

FUNDAMENTALS OF STRUCTURAL ANALYSIS

3rd Edition

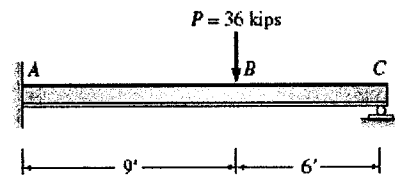
Kenneth M. Leet, Chia-Ming Uang, and Anne M. Gilbert

SOLUTIONS MANUAL

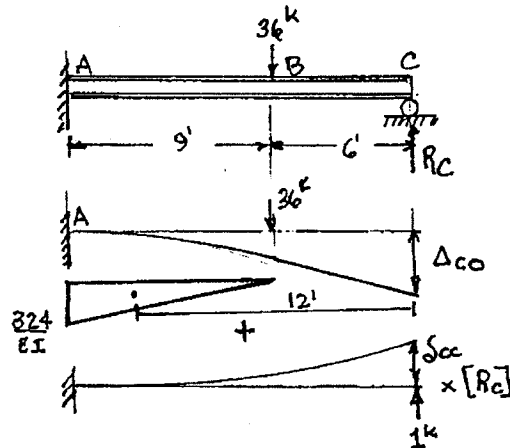
CHAPTER 11:

**ANALYSIS OF INDETERMINATE
STRUCTURES BY THE FLEXIBILITY METHOD**

P11.1. Compute the reactions, draw the shear and moment curves, and locate the point of maximum deflection for the beam in Figure P11.1. EI is constant.



P11.1



Δ_{CO} (USE MOMENT-AREA)

$$\Delta_{CO} = \frac{1}{2} \frac{32.4(9) \times 12}{EI} = \frac{17,496}{EI}$$

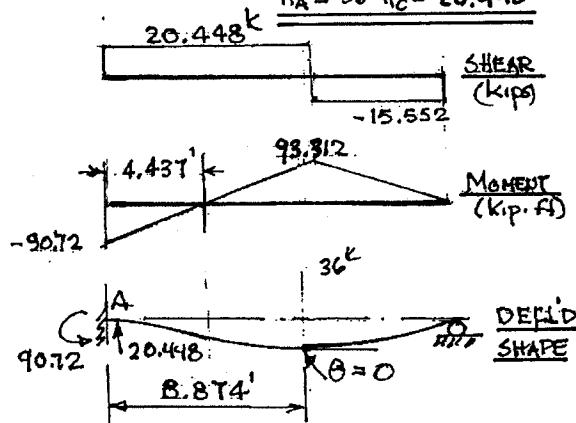
$$S_{CC} = \frac{PL^3}{3E} = \frac{1(15)^3}{3EI} = \frac{1125}{EI} \quad \text{SEE TABLE 9.1}$$

COMPATIBILITY EQ

$$\Delta_C = 0 : -\frac{17,496}{EI} + R_C \frac{1125}{EI} = 0$$

$$R_C = 15.552 \text{ kips}$$

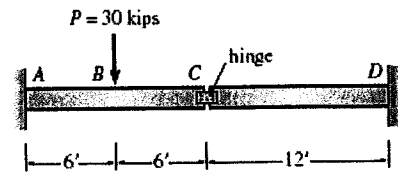
$$R_A = 36 - R_C = 20.448 \text{ k}$$



$$\theta_C = -\frac{1}{2} \frac{90.72}{EI} (4.437) + \frac{93.312}{EI} \frac{(10.563)}{2}$$

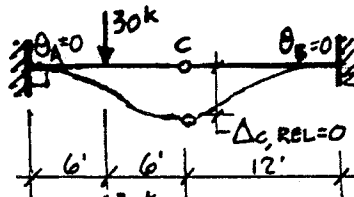
$$\theta_C = \frac{291.57}{EI}$$

P11.2. For the beam in Figure P11.2, compute the reactions, draw the shear and moment curves, and compute the deflection of the hinge at C. $E = 29,000$ ksi and $I = 180$ in⁴.

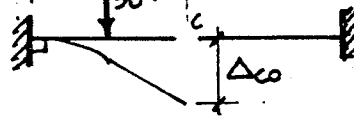


P11.2

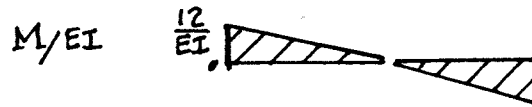
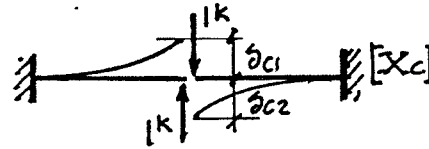
RELEASE STR.
W/ ORIGINAL
LOADS & V_C
AS REDUNDANT



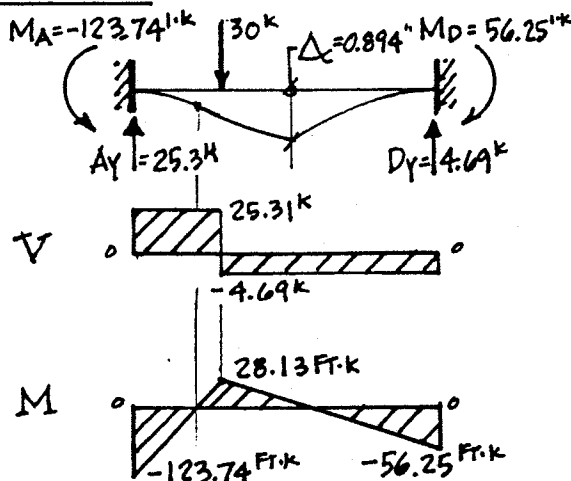
RELEASE STR.
W/ ORIGINAL
LOADS



RELEASE STR.
W/ UNIT VALUE
REDUNDANT



FINAL RESULTS:



INDETERMINATE TO 2ND DEGREE,
SELECT SHEAR V_C AS THE REDUNDANT.
RELATIVE DISPLACEMENT OF ENDS OF
BEAMS EACH SIDE OF HINGE EQUALS ZERO:
 $\Delta_{C,REL} = 0$; & NO HORIZ. LOADS.

COMPATIBILITY EQUATION:

$$\Delta_{C0} + \delta_{CC} [X_C] = 0$$

$$\text{WHERE } \delta_{CC} = \delta_{C1} + \delta_{C2}$$

$$X_C = V_C$$

$$\Delta_{C0} = t_{CA} = \frac{1}{2}(6') \left(\frac{-180}{EI} \right) (10') = \underline{\underline{-1.788''}}$$

$$\delta_{C1} = \delta_{C2} = \pm \frac{PL^3}{3EI} = \frac{1^k (12')^3 (1728)}{3(29,000)(180)} = \underline{\underline{0.1907''}}$$

$$\delta_{CC} = 2(0.1907'') = \underline{\underline{0.3814''}}$$

$$\Delta_{C0} + \delta_{CC} [X_C] = 0$$

$$-1.788'' + 0.3814'' [X_C] = 0$$

$$\underline{\underline{X_C = V_C = 4.688^k}}$$

DEFLECTION @ C:

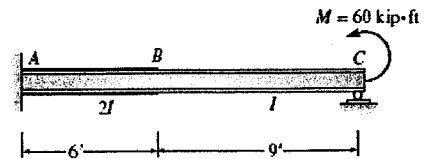
$$\Delta_C = \Delta_{C0} - 0.1907'' [X_C] = \underline{\underline{0.894'' \downarrow}}$$

REACTIONS:

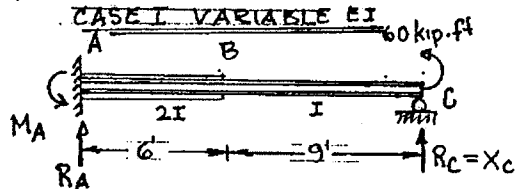
$$A_y = 30^k - 1^k [X_C] = \underline{\underline{25.31^k \uparrow}}$$

$$M_A = -180 \text{ FT}\cdot\text{k} + 12 \text{ FT}\cdot\text{k} [X_C] = \underline{\underline{-123.74 \text{ FT}\cdot\text{k}}}$$

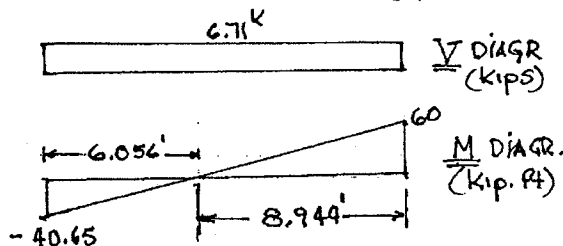
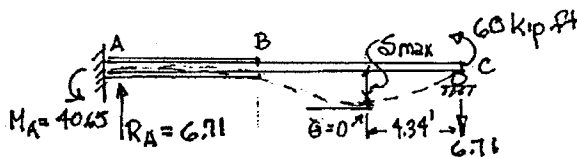
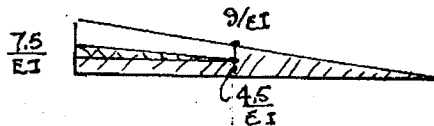
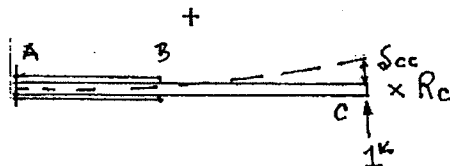
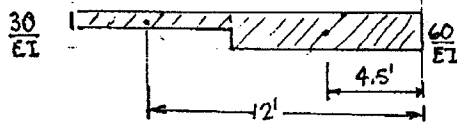
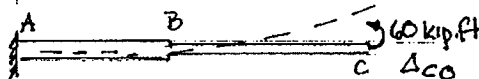
P11.3. Compute the reactions, draw the shear and moment curves, and locate the point of maximum deflection. Repeat the computation if I is constant over the entire length. E is constant. Express answer in terms of E , I , and L .



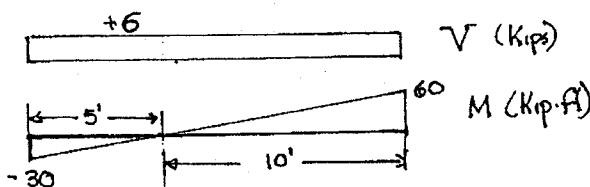
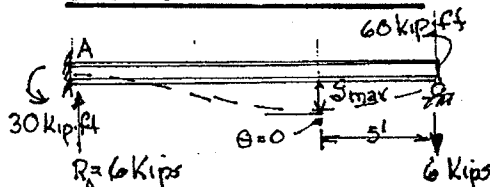
P11.3



SELECT R_C AS THE REDUNDANT



CASE II CONSTANT EI



USE MOMENT-AREA TO COMPUTE DEF'S

$$\Delta_{C0} = \frac{1}{2} \frac{9}{EI} \times 6$$

$$\Delta_{C0} = \frac{30 \times 6(12)}{EI} + \frac{60 \times 9(4.5)}{EI}$$

$$\Delta_{C0} = \frac{4590}{EI}$$

$$\Delta_{CC} = \frac{1}{2} \frac{9}{EI} \times 6 + \frac{4.5}{EI} \times 6(9+3)$$

$$+ \frac{1}{2} \frac{6}{EI} \times 3(9+4)$$

$$\Delta_{CC} = \frac{684}{EI}$$

COMPAT. EQ $\Delta_C = 0 = \Delta_{C0} + \Delta_{CC} R_C$

$$0 = \frac{4590}{EI} + \frac{684}{EI} R_C$$

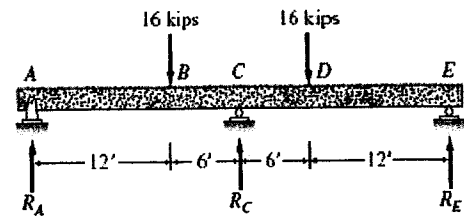
$$R_C = -6.71 \text{ kips} \downarrow$$

$$M_A = 60 - 6.71 \times 15 = 40.65 \text{ ft.kips}$$

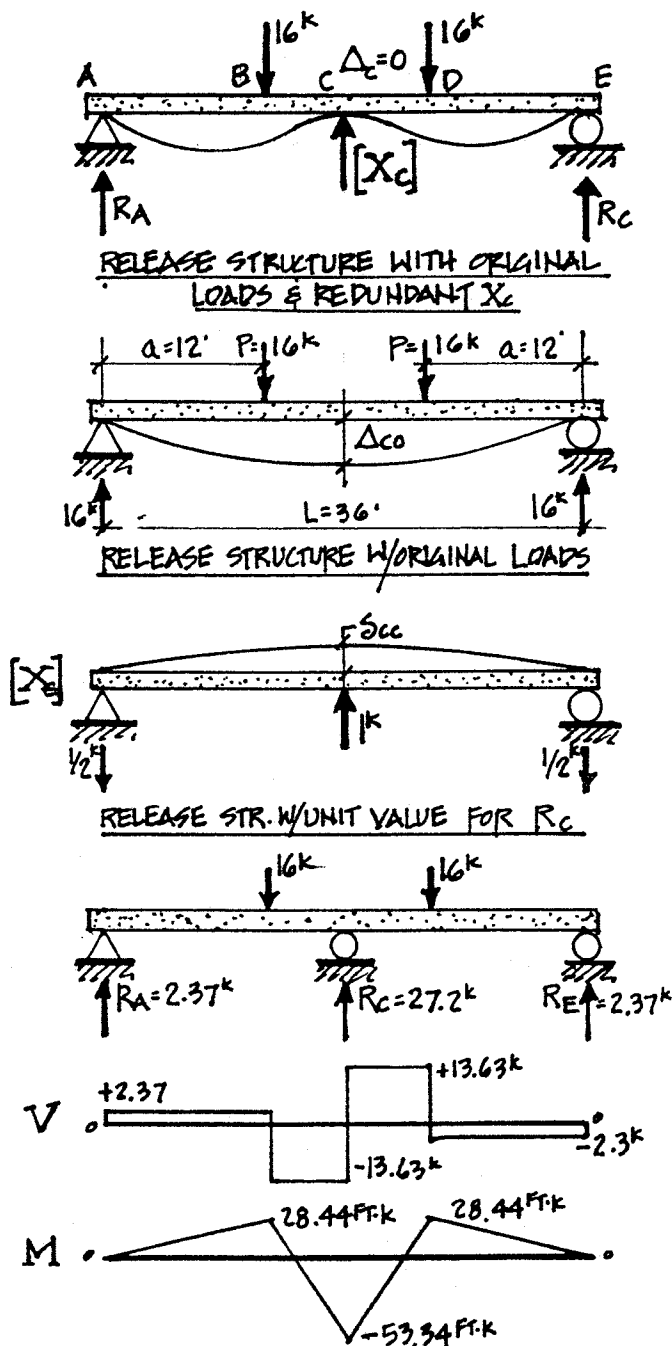
$$R_A = 6.71 \text{ kips} \uparrow$$

This example shows that increasing the stiffness of an indeterminate structure in the region between points A & B produces a 35% increase in the moment at support A.

P11.4. Compute the reactions and draw the shear and moment curves for the two-span continuous beam in Figure P11.4. EI is constant.



P11.4



INDETERMINATE TO 1ST DEGREE,
SELECT R_C AS THE REDUNDANT.

