

FUNDAMENTALS OF STRUCTURAL ANALYSIS

3rd Edition

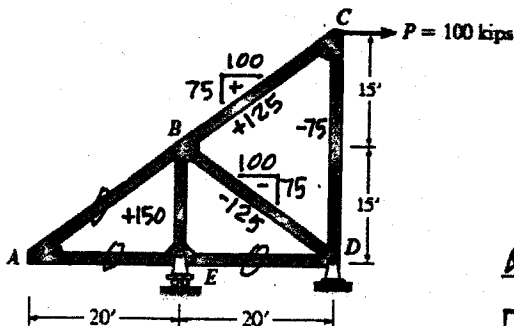
Kenneth M. Leet, Chia-Ming Uang, and Anne M. Gilbert

SOLUTIONS MANUAL

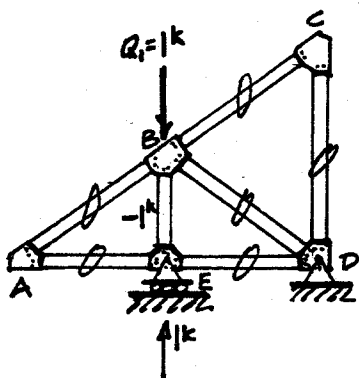
CHAPTER 10:

WORK-ENERGY METHODS **FOR COMPUTING DEFLECTIONS**

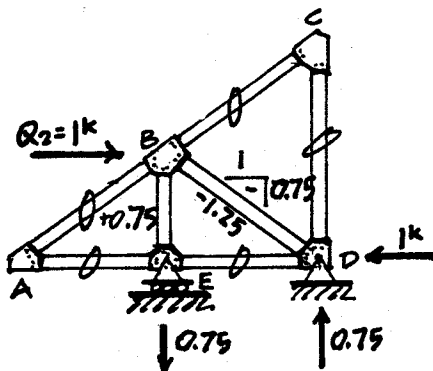
P10.1. For the truss in Figure P10.1, compute the horizontal and vertical components of displacement of joint B produced by the 100-kip load. The area of all bars = 4 in² and $E = 24,000$ kips/in².



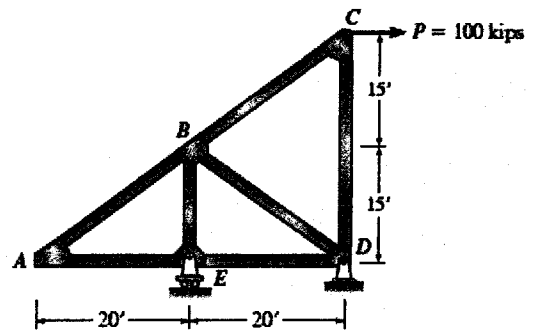
P-SYSTEM



Q1-SYSTEM FOR Δ_{BY}



Q2-SYSTEM FOR Δ_{BX}



P10.1

ALL BARS: $A = 4 \text{ in}^2$, $E = 24,000 \text{ ksi}$

BAR	F_P	F_{Q1} (Δ_{BY})	F_{Q2} (Δ_{BX})	L	$F_P F_{Q1} L$ (Δ_{BY})	$F_P F_{Q2} L$ (Δ_{BX})
AB	0	0	0	25'	—	—
BC	+125	0	0	25'	—	—
CD	-75	0	0	30'	—	—
DE	0	0	0	20'	—	—
AE	0	0	0	20'	—	—
BE	+150	-1	+0.75	15'	-2250	1687.5
BD	-125	0	-1.25	25'	—	3906.25

$\Sigma = -2250$ $\Sigma = 5593.75$

$$\underline{\Delta_{BY}}: \Sigma Q_1 \cdot \Delta_{BY} = \frac{\Sigma F_{Q1} F_P L}{AE}$$

$$1k \cdot \Delta_{BY} = \frac{-2250}{AE} = \frac{-2250(12'')}{4(24000)}$$

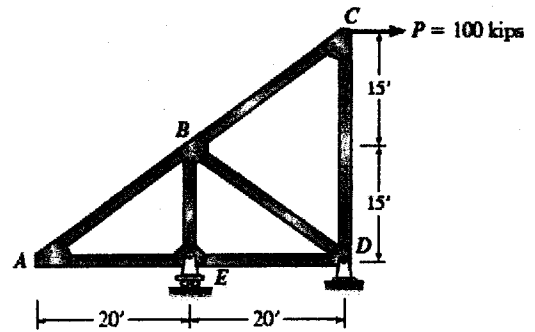
$$\underline{\Delta_{BY} = 0.281'' \uparrow}$$

$$\underline{\Delta_{BX}}: \Sigma Q_2 \cdot \Delta_{BX} = \frac{\Sigma F_{Q2} F_P L}{AE}$$

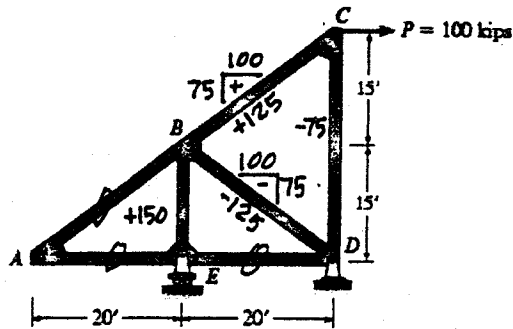
$$1k \cdot \Delta_{BX} = \frac{5593.75}{AE}$$

$$\underline{\Delta_{BX} = 0.70'' \rightarrow}$$

P10.2. For the truss in Figure P10.1, compute the vertical displacement of joint A and the horizontal displacement of joint C.



P10.1



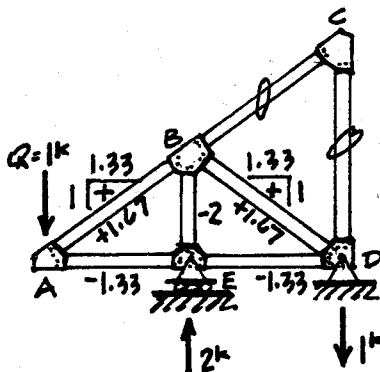
F-SYSTEM, & Q-SYSTEM FOR ΔCx

ALL BARS: $A = 4 \text{ in}^2$, $E = 24,000 \text{ ksi}$

BAR	F_P	F_Q (Δ_{AY})	L	$F_P F_Q L$ (Δ_{AY})	$F_P^2 L$ (Δ_{Cx})
AB	0	+1.67	25'	—	—
BC	+125	0	25'	—	390,625
CD	-75	0	30'	—	168,750
DE	0	-1.33	20'	—	—
AE	0	-1.33	20'	—	—
BE	+150	-2	15'	-4500	337,500
BD	-125	+1.67	25'	-5218.75	390,625

$$\Sigma = -9718.75$$

$$\Sigma = 1,287,500$$



Q-SYSTEM FOR Δ_{AY}

$$\Delta_{AY} : \Sigma Q \cdot \Delta_{AY} = \frac{\Sigma F_P F_Q L}{AE}$$

$$100 \cdot \Delta_{AY} = \frac{-9718.75 (12)}{4 (24000)}$$

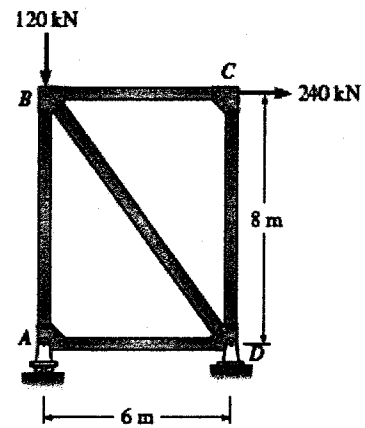
$$\Delta_{AY} = 1.214'' \uparrow$$

$$\Delta_{Cx} : \Sigma Q \cdot \Delta_{Cx} = \frac{\Sigma F_P^2 L}{AE}$$

$$100 \cdot \Delta_{Cx} = \frac{1,287,500 (12)}{4 (24,000)}$$

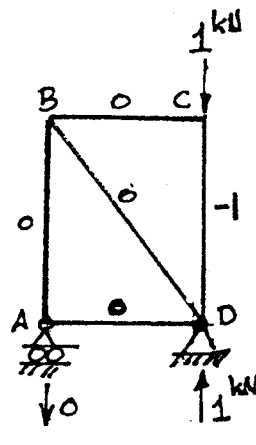
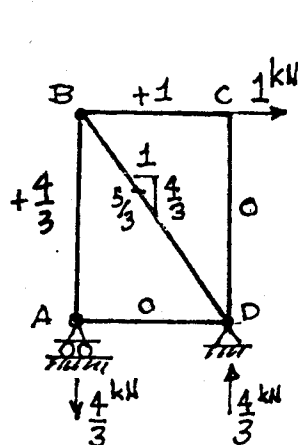
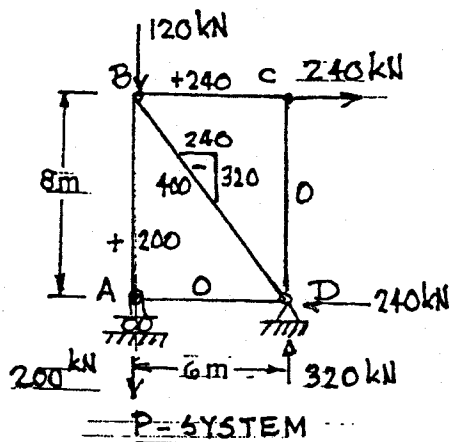
$$\Delta_{Cx} = 1.61'' \rightarrow$$

P10.3. For the truss in Figure P10.3, compute the horizontal and vertical components of the displacement of joint C. The area of all bars = 2500 mm², and E = 200 GPa.



P10.3

$$A = 2500 \text{ mm}^2, E = 200 \text{ GPa}$$



BAR	S_{CH} S_{CV}			L	S_{CH} S_{CV}		
	F_P	F_{Q1}	F_{Q2}		$F_P F_{Q1} L$	$F_P F_{Q2} L$	
AB	200	4/3	0	8	6400/3	0	
BC	240	1	0	6	1440	0	
CD	0	0	-1	8	0	0	
AD	0	0	0	6	0	0	
BD	-400	-5/3	0	10	20000/3	0	

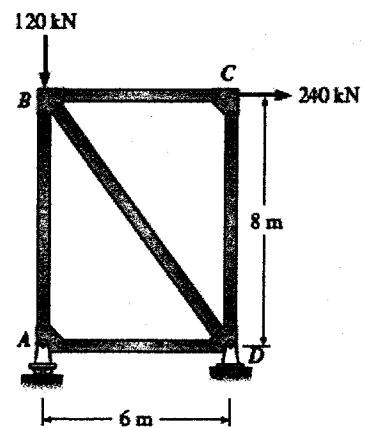
$$\Sigma = 10,240 \quad \Sigma = 0$$

$$\delta_{CH} = \frac{\Sigma Q_1 S_{CH}}{\Sigma F_{Q1} F_P L / AE} = \frac{10,240 \times 10^3}{2500 \times 10^{-6} (200 \times 10^9)}$$

$$\delta_{CH} = 0.02 \text{ m} \rightarrow$$

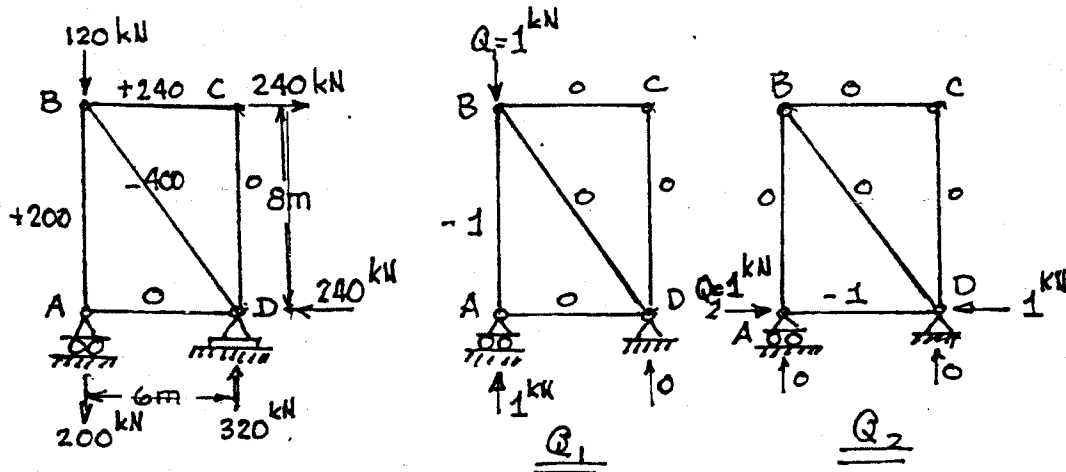
$$\delta_{CV} = \frac{\Sigma Q_2 S_{CV}}{\Sigma F_{Q2} F_P L / AE} = 0$$

P10.4. For the truss in Figure P10.3, compute the vertical displacement of joint *B* and the horizontal displacement of the roller at joint *A*.



P10.3

ALL BARS: $A = 2,500 \text{ mm}^2$, $E = 200 \text{ GPa}$



$$\delta_{BV} : \sum Q_i \cdot \delta_{BV} = \sum F_q F_p \frac{L}{AE}$$

$$1 \cdot \delta_{BV} = - \frac{1(200) 8 \times 10^3 (1000)}{200 \text{ GPa} \times 10^9 (2500 \times 10^6)}$$

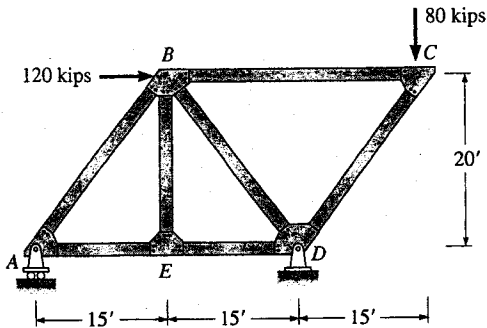
$$\delta_{BV} = - \underline{\underline{3.2 \text{ mm} \uparrow}}$$

$$\delta_{Ax} :$$

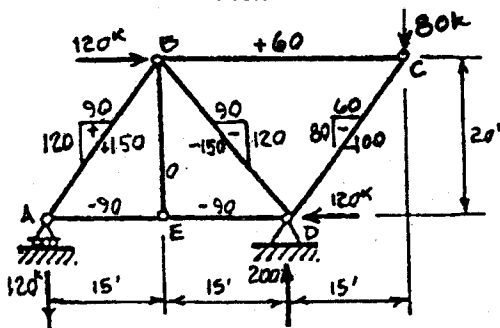
$$1 \cdot \delta_{Ax} = Q \quad (F_p = 0)$$

$$\underline{\underline{\delta_{Ax} = 0}}$$

P10.5. (a) In Figure P10.5 compute the vertical and horizontal components of displacement of joint E produced by the loads. The area of bars AB, BD, and CD = 5 in²; the area of all other bars = 3 in². $E = 30,000$ kips/in². (b) If bars AB and BD are fabricated $\frac{3}{4}$ in too long and support D settles 0.25 in, compute the vertical displacement of joint E. Neglect the 120-kip load.



P10.5



P. SYSTEM

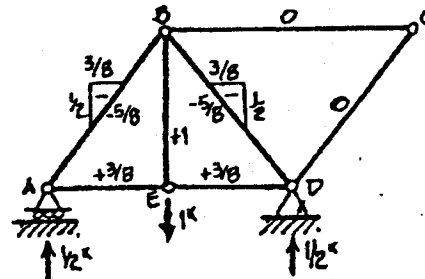
AREA OF BARS AB, BD, CD = 5 in²

AREA ALL OTHER BARS = 3 in²

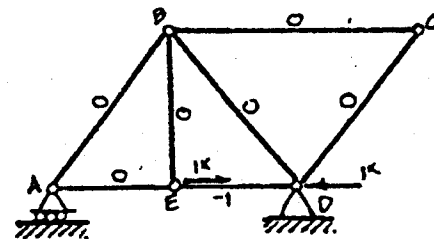
$E = 30,000$ ksi

MEMBER	F_P (K)	F_{Q1} (K)	F_{Q2} (K)	L (ft)	A (in ²)	$F_{Q1} F_P L / A$	$F_{Q2} F_P L / A$ (12)
AB	+150	-5/8	0	25	5	-5625	0
BC	+60	—	—	—	—	—	—
CD	+100	—	—	—	—	—	—
DE	-90	+3/8	-1	15	3	-2025	+5400
EA	-90	+3/8	0	15	3	-2025	0
BE	0	—	—	—	—	—	—
BD	-150	-5/8	0	25	5	+5625	0
$\Sigma = -4050$						+5400	

BARS AB AND BD FABRICATED
 $\frac{3}{4}$ " TOO LONG. SUPPORT D SETTLES 0.25"



Q₁-SYSTEM FOR Δ_{EV}



Q₂-SYSTEM FOR Δ_{EH}

(a) Δ_{EH}

$$1k \cdot \Delta_{EH} = \sum F_{Q2} F_P L / AE = \frac{5400}{30,000}$$

$$\Delta_{EH} = 0.18" \rightarrow$$

Δ_{EV}

$$1k \cdot \Delta_{EV} = \sum F_{Q1} F_P L / AE = \frac{-4050}{30,000}$$

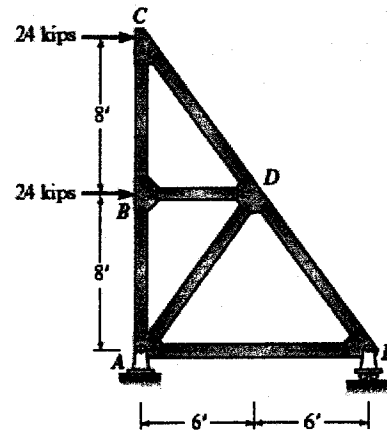
$$\Delta_{EV} = -0.135" \text{ DUE TO LOADS}$$

(b) Δ_{EV} DUE TO SETTLEMENT & FABRICATION ERROR:

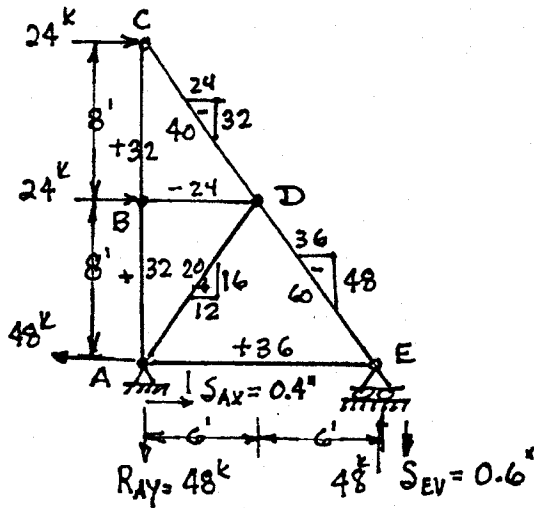
$$1k \cdot \Delta_{EV} - \frac{1}{2}k \left(\frac{1}{4} \right) = -\frac{5}{8} \left(\frac{3}{4} \right) \text{ 2 BARS}$$

$$\Delta_{EV} (\text{SETTLEMENT \& FAB ERR}) = -0.81"$$

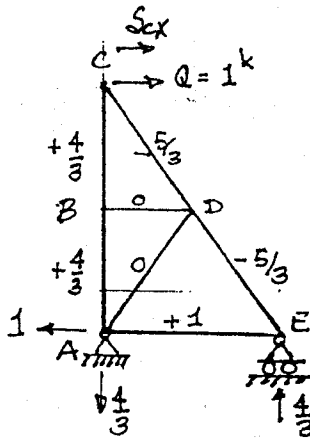
P10.6. When the truss in Figure P10.6 is loaded, the support at E displaces 0.6 in vertically downward and the support at A moves 0.4 in to the right. Compute the horizontal and vertical components of displacement of joint C. For all bars the area = 2 in² and E = 29,000 kips/in².



