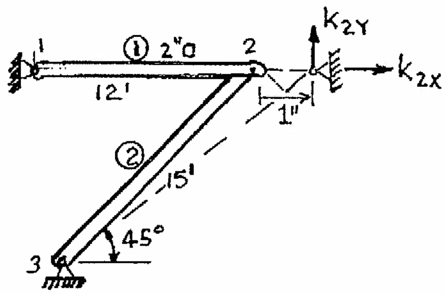


REACTIONS

P16.8 The pin-connected bar system in Figure P16.8 is stretched 1 in horizontally and connected to the pin support 4. Determine the horizontal and vertical components of force that the support must apply to the bars. Area of bar 1 = 2 in^2, area of bar 2 = 3 in^2, and E = 30,000 kips/in^2. K_{2x} and K_{2y} are stiffness coefficients.



P16.8



$\Delta L_1 \doteq 1"$; $\Delta L_2 = 0.707"$

$$F_1 = \frac{\Delta L\, A E}{L_1} = \frac{1"(2\text{ in}^2)\,30{,}000}{12\times 12\text{ in}} = 416.67 \text{ kips}$$

$$F_2 = \frac{\Delta L\, A E}{L_2} = \frac{0.707\,(3\text{ in}^2)\,30{,}000}{15\times 12} = 353.5 \text{ kips}$$

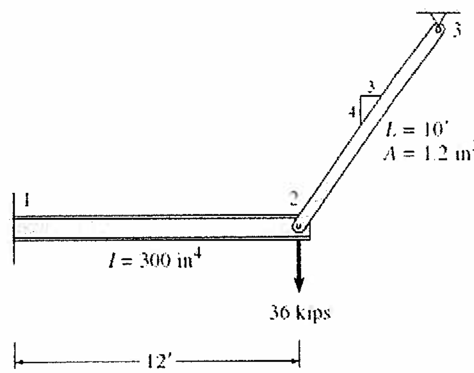
$F_{2x} = 0.707(353.5) = 249.93$ kips

$F_{2y} = 0.707(353.5) = 249.93$ kips

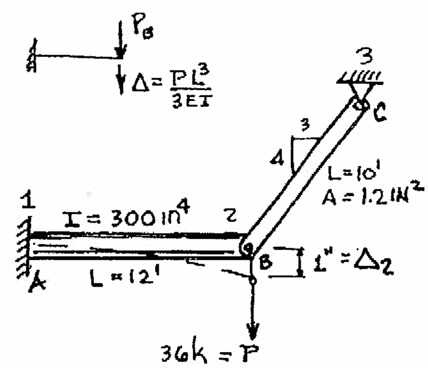
$K_{2x} = 416.67 + 249.93 = \underline{666.6 \text{ kips}}$

$\underline{K_{2y} = 249.93 \text{ kips}}$

P16.9 The cantilever beam in Figure P16.9 is connected to a bar at joint 2 by a pin. Compute all reactions. Given: E = 30,000 kips/in^2. Assume only vertical deflection at joint 2 is significant.



P16.9



BM FORCE FOR $\Delta_2 = 1''$

$$P_B = \frac{3\Delta EI}{L^3} = \frac{3 \times 1'' \times 30{,}000 \times 300}{(12\times 12)^3}$$

$$P_B = 9.04 \text{ KIPS}$$

FORCE IN BAR 2-3 FOR $\Delta_2 = 1''$

$$\Delta L_{2\text{-}3} = 1'' \times 4/5 = 0.8 \text{ IN}$$

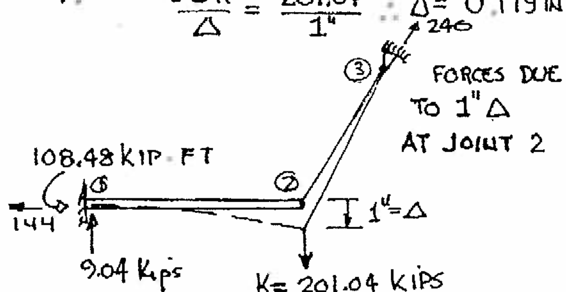
$$F_{2\text{-}3} = \frac{\Delta L\, AE}{L} = \frac{0.8(1.2)\,30{,}000}{10\times 12}$$