COMPATIBILITY EQUATION:

$$\Delta_c = 0$$

$$\Delta_{c0} + \delta_{cc} [X_c] = 0$$

$$\Delta_{c0} = \frac{-Pa(3L^2 - 4a^2)}{24EI}$$

$$= \frac{-16k(12')(3(36')^2 - 4(12')^2)}{24EI}$$

$$= -26,496/EI$$

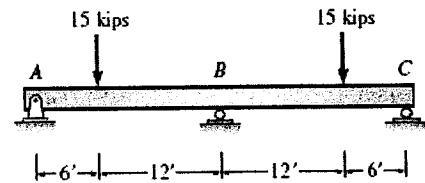
$$\delta_{cc} = \frac{PL^3}{48EI} = \frac{1k(36')^3}{48EI} = 972/EI$$

$$-26,496/EI + (972/EI)[X_c] = 0$$

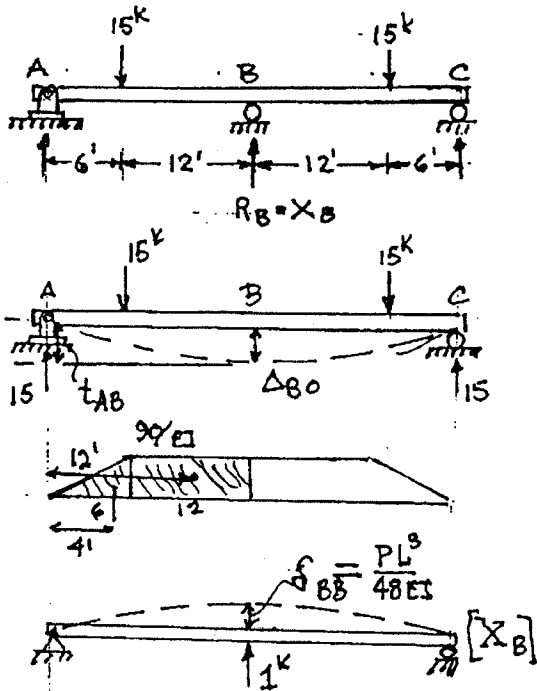
$$X_c = R_c = \underline{27.26k \uparrow}$$

$$R_A = R_E = \underline{2.37k \uparrow}$$

P11.5. Compute the reactions and draw the shear and moment curves for the beam in Figure P11.5. EI is constant.



P11.5



Δ_{B0} USE MOMENT-AREA

$$\Delta_{B0} = t_{AB} = 12 \left(\frac{90}{EI} \right) (2) + \frac{1}{2} 6 \left(\frac{90}{EI} \right) 4$$

$$\Delta_{B0} = -14,040/EI$$

$$S_{BB} = \frac{PL^3}{48EI} \text{ SEE FIG. 11.3 d}$$

$$S_{BB} = 1^3(36)/48EI = 972/EI$$

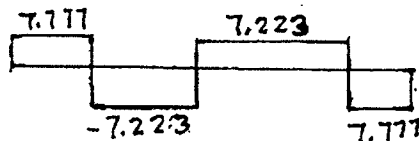
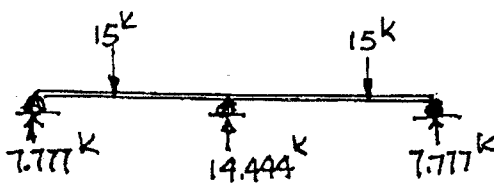
COMPATIBILITY E.Q.

$$\Delta_B = 0$$

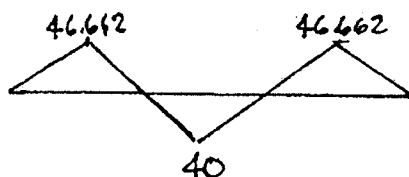
$$\Delta_{B0} + S_{BB} X_B = 0$$

$$-14,040/EI + 972/EI X_B = 0$$

$$X_B = \underline{\underline{14.444}}$$

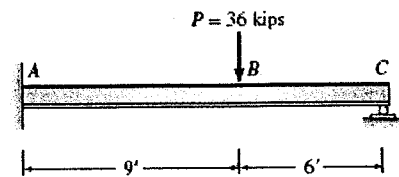


V
(kips)



M
(kip-ft)

P11.6. Solve Problem P11.1 for the loading shown if support C settles 0.25 in when the load is applied. $E = 30,000 \text{ kips/in}^2$ and $I = 320 \text{ in}^4$.



P11.1

$E = 30,000 \text{ kips/in}^2, I = 320 \text{ in}^4$
 support C settles 0.25"

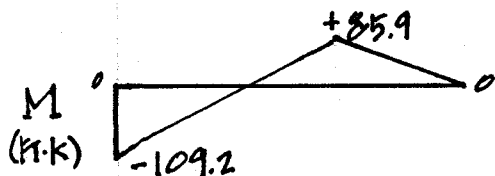
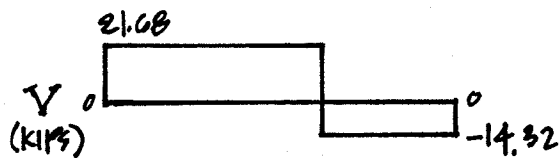
$\Delta_s = 0.25"$
 $R_c = X_c$

$\Delta_{c0} = \frac{17,496(1728)}{30,000(320)} = -3.15 \text{ in} \downarrow$

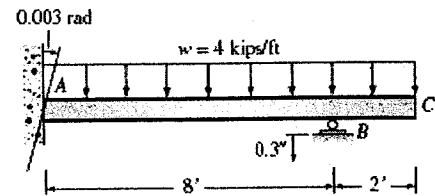
$X_c \uparrow \quad \Delta_{cc} = \frac{1125(1728)}{30,000(320)} = 0.2025 \text{ in up}$

$(\text{GAP}) \Delta_{c0} - 0.25 = 3.15 - 0.25 = 2.90 \text{ in}$
 $R_c = X_c = \frac{2.90}{0.2025} = \underline{\underline{14.32 \text{ K}}}$

$M_A = 109.2 \text{ kip}\cdot\text{ft}$
 $R_A = 21.68 \text{ kips}$
 $14.32 \text{ kips} = R_c$

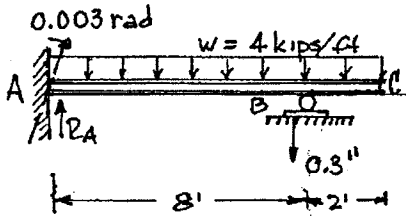


P11.7. Determine the reactions for the beam in Figure P11.7. When the uniform load is applied, the fixed support rotates clockwise 0.003 rad and support B settles 0.3 in. Given: $E = 30,000$ kips/in² and $I = 240$ in⁴.

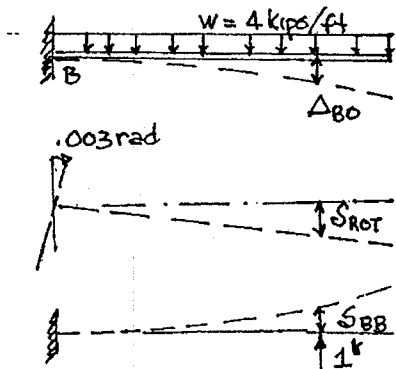


P11.7

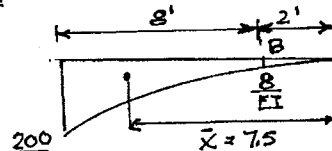
$$I = 240 \text{ in}^4, E = 30,000 \text{ kips/in}^2$$



SELECT R_B AS THE REDUNDANT



Δ_{B0} USE MOMENT-AREA METHOD



$$\Delta_{B0} = t_{B/A} = \frac{10 \times 200}{3EI} (7.5 - 2) - \frac{1}{3} (2 \times 8) \times \frac{2}{4}$$

$$= \frac{3664}{EI} = \frac{3664 (1728)}{30,000 \times 240} = 0.879'' \downarrow$$

$$s_{BB} = \frac{PL^3}{3EI} = \frac{1^k (8^3)}{3EI} = \frac{512 (1728)}{3 \times 30,000 \times 240}$$

$$s_{BB} = 0.041'' \uparrow$$

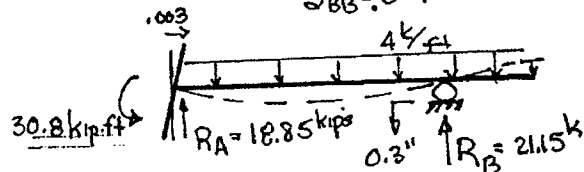
$$s_{ROT} = 0.003 (8 \times 12) = 0.288''$$

$$\text{GAP: } \text{OPENS} = .879 + 0.288 = 1.167''$$

$$\text{support mov} = -.3$$

$$\text{GAP} = 0.867''$$

$$R_B = \frac{0.867}{s_{BB} = .041} = 21.15 \text{ kips}$$



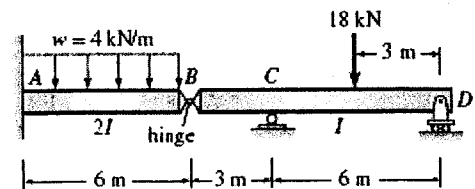
$$R_A + R_B - 4 \times 10 = 0$$

$$R_A = 40 - 21.15 = 18.85$$

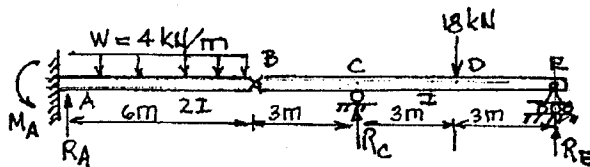
$$M_A = 4 \times 10 \times 5 - 21.15 \times 8$$

$$M_A = 30.8 \text{ Kip-ft}$$

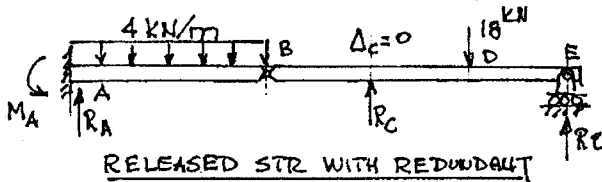
P11.8. Compute the reactions and draw the shear and moment curves in Figure P11.8. E is constant.



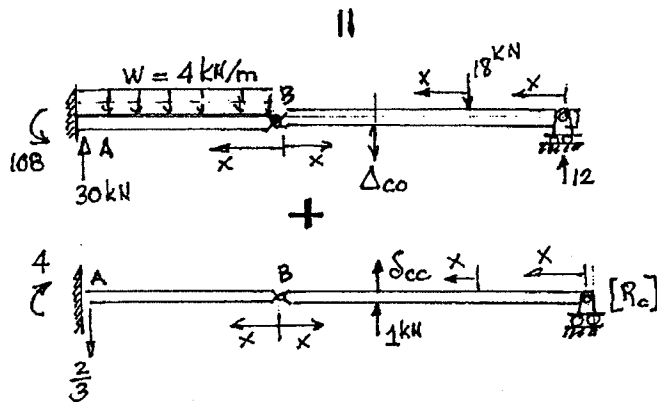
P11.8



INDENT TO 1ST DEGREE
SELECT R_C AS THE REDUNDANT



RELEASED STR WITH REDUNDANT



COMPUTE Δ_{c0} :

$$1^{\text{st}} \Delta_{c0} = \int M_Q M_P \frac{dx}{EI}$$

$$1. \Delta_{c0} = \int_0^6 \frac{2x}{3} (-6x - 4x \cdot \frac{x}{2}) \frac{dx}{EI} + \int_0^3 \frac{(-2x)(6x)}{EI} dx$$

$$+ \int_0^3 \frac{-x}{3} (12x) \frac{dx}{EI} + \int_0^3 \frac{(-\frac{1}{3})(3+x)[12(x+3) - 18x]}{EI} dx$$

$$\Delta_{c0} = \int_0^6 \frac{(-4x^2 - \frac{4x^3}{3})}{2EI} dx + \int_0^3 \frac{-4x^2}{EI} dx + \int_0^3 \frac{(-4x^2)}{EI} dx$$

$$+ \int_0^3 \frac{2x^2 - 6x - 36}{EI} dx$$

INTEGRATE AND SUBSTI LIMITS

$$\Delta_{c0} = -\frac{549}{EI}$$

COMPUTE S_{CC}

$$1^{\text{st}} S_{CC} = \int_0^6 \frac{(2x)^2}{2EI} dx + \int_0^3 \frac{(-2x)^2}{EI} dx + \int_0^3 \frac{(\frac{x}{3})^2}{EI} dx$$

$$S_{CC} = \frac{2}{9} \left[\frac{x^3}{3} \right]_0^6 \frac{1}{EI} + \frac{4}{9} \frac{x^3}{3EI} \Big|_0^3 + \frac{x^3}{27EI} \Big|_0^3$$

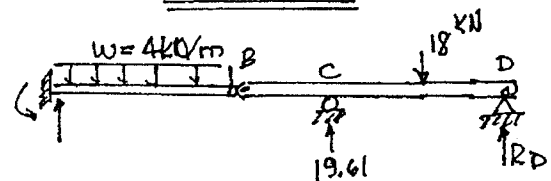
$$S_{CC} = \frac{28}{EI}$$

COMPAT. EQ: $\Delta_c + R_C S_{CC} = 0$

$$\Delta_{c0} + R_C S_{CC} = 0$$

$$-\frac{549}{EI} + R_C \frac{28}{EI} = 0$$

$$R_C = 19.61 \text{ kN}$$

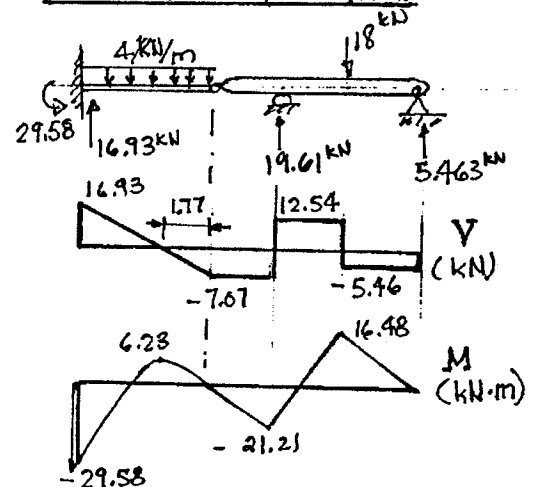


COMPUTE REACTIONS USING FBD
COMPUTE R_D USE BCD

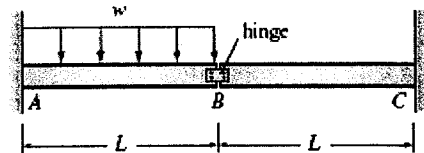
$$\sum M_B = 0 = -19.61 \times 3 + 18 \times 6 - 9R_D$$

$$R_D = 5.463 \text{ kN}$$

SHEAR & MOMENT DIAGRAMS

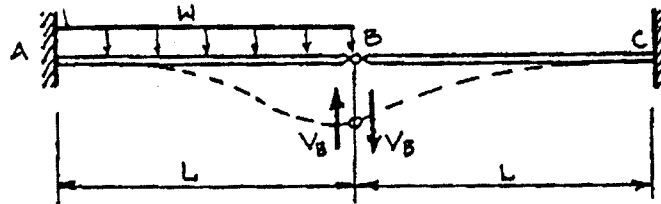


P11.9. Compute the reactions and draw the shear and moment curves for the beam in Figure P11.9. EI is constant. The bolted web connection at B may be assumed to act as a hinge. Express answer in terms of E , I , L , and w .