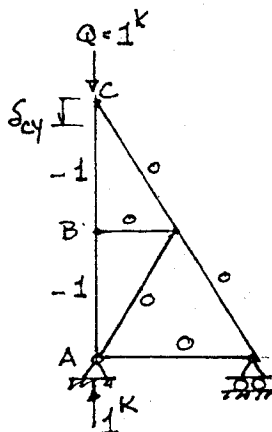
P10.6



P-SYSTEM



Q-SYSTEM S_{cx}



Q-SYSTEM S_{cy}

BAR	L	F _P	F _Q	$\sum \frac{F_Q F_P L}{AE}$	F _Q	F _P F _L
AB	8	32	4/3	341.33	-1	-256
BC	8	32	4/3	341.33	-1	-256
CD	10	-40	-5/3	666.67	0	0
DE	10	-60	-5/3	1000.0	0	0
BD	6	-24	0	0	0	0
DA	10	20	0	0	0	0
AE	12	36	1	432	0	0

$$\sum = 2781.3 \quad \sum = -512$$

S_{cx}

$$\sum Q \cdot S_P = \sum \frac{F_Q F_P L}{AE}$$

$$1 \cdot S_{cx} = 1(0.4) + \frac{4}{3}(0.6) = \frac{2781.3(12)}{2(29,000)}$$

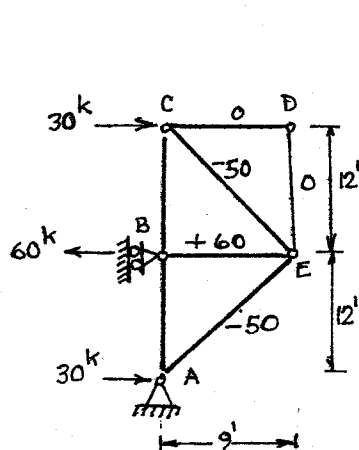
$$S_{cx} = 0.4 + 0.8 + 0.575 = 1.775" \rightarrow$$

S_{cy}

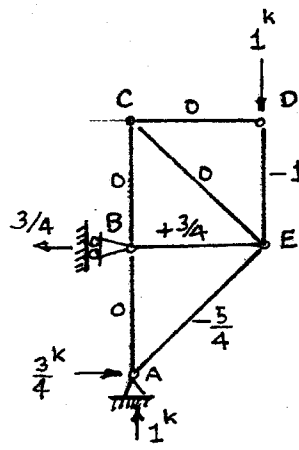
$$\sum Q \cdot S_P = \sum \frac{F_Q F_P L}{AE}$$

$$1 \cdot S_{cy} = \frac{-512(12)}{2 \times 29,000} = -0.106" \uparrow$$

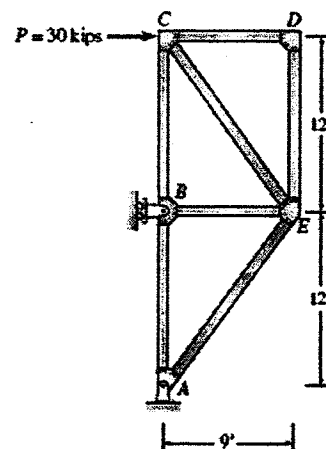
P10.7. (a) Compute the vertical deflection of joint D produced by the 30-kip load in Figure P10.7. For all bars area = 2 in² and $E = 9000$ kips/in², (b) Assume that the truss is unloaded. If bar AE is fabricated in too long, $\frac{8}{5}$ " how far to the right must the roller at B be displaced horizontally so that no vertical deflection occurs at joint D ?



P-SYSTEM



Q-SYSTEM



P10.7

$$(a) \sum Q \cdot \delta_{Dy} = \sum F_Q \frac{F_P L}{AE}$$

$$1 \cdot \delta_{Dy} = \left(\frac{3}{4}\right) \frac{60(9 \times 12)}{2 \times 9000} + \left(-\frac{5}{4}\right) \frac{(-50)(15 \times 12)}{2 \times 9000}$$

$$\delta_{Dy} = 0.27 + 0.625 = \underline{0.895 \text{ in}} \quad \text{ANS}$$

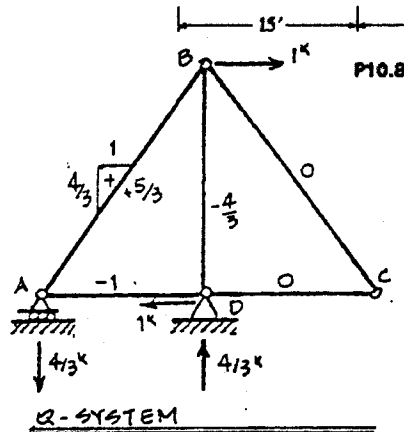
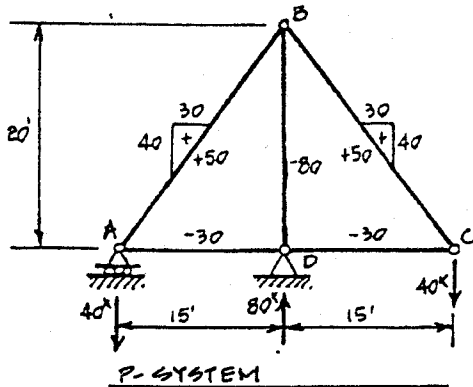
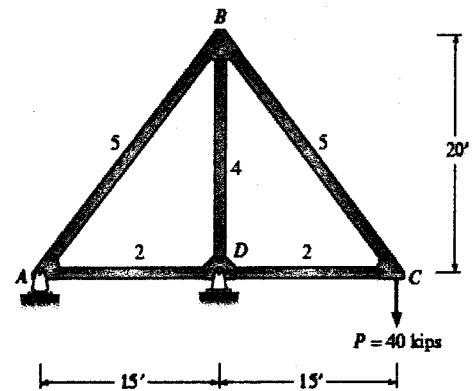
(b) BAR AE FABRICATED $\frac{8}{5}$ " TOO LONG

$$\sum Q \cdot \delta = \sum F_Q \cdot \Delta L$$

$$1 \times 0 + \frac{3}{4} \cdot \Delta_{BH} = -\frac{5}{4} \times \frac{8}{5}$$

$$\underline{\Delta_{BH} = \frac{8}{3} \text{ in}}$$

P10.8. (a) Find the horizontal deflection at joint B produced by the 40-kip load in Figure P10.8. The area of all bars in units of square inches is shown on the sketch of the truss; $E = 30,000$ kips/in². (b) To restore joint B to its initial position in the horizontal direction, how much must bar AB be shortened? (c) If the temperature of bars AB and BC increases 80°F , determine the vertical displacement of joint C . $\alpha_t = 6.5 \times 10^{-6}$ (in/in)/ $^\circ\text{F}$. The rocker at support A is equivalent to a roller.



MEMBER	F_P	F_Q	L	A	$(\sum F_P F_Q \frac{L}{A}) 12''/\text{FT.}$
AB	+50	+5/3	25	5	5000
BC	+50	0	25	5	0
CD	-30	0	15	2	0
DA	-30	-1	15	2	2700
BD	-80	-4/3	20	4	6400
					$\Sigma = 14100$

a) δ_{BH}
 $W_Q = U_Q$

$$1\text{K} \cdot \delta_{BH} = \sum F_P F_Q \frac{L}{AE} = \frac{14100}{E} = \frac{14100}{30,000}$$

$$\delta_{BH} = 0.47'' \rightarrow$$

b) ΔL_{AB} TO RESTORE B TO ORIGINAL HORIZONTAL POSITION.
 (WITH NO LOAD, MOVE 'D' 0.47" TO THE LEFT)

$$W_Q = U_Q$$

$$-1\text{K} \cdot (0.47'') = (+5/3) \Delta L_{AB}$$

$$\Delta L_{AB} = 0.28''$$

SHORTER

c) δ_{CV} USE P-SYSTEM AS Q-SYSTEM

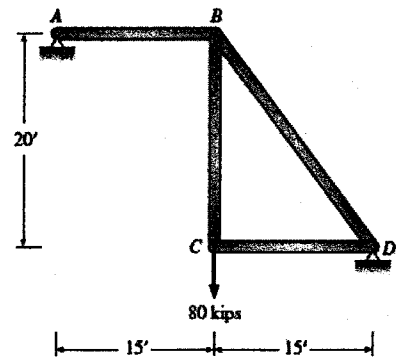
$$W_Q = U_Q$$

$$40 \delta_{CV} = \sum F_Q \cdot \Delta L = \sum F_Q (\alpha_t \Delta T L)$$

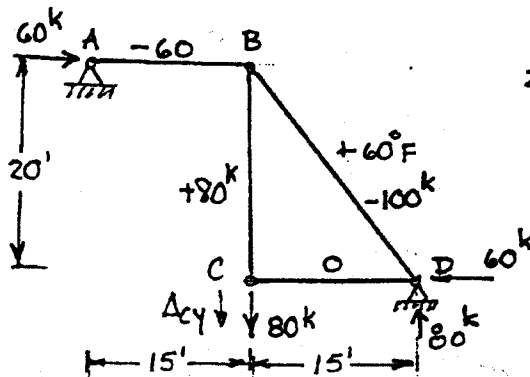
$$\delta_{CV} = 1/40 \cdot 2[50(6.5 \times 10^{-6}) 80(25 \times 12'')]$$

$$\delta_{CV} = 0.39'' \downarrow$$

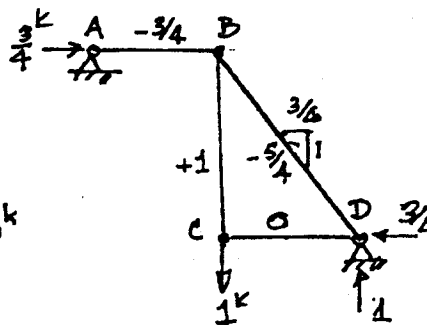
P10.9. Determine the horizontal and vertical deflection of joint C of the truss in Figure P10.9. In addition to the load at joint C, the temperature of member BD is subject to a temperature increase of 60°F. For all bars, $E = 29,000 \text{ kips/in}^2$, $A = 4 \text{ in}^2$, and $\alpha = 6.5 \times 10^{-6} \text{ (in/in)/}^\circ\text{F}$.



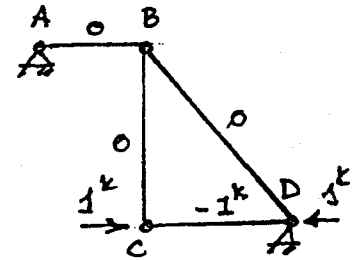
P10.9



P-SYSTEM



Q1-system



Q2-system

δ_{BD}^T due to $\Delta T = +60^\circ\text{F}$

$\delta_{BD}^T = \alpha \cdot \Delta T \cdot L = 6.5 \times 10^{-6} (60)(25 \times 12) = 0.117 \text{ in}$

BAR deformation due to 80 k LOAD AT "C"

$\Delta L_{AB} = \frac{PL}{AE} = \frac{-60(15 \times 12)}{EA} = -0.093''$

$\Delta L_{BC} = \frac{80(20 \times 12)}{EA} = 0.1655''$

$\Delta L_{BD} = \frac{-100(25 \times 12)}{EA} = -0.2586''$

Δ_{cy} : $Q_1 \cdot \Delta_{cy} = \sum F_{Q1} (\Delta L + \alpha \Delta T L)$

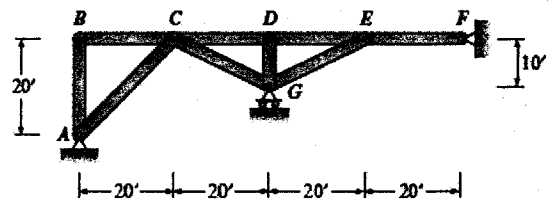
$\Delta_{cy} = \left[-\frac{3}{4}(-0.093) + 1(0.1655) + -\frac{5}{4}(-0.2586 + 0.117) \right]$

$\Delta_{cy} = 0.41'' \downarrow$

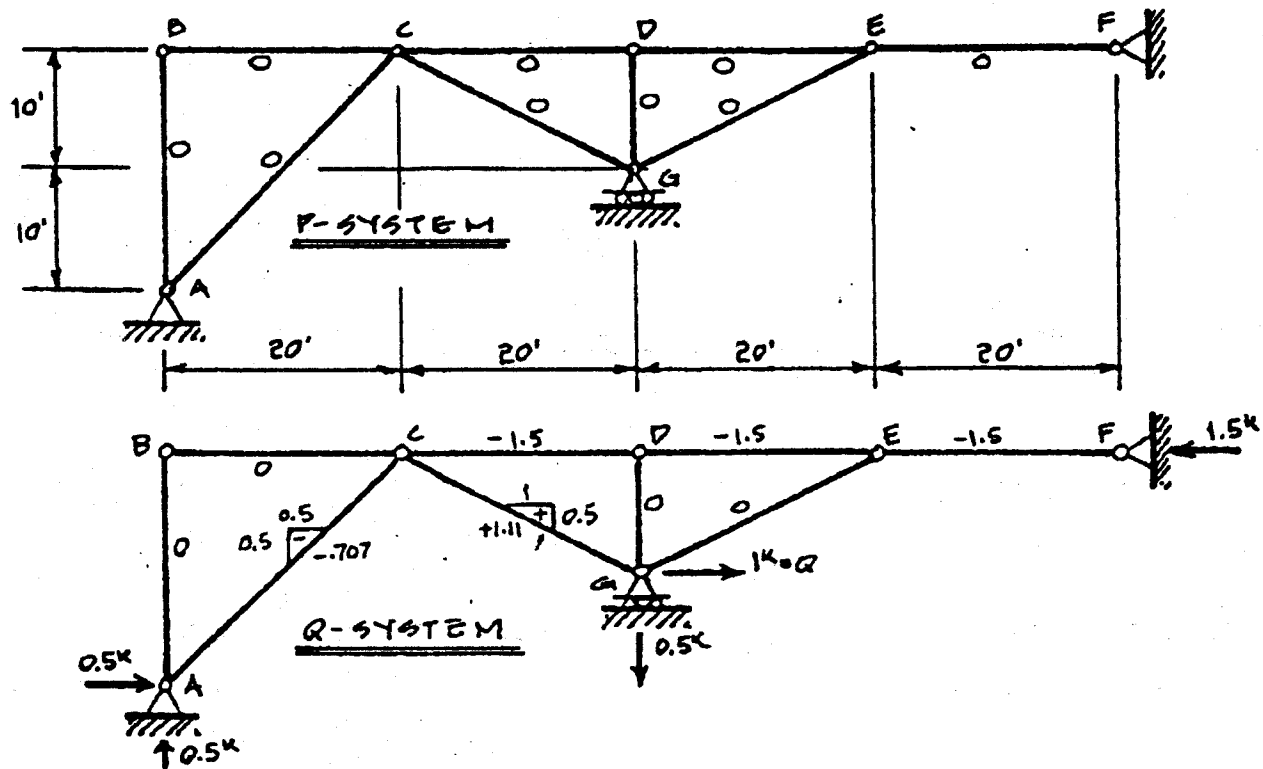
Δ_{cx} : $Q_2 \cdot \Delta_{cx} = \sum F_{Q2} (\Delta L + \alpha \Delta T L)$

$\Delta_{cx} = 1(0 + 0) = 0$

P10.10. In Figure P10.10 if support A moves horizontally 2 in to the right and support F settles 1 in vertically, compute the horizontal deflection of the roller at support G .



P10.10

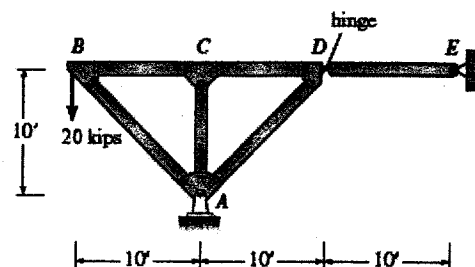


$$W_Q = \sum Q \cdot \Delta_Q; \quad \sum Q \cdot \delta_Q = \sum F_Q \Delta_L;$$

$$0.5(2') + 0.5(1') + 1K \delta_{GH} = 0$$

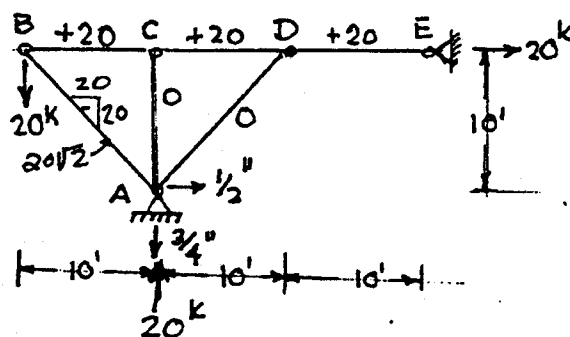
$$\delta_{GH} = 1" \leftarrow$$

P10.11. When the 20-kip load is applied to joint B of the truss in Figure P10.11, support A settles vertically downward in and displaces in horizontally to the right. Determine the vertical displacement of joint B due to all effects. The area of all bars = 2 in^2 ; $E = 30,000 \text{ kips/in}^2$.

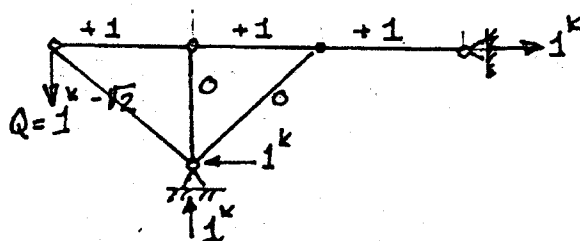


P10.11

$$A = 2 \text{ in}^2, E = 30,000 \text{ K/in}^2$$



P-SYSTEM



Q-SYSTEM

$$\sum Q \cdot S_p = \sum F_Q F_P \frac{L}{AE}$$

$$1^k \cdot S_{By} - \frac{1}{2} \times 1^k - \frac{3}{4} \times 1^k = \sum F_Q F_P \frac{L}{AE} \quad (1)$$

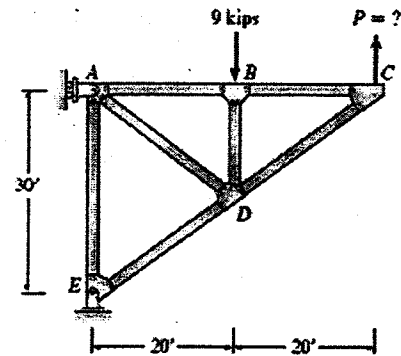
$$\sum F_Q F_P \frac{L}{AE} = \left[\frac{1^k \times 20^k (10 \times 12)}{2 \times 30,000} \right] 3 + \frac{(-\sqrt{2}) \times (20 \times \sqrt{2}) \times (10 \sqrt{2}) 12}{2 \times 30,000} \quad (2)$$

SUBSTN. (2) INTO (1)

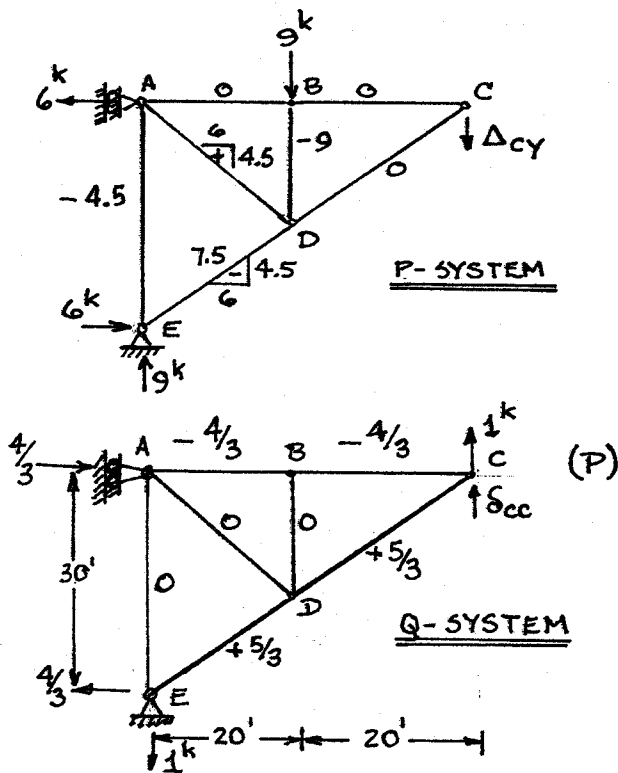
$$S_{By} = \frac{1}{2} + \frac{3}{4} + [0.04] 3 + 0.113$$

$$S_{By} = 1.483'' \downarrow$$

P10.12. Determine the value of the force P that must be applied to joint C of the truss in Figure P10.12 if the vertical deflection at C is to be zero. The area of all bars = 1.8 in^2 ; $E = 30,000 \text{ kips/in}^2$.