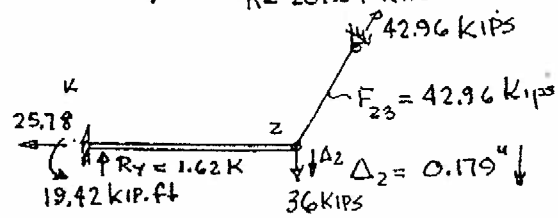
$$= 240 \text{ KIPS}$$

K STIFFNESS COEF = $240(.8) + 9.04$

K = 201.04 KIPS/1 IN

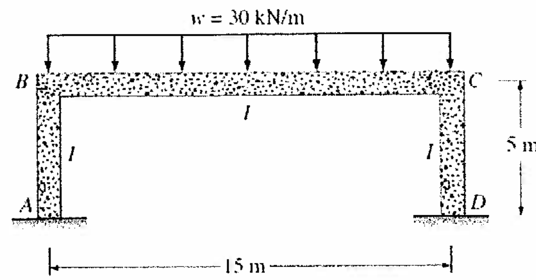
$$\therefore \quad \frac{36k}{\Delta} = \frac{201.04}{1''} \quad \therefore \Delta = 0.179 \text{ IN}$$



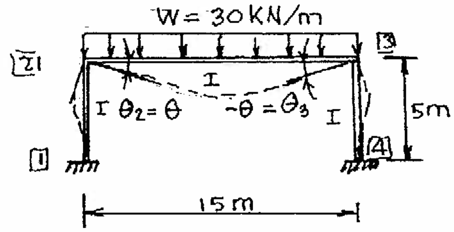


REACTIONS AND BAR FORCE BY 36KIP LOAD AT JOINT 2

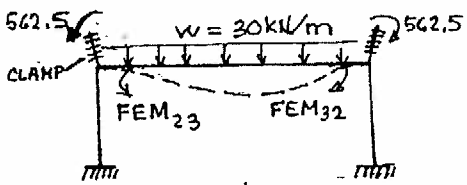
P16.10 Analyze the rigid frames in Figures P16.10 by the general stiffness method, using symmetry to simplify the analysis. Compute all reactions and draw the moment diagrams for all members. Also E is constant.



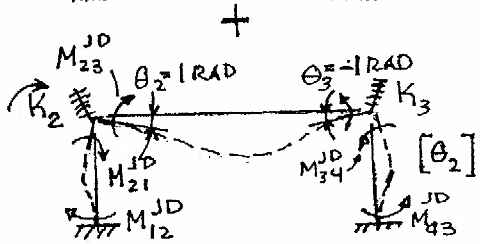
P16.10



SINCE STRUCTURE AND LOAD ARE symmetrical θ_2 and θ_3 are EQUAL IN MAGNITUDE.



+



RESTRAINED STRUCTURE.

$$FEM_{23} = -\frac{wL^2}{12} = -\frac{30(15)^2}{12} = -562.5 \text{ kN}\cdot\text{m}$$

$$FEM_{32} = \frac{wL^2}{12} = 562.5 \text{ kN}\cdot\text{m}$$

INTRODUCE UNIT ROTATIONS AT JOINTS 2 & 3. MULTIPLY THE RESULTING MOMENTS BY θ_2

$$M_{12}^{JD} = \frac{2EI}{5}(2(0)+1) = \frac{2EI}{5}$$

$$M_{21}^{JD} = \frac{2EI}{5}(2(1)+0) = \frac{4EI}{5}$$

$$M_{23}^{JD} = \frac{2EI}{15}\{2(1)+(-1)\} = \frac{2EI}{15}$$

$$K_2 = M_{21}^{JD} + M_{23}^{JD} = \frac{4EI}{5} + \frac{2EI}{15} = \frac{14EI}{15}$$