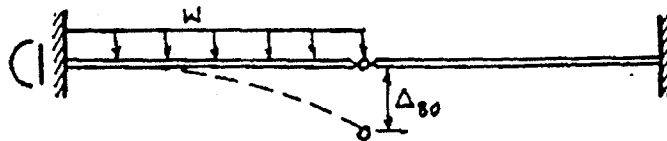


P11.9

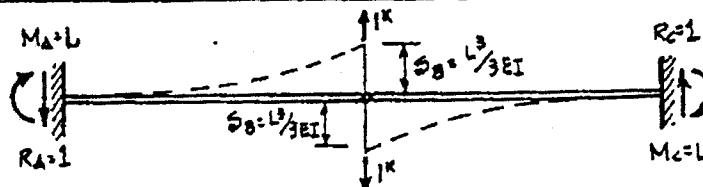
USE SHEAR AT HINGE AS REDUNDANT



LOAD APPLIED TO RELEASED STRUCTURE



$[V_B]$ UNIT VALUE OF REDUNDANT APPLIED TO RELEASED STRUCTURE

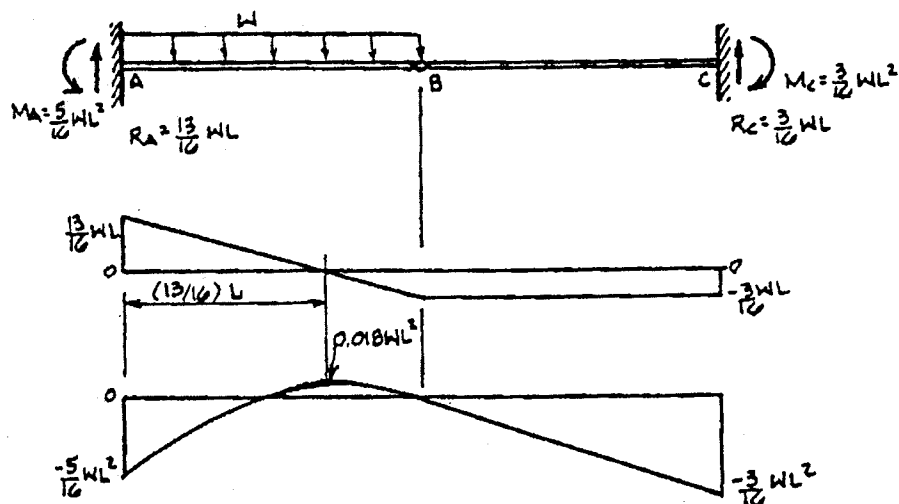


COMPATIBILITY EQUATION $\Delta_{B,REL} = 0$

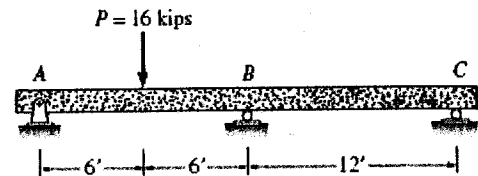
$$\Delta_{B0} + 2\Delta_B V_B = 0$$

$$-wL^4/8EI + 2L^3/3E[V_B] = 0$$

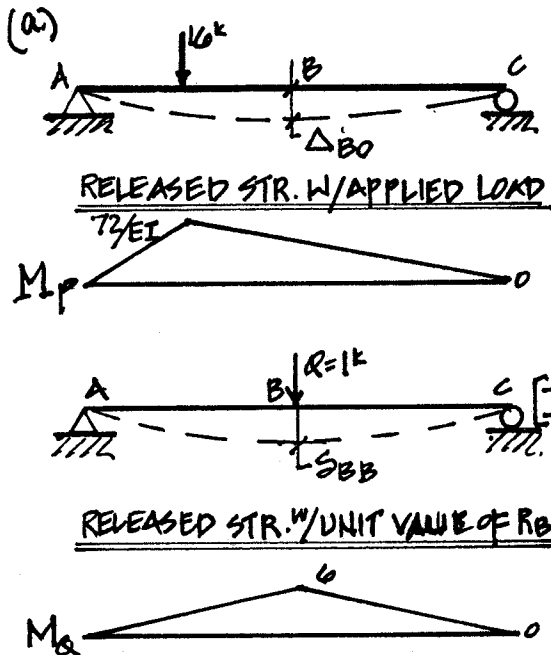
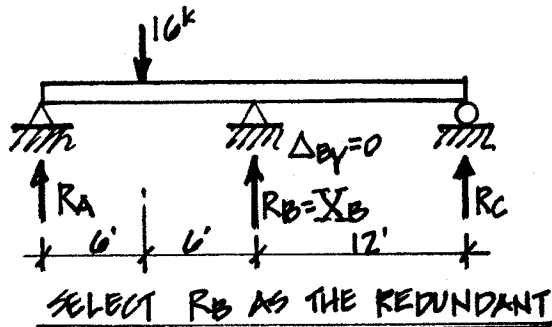
$$V_B = 5wL/16$$



P11.10. (a) Compute all reactions for the beam in Figure P11.10 assuming that the supports do not move; EI is constant. (b) Repeat computations given that support C moves upward a distance of $288/(EI)$ when the load is applied.



P11.10



COMPATIBILITY EQUATION:

$$\Delta_{B0} = 0 = \Delta_{B0} + \Delta_{BB} X_B = 0$$

VIRTUAL WORK:
USING TABLE 4 (VALUES OF PRODUCT INTEGRALS):

$$\Delta_{B0} = \frac{1}{EI} \left(\frac{1}{3} - \frac{(12-6)^2}{4(12)(18)} \right) (6)(72)(24)$$

$$= \frac{3168}{EI} \downarrow$$

$$\Delta_{BB} = \frac{PL^3}{48EI} = \frac{1(24)^3}{48EI}$$

$$= \frac{288}{EI} \uparrow$$

$$\Delta_{B0} + \Delta_{BB} X_B = 0$$

$$-\frac{3168}{EI} + \frac{288}{EI} X_B = 0$$

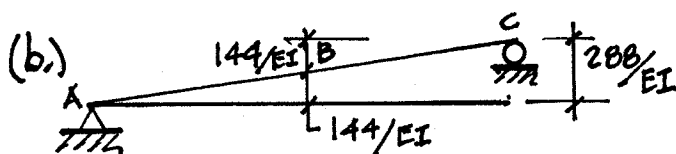
$$X_B = R_B = 11^k \uparrow$$

REMAINING REACTIONS BY STATICS:

$$\sum \vec{M}_C = 0; \quad 24(R_A) + 12(11^k) - 18(16) = 0$$

$$R_A = 6.5^k \uparrow$$

$$\uparrow \sum F_y = 0; \quad R_C = 11 + 6.5 - 16^k = 1.5^k \downarrow$$



(b.) COMPATIBILITY EQUATION:

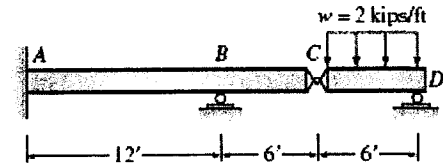
$$\Delta_{B0} + \Delta_{BB} X_B = -144/EI$$

$$-\frac{3168}{EI} + \frac{288}{EI} X_B = -144/EI$$

$$X_B = R_B = 10.5^k \uparrow$$

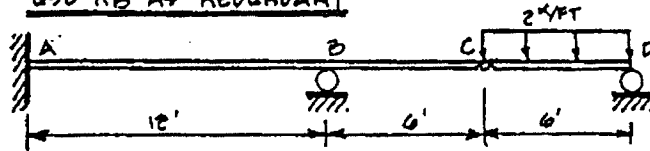
$$R_A = 6.75^k \uparrow; \quad R_C = 1.25^k \downarrow$$

P11.11. (a) Determine the reactions and draw the shear and moment curves for the beam in Figure P11.11. Given: EI is constant, $E = 30,000$ kips/in², and $I = 288$ in⁴. (b) Repeat the computations if the applied loads also produce a settlement of 0.5 in at support B and 1 in at support D .

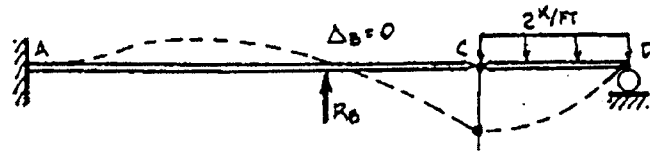


P11.11

(a) USE R_D AS REDUNDANT

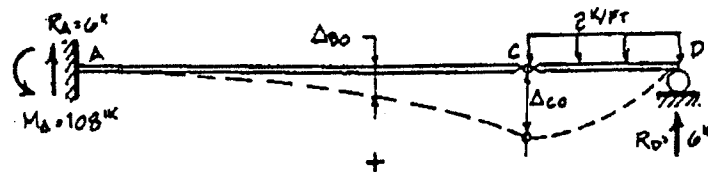


REDUNDANT APPLIED AS FORCE

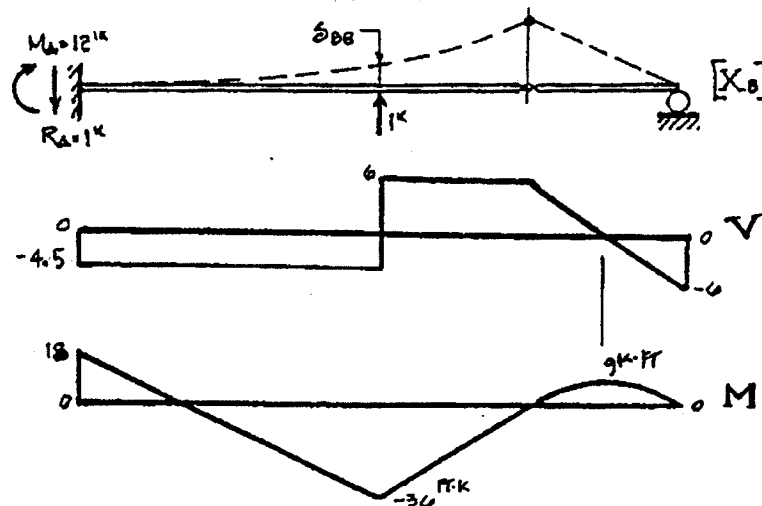


||

RELEASE STRUCTURE WITH ORIGINAL LOADS



RELEASE STRUCTURE W/ UNIT VALUE OF R_D



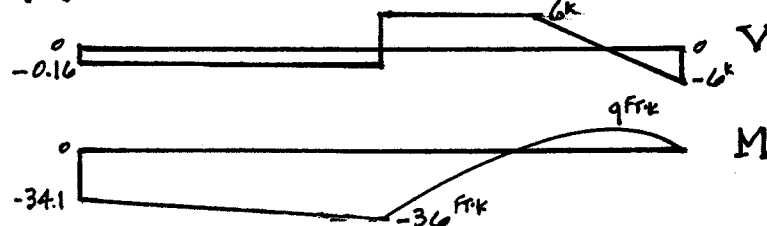
GEOMETRY $\Delta_{B0} = 0$

$$0 = \Delta_{B0} + \delta_{BB} [X_B]$$

$$0 = -1.21 + 0.115 X_B$$

$$\underline{X_B = 10.5k}$$

(b)



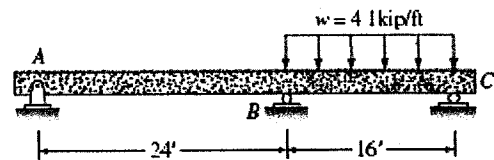
$$R_{AY} = -0.16k$$

$$M_A = -34.1 \text{ ft-k}$$

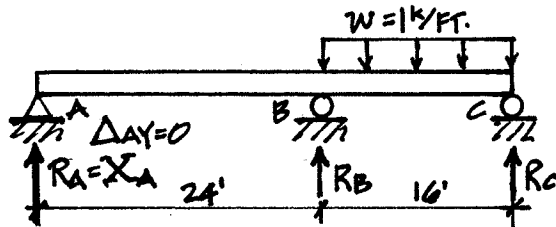
$$R_{BY} = 6.16k$$

$$R_{DY} = 6k$$

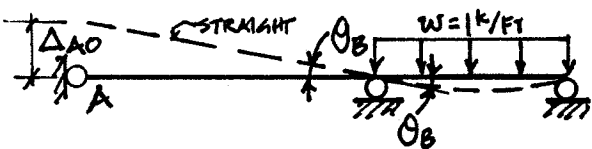
P11.12. Determine all reactions and draw the shear and moment curves. EI is constant.



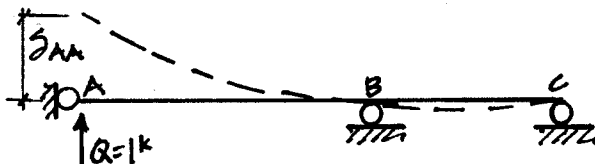
P11.12



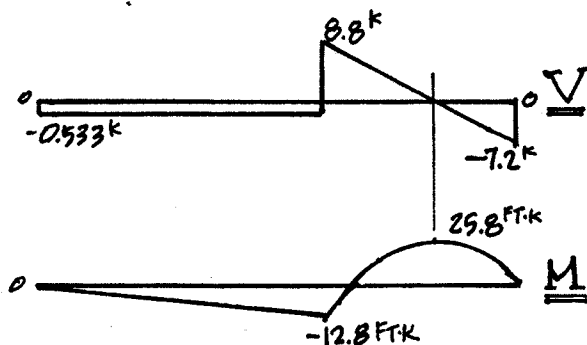
SELECT R_A AS THE REDUNDANT



RELEASED STRUCT. W/APPLIED LOADS.



RELEASED STR. W/UNIT VALUE OF R_A



COMPATIBILITY EQUATION:

$$\Delta_{A0} + S_{AA} X_A = 0$$

θ_B : FROM FIGURE 11.3(C) [DISPLACEMENTS OF PRISMATIC BEAMS]:

$$\theta_B = \frac{WL^3}{24EI} = \frac{1(16)^3}{24EI} = \frac{170.7}{EI}$$

$$\Delta_{A0} = \theta_B (24') = \frac{4096}{EI} \uparrow$$

S_{AA} : FROM TABLE 1, CASE (5) [MOMENT DIAGRAMS & EQNS. FOR MAX. DISPLACEMENTS]

$$S_{AA} = \frac{Pa^2}{3EI} (L+a) = \frac{1(24)^2}{3EI} (16+24) = \frac{7680}{EI} \uparrow$$

$$\Delta_{A0} + S_{AA} X_A = 0$$

$$\frac{4096}{EI} + \frac{7680}{EI} [X_A] = 0$$

$$X_A = R_A = -0.533k \downarrow$$

REMAINING REACTIONS BY STATICS:

$$\sum \overset{\uparrow}{M}_B = 0;$$

$$-0.533(24') + (1)(16')(8') - 16'R_C = 0$$

$$R_C = 7.2k \uparrow$$

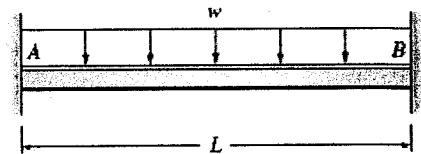
$$\sum F_y = 0;$$

$$-0.533 + R_B + 7.2k - 1k/ft(16) = 0$$

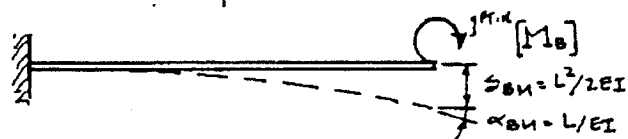
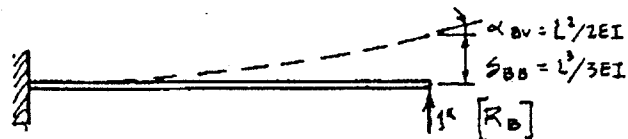
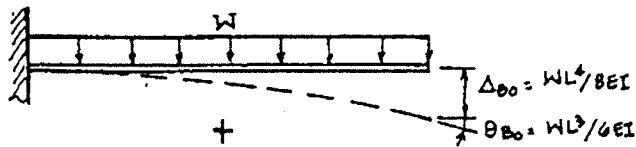
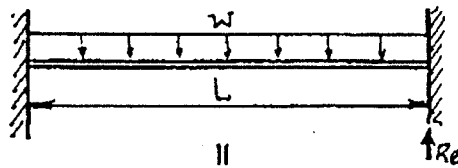
$$R_B = 9.33k \uparrow$$

P11.13. Compute the reactions and draw the shear and moment curves for the beam in Figure P11.13. Given: EI is constant.