P10.12



COMPUTE Δ_{cy}

$$\sum Q \cdot \delta_P = \sum F_Q \frac{F_P L}{AE}$$

$$1^k \cdot \Delta_{cy} = \sum \frac{5}{3} \cdot \frac{(-7.5)(25)}{AE}$$

$$\Delta_{cy} = \underline{\underline{-\frac{312.5}{AE}}}$$

COMPUTE δ_{cc}

$$\sum Q \cdot \delta_P = \sum F_Q^2 \frac{L}{AE}$$

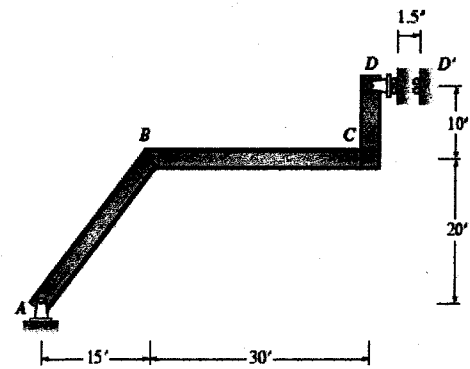
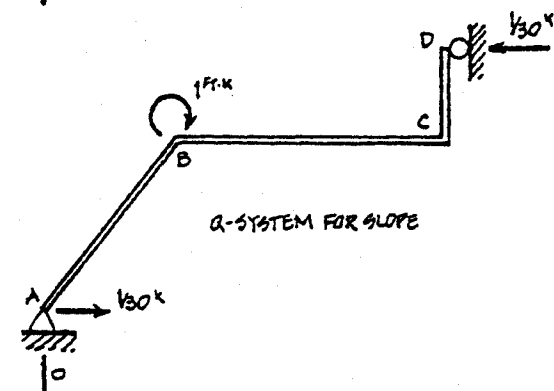
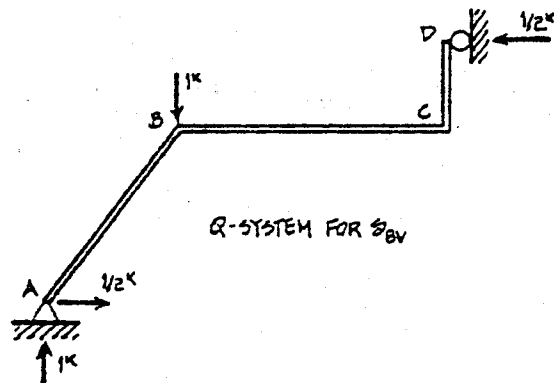
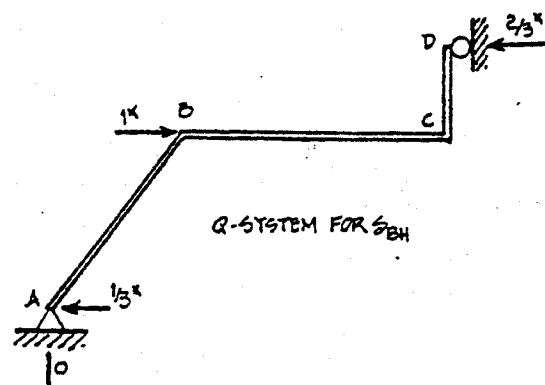
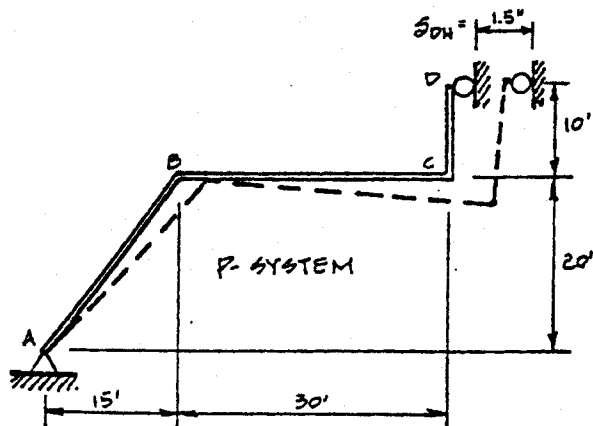
$$1 \cdot \delta_{cc} = \frac{(-\frac{4}{3})^2 20(2)}{AE} + \frac{(\frac{5}{3})^2 25(2)}{AE}$$

$$\delta_{cc} = \frac{71.11}{AE} + \frac{138.89}{AE} = \underline{\underline{\frac{210}{AE}}}$$

$$\Delta_{cy} = 0 = -\frac{312.5}{AE} + \frac{210}{AE} P$$

$$P = \underline{\underline{1.488 \text{ kips} \text{ ANS}}}$$

P10.13. In Figure P10.13 support D is constructed 1.5 in to the right of its specified location. Using Bernoulli's principle in Section 10.8, compute (a) the horizontal and vertical components of the displacement of joint B and (b) the change in slope of member BC .



P10.13

$$W_Q = U_Q$$

$$-2/3K(1.5'') + 1K \delta_{BH} = 0$$

$$\delta_{BH} = 1'' \rightarrow$$

$$W_Q = U_Q$$

$$-1/2K(1.5'') + 1K \delta_{BY} = 0$$

$$\delta_{BY} = 3/4'' \downarrow$$

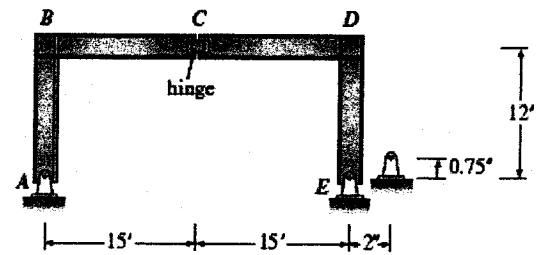
$$W_Q = U_Q$$

$$-1/30K(1.5'') + 1Ft.K \Delta\theta_{BC} = 0$$

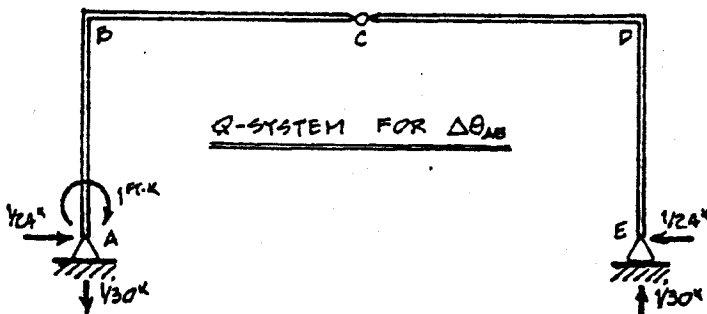
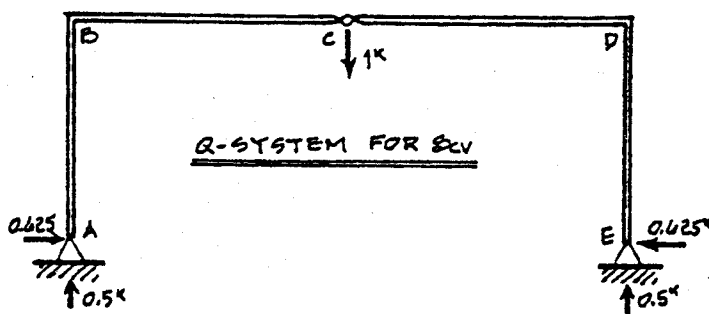
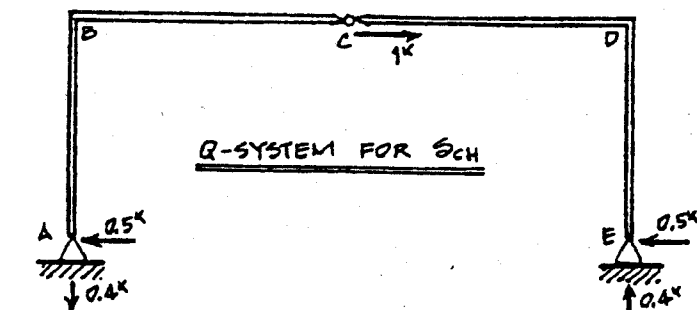
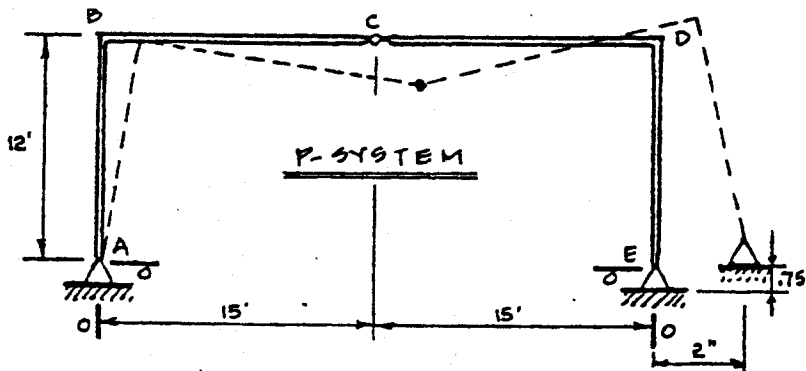
$$\Delta\theta_{BC} = \frac{1.5(1/30)}{12''}$$

$$\Delta\theta_{BC} = 0.004167 \text{ RADIANS } \curvearrowright$$

P10.14. If supports A and E in Figure P10.14 are constructed 30 ft and 2 in apart instead of 30 ft apart, and if support E is also 0.75 in above its specified elevation, determine the vertical and horizontal components of deflections of the hinge at C and the slope of member AB when the frame is erected.



P10.14

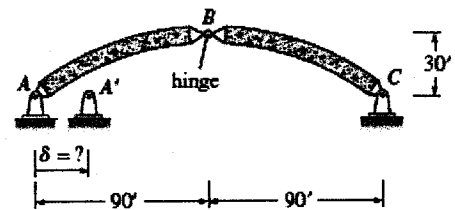


$$\begin{aligned} \delta_{CH} \quad W_Q &= U_Q \\ 0.5k(-2'') + 0.4k(0.75'') + 1k\delta_{CH} &= 0 \\ \delta_{CH} &= 0.7'' \rightarrow \end{aligned}$$

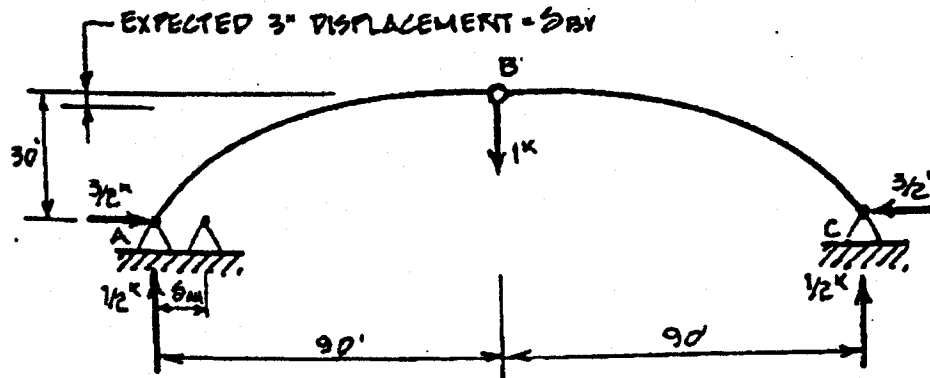
$$\begin{aligned} \delta_{CV} \quad W_Q &= U_Q \\ 0.625k(-2'') + 0.5k(0.75'') + 1k\delta_{CV} &= 0 \\ \delta_{CV} &= 0.875'' \downarrow \end{aligned}$$

$$\begin{aligned} \Delta\theta_{AB} \quad W_Q &= U_Q \\ 1ft-k(\Delta\theta_{AB}) - \frac{1}{24}k(2'') + \frac{1}{30}k(0.75'') &= 0 \\ \Delta\theta_{AB} &= 0.00486 \text{ RADIANS } \curvearrowright \end{aligned}$$

P10.15. Under the dead load of the arch in Figure P10.15, the hinge at B is expected to displace 3 in downward. To eliminate the 3-in displacement, the designers will shorten the distance between supports by moving support A to the right. How far should support A be moved?



P10.15



$$\delta_{BY} = 3''$$

$$\delta_{AH} = ?$$

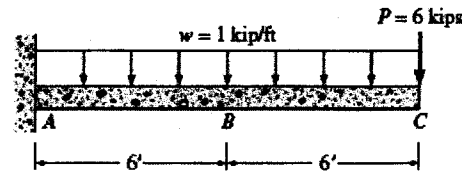
$$W_Q = U_Q$$

$$\sum Q \cdot \delta_p = 0$$

$$\frac{3}{2} k \delta_{AH} - 1 k (3'') = 0$$

$$\delta_{AH} = 2'' \rightarrow$$

P10.16. (a) Compute the vertical deflection and slope of the cantilever beam at points B and C in Figure P10.16. Given: EI is constant throughout, $L = 12$ ft, and $E = 4000$ kips/in². What is the minimum required value of I if the deflection of point C is not to exceed 0.4 in?



P10.16

COMPUTE S_C

$P_x + w \cdot x \cdot \frac{x}{2} = M_p$

$1^k = q$

$\int_0^L x = M_q$

$\delta_C = \int_0^L M_p M_q \frac{dx}{EI}$

$\delta_C = \int_0^L (P_x + \frac{wx^2}{2}) x \frac{dx}{EI}$

$\delta_C = \frac{1}{EI} \int_0^L (Px^2 + \frac{wx^3}{2}) dx$

$\delta_C = \left[\frac{Px^3}{3EI} + \frac{wx^4}{8EI} \right]_0^L$

$\delta_C = \frac{PL^3}{3EI} + \frac{wL^4}{8EI} \quad (1)$

COMPUTE $I_{\min}$ IF $S_C \leq 0.4$ in

$0.4 = \frac{6(12)^3}{3(4000)I} + \frac{1(12)^4}{8(4000)I}$

$\min. I = 6531.8 \text{ in}^4$

COMPUTE θ_C

P-SYSTEM

$M_p = Px + \frac{wx^2}{2}$

Q-SYSTEM

$M_q = 1$

$1^k \cdot ft$

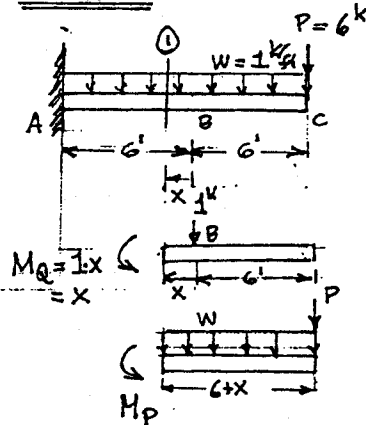
$\theta_C = \int_0^L 1 \cdot (Px + \frac{wx^2}{2}) \frac{dx}{EI}$

$\theta_C = \frac{1}{EI} \left[\frac{Px^2}{2} + \frac{wx^3}{6} \right]_0^L$

$\theta_C = \frac{PL^2}{2EI} + \frac{wL^3}{6EI}$

$\theta_C = \frac{6(12)^2}{2EI} + \frac{1(12)^3}{6EI} = \frac{720}{EI}$

DEFL. AT B



$M_p = w(6+x)(6+x) + P(6+x)$

$= \frac{w}{2}(36 + 12x + x^2) + 6P + Px$

$\delta_B = \frac{1}{EI} \int_0^6 M_q M_p dx$

$\delta_B = \frac{1}{EI} \int_0^6 (x)(18w + 6xw + \frac{wx^2}{2} + 6P + Px) dx$

$\delta_B = \frac{1}{EI} \int_0^6 (18wx + 6wx^2 + \frac{wx^3}{2} + 6Px + Px^2) dx$

$\delta_B = \frac{1}{EI} \left[\frac{18w}{2}x^2 + \frac{6w}{3}x^3 + \frac{wx^4}{8} + 3Px^2 + \frac{Px^3}{3} \right]_0^6$

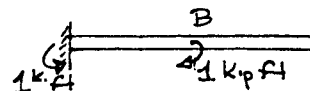
$= \frac{1}{EI} [9w36 + 2w216 + 162w + 108P + 72P]$

$\delta_B = \frac{1}{EI} [324w + 432w + 162w + 180P]$

$\delta_B = \frac{918w + 180P}{EI} = \frac{918 + 180 \times 6}{EI}$

$\delta_B = \frac{1998}{EI}$

θ_B



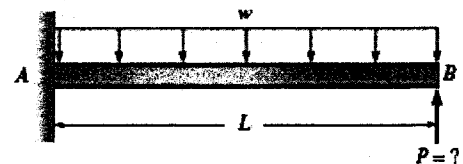
Q-SYSTEM
P-SYSTEM ABCD

$\theta_B = \frac{1}{EI} \int_0^6 M_q M_p dx$

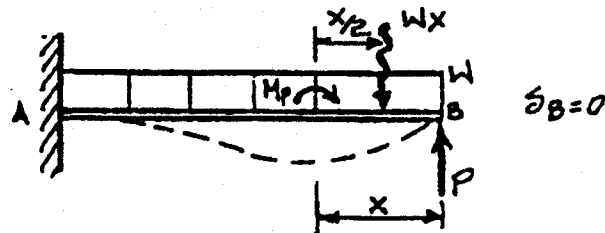
$\theta_B = \frac{1}{EI} \int_0^6 (1)(18w + 6wx + \frac{wx^2}{2} + 6P + Px) dx$

$\theta_B = \frac{576}{EI}$

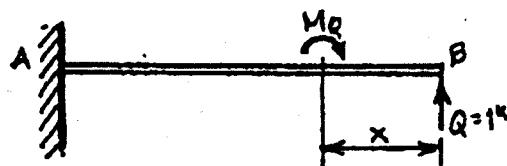
P10.17. Determine the magnitude of the vertical force P that must be applied at the tip of the cantilever beam in Figure P10.17 if the deflection at B is to be zero. EI is constant. Express the answer in terms of w and L .



P10.17



P-SYSTEM



Q-SYSTEM

$$\Sigma Q \cdot \delta_P = \int_{x=0}^{x=L} M_Q M_P dx / EI$$

$$\text{WHERE } M_Q = 1^k x ; M_P = Px - wx^2$$

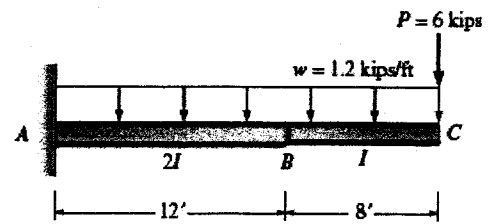
$$Q \cdot \delta_B = \int_0^L x (Px - wx^2/2) dx / EI$$

$$1^k \cdot 0 = \int_0^L (Px^2 - wx^3/2) dx / EI$$

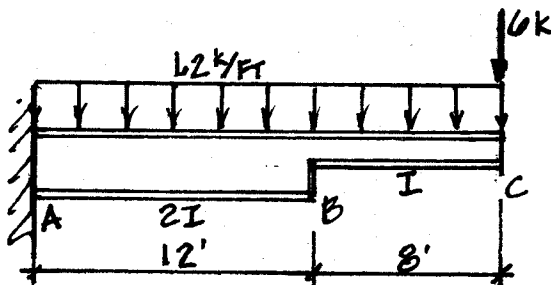
$$0 = PL^3/3 - WL^4/8$$

$$\underline{P = 3WL/8}$$

P10.18. Compute the vertical deflection of point C in Figure P10.18. Given: $I = 1200 \text{ in}^4$, $E = 29,000 \text{ kips/in}^2$.