JOINT EQUIL. EQ. $\Sigma M_2 = 0$

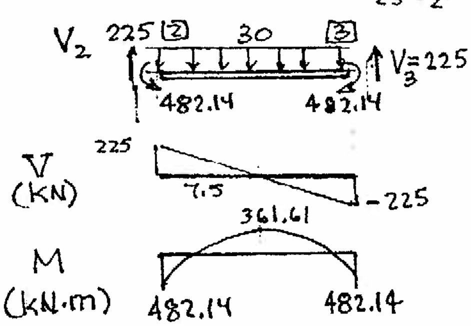
$$-562.5 + K_2\theta_2 = 0$$

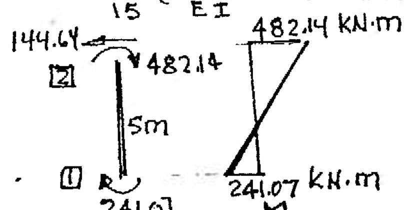
$$\theta_2 = \frac{562.5}{K_2} = \frac{562.5}{14EI/15} = \frac{602.68}{EI}$$

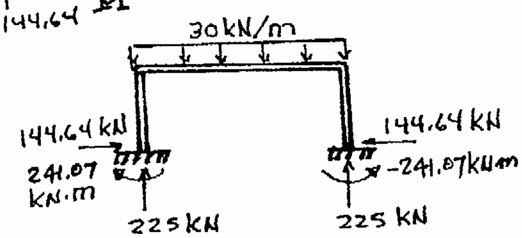
$$M_{12} = 0 + M_{12}^{JD}\theta_2 = \frac{2EI}{5}\left(\frac{602.68}{EI}\right) = 241.07 \text{ kN}\cdot\text{m}$$

$$M_{21} = 0 + M_{21}^{JD}\theta_2 = \frac{4EI}{5} \times \frac{602.68}{EI} = 482.14 \text{ kN}\cdot\text{m}$$

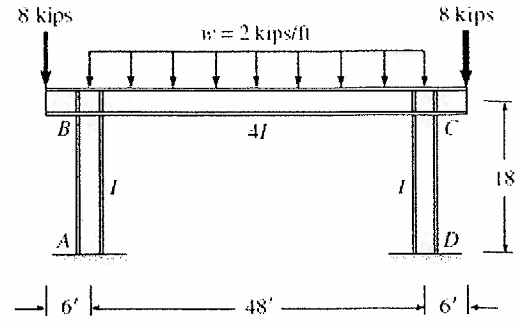
$$M_{23} = -562.5 + M_{23}^{JD}\theta_2 = -562.5 + \frac{2EI}{15}\left(\frac{602.68}{EI}\right) = -482.14 \text{ kN}\cdot\text{m}$$



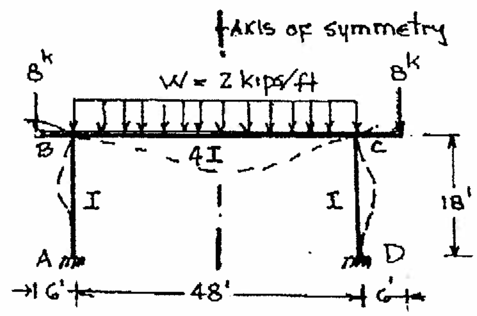




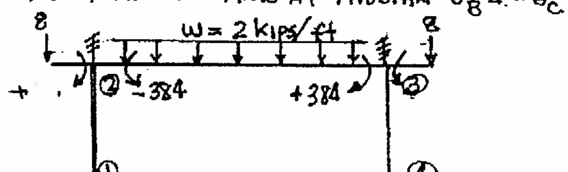
P16.11 Analyze the rigid frames in Figures P16.11 by the general stiffness method, using symmetry to simplify the analysis. Compute all reactions and draw the moment diagrams for all members. Also E is constant.