COMPUTE REACTIONS USING M_B AND R_B AS REDUNDANTS. DEFLECTIONS COMPUTED BY MOMENT-AREA OR BY EQNS. ON P.209.



P11.13



COMPATIBILITY EQUATIONS

$$\Delta_B = 0 = \Delta_{B0} + \Delta_{B1} R_B + \Delta_{B2} M_B$$

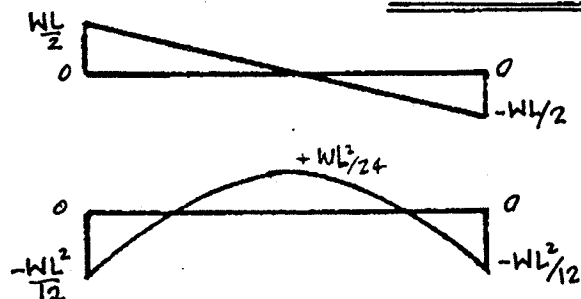
$$\theta_B = 0 = \theta_{B0} + \theta_{B1} R_B + \theta_{B2} M_B$$

IN TERMS OF DISPLACEMENTS

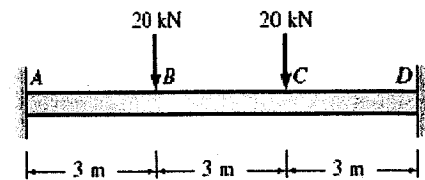
$$0 = -\frac{WL^4}{8EI} + \frac{L^3}{3EI} R_B - \frac{L^2}{2EI} M_B$$

$$0 = \frac{WL^3}{6EI} - \frac{L^2}{2EI} R_B + \frac{L}{EI} M_B$$

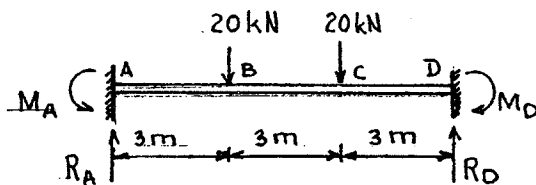
SOLVING GIVES: $R_B = WL/2$
 $M_B = WL^2/12$



P11.14. Compute the reactions and draw the shear and moment curves for the beam in Figure P11.14. Given: EI is constant.



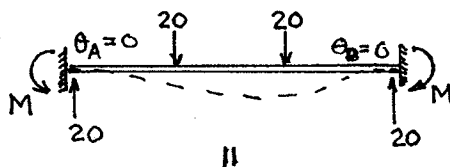
P11.14



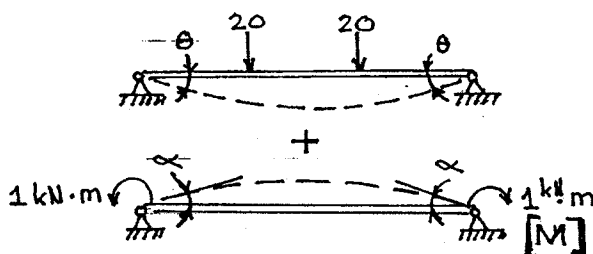
USE SYMMETRY TO SIMPLIFY

1. $R_A = R_D = 20 \text{ kN}$
2. $M_A = M_D = M$

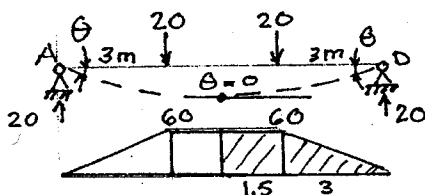
SINCE NO HORIZ. LOADS, STRUCTURE INDETERMINATE TO 1°



SELECT "M" AS REDUNDANT



COMPUTE θ . USE MOMENT-AREA

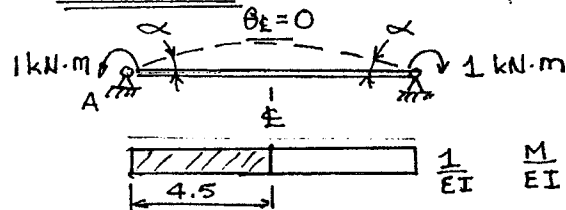


$$\theta = \theta_A + \Delta\theta_{A-D}$$

$$= 0 + \frac{60}{EI}(1.5) + \frac{60}{EI}\left(\frac{3}{2}\right)$$

$\theta = -\frac{180}{EI}$ · USE MINUS SIGN.
SINCE OPPOSITE
IN DIRECTION
TO REDUNDANT.

COMPUTE α USE MOMENT-AREA



$$\alpha = \theta_A + \Delta\theta_{A-D}$$

$$\alpha = 0 + \frac{1}{EI}(4.5) = \frac{4.5}{EI}$$

COMPATIBILITY EQ.

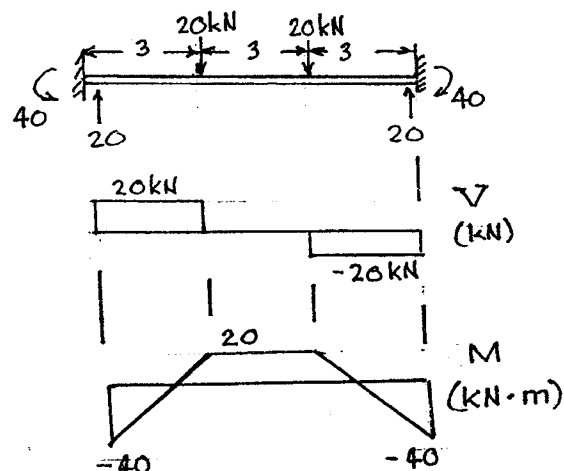
$$\theta_A = 0$$

$$\theta + \alpha M = 0$$

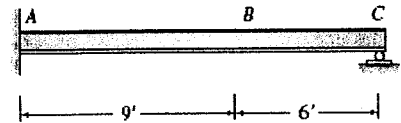
$$-\frac{180}{EI} + \frac{4.5}{EI}(M) = 0$$

$$\underline{M = 40 \text{ kN}\cdot\text{m}}$$

SHEAR AND MOMENT DIAGRAMS

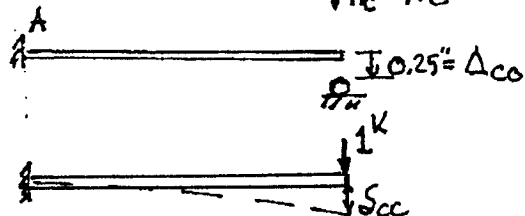
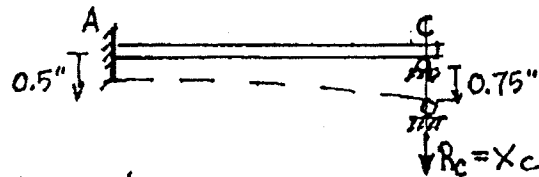


P11.15. Assuming that no load acts, compute the reactions and draw the shear and moment curves for the beam in Figure P11.1 if support *A* settles 0.5 in and support *C* settles 0.75 in. Given: $E = 29,000$ kips/in² and $I = 150$ in⁴.



P11.1

$$E = 29,000 \text{ ksi}, I = 150 \text{ in}^4$$



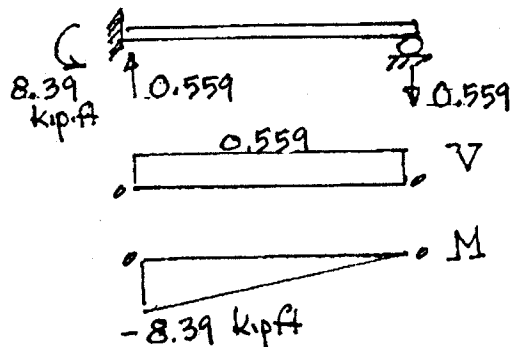
$$\Delta_{CC} = \frac{PL^3}{3EI} = \frac{1(15^3)1728}{3 \times 29,000 \times 150}$$

$$\Delta_{CC} = \underline{\underline{0.447}}$$

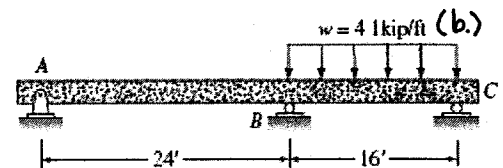
$$\Delta_{C0} + X_C \Delta_{CC} = 0$$

$$0.25 + X_C (0.447) = 0$$

$$X_C = \underline{\underline{0.559 \text{ kips}}}$$

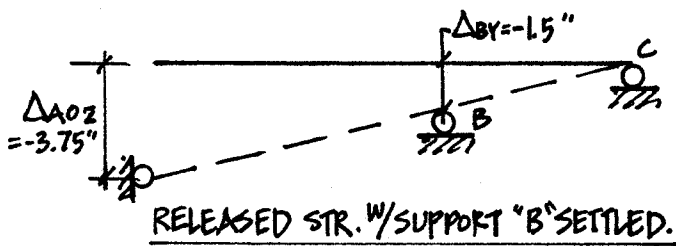
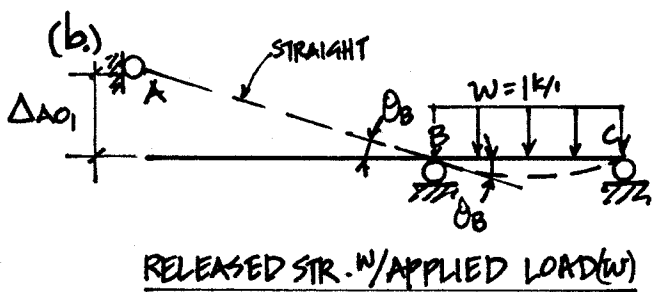
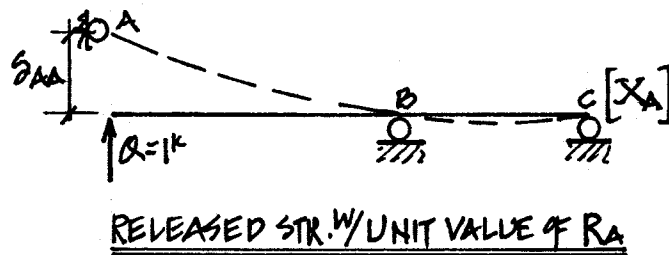
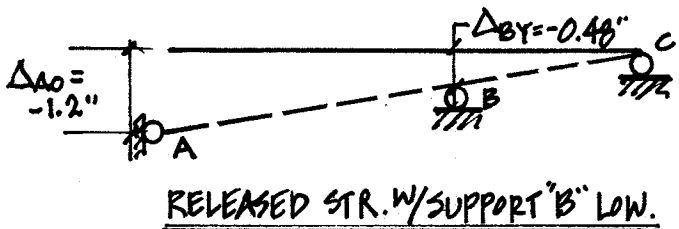
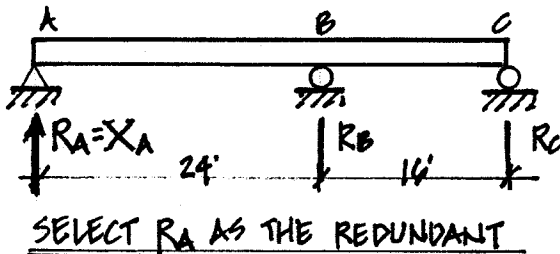


P11.16. (a) Assuming that no loads act in Figure P11.12, compute the reactions if support B is constructed 0.48 in too low. Given: $E = 29,000$ kips/in², $I = 300$ in⁴. (b) If support B (Figure P11.12) settles 1.5 in under the applied loads, compute the reactions.



P11.12

(a.)



COMPATIBILITY EQUATION:

$$\Delta_{A0} = -1.2'' \downarrow$$

$$\Delta_{A0} + \delta_{AA} X_A = 0$$

δ_{AA} : FROM TABLE 1, CASE (5) [MOMENT DIAGRAMS & EQNS FOR MAX. DISPL'CMTS]

$$\delta_{AA} = \frac{Pa^2}{3EI} (L+a)$$

$$= \frac{1(24)^2(16+24)}{3EI}$$

$$= \frac{7680(1728)}{(29,000)(300)} = 1.525 \text{ in/k}$$

COMPATIBILITY EQN

$$-1.2'' + (1.525) X_A = 0$$

$$X_A = R_A = 0.787 \text{ k} \uparrow$$

REMAINING RXNS BY STATICS:

$$\sum M_B = 0; 0.787(24) - R_C(16) = 0$$

$$R_C = 1.18 \text{ k} \uparrow$$

$$\uparrow \sum F_y = 0; R_B = 1.967 \text{ k} \downarrow$$

(b.) COMPATIBILITY EQN:

$$\Delta_{A01} + \Delta_{A02} + \delta_{AA} X_A = 0$$

(LOAD) (SETTLEMENT)

θ_B : FROM FIG. 11.3(C.) [DISPLACEMENTS OF PRISMATIC BEAMS]

$$\theta_B = \frac{WL^3}{24EI} = \frac{(1)(16)^3(144)}{24(29,000)(300)} = 2.825(10^{-3}) \text{ RAD.}$$

$$\Delta_{A01} = \theta_B(24' \times 12'') = 0.814'' \uparrow$$

$$\Delta_{A02} = -1.5 + \frac{-1.5}{16}(24) = -3.75'' \downarrow$$

$$\text{COMPAT. EQN: } 0.814'' - 3.75 + 1.525 X_A = 0$$

$$X_A = R_A = 1.925 \text{ k} \uparrow$$

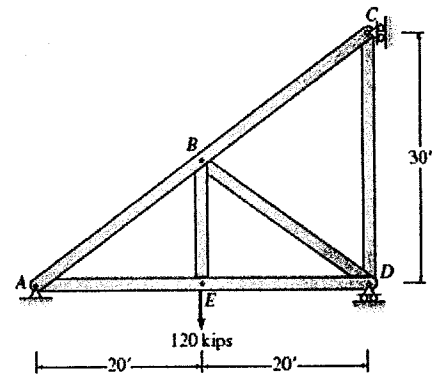
REMAINING RXNS BY STATICS

$$R_C = 10.89 \text{ k} \uparrow$$

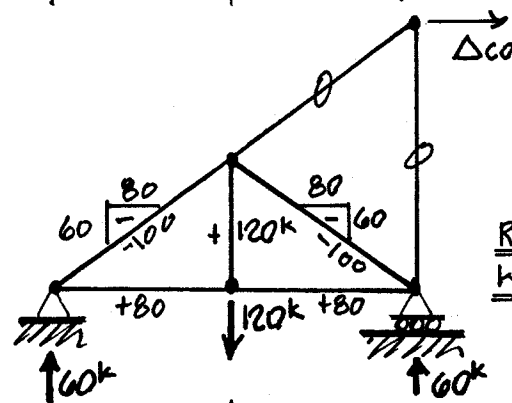
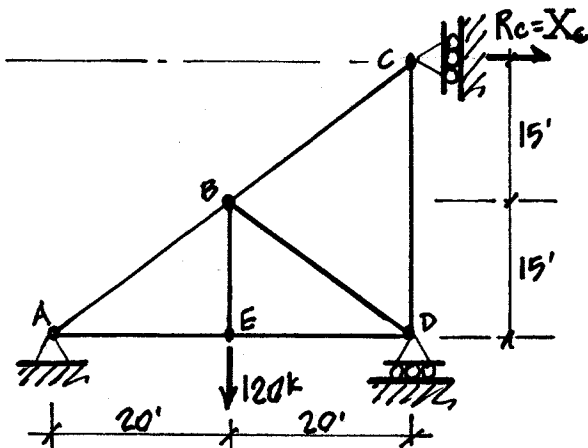
$$R_B = 3.187 \text{ k} \uparrow$$

P11.17. Compute the reactions and bar forces in all truss members. The area of all bars is 5 in² and $E = 30,000 \text{ kips/in}^2$.