P10.18



SEGMENT "BC"

Q-SYSTEM

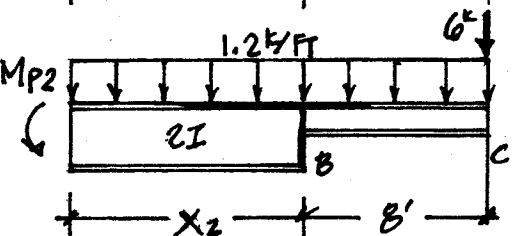
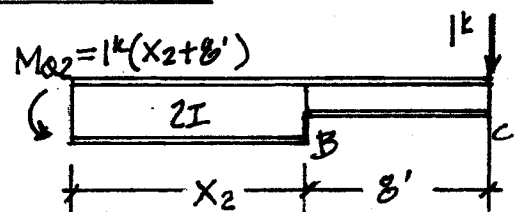
$$M_{Q1} = 1^k X_1$$

P-SYSTEM

$$M_{P1} = \frac{1.2 X_1^2}{2} + 6 X_1$$

$$= 0.6 X_1^2 + 6 X_1$$

SEGMENT "AB"



$$M_{P2} = \frac{1.2 (X_2 + 8)^2}{2} + 6^k (X_2 + 8')$$

$$\delta_C = 1^k = \sum \int M_Q M_P dx / EI$$

$$\delta_C = \int_0^{8'} X_1 (0.6 X_1^2 + 6 X_1) dx / EI + \int_0^{12'} (X_2 + 8') [0.6 (X_2 + 8)^2 + 6 (X_2 + 8)] dx / 2EI$$

$$\delta_C = \int_0^{8'} (0.6 X_1^3 + 6 X_1^2) dx / EI + \int_0^{12'} 0.6 X_2^3 + 20.4 X_2^2 + 211.2 X_2 + 691.2 dx / 2EI$$

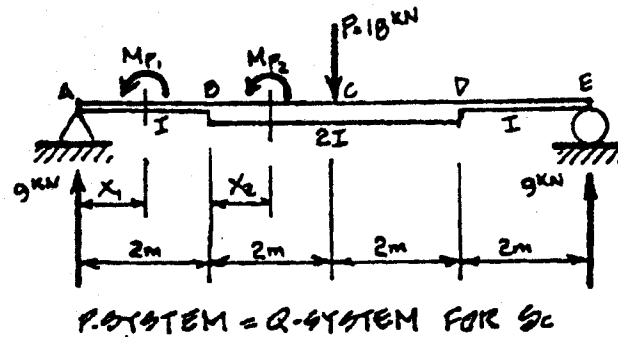
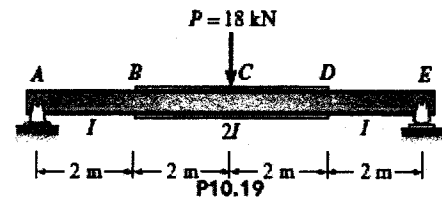
$$\delta_C = \frac{0.15 X_1^4 + 2 X_1^3}{EI} \Big|_0^{8'} + \frac{0.15 X_2^4 + 6.8 X_2^3 + 105.6 X_2^2 + 691.2 X_2}{2EI} \Big|_0^{12'}$$

$$= 1638.4 / EI + 19180.8 / EI = 20,819.2 / EI$$

$$= \frac{20,819.2 (1728)}{29000 (1200)}$$

$$\delta_C = 1.034 \text{ in. } \downarrow$$

P10.19. Compute the deflection at midspan of the beam in Figure P10.19. Given: $I = 46 \times 10^6 \text{ mm}^4$, $E = 200 \text{ GPa}$. Treat rocker at E as a roller.



$$I = 46 \times 10^6 \text{ mm}^4 \quad E = 200 \text{ GPa}$$

SEGMENT	I	ORIGIN	RANGE	$M_P = M_Q$
AB = DE	I	A	$0 \leq x_1 \leq 2$	$9x$
BC = CD	2I	B	$0 \leq x_2 \leq 2$	$9(x+2)$

BY SYMMETRY, DOUBLE U_Q FROM 'A' TO 'C'

$$\Sigma Q \cdot \delta_C = 2 \int_A^B M_P^2 dx / EI_{AB} + 2 \int_B^C M_P^2 dx / EI_{BC}$$

$$18 \delta_C = 2 \int_0^2 (9x)^2 dx / EI_{AB} + 2 \int_0^2 [9(x+2)]^2 dx / EI_{BC}$$

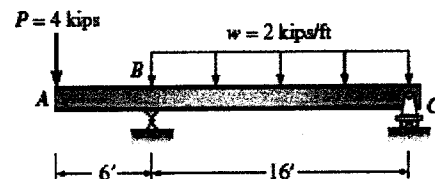
$$= 2 \left[\frac{81x^3}{3EI} \right]_0^2 + \frac{(2)81}{EI} \left[\frac{x^3}{3} + \frac{4x^2}{2} + 4x \right]_0^2$$

$$\delta_C = 108 / EI$$

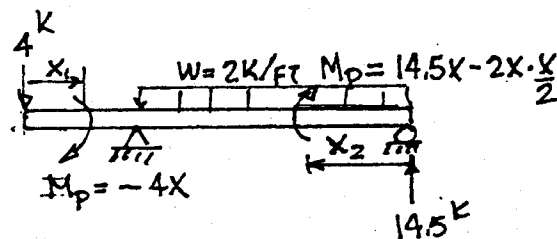
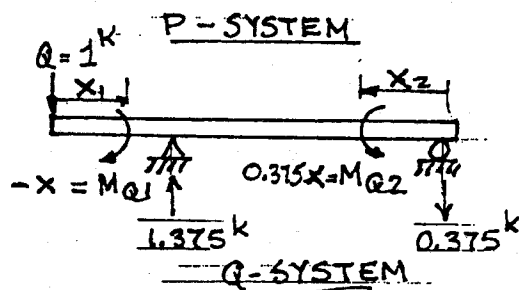
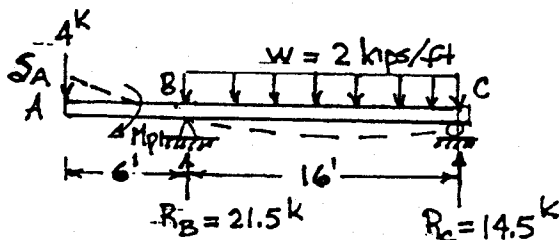
$$\delta_C = \frac{108 (1000)^3}{(46 \times 10^6) (200 \times 10^9)}$$

$$\delta_C = \underline{\underline{11.74 \text{ mm}}}$$

P10.20. What is the minimum required value of I for the beam in Figure P10.20 if point A is not to deflect more than 0.3 in? Given: EI is constant, $E = 29,000$ kips/in².



P10.20



$$1 \times S_A = \int \frac{M_Q M_P}{EI} dx$$

$$1 \cdot S_A = \int_0^6 \frac{(-x)(-4x)}{EI} dx + \int_0^{16} \frac{(-0.375x)(14.5x - 2x \cdot \frac{x}{2})}{EI} dx$$

$$S_A = \int_0^6 \frac{4x^2}{EI} dx + \int_0^{16} \frac{(-5.4375x^2 + 0.375x^3)}{EI} dx$$

$$S_A = \frac{4x^3}{3EI} \Big|_0^6 + \left[-\frac{5.4375x^3}{3EI} + \frac{0.375x^4}{4EI} \right] \Big|_0^{16}$$

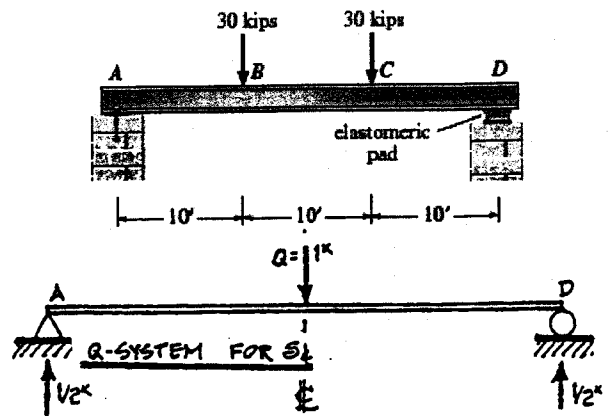
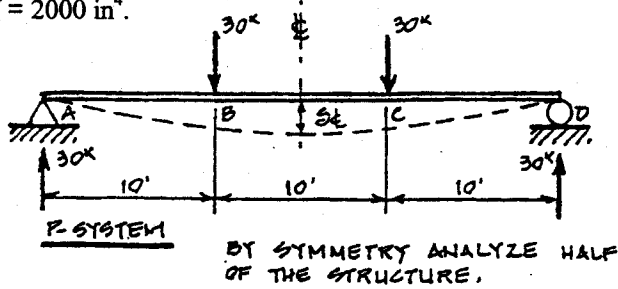
$$S_A = \frac{288}{EI} = \frac{1424}{EI} + \frac{6144}{EI} = \frac{992}{EI} \uparrow$$

COMPUTE I_{min} if $S_A \leq 0.3$ in

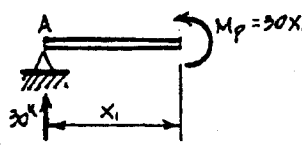
$$0.3 = \frac{992(1728)}{EI}$$

$$I_{MIN} = \frac{992(1728)}{29,000(0.3)} = \underline{\underline{197 \text{ in}^4}}$$

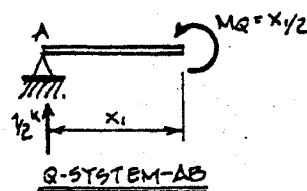
P10.21. Compute the deflection at midspan and the slope at *A* in Figure P10.21. *EI* is constant. Express the slope in degrees and the deflection in inches. Assume a pin support at *A* and a roller at *D*. $E = 29,000 \text{ kips/in}^2$, $I = 2000 \text{ in}^4$.



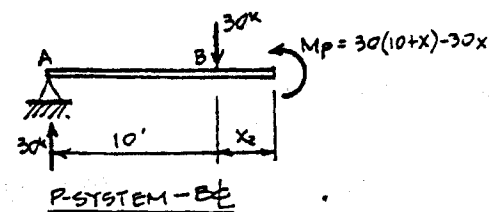
COMPUTE δ_k (MIDSPAN)
DIAGRAM

	SEGMENT	ORIGIN	RANGE	M_P	M_Q
	AB (=CD)	A	$0 \leq x_1 \leq 10$	$30x_1$	$x_1/2$
	BC (=EC)	B	$0 \leq x_2 \leq 5$	300	$5 + x_2/2$

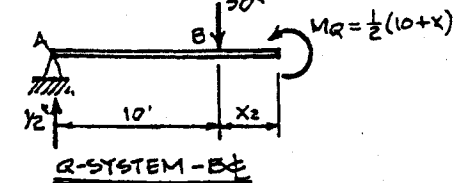
P-SYSTEM-AB



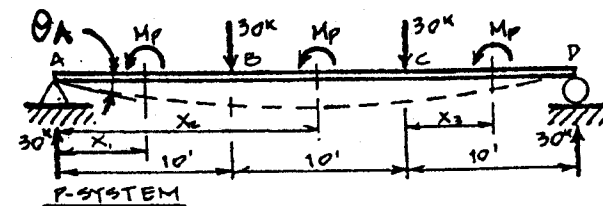
Q-SYSTEM-AB



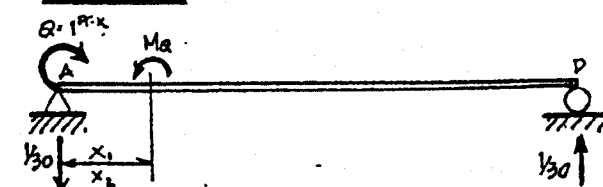
P-SYSTEM-BC



Q-SYSTEM-BC



P-SYSTEM



Q-SYSTEM FOR θ_A

$$W_Q = \int Q$$

$$1^k. \delta_k = \int M_Q M_P dx / EI$$

$$\delta_k = 2 \int_{x=0}^{x=10} \frac{x}{2} (30x) \frac{dx}{EI} + 2 \int_{x=0}^{x=5} (5 + \frac{x}{2}) 300 \frac{dx}{EI}$$

$$\delta_k = \frac{30}{EI} \left[\frac{x^3}{3} \right]_0^{10} + \frac{600}{EI} \left[5x + \frac{x^2}{4} \right]_0^5$$

$$\delta_k = \frac{10,000}{EI} + \frac{18,750}{EI}$$

$$\delta_k = \frac{28,750 (1728)}{29,000 (2,000)}$$

$$\delta_k = 0.86 \text{ in.} \downarrow$$

COMPUTE θ_A :

ORIGIN	RANGE	M_P	M_Q
A	$0 \leq x_1 \leq 10$	$30x_1$	$1 - x_1/30$
A	$10 \leq x_2 \leq 20$	300	$1 - x_2/30$
C	$0 \leq x_3 \leq 10$	$30x_3$	$x_3/30$

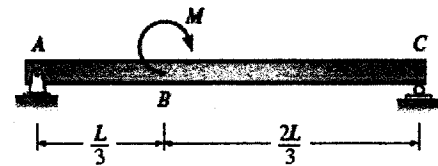
$$\theta_A = \int_0^{10} (30x_1)(1 - x_1/30) \frac{dx_1}{EI} + \int_{10}^{20} (300)(1 - x_2/30) \frac{dx_2}{EI} + \int_0^{10} (30x_3)(x_3/30) \frac{dx_3}{EI}$$

INTEGRATING GIVES:

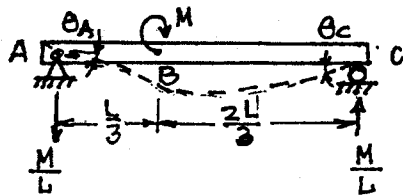
$$\theta_A = \frac{3000}{EI} = \frac{3000(144)}{29,000(2,000)} = 0.00745 \text{ RADIANS}$$

$$\theta_A = 0.43 \text{ DEGREES}$$

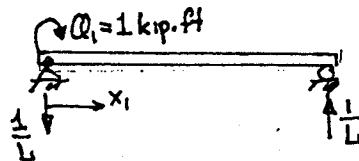
P10.22. Compute the slope at supports A and C in Figure P10.22. EI is constant. Express your answer in terms of E , I , L , and M .



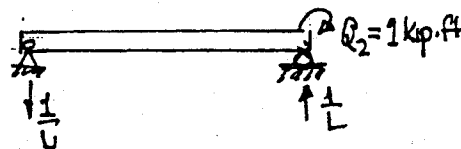
P10.22



P-SYSTEM



Q1 SYSTEM



Q2 SYSTEM

COMPUTE θ_A

$$\frac{1}{EI} \theta_A = \int_0^{L/3} \left(1 - \frac{1}{L} x_1\right) \left(-\frac{M}{L} x_1\right) \frac{dx_1}{EI} + \int_{L/3}^L \left(1 - \frac{1}{L} x_2\right) \left(M - \frac{M}{L} x_2\right) \frac{dx_2}{EI}$$

$$\theta_A = \left[-\frac{1}{2L} x_1^2 + \frac{1}{3L^2} x_1^3 \right]_0^{L/3} + \left[x_2 \frac{x_2^2}{L} + \frac{x_2^3}{3L^2} \right]_{L/3}^L \left(\frac{M}{EI} \right)$$

$$\theta_A = \frac{M}{EI} \left(-\frac{L}{18} + \frac{L}{81} + \frac{8L}{81} \right) = \frac{ML}{18EI} \text{ RAD } \curvearrowright$$

COMPUTE θ_B

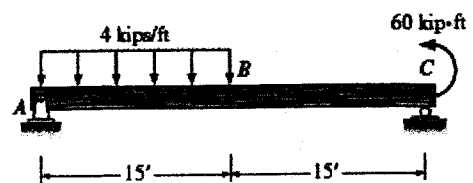
$$Q_2^{11} \theta_B = \int_0^{L/3} \left(-\frac{1}{L} x_1\right) \left(-\frac{M}{L} x_1\right) \frac{dx_1}{EI} + \int_{L/3}^L \left(-\frac{1}{L} x_2\right) \left(M - \frac{M}{L} x_2\right) \frac{dx_2}{EI}$$

$$\theta_B = \frac{M}{EI} \left(\left[\frac{1}{3L^2} x_1^3 \right]_0^{L/3} + \left[-\frac{1}{2L} x_2^2 + \frac{1}{3L^2} x_2^3 \right]_{L/3}^L \right)$$

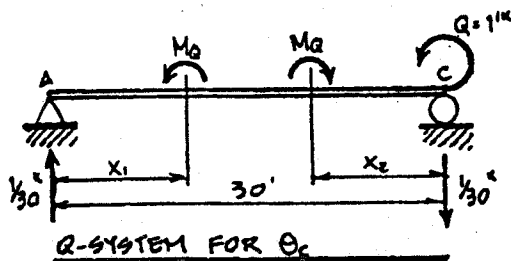
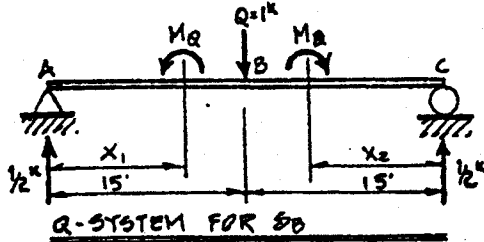
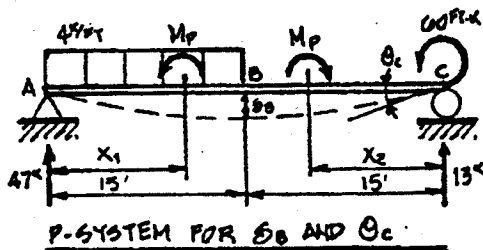
$$\theta_B = \frac{M}{EI} \left[\frac{L}{81} + \left(-\frac{L}{2} + \frac{L}{3} \right) - \left(-\frac{1}{18} L + \frac{L}{81} \right) \right]$$

$$\theta_B = -\frac{ML}{9EI} \text{ RAD } \curvearrowleft$$

P10.23. Compute the deflection at B and the slope at C in Figure P10.23. Given: EI is constant.



P10.23



COMPUT'G	SEGMENT	ORIGIN	RANGE	M_P	M_Q
δ_B	AB	A	$0 \leq x_1 \leq 15$	$47x - 2x^2$	$\frac{x}{2}$
	BC	C	$0 \leq x_2 \leq 15$	$60 + 13x$	$\frac{x}{2}$
θ_C	AB	A	$0 \leq x_1 \leq 15$	$47x - 2x^2$	$\frac{x}{30}$
	BC	C	$0 \leq x_2 \leq 15$	$60 + 13x$	$1 - \frac{x}{30}$

δ_B

$$Q \cdot \delta_B = \int M_Q M_P dx / EI$$

$$1^k \cdot \delta_B = \int_{x=0}^{x=15} \frac{x}{2} (47x - 2x^2) \frac{dx}{EI} + \int_{x=0}^{x=15} \frac{x}{2} (60 + 13x) \frac{dx}{EI}$$

$$\delta_B = \int_0^{15} (23.5x^2 - x^3) \frac{dx}{EI} + \int_0^{15} (30x + 6.5x^2) \frac{dx}{EI}$$

$$\delta_B = \frac{1}{EI} \left[\frac{23.5x^3}{3} - \frac{x^4}{4} \right]_0^{15} + \frac{1}{EI} \left[\frac{30x^2}{2} + \frac{6.5x^3}{3} \right]_0^{15}$$

$$\delta_B = 24,468.7 / EI$$

θ_C

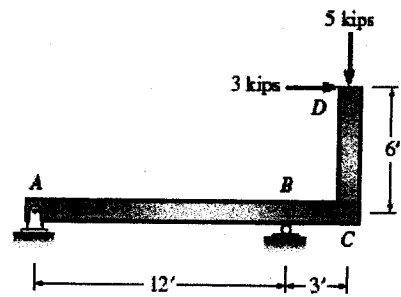
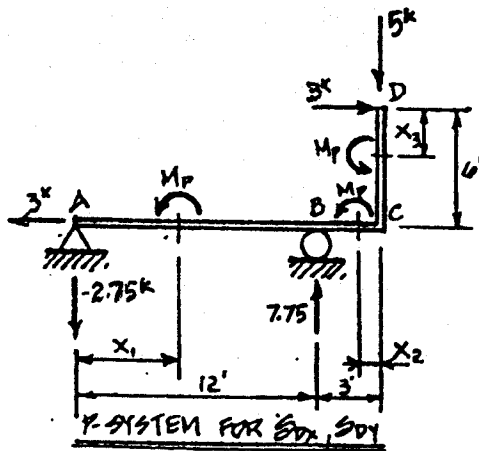
$$Q \cdot \theta_C = \int M_Q M_P dx / EI$$

$$1^{k \cdot ft} \theta_C = \int_0^{15} \frac{x}{30} (47x - 2x^2) \frac{dx}{EI} + \int_0^{15} \left(1 - \frac{x}{30}\right) (60 + 13x) \frac{dx}{EI}$$

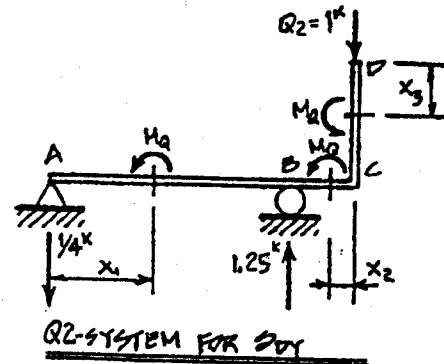
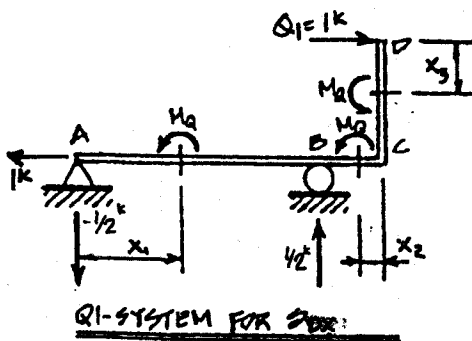
$$\theta_C = \frac{1}{EI} \left[\frac{47x^3}{90} - \frac{x^4}{60} \right]_0^{15} + \frac{1}{30EI} \left[1800x + \frac{330x^2}{2} - \frac{13x^3}{3} \right]_0^{15}$$

$$\theta_C = 2568.5 / EI$$

P10.24. Compute the horizontal and vertical components of deflection at point D in Figure P10.24. EI is constant, $I = 120 \text{ in}^4$, $E = 29,000 \text{ kips/in}^2$.