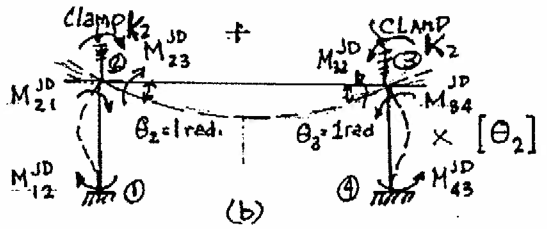
P16.11



SINCE STRUCTURE AND LOADS ARE SYMMETRICAL ABOUT VERTICAL AXIS AT MIDSPAN $\theta_B = -\theta_C$



(a)



(b)

(a) RESTRAINED STRUCTURE CLAMPS AT ② AND ③

$$FEM_{23} = -FEM_{32} = \pm \frac{wL^2}{12} = \frac{2(48)^2}{12} = \pm 384^{k\cdot ft}$$

CANTILEVERS: $M = 8 \times 6 = 48$ Kip-ft

(b) INTRODUCE UNIT ROTATIONS AT JOINTS 2 AND 3 AND CLAMP.

$$M_{12}^{JD} = \frac{2EI}{18}[2\times 0 + 1) = EI/9$$

$$M_{21}^{JD} = \frac{2EI}{18}[2\times 1 + 0) = 2EI/9$$

$$M_{23}^{JD} = \frac{2E(4I)}{48}[(2\times 1) - 1) = EI/6$$

MOMENT IN CLAMPS AT JOINTS 2 & 3

$$\underline{K_2} = M_{21}^{JD} + M_{23}^{JD} = 2EI/9 + EI/6 = \frac{7EI}{18}$$

JOINT EQUIL AT ②: $\Sigma M_2 = 0$

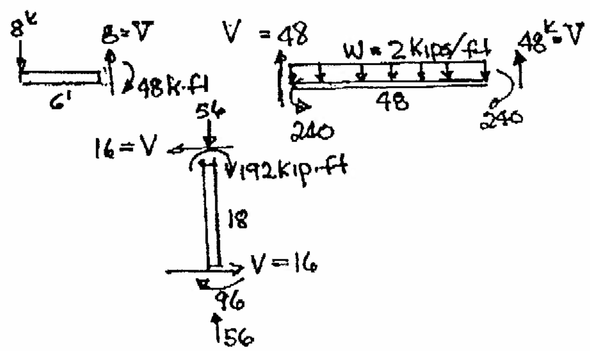
$$48 - 384 + \theta_2 K_2 = 0$$

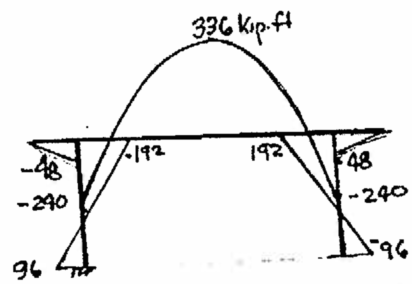
$$\theta_2 = 336/K_2 = 336/\frac{7EI}{18} = 864/EI$$

$$M_{12} = 0 + EI/9\,\theta_2 = \frac{EI}{9} \times \frac{864}{EI} = 96 \text{ Kip-ft}$$

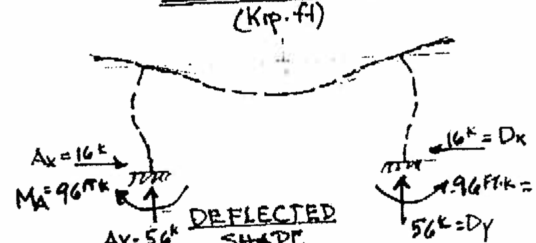
$$M_{21} = 0 + 2EI/9\,\theta_2 = \frac{2EI}{9} \times \frac{864}{EI} = 192 \text{ kip-ft}$$

$$M_{23} = -384 + \frac{EI}{6} \times \frac{864}{EI} = -240 \text{ kip-ft}$$





MOMENT DIAGRAM (Kip·ft)



DEFLECTED SHAPE & REACTIONS