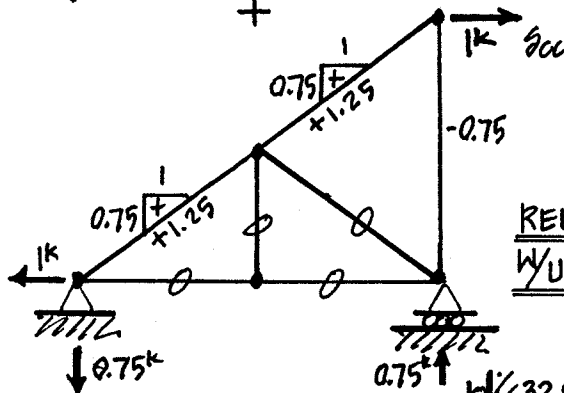
SELECT R_C AS THE REDUNDANT
ALL BARS $E = 30,000 \text{ ksi}$ & $A = 5 \text{ in}^2$



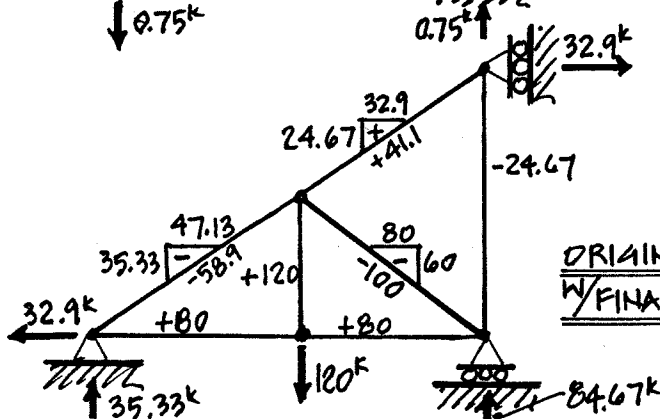
P11.17



RELEASED STR.
W/APPLIED LOAD



RELEASED STR.
W/UNIT VALUE OF R_C



ORIGINAL STR
W/FINAL RESULTS

GEOMETRY $\delta_{CX} = 0$

Δ_{CO} : $W_Q = U_Q = \sum F_Q F_P L / AE$

$$1^k \cdot \Delta_{CO} = 1.25 \left[\frac{-100(25 \times 12)}{AE} \right]$$

$$\Delta_{CO} = -0.25$$

δ_{CC} : $W_Q = U_Q = \sum F_Q^2 L / AE$

$$1^k \cdot \delta_{CC} = \frac{2(1.25)^2 25 \times 12}{AE} + \frac{(-0.75)^2 30 \times 12}{AE}$$

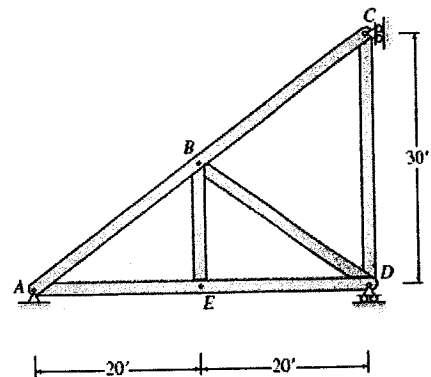
$$\delta_{CC} = 0.0076$$

$\Delta_{CO} + \delta_{CC} X_C = 0$

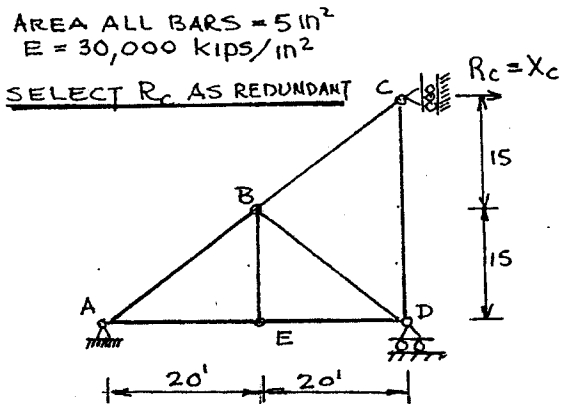
$$-0.25 + 0.0076 X_C = 0$$

$$\underline{X_C = R_C = 32.9^k \rightarrow}$$

P11.18. Assuming that the 120-kip load is removed from the truss in Figure P11.17, compute the reactions and bar forces if the temperature of bars AB and BC increases 60°F ; the coefficient of temperature expansion $\alpha = 6 \times 10^{-6} \text{ (in/in)/}^\circ\text{F}$.



P11.17

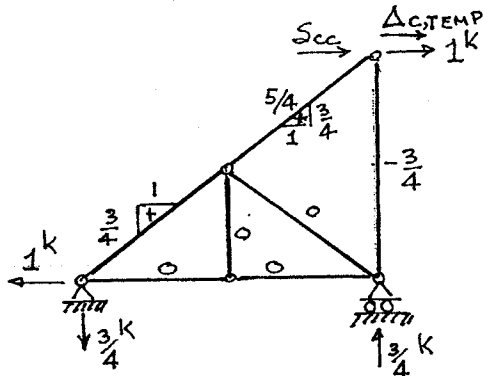


BARS AB & BC INCREASE IN TEMP. 60°F
 COEF. OF TEMP. EXPANSION, $\alpha = 6 \times 10^{-6} \text{ in/in/}^\circ\text{F}$
 DETERMINE BAR FORCES AND REACTIONS

$$\Delta L_{AB} = \alpha L \Delta T = 6 \times 10^{-6} \times 25 \times 12$$

$$\Delta L_{BC} = \Delta L_{AB} = 0.108 \text{ in}$$

Δ_{CC} IN RELEASED STRUCTURE



$\Delta_{C,TEMP}$

$$1 \text{ k} \times \Delta_{C,TEMP} = \sum F_Q \Delta L_P$$

$$\Delta_{C,TEMP} = \frac{5}{4} (0.108) \frac{2}{3} = 0.27 \text{ in}$$

S_{CC} SEE 11.17 $S_{CC} = \frac{1140}{AE}$

PREVIOUSLY
COMPUTED

$$= \frac{1140}{5 \times 30,000} = 0.0076 \text{ in}$$

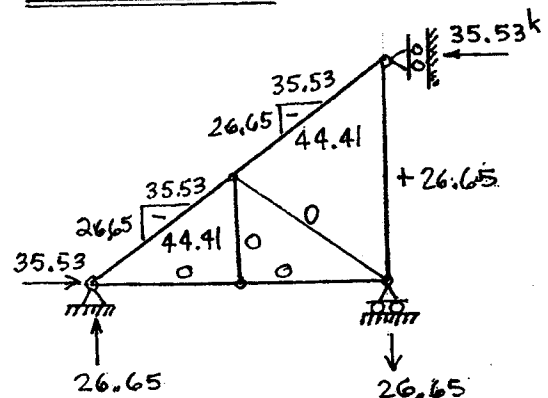
COMPATIBILITY EQ.

$$-\Delta_{C,TEMP} + S_{CC} R_c = 0$$

$$0.27 \text{ in} + 0.0076 \text{ in} R_c = 0$$

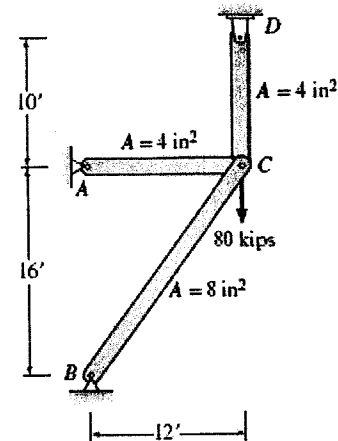
$$R_c = 35.526 \text{ kips}$$

FINAL RESULTS

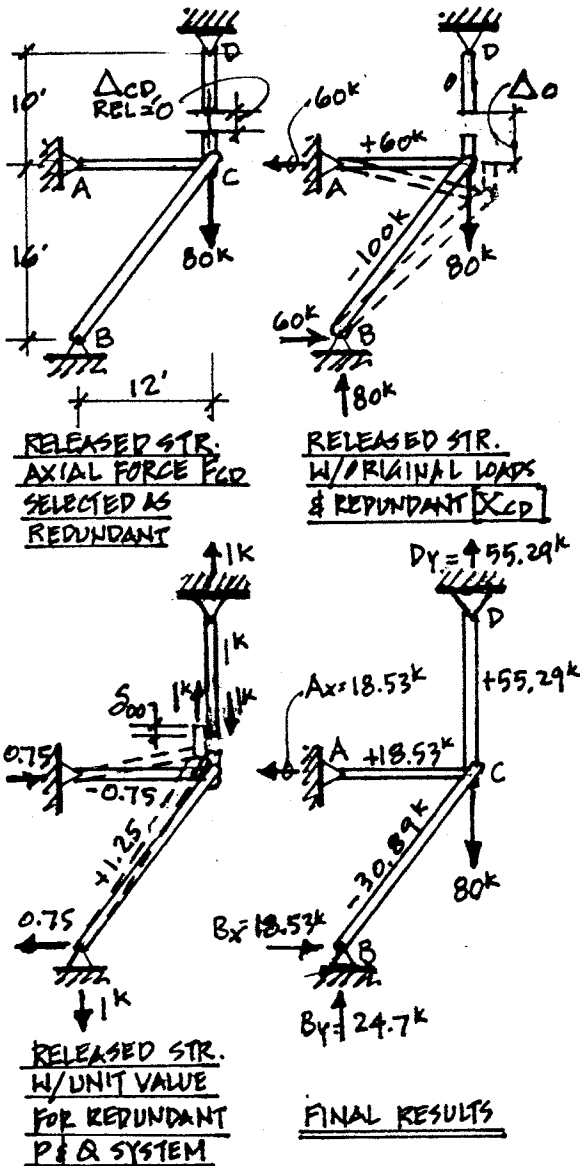


ALL FORCES IN KIPS

P11.19. Determine all reactions and bar forces for the truss in Figure P11.19. $E = 30,000 \text{ kips/in}^2$.



P11.19



INDETERMINATE TO THE 1ST DEGREE, SELECT AXIAL FORCE IN BAR CD AS THE REDUNDANT COMPATIBILITY EQUATION:

$$\Delta_{CD,REL} = 0$$

$$\Delta_0 + \delta_{00}[X_{CD}] = 0 \quad \text{WHERE } X_{CD} = F_{CD}$$

Δ_0 IN BAR CD DUE TO ACTUAL LOADS IN RELEASE

$$\text{STRUCTURE: } W_Q = \sum (F_Q F_P L / AE)$$

$$1^k(\Delta_0) = \frac{-0.75(+60)(12' \times 12')}{4 \text{ in}^2(30,000 \text{ ksi})} + \frac{1.25(-100)(20' \times 12')}{8 \text{ in}^2(30,000 \text{ ksi})}$$

$$\Delta_0 = -0.054" + 0.125" = -0.179"$$

δ_{00} IN BAR CD DUE TO UNIT LOAD IN RELEASE

STRUCTURE, P & Q SYSTEM ARE THE SAME.

$$W_Q = \sum (F_Q^2 L / AE)$$

$$\delta_{00} = 1^k \delta_1 + 1^k \delta_2 = \frac{(-0.75)^2(12' \times 12')}{4 \text{ in}^2(30,000)} + \frac{(1.25)^2(20' \times 12')}{8 \text{ in}^2(30,000)} + \frac{(1)^2(10' \times 12')}{4(30,000)}$$

$$= 0.003238"$$

$$\Delta_0 + \delta_{00}[X_{CD}] = 0$$

$$-0.179" + 0.003238[X_{CD}] = 0$$

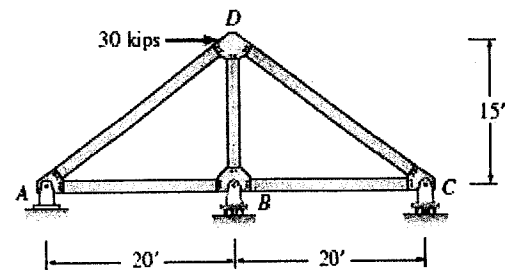
$$X_{CD} = F_{CD} = 55.29k$$

BAR FORCES BY SUPERPOSITION:

$$F_{BC} = -100(1.25)[55.29] = -30.89k$$

$$F_{AC} = 60(-0.75)[55.29] = 18.53k$$

P11.20. (a) Determine all reactions and bar forces produced by the applied load in Figure P11.20. (b) If support B settles 1 in and support C settles 0.5 in while the load acts, recompute the reactions and bar forces. For all bars the area = 2 in² and E = 30,000 kips/in².



P11.20

$A = 2 \text{ in}^2$ $E = 30,000 \text{ ksi}$

GEOMETRY $\Delta_{CV} = 0$

DEFLECTIONS

Δ_{CO} $W_Q = U_Q = \sum F_Q F_P L / AE$
 $1^k \Delta_{CO} = 2 \left[\frac{-22.5(15 \times 12)}{AE} \right] + \left(\frac{5}{3} \right) \left[\frac{37.5(25 \times 12)}{AE} \right]$
 $\Delta_{CO} = 0.448'' \downarrow$

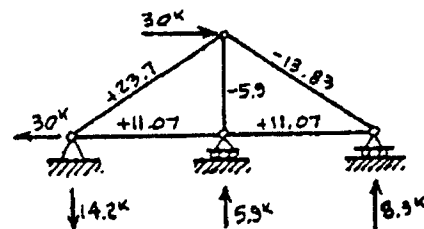
δ_{CC} $W_Q = U_Q = \sum F_Q^2 L / AE$
 $1^k \delta_{CC} = \left(\frac{5}{3} \right)^2 \frac{(25 \times 12)(2)}{AE} + (2)^2 \frac{(15 \times 12)}{AE}$
 $+ \left(\frac{4}{3} \right)^2 \frac{(20 \times 12)(2)}{AE}$
 $\delta_{CC} = 0.054'' \uparrow$

$\Delta_{CO} + \delta_{CC} X_C = 0$

$-0.448 + 0.054 X_C = 0$

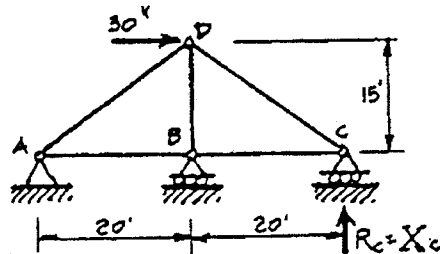
$X_C = R_C = 8.3^k \uparrow$

ORIGINAL STRUCTURE W/ FINAL RESULTS

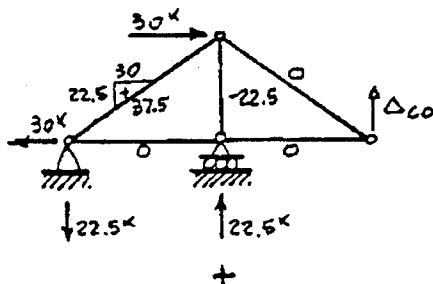


A.)