P10.24



IGNORE AXIAL DEFORMATIONS, $E = 29,000 \text{ ksi}$ $I = 120 \text{ in}^4$

SEGMENT	ORIGIN	RANGE	M_{Q1}	M_{Q2}	M_P
AB	A	$0 \leq x_1 \leq 12'$	$0.5x_1$	$0.25x_1$	$2.75x_1$
BC	C	$0 \leq x_2 \leq 3'$	6	$1x_2$	$18 + 5x_2$
DC	D	$0 \leq x_3 \leq 6'$	$1x_3$	0	$3x_3$

δ_{Dx}

$$\sum Q \cdot \delta_P = \sum \int M_Q M_P dx / EI$$

$$1k \cdot \delta_{Dx} = \int_0^{12'} (0.5x_1)(2.75x_1) dx / EI + \int_0^3 6(18 + 5x_2) dx / EI + \int_0^6 x_3(3x_3) dx / EI$$

$$= 0.4583x_1^3 \Big|_0^{12'} + (108x_2 + 15x_2^2) \Big|_0^3 + (x_3^3) \Big|_0^6$$

$$= 791.94/EI + 459/EI + 216/EI$$

$$= 1466.9 (1728) / 29000 (120) = \underline{0.728'' \rightarrow}$$

δ_{Dy}

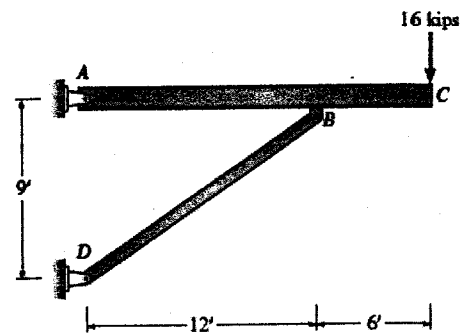
$$\sum Q \cdot \delta_P = \sum \int M_Q M_P dx / EI$$

$$1k \cdot \delta_{Dy} = \int_0^{12'} (0.25x_1)(2.75x_1) dx / EI + \int_0^3 (x_2)(18 + 5x_2) dx / EI$$

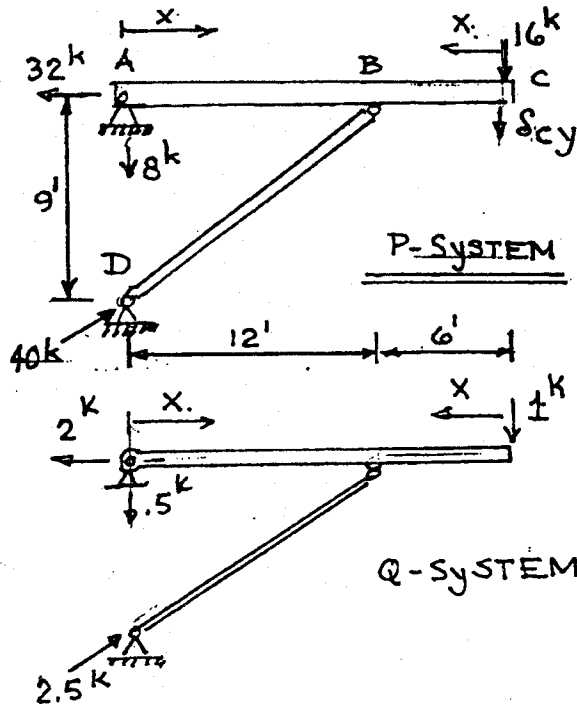
$$= 0.2292x_1^3 \Big|_0^{12'} + (9x_2^2 + 1.667x_2^3) \Big|_0^3$$

$$= 522.07 (1728) / 29000 (120) = \underline{0.26'' \downarrow}$$

P10.25. Compute the vertical deflection of joint C in Figure P10.25. In member ABC consider only the strain energy associated with bending. Given: $I_{AC} = 340 \text{ in}^4$ and $A_{BD} = 5 \text{ in}^2$. How much should bar BD be lengthened to eliminate the vertical deflection of point C when the 16-kip load acts?



P10.25



COMPUTE: S_{cy}

$$W_Q = \int \frac{M_Q M_P}{EI} dx + F_Q \cdot \frac{F_P L}{AE}$$

$$1^k \cdot S_{cy} = \int_0^6 \frac{(1 \cdot x)(16x)}{EI_{AC}} dx + \int_0^{12} \frac{(0.5x)(-8x)}{EI} dx + \frac{(2.5)(40)(15)}{AE}$$

$$S_{cy} = \left[\frac{16x^3}{3EI} \right]_0^6 + \left[\frac{4x^3}{3EI} \right]_0^{12} + \frac{2.5 \times 40 \times 15}{5E}$$

$$\underline{S_{cy} = 0.73 \text{ in}}$$

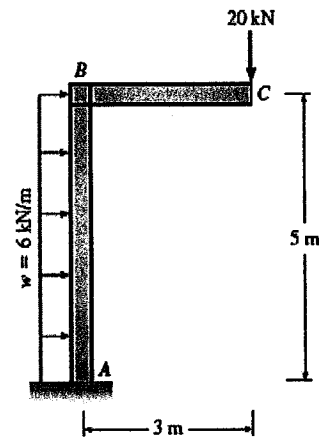
COMPUTE ΔL_{BD} TO ELIMINATE
DEFL AT C

$$W_Q = U_Q$$

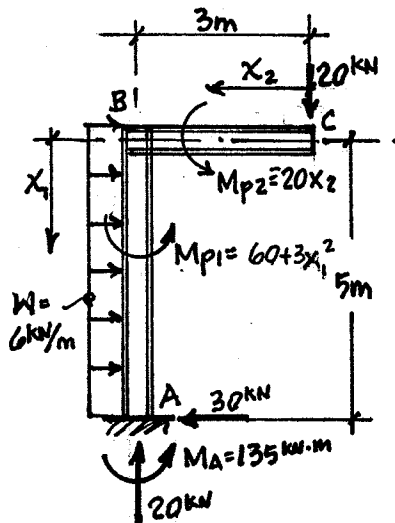
$$1^k \cdot 0.73 \text{ in} = F_Q \cdot \Delta L$$

$$\underline{0.292'' = \Delta L \text{ INCREASE IN LENGTH}}$$

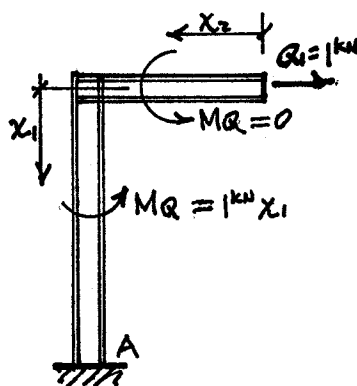
P10.26. Compute the horizontal and vertical components of the deflection at C in Figure P10.26.
 $E = 200 \text{ GPa}$, $I = 240 \times 10^6 \text{ mm}^4$.



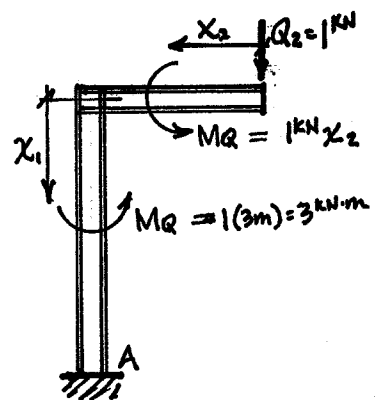
P10.26



P-SYSTEM



Q1-SYSTEM FOR δ_{Cx}



Q2-SYSTEM FOR δ_{Cy}

VIRTUAL WORK

COMPUTE δ_{Cx} :

$$Q_1 \cdot \delta_{Cx} = \int_0^x M_{Q1} M_P dx / EI$$

$$1 \text{ kN} \cdot \delta_{Cx} = \int_0^5 x (60 + 3x^2) dx / EI$$

$$= \frac{1}{EI} (30x^2 + \frac{3}{4} x^4) \Big|_0^5 = \frac{1218.75}{EI}$$

$$\delta_{Cx} = 25.4 \text{ mm} \rightarrow$$

COMPUTE δ_{Cy} :

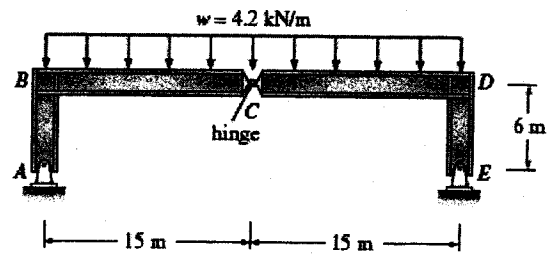
$$Q_2 \cdot \delta_{Cy} = \int_0^x M_{Q2} M_P dx / EI$$

$$1 \text{ kN} \cdot \delta_{Cy} = \int_0^5 (3 \text{ kN·m}) (60 + 3x^2) dx / EI + \int_0^3 x (20x) dx / EI$$

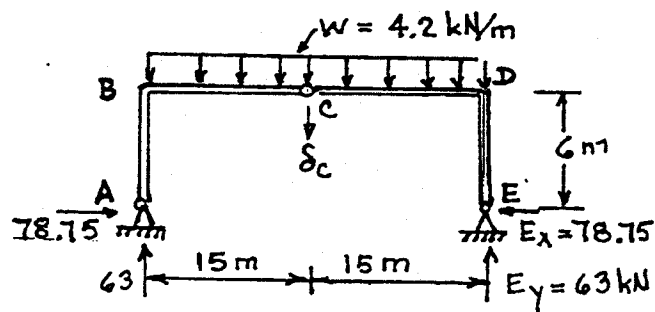
$$= \frac{1}{EI} (180x + 3x^3) \Big|_0^5 + 6.67x^3 \Big|_0^3 = \frac{1455}{EI}$$

$$\delta_{Cy} = 30.3 \text{ mm} \downarrow$$

P10.27. Compute the vertical displacement of the hinge at C in Figure P10.27. EI is constant for all members, $E = 200 \text{ GPa}$, $I = 1800 \times 10^6 \text{ mm}^4$.

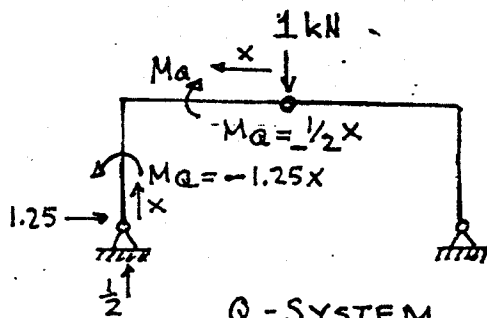
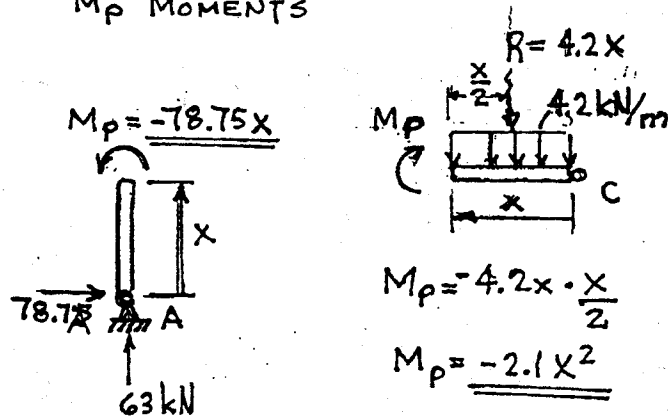


P10.27



P-SYSTEM

M_P MOMENTS



Q-SYSTEM

VIRTUAL WORK (AB + ED)

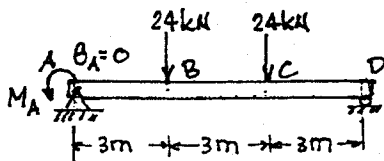
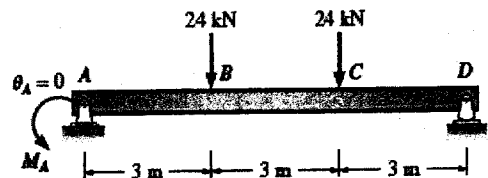
$$1 \text{ kN} \cdot \delta_c = 2 \int_0^6 \frac{(-1.25x)(-78.75x)}{EI} dx + 2 \int_0^{15} \frac{(-0.5x)(-2.1x^2)}{EI} dx$$

$$\delta_c = \frac{2}{EI} \left[\left(\frac{98.44}{3} \right) x^3 \right]_0^6 + \frac{2}{EI} \left[\frac{1.05x^4}{4} \right]_0^{15}$$

$$= \frac{2}{EI} [7088 + 13290]$$

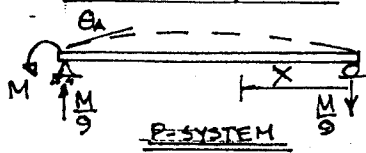
$$\delta_c = \frac{40,756 \text{ kN} \cdot \text{m}}{200 \times 10^6 \times 1800 \times 10^6} = 0.113 \text{ m}$$

P10.28. Determine the value of moment that must be applied to the left end of the beam in Figure P10.28 if the slope at A is to be zero. EI is constant. Assume rocker at support D acts as a roller.

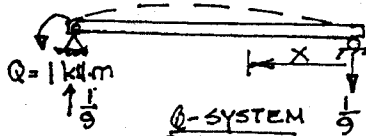


USE SUPERPOSITION

θ_A PRODUCED BY M_A



P-SYSTEM



Q-SYSTEM

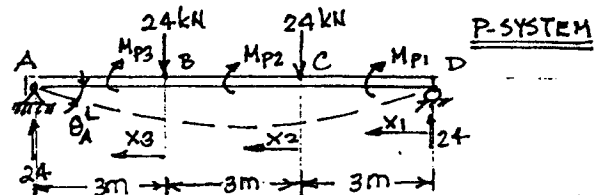
$$I \cdot \theta_A^M = \int_0^9 \frac{M_A M_P}{EI} dx$$

$$\theta_A^M = \int_0^9 \frac{(x)(M/9)}{EI} dx$$

$$= \left[\frac{Mx^3}{243EI} \right]_0^9$$

$$\theta_A^M = \frac{M \cdot 729}{243EI} = \frac{3M}{EI}$$

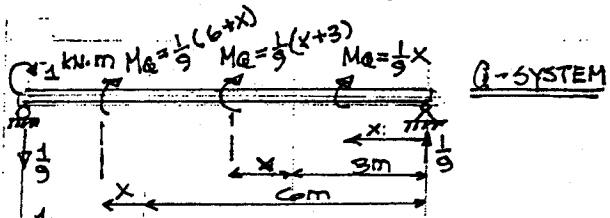
P10.28
 θ_A PRODUCED BY DESIGN LOADS OF 24 kN



$$M_{P1} = 24 \text{ k}$$

$$M_{P2} = 24(3+x_2) - 24x_2$$

$$M_{P3} = 24(6+x_3) - 24(3+x_3) - 24x_3$$



Q-SYSTEM

$$M_Q \theta_A^L = \int_0^9 \frac{M_Q M_P}{EI} dx$$

$$\theta_A^L = \int_0^3 \frac{x}{9} (24x) \frac{dx}{EI} + \int_0^3 \frac{(x+3)}{9} [24(3+x) - 24x] \frac{dx}{EI} + \int_0^3 \frac{(6+x)}{9} [24(6+x) - 24(3+x) - 24x] \frac{dx}{EI}$$

$$\theta_A^L = \int_0^3 \frac{8x^2}{9} \frac{dx}{EI} + \int_0^3 \frac{x+3}{9} (72 - 24x) \frac{dx}{EI} + \int_0^3 \frac{6+x}{9} (144 + 24x - 72 - 24x - 24x) \frac{dx}{EI}$$

$$\theta_A^L = \frac{1}{EI} \left[\frac{8x^3}{9} \right]_0^3 + \int_0^3 \frac{x+3}{9} (72) \frac{dx}{EI} + \int_0^3 \frac{x+6}{9} (72 - 24x) \frac{dx}{EI}$$

$$\theta_A^L = \frac{24}{EI} + \int_0^3 (x+3)(24) \frac{dx}{3EI} + \int_0^3 \frac{x+6}{9} (24 - 8x) \frac{dx}{EI}$$

$$\theta_A^L = \frac{24}{EI} + \int_0^3 (72 + 24x) \frac{dx}{3EI} + \int_0^3 \frac{(144 - 24x - 8x^2)}{3EI} \frac{dx}{EI}$$

$$= \frac{24}{EI} + \left[72x + \frac{24x^2}{2} \right]_0^3 \left(\frac{1}{3EI} \right) + \left[432x - 108x - 72x^2 \right] \frac{1}{3EI}$$

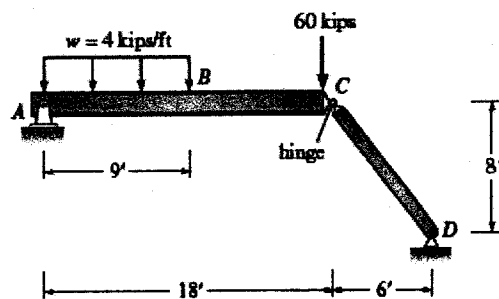
$$\theta_A^L = \frac{24}{EI} + \frac{108}{EI} + \frac{84}{EI} = \frac{216}{EI}$$

FOR SLOPE TO EQUAL ZERO

$$\theta_A^M = \theta_A^L$$

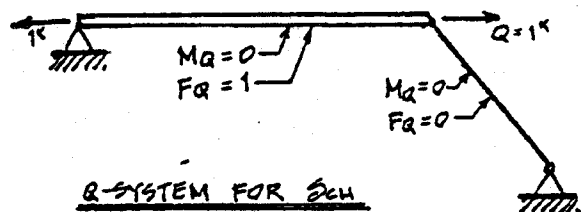
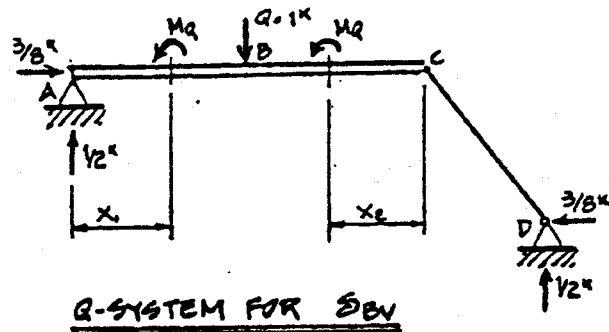
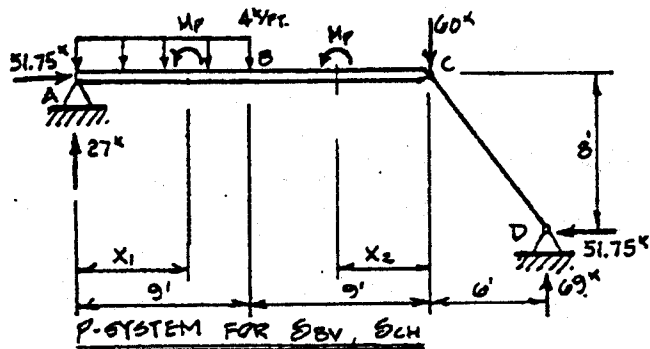
$$\frac{3M}{EI} = \frac{216}{EI} \quad \text{AND} \quad M = 216/3 = 72 \text{ kN}\cdot\text{m}$$

P10.29. Compute the vertical deflection at B and the horizontal deflection at C in Figure P10.29. Given: $ACD = 3 \text{ in}^2$, $I_{AC} = 160 \text{ in}^4$, $A_{AC} = 4 \text{ in}^2$, and $E = 29,000 \text{ kips/in}^2$. Consider the strain energy produced by both axial and flexural deformations.