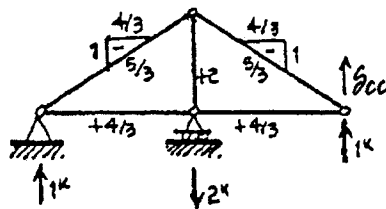
SELECT R_C AS REDUNDANT



RELEASED STRUCTURE W/ APPLIED LOAD

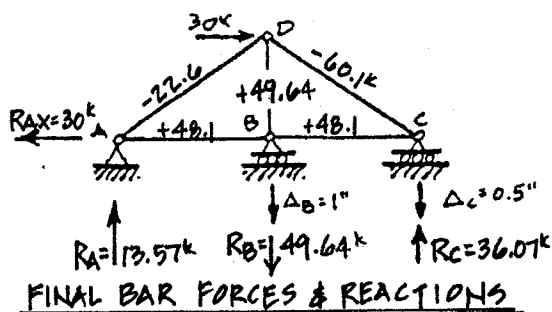


RELEASE STRUCTURE W/ UNIT VALUE OF R_C
ALSO USE AS Q-SYSTEM FOR Δ_{CO} AND δ_{CC}



B.)

ADD TO THE ABOVE ANALYSIS SUPPORT SETTLEMENT



FINAL BAR FORCES & REACTIONS

B.) RECOMPUTE BAR FORCES & REACTIONS W/ SUPPORT SETTLEMENTS:

$\sum Q \cdot \delta_P = \sum F_Q \Delta L_P = \sum F_Q F_P L / AE$

COMPATIBILITY EQU: $\Delta_{CO} + \delta_{CC} X_C = \Delta_C$

$1^k \cdot \Delta_{CO} + (-2^k)(1'') = -0.448$

$\Delta_{CO} = -2.448''$

$\delta_{CC} = 0.054'' \uparrow$

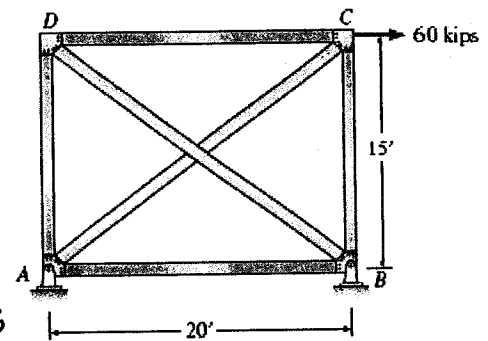
$-2.448 + 0.054 X_C = -0.5''$

$R_C = X_C = 36.07^k \uparrow$

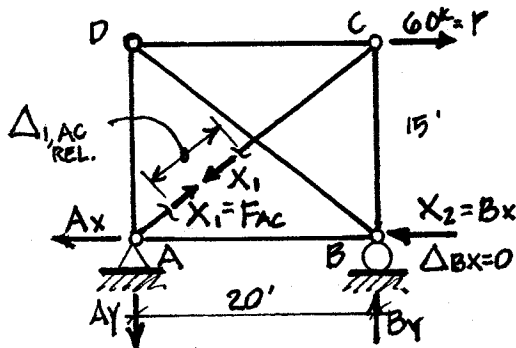
$R_B = 49.64^k \downarrow$

$R_A = 13.57^k \uparrow$

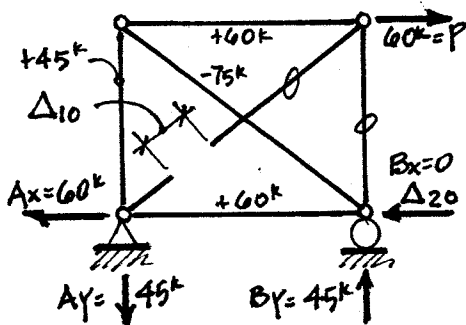
P11.21. Determine all bar forces and reactions for the truss in Figure P11.21. Given: area of bar $BD = 4 \text{ in}^2$, all other bars $= 2 \text{ in}^2$, and $E = 30,000 \text{ kips/in}^2$.



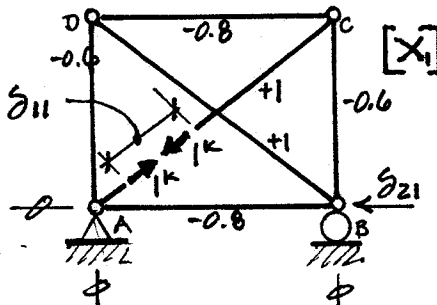
P11.21



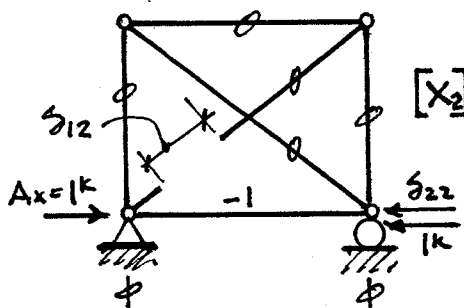
RELEASED STR.
LOADED BY APPLIED
LOAD 60k & REDUNDANTS
 $X_1 = F_{AC}$ & $X_2 = B_X$



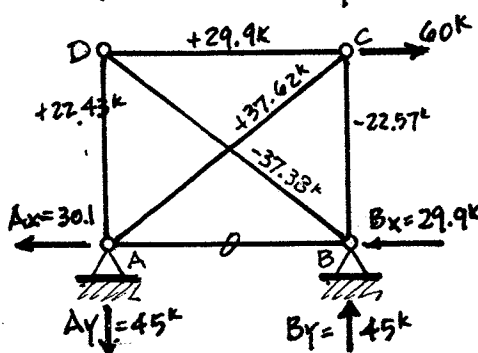
RELEASED STR.
W/APPLIED LOAD.
P-SYSTEM FOR
 Δ_{10}, Δ_{20}



RELEASED STR.
W/UNIT VALUE FOR
REDUNDANT $X_1 = F_{AC} = 1\text{k}$
Q-SYSTEM FOR $\Delta_{10}, \hat{s}_{11}$
P-SYSTEM FOR $\hat{s}_{11}, \hat{s}_{21}$



RELEASED STR.
W/UNIT VALUE FOR
REDUNDANT $X_2 = B_X = 1\text{k}$
Q-SYSTEM FOR $\Delta_{20}, \hat{s}_{21}, \hat{s}_{22}$
P-SYSTEM FOR $\hat{s}_{22}$



FINAL BAR
FORCES &
REACTIONS.

COMPATIBILITY EQUATIONS:

$$\Delta_{1, AC \text{ REL}} = 0 = \Delta_{10} + \hat{s}_{11} X_1 + \hat{s}_{12} X_2$$

$$\Delta_{BX} = 0 = \Delta_{20} + \hat{s}_{21} X_1 + \hat{s}_{22} X_2$$

VIRTUAL WORK:

$$\Delta_{10}: \sum Q_i \delta P_i = \sum F_i \delta P_i L / AE$$

$$1\text{k} \cdot \Delta_{10} = \frac{(-0.8)(60)(20 \times 12)}{2(30,000)} + \frac{(-0.6)(45)(15 \times 12)}{2(30,000)} + \frac{(1)(75)(25 \times 12)}{4(30,000)}$$

$$\Delta_{10} = -0.6525" \text{ GAP OPENS}$$

$$\Delta_{20}: 1\text{k} \cdot \Delta_{20} = \frac{-(60)(20 \times 12)}{2(30,000)}$$

$$\Delta_{20} = -0.24" \rightarrow$$

$$\hat{s}_{11}: F_Q = F_P; Q \cdot \hat{s}_{11} = F_Q^2 L / AE$$

$$1\text{k} \cdot \hat{s}_{11} = \frac{(-0.8)^2(20 \times 12)}{2(30,000)} + \frac{(-0.6)^2(15 \times 12)}{2(30,000)} + \frac{(1)^2(25 \times 12)}{4(30,000)}$$

$$\hat{s}_{11} = +0.0148" \text{ GAP CLOSERS}$$

$$\hat{s}_{21}: 1\text{k} \cdot \hat{s}_{21} = \frac{(-1)(-0.8)(20 \times 12)}{2(30,000)}$$

$$\hat{s}_{21} = 0.0032" \leftarrow$$

$$\hat{s}_{12}: 1\text{k} \cdot \hat{s}_{12} = \frac{(-0.8)(-1)(20 \times 12)}{2(30,000)}$$

$$\hat{s}_{12} = 0.0032" \leftarrow$$

$$\hat{s}_{22}: 1\text{k} \cdot \hat{s}_{22} = \frac{(-1)(-1)(20 \times 12)}{2(30,000)}$$

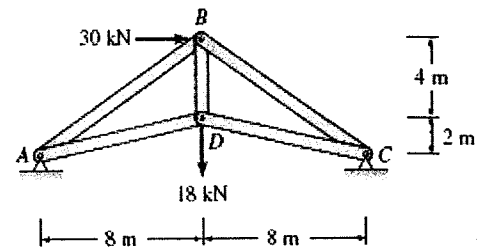
$$\hat{s}_{22} = 0.004" \leftarrow$$

SUBSTITUTE DISPLACEMENTS INTO
COMPATIBILITY EQUATIONS:

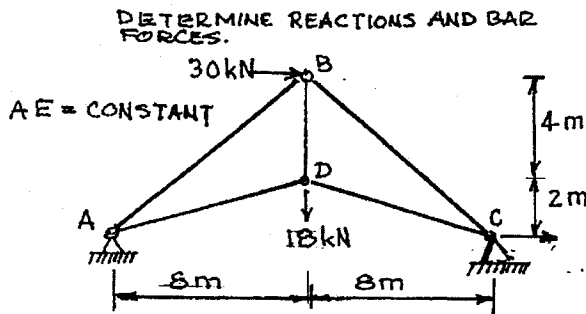
$$X_1 = 37.62\text{k}$$

$$X_2 = 29.9\text{k}$$

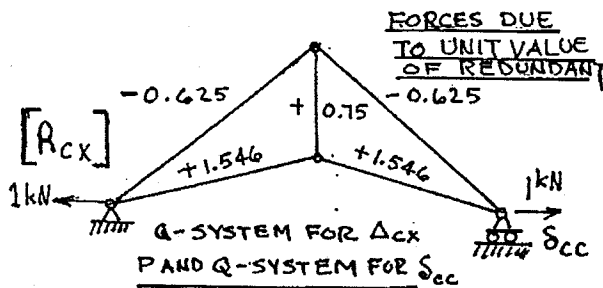
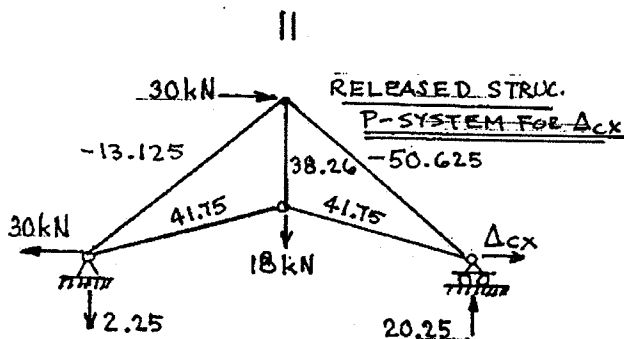
P11.22. For the truss in Figure P11.22, compute the reactions and bar forces produced by the applied loads. Given: $AE =$ constant, $A = 1000 \text{ mm}^2$, and $E = 200 \text{ GPa}$.



P11.22



TAKE C_x AS THE REDUNDANT



COMPUTE Δ_{cx}

$$1 \text{ kN} \cdot \Delta_{cx} = \sum F_Q F_P \frac{L}{AE} = \frac{1}{AE} \sum F_Q F_P L$$

$$\Delta_{cx} = \frac{1}{AE} \left[-0.625(-13.125) \cdot 10 + (-0.625)(-50.625) \cdot 10 + (0.75)(38.26) \cdot 4 + (1.546)(41.75)(8.246 \times 2) \right]$$

$$\Delta_{cx} = \frac{1577.67}{AE} \rightarrow$$

COMPUTE S_{cc}

$$1 \text{ kN} \cdot S_{cc} = \frac{1}{AE} \left[(0.75^2 \times 4) + (-0.625)^2 \cdot 10 \times 2 + (1.546)^2 \cdot 8.246 \times 2 \right]$$

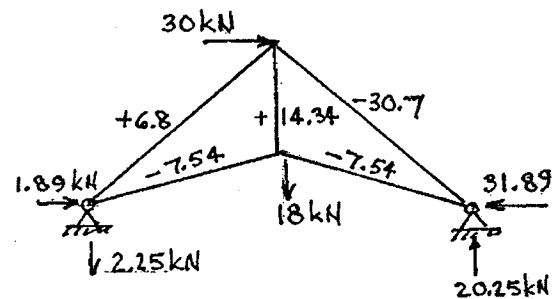
$$S_{cc} = \frac{49.48}{AE} \rightarrow$$

COMPATABILITY EQ.

$$\Delta_{cx} + S_{cc} R_{cx} = 0$$

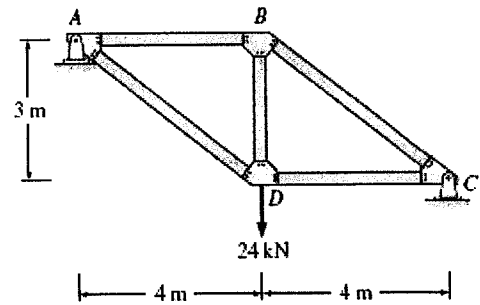
$$\frac{1577.67}{AE} + \frac{49.48}{AE} R_{cx} = 0$$

$$R_{cx} = -31.89 \text{ kN} \leftarrow$$

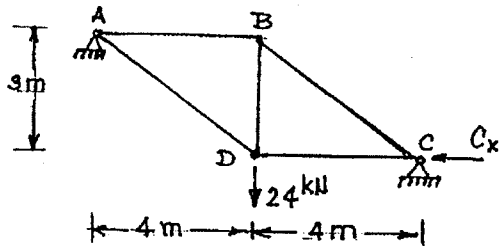


FINAL RESULTS. (kN)

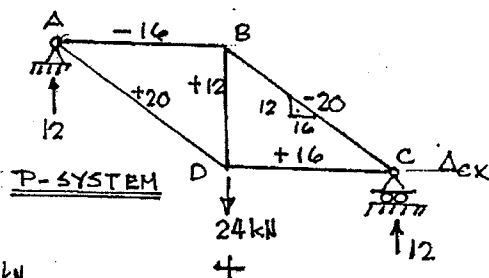
P11.23. For the truss in Figure P11.23, compute the reactions and bar forces produced by the applied loads. Given: $AE =$ constant, $A = 1000 \text{ mm}^2$, and $E = 200 \text{ GPa}$.



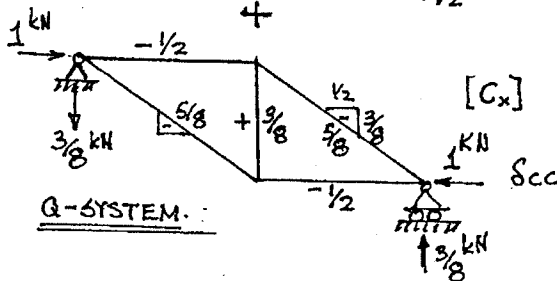
P11.23



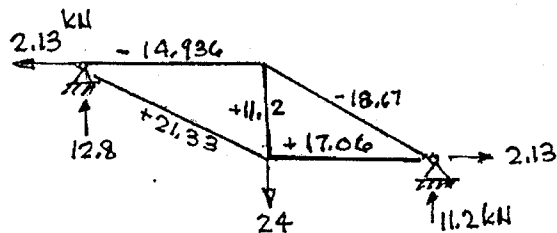
SELECT C_x AS THE REDUNDANT



P-SYSTEM



Q-SYSTEM



FINAL RESULTS

$$1. \Delta_{Cx} = \sum F_Q F_P \frac{L}{AE}$$

$$\Delta_{Cx} = \frac{1}{AE} \left[\left(-\frac{1}{2}\right)(-16)(4) + \left(-\frac{1}{2}\right)(12)(4) \right. \\ \left. + (20)\left(\frac{5}{8}\right)(5) + (-20)\left(-\frac{5}{8}\right)(5) \right. \\ \left. + \left(\frac{9}{8}\right)(12)(3) \right]$$

$$\Delta_{Cx} = 13.5/AE \leftarrow$$

$$1. S_{Cx} = \frac{1}{AE} \left[\left(-\frac{1}{2}\right)^2 (4)(2) - \left(\frac{5}{8}\right)^2 (5)(2) \right. \\ \left. + \left(\frac{9}{8}\right)^2 (3) \right]$$

$$S_{Cx} = 6.528/AE \leftarrow$$

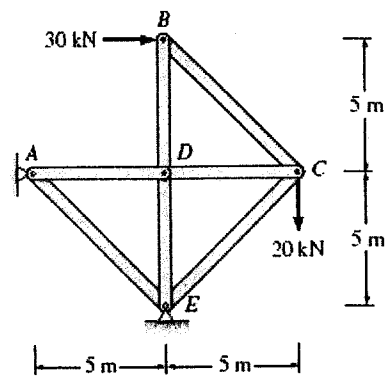
COMPATIBILITY EQ

$$\Delta_{Cx} + C_x S_{Cx} = 0$$

$$13.5/AE + C_x (6.528/AE) = 0$$

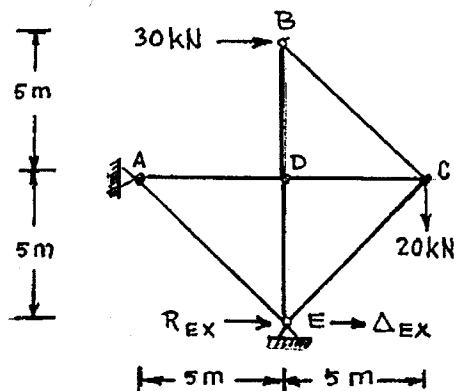
$$C_x = -2.13 \text{ kN} \rightarrow$$

P11.24. For the truss in Figure P11.24, compute the reactions and bar forces produced by the applied loads. Given: $AE =$ constant, $A = 1000 \text{ mm}^2$, and $E = 200 \text{ GPa}$.

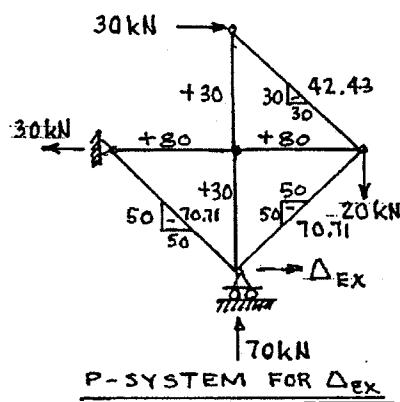


P11.24

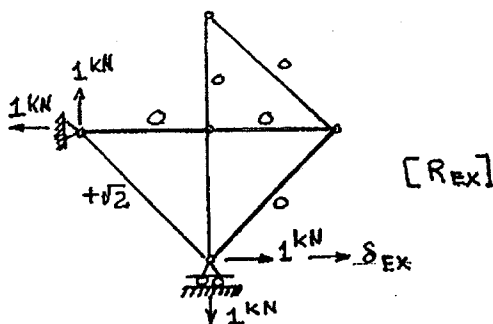
$AE = \text{CONSTANT}$



SELECT R_{EX} AS THE REDUNDANT



+



COMPUTE Δ_{EX} BY VIRT. WORK

$$1 \text{ kN} \cdot \Delta_{EX} = (\sqrt{2})(50\sqrt{2}) \frac{5\sqrt{2}}{AE}$$

$$\Delta_{EX} = \frac{-500\sqrt{2}}{AE}$$

COMPATIBILITY EQ.

$$\Delta_{EX} + S_{EX} R_{EX} = 0$$

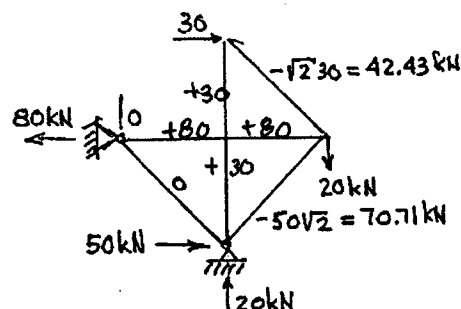
$$-\frac{500\sqrt{2}}{AE} + \frac{10\sqrt{2}}{AE} R_{EX} = 0$$

$$\underline{R_{EX} = 50 \text{ kN}}$$

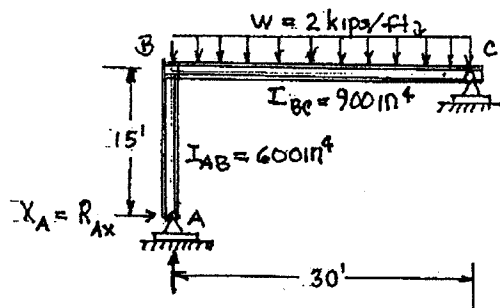
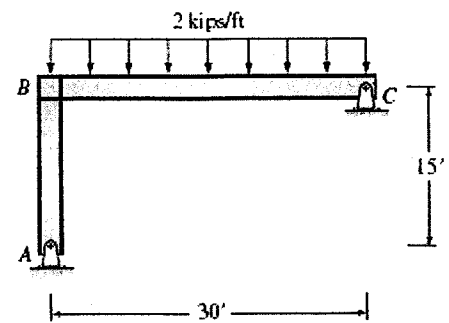
COMPUTE S_{EX} BY VIRT. WORK

$$1 \text{ kN} \cdot S_{EX} = \sum F_a F_p \frac{L}{AE}$$

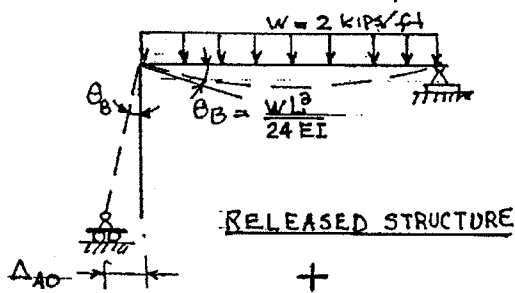
$$S_{EX} = \frac{\sqrt{2} \cdot \sqrt{2} \cdot 5\sqrt{2}}{AE} = \frac{10\sqrt{2}}{AE}$$



P11.25. Determine all reactions for the frame in Figure P11.25 given $I_{AB} = 600 \text{ in}^4$, $I_{BC} = 900 \text{ in}^4$, and $E = 29,000 \text{ kips/in}^2$. Neglect axial deformations.

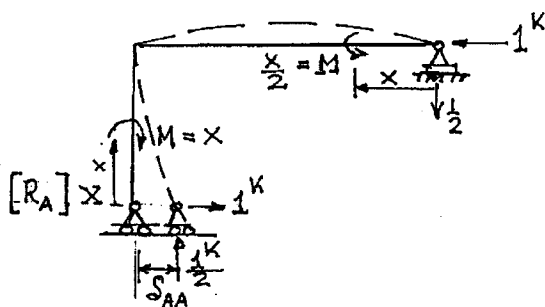


SELECT R_{Ax} AS REDUNDANT



$$\Delta_{A0} = \theta_B 15' \quad (\text{SEE Fig 11.3b})$$

$$\Delta_{A0} = \frac{WL^3}{24EI} = \frac{2(30)^3}{24E900} = \frac{-37.5}{E}$$



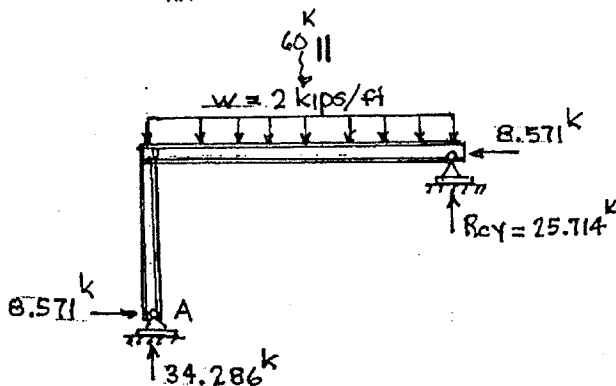
$$\delta_{AA} = \int_0^{15} \frac{M_a^2}{EI_c} dx + \int_0^{30} \frac{M_a^2}{EI_g} dx$$

$$= \int_0^{15} \frac{x^2}{EI_c} dx + \int_0^{30} \frac{(x/2)^2}{EI_g} dx$$

$$= \left[\frac{x^3}{3EI_{600}} \right]_0^{15} + \left[\frac{x^3}{12EI_{900}} \right]_0^{30}$$

$$= \frac{3375}{1800E} + \frac{27,000}{10800E}$$

$$\delta_{AA} = \frac{1.875}{E} + \frac{2.5}{E} = \frac{4.375}{E}$$



COMPATABILITY EQ.

$$\Delta_A = 0 = \Delta_{A0} + R_A \delta_{AA}$$

$$0 = -\frac{37.5}{E} + R_A \frac{4.375}{E}$$

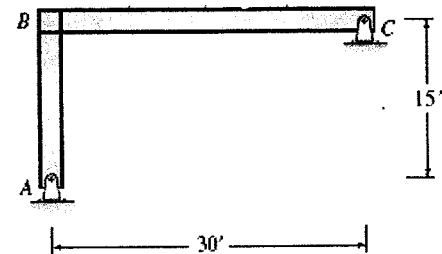
$$R_A = 8.571 \text{ kips}$$

$$\sum M_A = 0 = -8.571 \times 15 + 60 \times 15$$

$$= R_{cy} 30$$

$$R_{cy} = 25.714 \text{ kips}$$

P11.26. Assuming that the load is removed, compute all reactions for the frame in Figure P11.25 if member BC is fabricated 1.2 in too long.



P11.25

BAR BC FABRICATED 1.2" TOO LONG.

$I = 900 \text{ in}^4$
 $I = 600 \text{ in}^4$

SELECT R_{AX} AS REDUNDANT.

RELEASED STRUCTURE

$S_{AA} = \frac{4.375}{E}$ (SEE 11.25) FOR EVALUATION OF S_{AA}

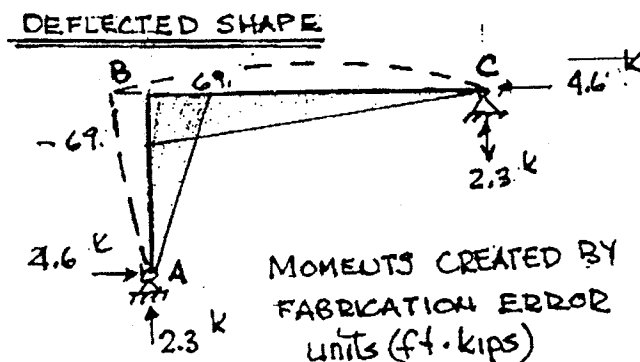
$= \frac{4.375(12)^3}{29,000}$

$= 0.261 \text{ in} / 1 \text{ k HORIZ}$

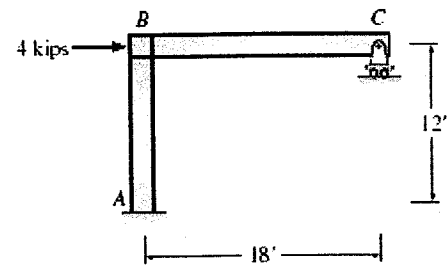
$R_{AX} = \frac{1.2}{0.261} = 4.6 \text{ kips}$

$\sum M_C: R_{AY} 30 - R_{AX} 15 = 0$

$R_{AY} = \frac{1}{2} R_{AX} = 2.3 \text{ kips}$

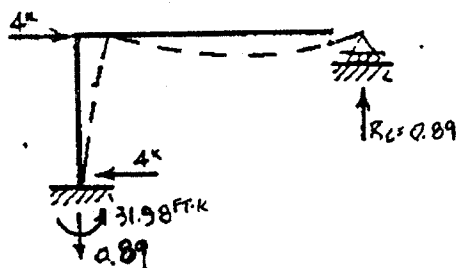
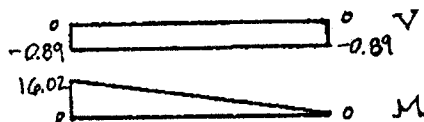
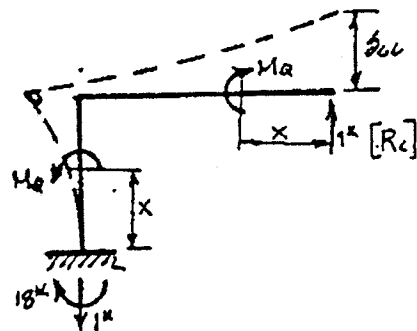
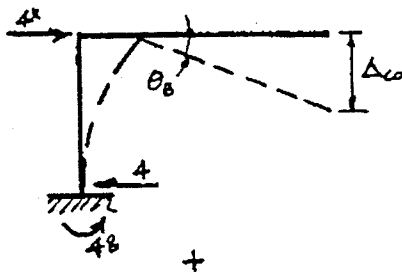
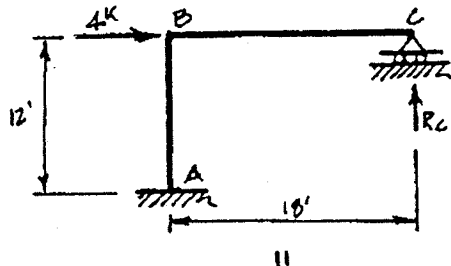


P11.27. Determine all reactions and draw the shear and moment diagrams for beam BC in Figure P11.27. EI is constant.