P10.29

COMPUTING	SEGMENT	ORIGIN	RANGE (FT.)	M_Q (FT.-K)	M_P (FT.-K)	F_Q (K)	F_P (K)	L (FT.)	A (IN ²)	I (IN ⁴)
δ_{BV}	AB	A	$0 \leq x_1 \leq 9$	$x_1/2$	$27x_1 - \frac{4x_1^2}{2}$	$-3/8$	-51.75	9'	4	160
	BC	C	$0 \leq x_2 \leq 9$	$x_2/2$	$9x_2$	$-3/8$	-51.75	9'	4	160
	CD	—	—	0	0	$-5/8$	-86.25	10'	3	—



$$Q \cdot \delta_{CH} = \sum F_Q \cdot F_P \cdot L / AE$$

$$1^k \cdot \delta_{CH} = \frac{(1)(-51.75)(18)(12)}{4(29,000)} = -0.096 \text{ in}$$

$$1^k \cdot \delta_{BV} = U_Q = \sum F_Q F_P L / AE + \sum \int M_Q M_P dx / EI$$

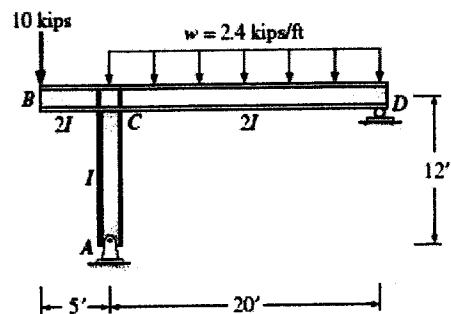
$$\delta_{BV} = \left(\frac{-5}{8} \right) \frac{(-86.25)(10)(12)}{3E} + \left(\frac{-3}{8} \right) \left[\frac{-51.75(18)(12)}{4E} \right] + \int_0^9 \left(\frac{x}{2} \right) (27x - 2x^2) \frac{dx}{EI} + \int_0^9 \left(\frac{x}{2} \right) 9x \frac{dx}{EI}$$

$$\delta_{BV} = \frac{2156.25}{E} + \frac{1047.94}{E} + \frac{13.5(9)^3(1728)}{3(160)E} + \frac{4.5(9)^3(1728)}{3(160)E} - \frac{(9)^4(1728)}{4(160)E} =$$

$$\delta_{BV} = -32728/E$$

$$\delta_{BV} = -1.13 \text{ in} \downarrow$$

P10.30. Compute the vertical and horizontal deflection at B and at the midspan of member CD in Figure P10.30. Consider both axial and bending deformations. Given: $E = 29,000$ kips/in², $I = 180$ in⁴, area of column = 6 in², area of girder = 10 in².



P10.30

$I = 180 \text{ in}^4$, $A_{AC} = 6 \text{ in}^2$, $A_{BCD} = 10 \text{ in}^2$

COMPUTE δ_{BX}

$$Q \cdot \delta_{BX} = \int M_Q M_P \frac{dx}{EI} + \frac{F_Q F_P L}{AE}$$

$$\delta_{BX} = \int_0^{20} (0.6x)(21.5x - 2.4 \cdot \frac{x^2}{2}) \frac{dx}{2EI} + \frac{F_Q F_P L}{AE}$$

$$= \int_0^{20} (12.9x^2 - 0.72x^3) \frac{dx}{2EI} + \frac{(0.6)(36.5)(12 \times 12)}{6 \times 29,000}$$

$$= \frac{1}{2EI} \left[\frac{12.9x^3}{3} - \frac{0.72x^4}{4} \right]_0^{20} + -0.018$$

$$= \frac{5600(1728)}{29,000 \times 360} - 0.018$$

$$\delta_{BX} = 0.927 - 0.018 = \underline{0.909 \text{ in}} \rightarrow$$

SINCE NO AXIAL LOAD DEVELOPS IN BEAM BCD ALL SECTIONS OF BEAM DEFLECT $0.909 \text{ in} \rightarrow$

P10.30 Continued

P10.30 continued

COMPUTE S_{BY}

$$Q \cdot S_{BY} = \int_0^{20} (21.5x - 1.2x^2)(-2.5x) \frac{dx}{2EI} + \int_0^5 (-x)(-10x) \frac{dx}{2EI} + (-\frac{5}{4})(-36.5)(12 \times 12)$$

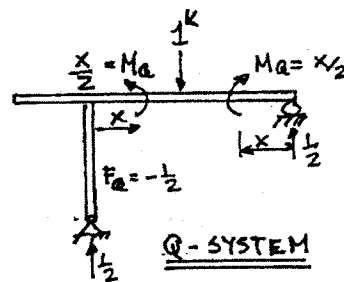
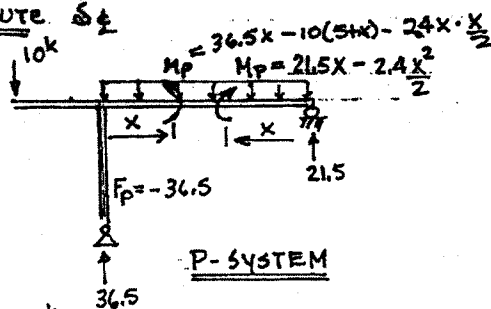
$$S_{BY} = \int_0^{20} (-5.375x + 0.3x^3) \frac{dx}{2EI} + \int_0^5 10x^2 \frac{dx}{2EI} + 0.0378$$

$$S_{BY} = \left[-\frac{5.375x^2}{2} + \frac{0.3x^4}{4} \right] \frac{dx}{2EI} + \left[\frac{10x^3}{3} \right] \frac{dx}{2EI} + 0.0378$$

$$S_{BY} = -\frac{2333.33(1728)}{360 \times 29,000} + \frac{10(5)^3(1728)}{3 \times 29,000 \times 360} + 0.0378$$

$$S_{BY} = -0.279" \uparrow$$

COMPUTE S_L



$$Q \cdot S_L = \int \int M_Q M_P \frac{dx}{EI} + F_Q F_P L / AE$$

$$S_L = \int_0^{10} \frac{x}{2} (21.5x - \frac{2.4x^2}{2}) \frac{dx}{2EI} + \int_0^{10} (36.5x - 10(5+x) - 2.4x \cdot \frac{x}{2}) \frac{dx}{2EI} \left(\frac{x}{2} \right)$$

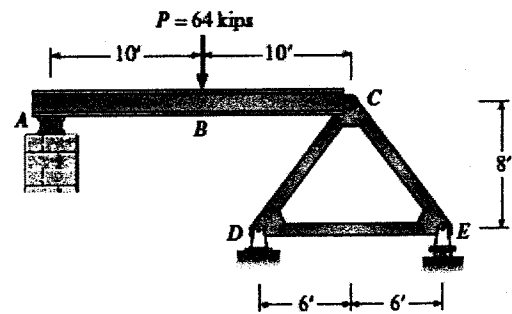
$$+ (-\frac{1}{2})(-36.5)(12 \times 12)$$

$$= \int_0^{10} (10.75x^2 - 0.6x^3) \frac{dx}{2EI} + \int_0^{10} (-25x + 13.25x^2 - 0.6x^3) \frac{dx}{2EI} + 0.015$$

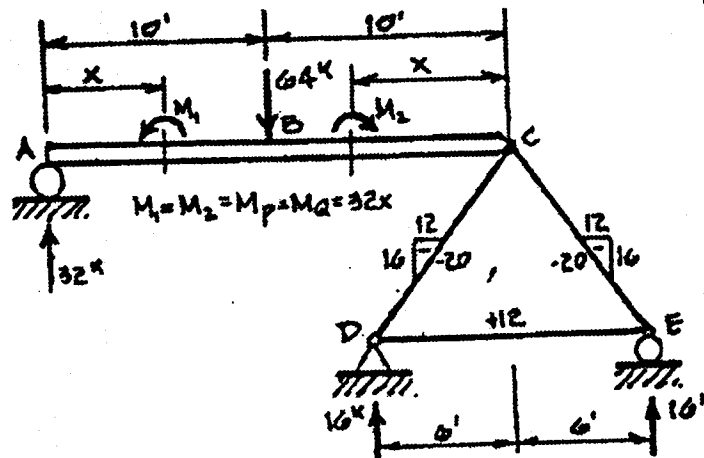
$$= \frac{1}{2EI} \left[\frac{10.75x^3}{3} - \frac{0.6x^4}{4} \right]_0^{10} + \frac{1}{2EI} \left[-\frac{25x^2}{2} + \frac{13.25x^3}{3} - \frac{0.6x^4}{4} \right]_0^{10} + 0.015$$

$$= 0.3448 + 0.2759 + 0.015 = \underline{0.636 in} \downarrow \text{Ans.}$$

P10.31. Beam ABC is supported by a three-bar truss at point C and at A by an elastomeric pad that is equivalent to a roller. (a) Compute the vertical deflection of point B in Figure P10.31 due to the applied load. (b) Compute the change in length of member DE required to displace point B upward 0.75 in. Is this a shortening or lengthening of the bar? Given: $E = 29,000$ kips/in², area of all truss bars = 1 in², area of beam = 16 in², I of beam = 1200 in⁴.



P10.31



P- AND Q-SYSTEM

$$Q \cdot \delta_{BV} = \sum F_P F_Q L / AE + \sum \int M_Q M_P dx / EI$$

$$64 \delta_{BV} = 2 \left[\frac{(-20)^2 (10 \times 12)}{1E} \right] + \left[\frac{(12)^2 (12 \times 12)}{1E} \right] + 2 \int_0^{10} \frac{(32x)^2 dx}{EI}$$

$$= \frac{96000}{E} + \frac{20736}{E} + \frac{2048}{EI} \left[\frac{x^3}{3} \right]_0^{10} (1728)$$

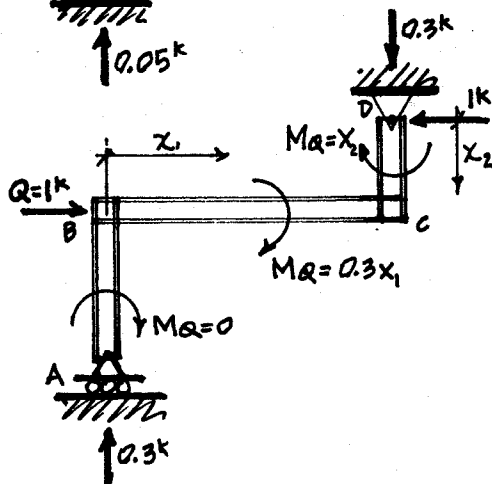
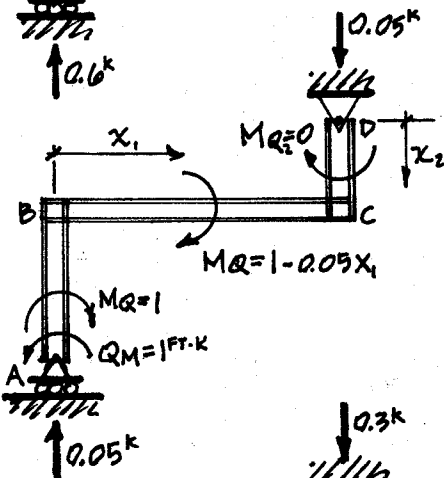
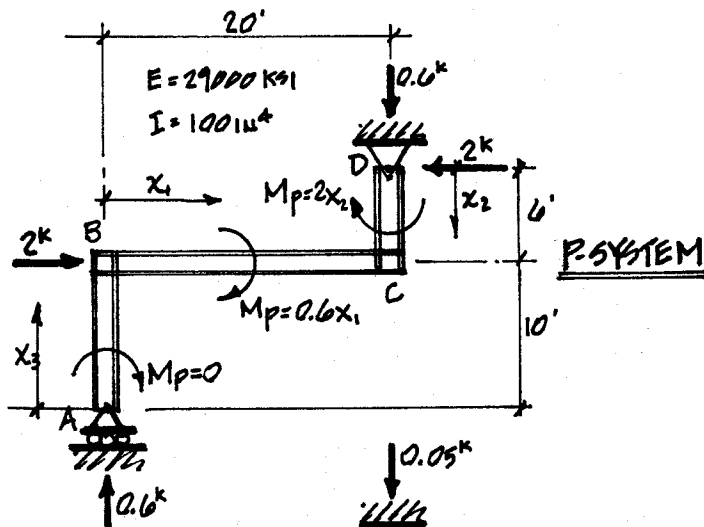
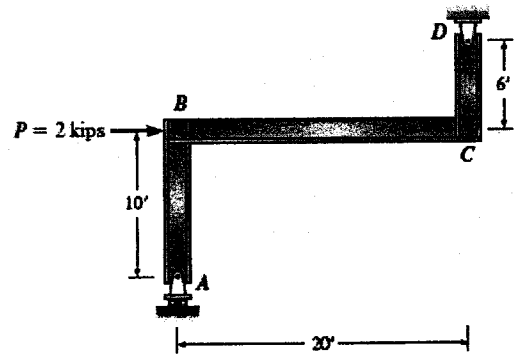
$$\delta_{BV} = 0.59 \text{ in.}$$

$$Q \cdot \delta_{BV} = F_Q (\Delta L_{DE})$$

$$-64 (0.75 \text{ in.}) = 12 (\Delta L_{DE})$$

$$\Delta L_{DE} = -4.0 \text{ in. (SHORTEN)}$$

P10.32. (a) Compute the slope at A and the horizontal displacement of joint B in Figure P10.32. EI is constant for all members. Consider only bending deformations. Given: $I = 100 \text{ in}^4$, $E = 29,000 \text{ kips/in}^2$. (b) If the horizontal displacement at joint B is not to exceed $\frac{3}{8} \text{ in.}$, what is the minimum required value of P ?



VIRTUAL WORK

(a) COMPUTE θ_A : $Q \cdot \theta_A = \sum \int \frac{M_Q M_P dx}{EI}$

$$1 \text{ ft-k} \cdot \theta_A = \int_0^{20} (1 - 0.05x)(0.6x) dx / EI$$

$$= \left(\frac{0.6x^2}{2} - \frac{0.03x^3}{3} \right) \Big|_0^{20}$$

$$= 40 / EI$$

$$\theta_A = 0.00199 \text{ RAD}$$

COMPUTE δ_{Bx} : $Q \cdot \delta_{Bx} = \sum \int \frac{M_Q M_P dx}{EI}$

$$1 \text{ k} \cdot \delta_{Bx} = \int_0^{20} (0.3x)(0.6x) dx / EI + \int_0^6 x(2x) dx / EI$$

$$= \left(\frac{0.06x^3}{3} \right) \Big|_0^{20} + \frac{0.66x^3}{3} \Big|_0^6$$

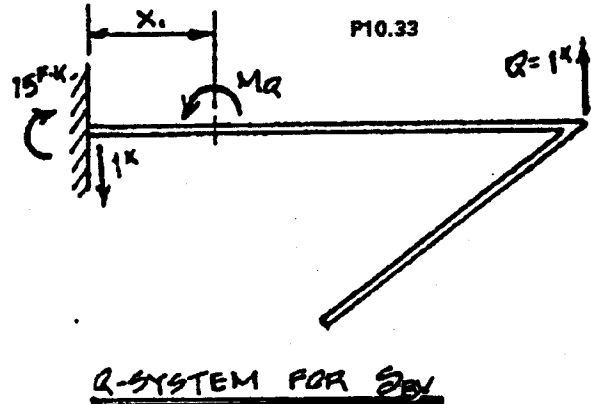
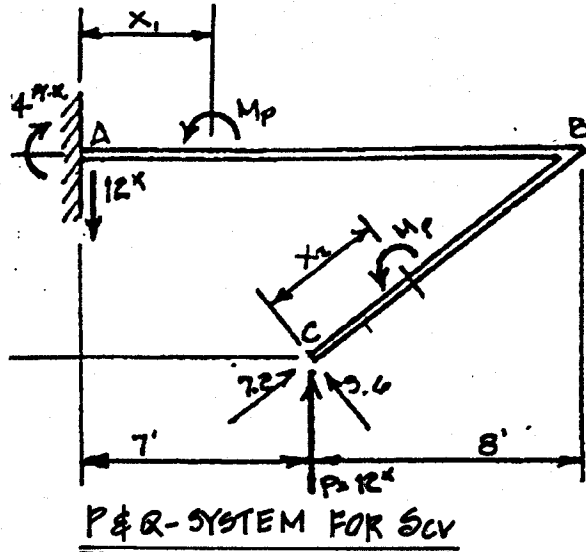
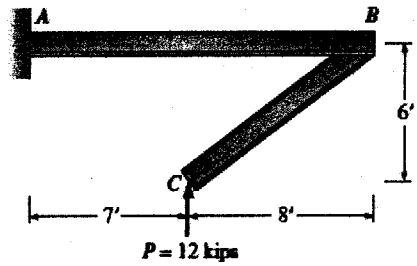
$$= 624 / EI$$

$$\delta_{Bx} = 0.372 \text{ in.}$$

(b) DETERMINE $I_{REQ'D}$ FOR $\delta_{Bx} = \frac{3}{8} \text{ in.}$

$$I_{REQ'D} = \frac{624 (1728)}{29000 (3/8)} = 99.2 \text{ in}^4$$

P10.33. Compute the vertical displacement of joints B and C for the frame shown in Figure P10.33. Given: $I = 360 \text{ in}^4$, $E = 30,000 \text{ kips/in}^2$. Consider only flexural deformations.



$$I = 360 \text{ in}^4 ; \quad E = 30,000 \text{ ksi}$$

COMPUTE	ELEMENT	ORIGIN	RANGE	M_P	M_Q
δ_{CV} ($M_Q = M_P$) ($\delta_{CV} = \delta_{CV}$)	AB	A	$0 \leq x \leq 15$	$84 - 12x$	$84 - 12x$
	CB	C	$0 \leq x \leq 10$	$9.6x$	$9.6x$
δ_{BV}	AB	A	$0 \leq x \leq 15$	$84 - 12x$	$15 - 1x$
	CB	—	—	—	0

$$\delta_{CV} \quad Q \cdot \delta_P = \sum \int M_Q M_P dx / EI = \sum \int M_P^2 dx / EI$$

$$12 \cdot \delta_{CV} = \int_0^{15} (84 - 12x)^2 dx / EI + \int_0^{10} (9.6x)^2 dx / EI$$

$$\delta_{CV} = \left[\frac{16}{12EI} \left[441x - \frac{126x^2}{2} + \frac{9x^3}{3} \right]_0^{15} + \frac{92.16}{3EI} \left[\frac{x^3}{3} \right]_0^{10} \right] (1728)$$

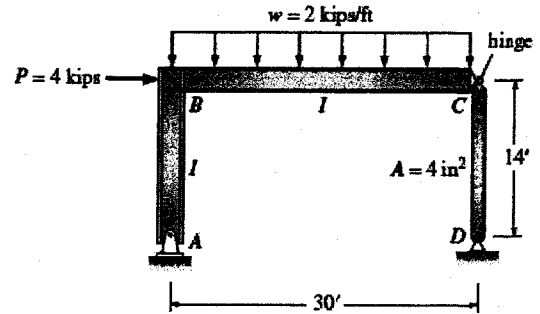
$$\delta_{CV} = 0.96''$$

$$\delta_{BV} \quad 1k \cdot \delta_{BV} = \int_0^{15} (15 - x)(84 - 12x) dx / EI$$

$$\delta_{BV} = \frac{4}{EI} \left[315x - 66x^2 + 3x^3/3 \right]_0^{15} (1728)$$

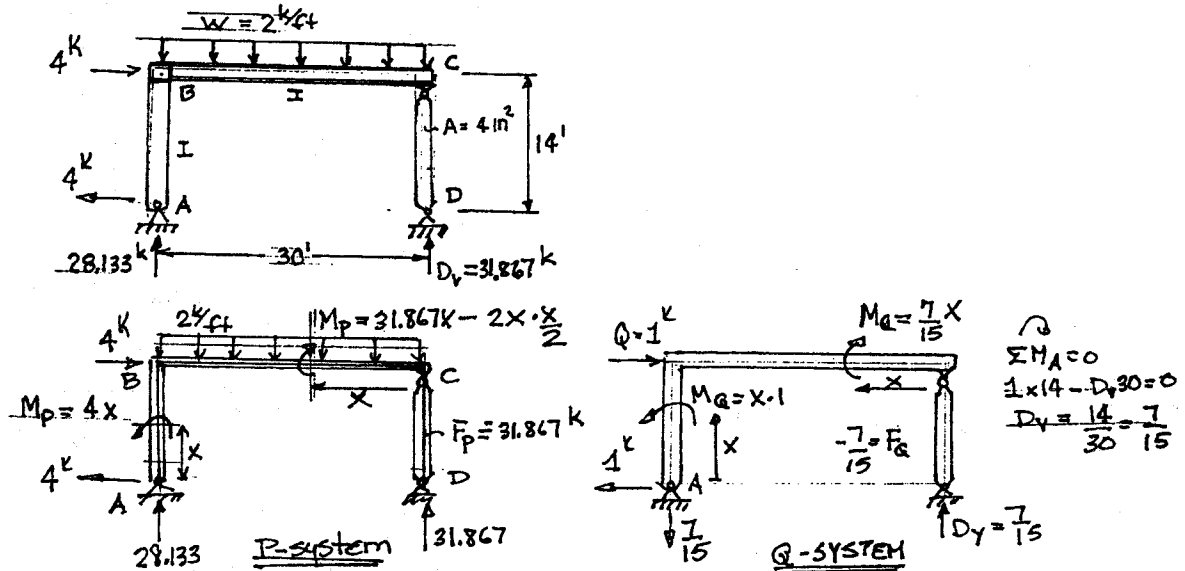
$$\delta_{BV} = 0.432''$$

P10.34. If the horizontal displacement of joint B of the frame in Figure P10.34 is not to exceed 0.36 in, what is the required I of the members? Bar CD has an area of 4 in^2 , and $E = 29,000 \text{ kips/in}^2$. Consider only the bending deformations of members AB and BC and the axial deformation of CD.



P10.34

Compute I , $\delta_{BH} \leq 0.36 \text{ in}$.



$$Q \cdot \delta_{BH} = \int \frac{M_a M_p}{EI} dx + \frac{F_a F_p L}{AE}$$

$$1 \cdot \delta_{BH} = \int_0^{14} \frac{x \cdot 4x}{EI} dx + \int_0^{30} \frac{(1x)(31.867x - x^2)}{EI} dx + \frac{(-7/15)(-31.867)(14 \times 12)}{AE}$$

$$\delta_{BH} = \frac{1}{EI} \left[\frac{4x^3}{3} \right]_0^{14} + \int_0^{30} (14.871x^2 - 0.467x^3) \frac{dx}{EI} + \frac{2500.16}{4 \times 29,000}$$

$$= \frac{3658.67(1728)}{29,000 I} + \left[\frac{14.871x^3}{3} - \frac{0.467x^4}{4} \right]_0^{30} \frac{1}{EI} + .0216''$$

$$0.36'' = \frac{218}{I} + \frac{(133,839 - 94,567.5)(1728)}{29,000 I} + .0216''$$

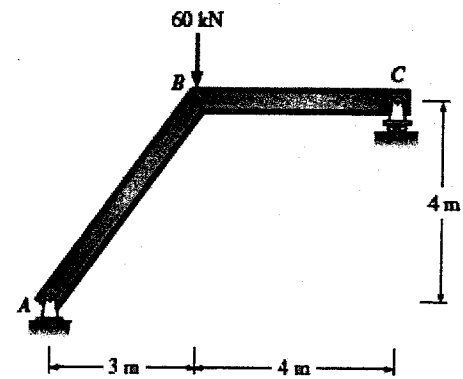
$$.36 = \frac{218}{I} + \frac{2340.1}{I} + .0216''$$

$$.338 = \frac{2558.07}{I}$$

$$\text{REQ'D } I = 7568.3 \text{ in}^4$$

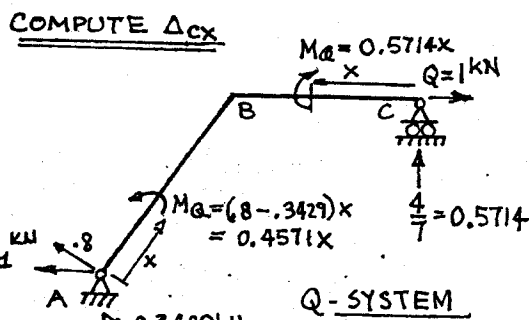
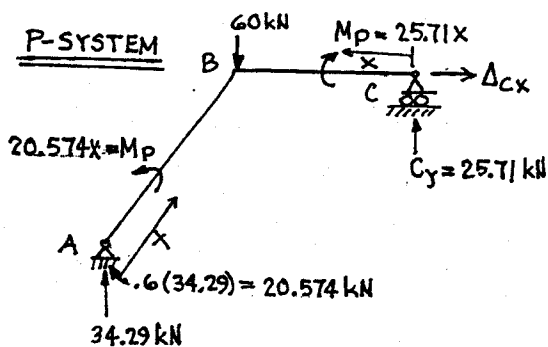
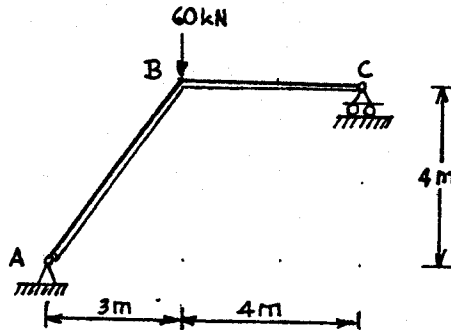
NOTE: IF joint C is rigid, I can be reduced APPROX 7. For D.

P10.35. For the steel rigid frame in Figure P10.35, compute the rotation of joint B and the horizontal displacement of support C . Consider only the deflections produced by bending moments. Given: $E = 200 \text{ GPa}$, $I = 80 \times 10^6 \text{ mm}^4$.



P10.35

$E = 200 \text{ GPa}$ $I = 80 \times 10^6 \text{ mm}^4$



$$1 \text{ kN} \cdot \Delta_{cx} = \int \int M_a M_p \frac{dx}{EI}$$

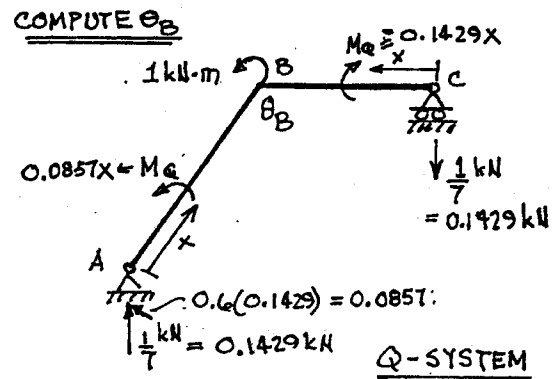
$$\Delta_{cx} = \int_0^4 \frac{(0.5714x)(25.71x)}{EI} dx + \int_0^5 \frac{(0.4571x)(20.574x)}{EI} dy$$

$$= \int_0^4 14.691x^2 \frac{dx}{EI} + \int_0^5 9.404x^2 \frac{dy}{EI}$$

$$= \frac{313.408}{EI} + \frac{391.833}{EI}$$

$$= \frac{705.241 \text{ kN} \cdot \text{m}^3}{200 \times 10^6 \frac{\text{kN}}{\text{m}^2} \times 80 \times 10^6 \frac{\text{mm}^4}{10^3 \text{ mm}^4}}$$

$$\Delta_{cx} = 0.0441 \text{ m} = 44.1 \text{ mm}$$



$$1 \text{ kN} \cdot \text{m} \theta_B = \int M_a M_p \frac{dx}{EI}$$

$$\theta_B = \int_0^4 \frac{(0.1429x)(25.71x)}{EI} dx + \int_0^5 \frac{(0.0857x)(20.574x)}{EI} dy$$

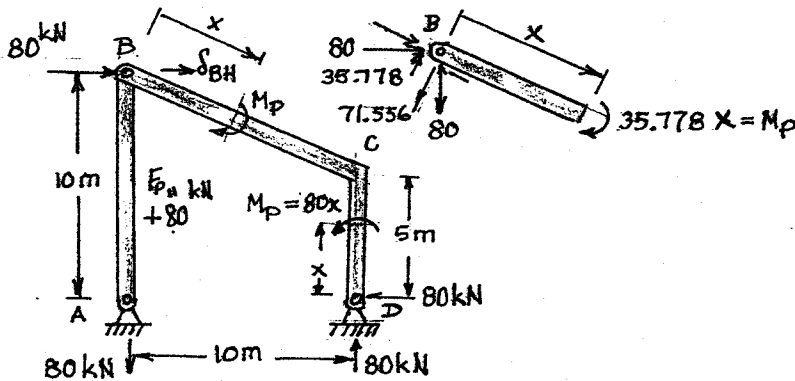
$$= \int_0^4 -3.674x^2 \frac{dx}{EI} + \int_0^5 -1.763x^2 \frac{dy}{EI}$$

$$= \left[\frac{-78.379}{EI} + \frac{-73.458}{EI} \right]$$

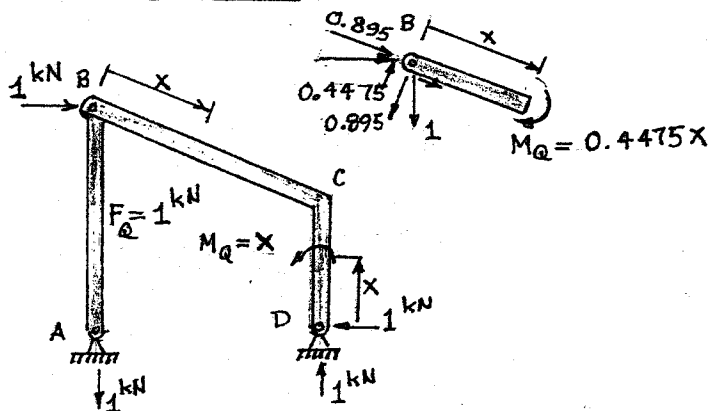
$$\theta_B = \frac{-4.921 \text{ kN} \cdot \text{m}^3}{200 \times 10^6 \times 80 \times 10^6 (10^{-12})}$$

$$\theta_B = -0.00031 \text{ RADIANS}$$

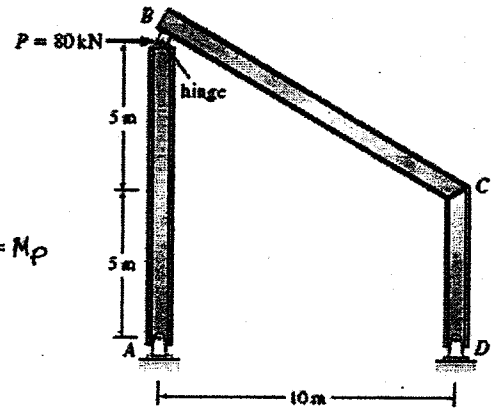
P10.36. For the steel frame in Figure P10.36, compute the horizontal displacement of joint B. For member BCD, $E = 200 \text{ GPa}$ and $I = 600 \times 10^6 \text{ mm}^4$. For member AB, area = $1,500 \text{ mm}^2$.



P SYSTEM



Q SYSTEM



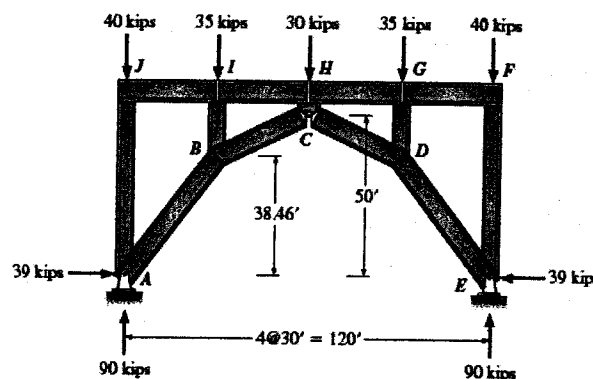
P10.36

NOTE: THE CONTRIBUTION OF AXIAL DEFORMATIONS IN MEMBER BCD IS SMALL AND NOT INCLUDED.

VIRTUAL WORK

$$\begin{aligned}
 1 \cdot \delta_{BH} &= \sum \int M_P M_Q \frac{dx}{EI} + \sum \frac{F_Q F_P L}{AE} \\
 \delta_{BH} &= \int_0^5 (x)(80x) \frac{dx}{EI} + \int_0^{11.18} (35.778x)(0.4475x) \frac{dx}{EI} + \frac{1 \times 80 \times 10}{AE} \\
 &= \left[\frac{80x^3}{3EI} \right]_0^5 + \left[\frac{16.011x^3}{3EI} \right]_0^{11.18} + \frac{800}{AE} \\
 &= \frac{3333.33}{EI} + \frac{7458}{EI} + \frac{800}{AE} \\
 &= \frac{10,791.33}{200 \times 10^6 \times 600 \times 10^6} + \frac{800 (10^6)}{1500 \times 200 \times 10^6} \\
 &= 0.0899 + 0.0026 = 0.0925 \text{ m} = \underline{\underline{92.5 \text{ mm}}}
 \end{aligned}$$

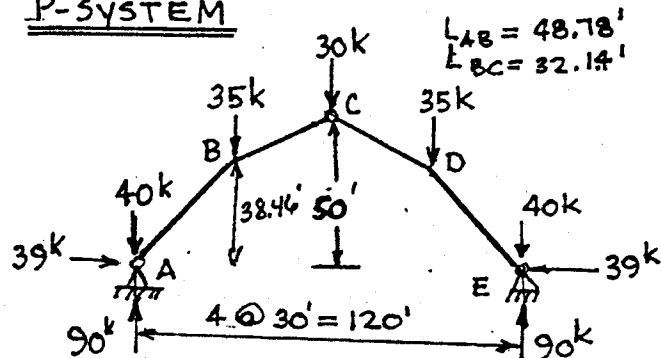
P10.37. Compute the vertical displacement of the hinge at C for the funicular loading shown in Figure P10.37. The funicular loading produces direct stress on all sections of the arch. Columns transmit only axial load from the roadway beams to the arch. Also assume that the roadway beams and the columns do not restrain the arch. All reactions are given. For all segments of the arch $A = 20 \text{ in}^2$, $I = 600 \text{ in}^4$, and $E = 30,000 \text{ kips/in}^2$.



P10.37

ARCH SUPPORTS A FUNICULAR LOADING, THEREFORE $M_p = 0$

P-SYSTEM

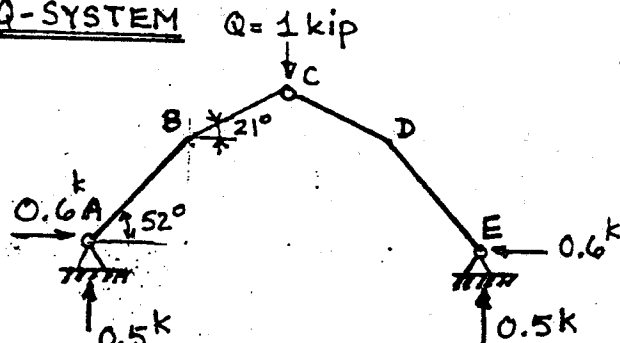


$$F_{DE} = \sqrt{39^2 + 50^2} = 63.41 \text{ kips}$$

$$F_{BC} = \sqrt{15^2 + 39^2} = 41.79 \text{ kips}$$

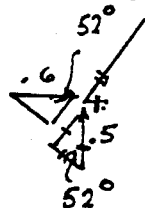
$M_p = 0$ ALL MEMBERS.

Q-SYSTEM



Compute AXIAL FORCES IN BARS

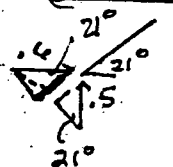
BAR AB



M_Q does not have to be considered, $M_p = 0$

$$\begin{aligned} F_Q &= 0.6 \cos 52^\circ + 0.5 (\sin 52^\circ) \\ &= 0.6 (.62) + 0.5 (.79) \\ &= 0.370 + 0.39 \\ &= -0.763 \text{ kips} \end{aligned}$$

BAR BC



$$\begin{aligned} F_Q &= 0.6 \cos 21^\circ + 0.5 \sin 21^\circ \\ F_Q &= 0.6 (.933) + 0.5 (.358) \\ &= 0.56 + 0.179 = 0.739 \text{ k} \end{aligned}$$

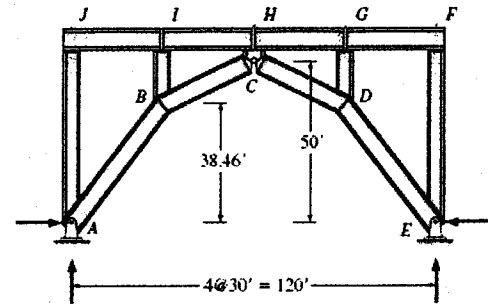
$$1 \cdot \delta_{cy} = \sum F_Q F_P L_{AE}$$

$$\delta_{cy} = 2 \left[\frac{(-0.767)(-63.41) 48.78 \times 12}{20 \times 30,000} \right]$$

$$+ 2 \left[\frac{0.739 (-41.79) 32.14 \times 12}{20 \times 30,000} \right]$$

$$\delta_{cy} = 0.134 \text{ in } \downarrow$$

P10.38. Determine the horizontal and vertical deflection of the hinge at point C of the arch in Figure P10.37 for a single concentrated load of 60 kips applied at joint B in the vertical direction. See Problem P10.37 for properties of the arch.

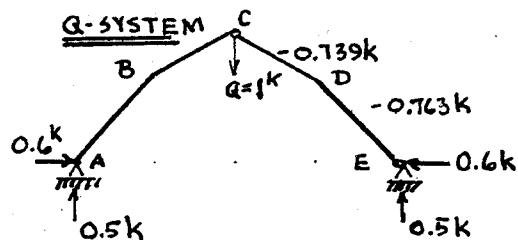
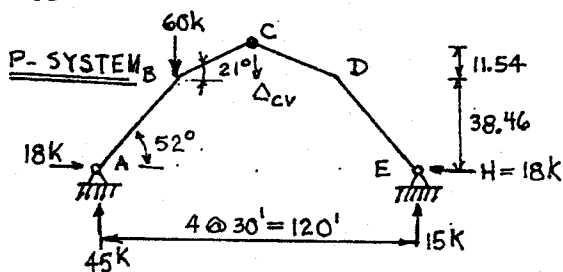


P10.37

$$E = 30,000 \text{ ksi}, A = 20 \text{ in}^2, I = 600 \text{ in}^4$$

$$L_{AB} = 48.78' \\ L_{BC} = 32.14'$$

VERTICAL DEFL. OF JOINT C



$$F_{AB}^P = -45 \sin 52^\circ - 18 \cos 52^\circ = -46.55 \text{ k} \\ M_{AB}^P = (45 \cos 52^\circ - 18 \sin 52^\circ)x = 13.52x$$

$$F_{BC}^P = 15 \sin 21^\circ - 18 \cos 21^\circ = -11.23 \text{ k} \\ M_{BC}^P = 659.5 - 20.52x$$

$$F_{ED}^P = -15 \sin 52^\circ - 18 \cos 52^\circ = -22.9 \text{ k} \\ M_{ED}^P = -4.95x$$

$$F_{OC}^P = -15 \sin 52^\circ - 18 \cos 21^\circ = -22.18 \text{ k} \\ M_{OC}^P = -241.46 + 7.51x$$

$$F_{AB}^Q = F_{DE}^Q = -0.763$$

$$M_{AB}^Q = M_{DE}^Q = -0.165x$$

$$F_{BC}^Q = F_{CD}^Q = -0.739 \text{ k}$$

$$M_{BC}^Q = M_{CD}^Q = -8.05 + 0.25x$$

$$1^k \cdot \Delta_{CV} = \sum \frac{F_a F_p L}{AE} + \sum \int \frac{M_a M_p dx}{EI}$$

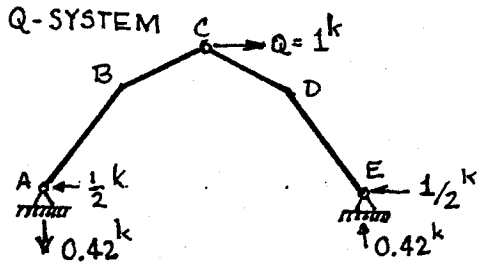
$$1^k \cdot \Delta_{CV} = \frac{1}{AE} \left[(-46.55)(-0.763)(48.78 \times 12) + (-11.23)(-0.739)(32.14 \times 12) \right. \\ \left. + (-22.9)(-0.763)(48.78 \times 12) + (-22.18)(-0.739)(32.14 \times 12) \right] \\ + \int_0^{48.78} \frac{(13.52x)(-0.165x)}{EI} dx + \int_0^{32.14} \frac{(659.5 - 20.52x)(-8.05 + 0.25x)}{EI} dx \\ + \int_0^{48.78} \frac{(-4.95x)(-0.165x)}{EI} dx + \int_0^{32.14} \frac{(-241.46 + 7.51x)(-8.05 + 0.25x)}{EI} dx$$

$$\Delta_{CV} = \frac{40600}{20 \times 30,000} + \frac{-90,792 (12)^3}{30,000 \times 600} = 0.068'' - 8.71'' = \underline{\underline{-8.6'' \uparrow}}$$

P10.38 Continued

P10.38 continued

HORIZONTAL DISPLACEMENT



$$\begin{aligned}
 F_{AB}^Q &= 0.42 \sin 52^\circ + 0.5 \cos 52^\circ = 0.64^k; & M_{AB}^Q &= 0.1354x \\
 F_{BC}^Q &= 0.42 \sin 21^\circ + 0.5 \cos 21^\circ = 0.62^k \\
 M_{BC}^Q &= 6.61 - 0.206x \\
 F_{DE}^Q &= -0.42 \sin 52^\circ - 0.5 \cos 52^\circ = -0.64^k \\
 M_{DE}^Q &= -0.1354x \\
 F_{CD}^Q &= -0.62^k \\
 M_{CD}^Q &= -6.61 + 0.206x
 \end{aligned}$$

HORIZONTAL DISPL. OF JOINT C

$$\Delta_{CH} \cdot 1^k = \sum F_Q \frac{F_P L}{AE} + \sum \int M_Q M_P \frac{dx}{EI}$$

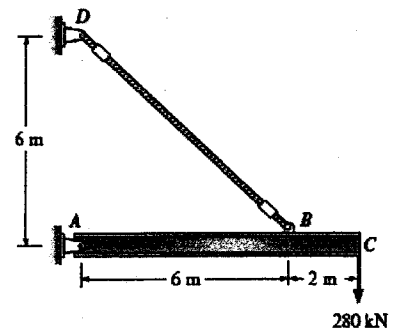
$$\begin{aligned}
 \Delta_{CH} = \frac{1}{AE} & \left[-46.55(0.64)(48.78 \times 12) + (-11.23)(0.62)(32.14 \times 12) \right. \\
 & + (-0.64)(-22.9)(48.78 \times 12) + (-22.18)(-0.62)(32.14 \times 12) \Big] \\
 & + \int_0^{48.78} \frac{x(13.52)(0.1354x)}{EI} dx + \int_0^{32.14} \frac{(6.61 - 0.206x)(659.5 - 20.52x)}{EI} dx \\
 & + \int_0^{48.78} \frac{(-0.1354x)(-4.95x)}{EI} dx + \int_0^{32.14} \frac{(-6.61 + 0.206x)(-241.46 + 7.51x)}{EI} dx
 \end{aligned}$$

$$\Delta_{CH} = \frac{-6240}{20 \times 30,000} + \frac{160,512.5(12)^3}{30,000 \times 600}$$

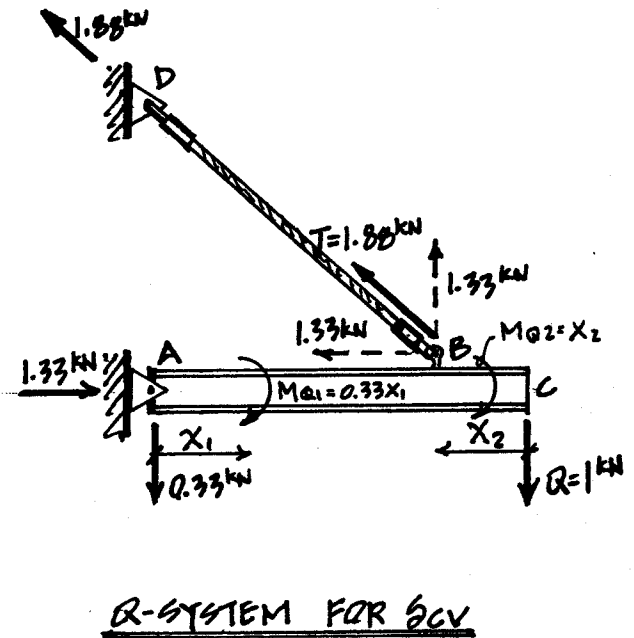
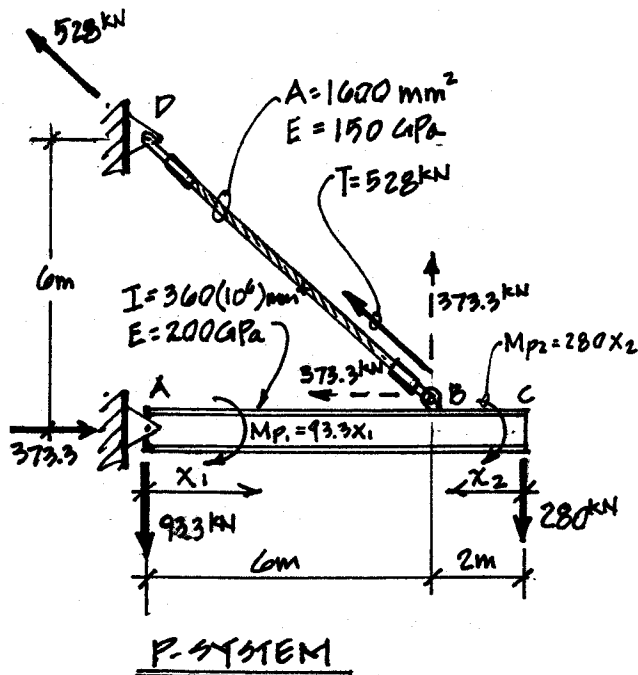
$$\Delta_{CH} = -0.0104'' + 15.41'' = \underline{\underline{15.4 \text{ in}}}$$

- ⊛ AS YOU CAN SEE IN COMPUTATIONS OF DEFLECTIONS, THE EFFECTS OF AXIAL LOAD ARE VERY SMALL COMPARED TO THE DEFLECTIONS DUE TO MOMENT; THEREFORE, WE TYPICALLY NEGLECT THE EFFECT OF AXIAL LOAD WHEN ANALYZING BEAMS, FRAMES, AND ARCHES THAT DO NOT SUPPORT A FUNICULAR LOADING.

P10.39. Compute the vertical displacement of point C for the beam in Figure P10.39. For the beam $I = 360 \times 10^6 \text{ mm}^4$ and $E = 200 \text{ GPa}$. For the cable $A = 1600 \text{ mm}^2$ and $E = 150 \text{ GPa}$.



P10.39



VIRTUAL WORK

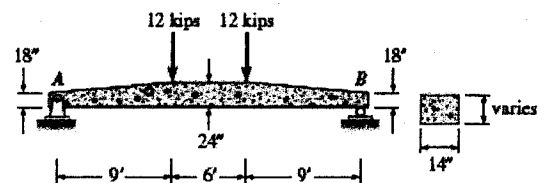
$$Q \cdot \delta_{CV} = \sum \int M_Q M_P dx / EI + \sum F_Q F_P L / AE$$

$$\begin{aligned} 1 \text{ kN} \cdot \delta_{CV} &= \int_0^6 (0.33x_1)(93.3x_1) dx / EI + \int_0^2 (x_2)(280x_2) \frac{dx}{EI} + \left(\frac{528(1.88)(6\sqrt{2})}{AE} \right) \\ &= \frac{2217.4}{EI} + \frac{746.47}{EI} + \frac{8422.35}{AE} \end{aligned}$$

NOTE: AXIAL DEFORMATION IS IGNORED IN THE BEAM

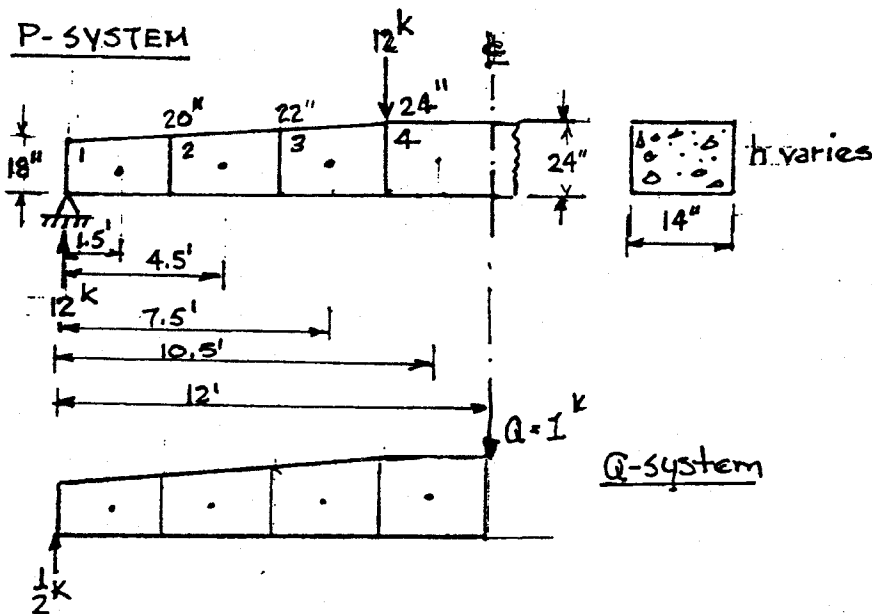
$$\underline{\delta_{CV} = 76.3 \text{ mm}}$$

P10.40. Using a finite summation, compute the initial deflection at midspan for the beam in Figure P10.40. Given: $E = 3000 \text{ kips/in}^2$. Use 3-ft segments. Assume $I = 0.5 I_G$.



P10.40

$E = 3000 \text{ kips/in}^2$. Use $I = 0.5 I_G$

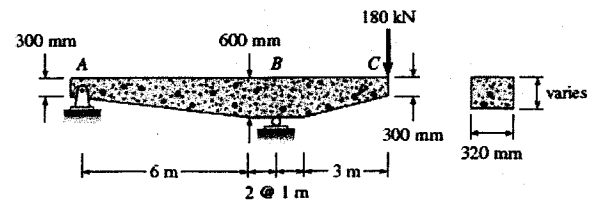


SEGMENT No.	M_p FT-K	M_Q FT-K	DEPTH, h IN.	$\frac{5I}{4} = \frac{bh^3}{12} (\frac{1}{2})$ IN ⁴	$\frac{M_Q M_p}{I}$
1	18	0.75	19"	4001.08	0.00337
2	54	2.25	21"	5402.25	0.02249
3	90	3.75	23"	7097.42	0.04755
4	126	5.25	24"	8064.00	0.08203

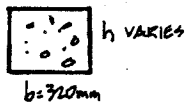
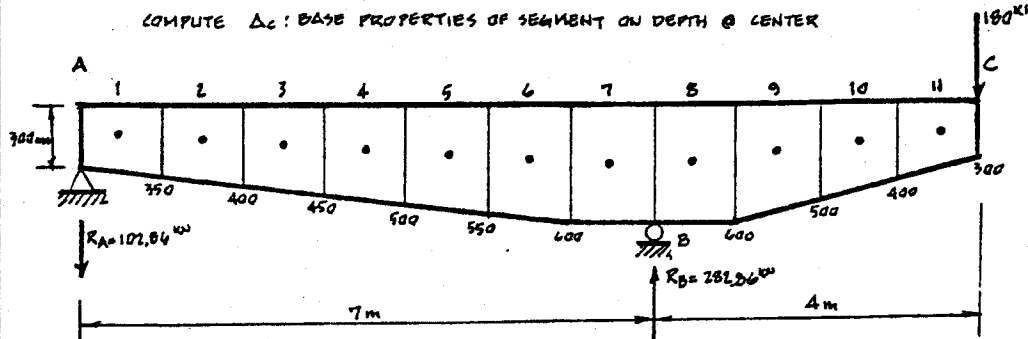
$\Sigma = 0.1554 \frac{\text{FT}^2 \cdot \text{Kips}}{\text{IN}^4}$

$W_Q = \Sigma Q$
 $\frac{1}{2} \cdot S_k = \frac{\Delta x n}{E} \sum \frac{M_Q M_p}{I}$ symmetry
 $= \frac{3 \text{ ft}}{3000} 0.1554 \times 2 (1728)$
 $S_k = \underline{\underline{0.537 \text{ in}}}$

P10.41. Using a finite summation, compute the initial deflection at point C for the tapered beam in Figure P10.41. $E = 3500 \text{ kips/in}^2$. Base your analysis on the properties of $0.5I_0$.



P10.41



$$E = 3500 \text{ K/IN}^2 = 24,100 \text{ MPa}$$

$$\Delta x = 1 \text{ m}$$

TABLE 1

SEGMENT No.	M_p kN·m	h mm	$I = bh^3/12$ mm ⁴ × 10 ⁶	M_p^2/I (10 ⁻⁶)
1	51.43	325	915.41	2.889
2	154.29	375	1406.25	16.928
3	257.15	425	2047.08	32.303
4	360.01	475	2857.92	45.350
5	462.87	525	3858.75	55.923
6	565.73	575	5069.58	63.132
7	668.59	600	5760.00	77.606
8	630.00	600	5760.00	68.906
9	450.00	550	4436.67	45.642
10	270.00	450	2430.00	30.00
11	90.00	350	1143.33	7.085

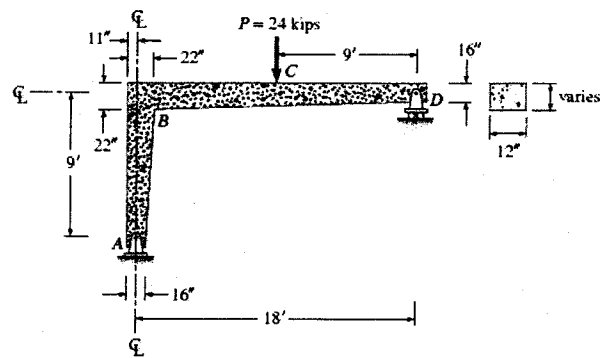
$$\Sigma = 445.4 \times 10^{-6}$$

USE P-SYSTEM AS Q-SYSTEM ($M_Q = M_P$)

$$180 \text{ kN} \cdot \Delta_c = \Sigma M_Q M_P \Delta x / EI = (\Delta x / E) \Sigma M_P^2 / 0.5 I$$

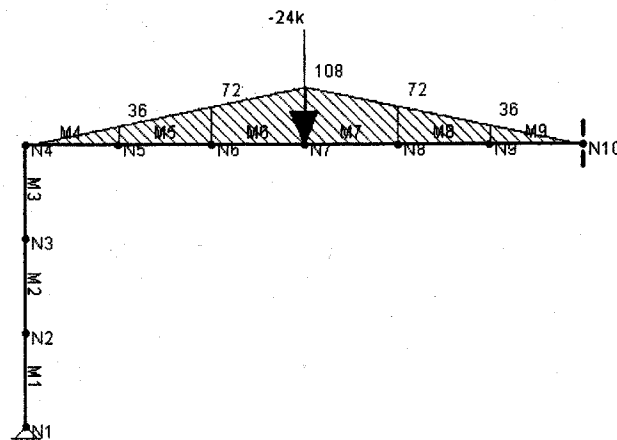
$$\Delta_c = (1/24100) [(445.4 \times 10^{-6}) / 0.5] (10^6 / 180) = \underline{\underline{205 \text{ mm}}}$$

P10.42. Computer study—Influence of supports on frame behavior. (a) Using the RISA-2D computer program, compute the initial elastic deflection at midspan of the girder in Figure P10.42, given that the support at *D* is a roller. For the computer analysis, replace the tapered members by 3 ft-long segments of constant depth whose properties are based on each segment's midspan dimensions; that is, there will be 9 members and 10 joints. When you set up the problem, specify in GLOBAL that forces are to be computed at three sections. This will produce values of forces at both ends and at the center of each segment. To account for cracking of the reinforced concrete, assume for girder *BCD* that $I_e = 0.35I_g$; for column *AB* assume $I_e = 0.7I_g$ (compression forces in columns reduce cracking). Since deflections of beams and one-story rigid frames are due almost entirely to moment and not significantly affected by the area of the member's cross section, substitute the gross area in the Member Properties Table. (b) Replace the roller at support *D* in Figure P10.42 by a pin to prevent horizontal displacement of joint *D*, and repeat the analysis of the frame. The frame is now an *indeterminate structure*. Compare your results with those in part (a), and briefly discuss differences in behavior with respect to the magnitude of deflections and moments.



P10.42

(a) Roller Support at D



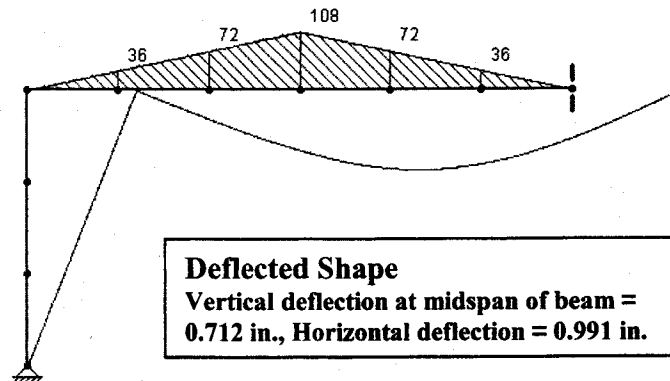
Member Properties:

Members									
	Label	I Joint	J Joint	Area[in ²]	Inertia[in ⁴]	Young's[ksi]	I Relea...	J Relea...	Length...
	M1	N1	N2	204	3439.1	3000			3
	M2	N2	N3	228	4801.3	3000			3
	M3	N3	N4	252	6482.7	3000			3
	M4	N4	N5	21.5	3478.4	3000			3
	M5	N5	N6	246	3015.3	3000			3
	M6	N6	N7	234	2595.2	3000			3
	M7	N7	N8	222	2216.1	3000			3
	M8	N8	N9	210	1875.8	3000			3
	M9	N9	N10	198	1572.2	3000			3

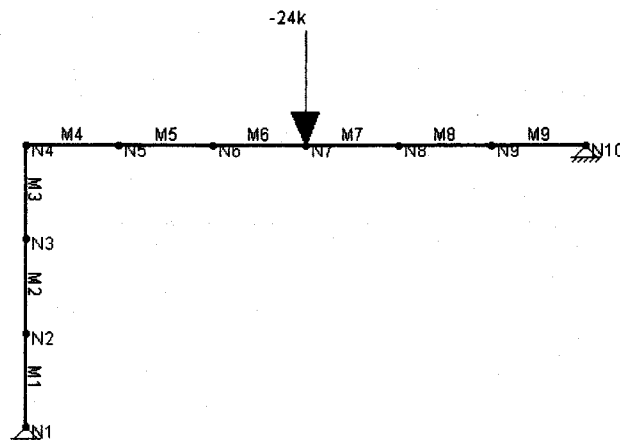
P10.42 Continued

P10.42 continued -Part (a)

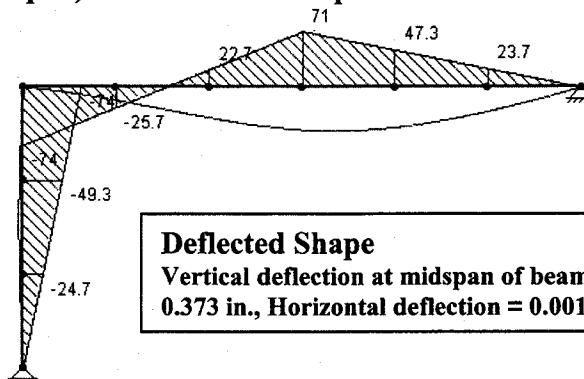
Moment Diagram (kip-ft) and Deflected Shape:



(b) Pin Support at D



Moment Diagram (kip-ft) and Deflected Shape:



When the support at D is a roller, the horizontal equilibrium of the entire structure requires that the reaction at support A (i.e., the shear of column AB) be equal to zero. As a result, column AB is subjected to an axial load only and beam BD acts as a simply supported beam. Since end B of the beam rotates, column AB has to sway to the right in order to maintain the 90-degree angle with the beam and, at the same time, the column has to remain straight (i.e., no moment in the column.) Once the roller is replaced by a pin support, the horizontal reaction at A produces a moment at the top end of the column, which restrains the beam and reduces the vertical deflection at midspan by about a half; this restraining effect also reduces the beam moment at midspan by 34%.