P11.27

SELECT R_C AS REDUNDANT, EI CONST.



$$\Delta_{C0} = 18 \theta_B$$

$$\theta_B = \theta_A + \Delta\theta_{AB} = 0 + \frac{1}{2} (12) \left(\frac{48}{EI} \right)$$

$$\theta_B = 288/EI$$

$$\Delta_{C0} = 5184/EI$$

Δ_{C1} BY VIRTUAL WORK

1. $\Delta_{C1} = \sum M_Q M_P dx/EI$ WHERE $M_Q = M_P$

$$\Delta_{C1} = \int_0^{18} x^2 dx/EI + \int_0^{12} (18)^2 dx/EI$$

$$= \frac{1}{3EI} [x^3]_0^{18} + \frac{324}{EI} [x]_0^{12}$$

$$\Delta_{C1} = 5832/EI$$

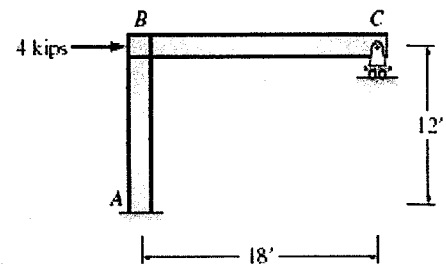
COMPATIBILITY EQUATION $\Delta_{Cv} = 0$

$$\Delta_{C0} + \Delta_{C1} R_C = 0$$

$$-5184/EI + 5832/EI R_C = 0$$

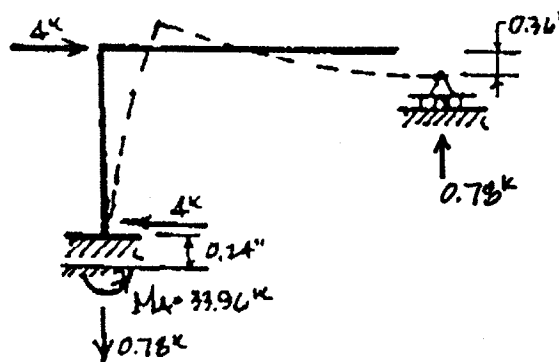
$$R_C = 0.89k$$

P11.28. Recompute the reactions for the frame in Figure P11.27 if support *C* settles 0.36 in when the load acts and support *A* is constructed 0.24 in above its intended position. $E = 30,000$ kips/in², $I = 60$ in⁴.



P11.27

REFER TO SOLUTION FOR P11.27



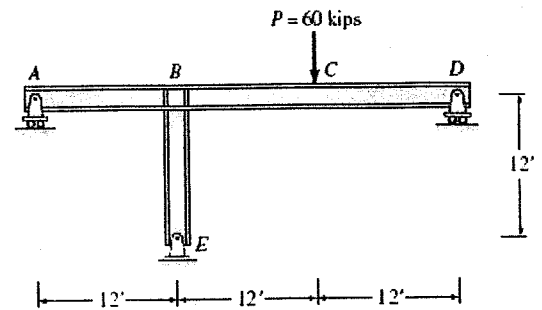
COMPATIBILITY EQUATION $\Delta_{CV} = -0.36''$

$$\Delta_{C0} + \Delta_C \text{ DUE TO A} + \delta_{CC} R_C = -0.36''$$

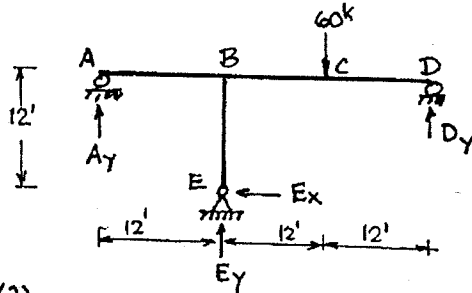
$$\frac{-5184(1728)}{30000(60)} + 0.24 + \frac{5832(1728)}{30000(60)} R_C = -0.36''$$

$$\underline{R_C = 0.78^k}$$

P11.29. (a) Determine the reactions and draw the shear and moment curves for all members of the frame in Figure P11.29. Given: $EI = \text{constant}$. (b) Compute the vertical deflection of the girder at point C produced by the 60-kip load.

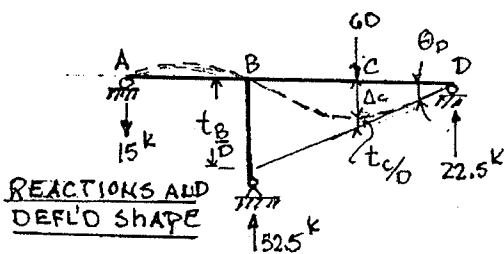
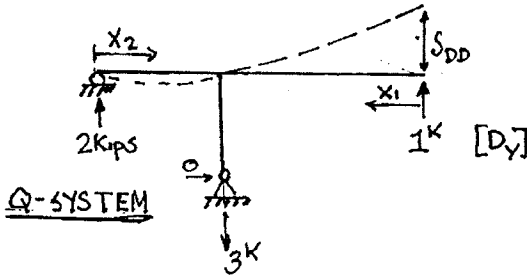
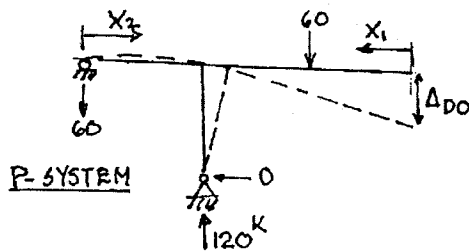


P11.29

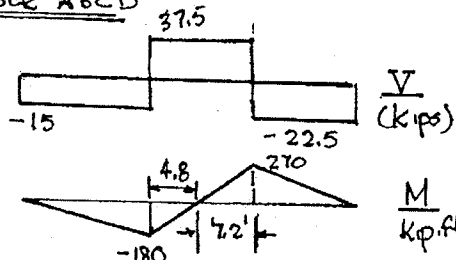


(a)

SELECT D_y AS REDUNDANT



MEMBER ABCD



$$\Delta_{D0} = -1 \cdot \Delta_{D0} = \frac{1}{EI} \int_{12}^{24} 60(x_1 - 12)(x_1) dx_1 + \int_0^{12} (-60x_2)(2x_2) dx_2$$

$$-\Delta_{D0} = \frac{1}{EI} \left[60 \left(\frac{x_1^3}{3} + 6x_1^2 \right) \right]_{12}^{24} - 120 \left(\frac{x_2^3}{3} \right) \Big|_0^{12}$$

$$\Delta_{D0} = -\frac{155,520}{EI}$$

$$S_{DD} = 1 \cdot S_{DD} = \int_0^{24} (1 \cdot x_1^2) dx_1 + \int_0^{12} 2x_2^2 dx_2$$

$$S_{DD} = \frac{1}{EI} \left(\frac{x_1^3}{3} \Big|_0^{24} + \frac{4x_2^3}{3} \Big|_0^{12} \right)$$

$$S_{DD} = \left(\frac{4608}{EI} + \frac{2304}{EI} \right) = \frac{6912}{EI}$$

COMPATIBILITY EQ:

$$\Delta_{D0} - S_{DD} D_y = 0$$

$$-\frac{155,520}{EI} + \frac{6912}{EI} D_y = 0$$

$$D_y = 22.5 \text{ kips}$$

$$\text{REACTION } E_y = 120 - 3(22.5) \\ E_y = 52.5 \text{ kips}$$

(b)

$$\Delta_c = \frac{t_{B/D}}{2} - t_{C/D}$$

$$t_{B/D} = \frac{270}{EI} \times \frac{12}{2} \times 16 + \frac{270}{EI} \times \frac{7.2}{2} (9.6) - \frac{180}{EI} \left(\frac{4.8}{2} \right) \left(\frac{4.8}{3} \right) = \frac{34560}{EI}$$

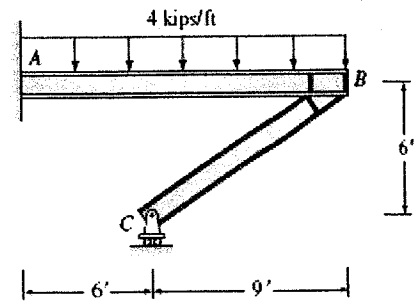
$$\theta_D = \frac{t_{B/D}}{24} = \frac{34560/EI}{24} = \frac{1440}{EI}$$

$$\Delta_c + t_{C/D} = 12 \theta_D = \frac{17280}{EI}$$

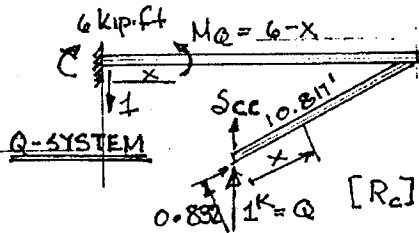
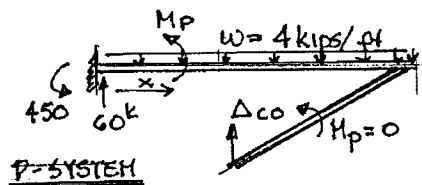
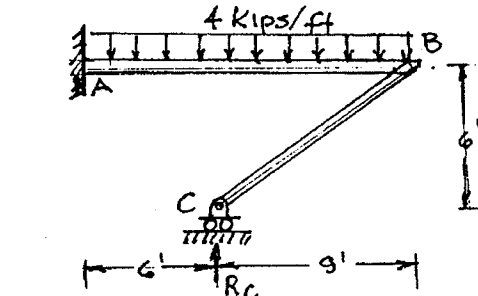
$$\Delta_c + \frac{270}{EI} \times 12 \times \frac{1}{2} (4) = \frac{17280}{EI}$$

$$\Delta_c = \frac{10800}{EI}$$

P11.30. (a) Compute the reaction at C in Figure P11.30. EI is constant. (b) Compute the vertical deflection of joint B.



P11.30



COMPATIBILITY EQUATION

$$\Delta_{CO} + S_{CC} R_C = 0$$

$$-\frac{5062.5}{EI} + \frac{606.95}{EI} R_C = 0$$

$$R_C = \frac{5062.5}{606.95} = 8.33 \text{ kips ANS.}$$

$$1 \cdot \Delta_{CO} = \int_0^{15} M_Q M_P \frac{dK}{EI}$$

$$\Delta_{CO} = \int_0^{15} \left[(6-x)X - 450 + 60X - 4X \cdot \frac{X}{2} \right] \frac{dK}{EI}$$

$$= \int_0^{15} \left[-2700 + 810X - 72X^2 + 2X^3 \right] \frac{dK}{EI}$$

$$= \left[-2700X + \frac{810X^2}{2} - \frac{72X^3}{3} + \frac{2X^4}{4} \right]_0^{15} \frac{1}{EI}$$

$$\Delta_{CO} = -\frac{5062.5}{EI}$$

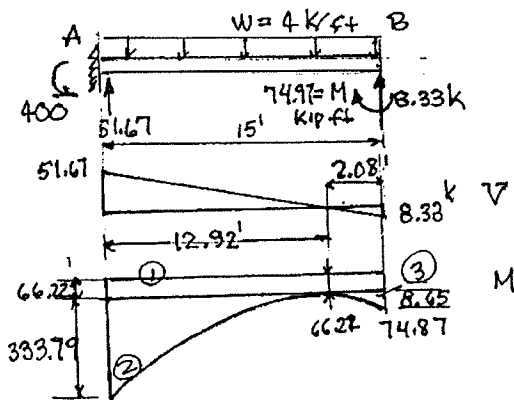
$$1 \cdot S_{CC} = \int_0^{10.817} M_Q^2 \frac{dK}{EI} + \int_0^{15} (6-x)^2 \frac{dK}{EI}$$

$$S_{CC} = \int_0^{10.817} (0.832x)^2 \frac{dK}{EI} + \int_0^{15} (36 - 12x + x^2) \frac{dK}{EI}$$

$$S_{CC} = \left[\frac{0.692x^3}{3} \right]_0^{10.817} \frac{1}{EI} + \left[36x - \frac{12x^2}{2} + \frac{x^3}{3} \right]_0^{15} \frac{1}{EI}$$

$$S_{CC} = \frac{606.95}{EI}$$

(b) COMPUTE Δ_B BY MOMENT-AREA



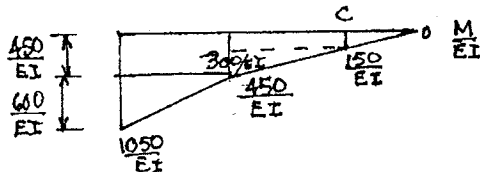
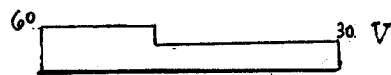
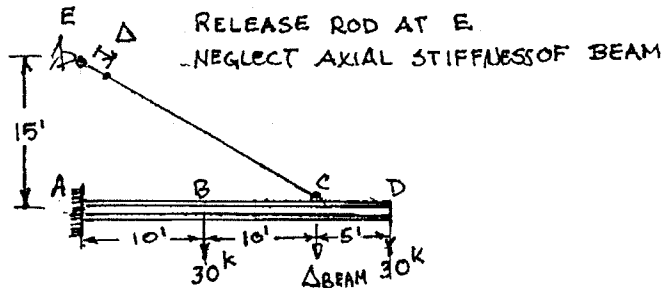
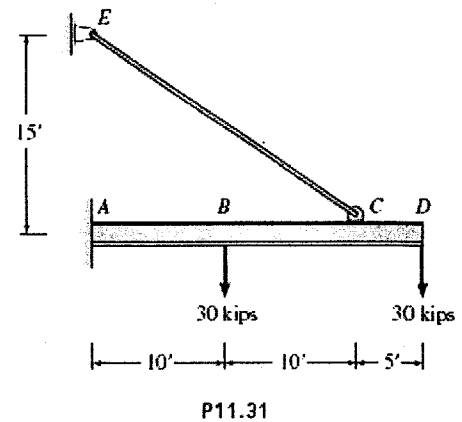
$$\Delta_B = \frac{I_B}{A}$$

$$= \frac{66.22 (15) 7.5}{EI} + \frac{1}{3} (12.92) \frac{333.79 (11.71)}{EI}$$

$$+ \frac{1}{3} (2.08) (8.65) \frac{(2.08)}{4}$$

$$\Delta_B = \frac{24,372.51}{EI} \downarrow \text{ANS.}$$

P11.31. Determine the reactions at supports *A* and *E* in Figure P11.31. Area of bar *EC* = 2 in², *I*_{AD} = 400 in⁴, and *A*_{AD} = 8 in²; *E* = 30,000 kips/in².

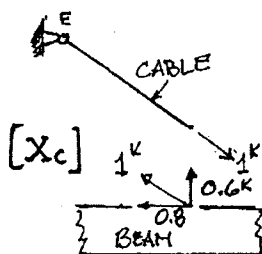


USING MOMENT-AREA, COMPUTE Δ_C (RELEASED)

$$\Delta_{C0} = t_{CA} = \frac{450(10)(15)}{EI} + \frac{600 \times 10}{EI} \left(\frac{16.67}{2} \right) + \frac{150(10)(5)}{EI} + \frac{300 \times 10}{EI} \times \frac{2}{3} \left(\frac{10}{2} \right)$$

$$\Delta_{C0} = \frac{135,043.3 (1728)}{30,000 \times 400} = 19.45''$$

Compute S_{CC} due to a unit value of redundant



CABLE: $S = \frac{FL}{AE} = \frac{1(25 \times 12)}{2 \times 30,000} = 0.005$ in

$S_V = \frac{0.005}{5} = \frac{0.005}{5}$
 $S_V = \frac{5}{3} (0.005)$
 $S_V = 0.00833$ in

BEAM:

DISPLACEMENT OF BEAM DUE TO VERTICAL COMPONENT OF UNIT LOAD

$$S_{VC} = \frac{PL^3}{3EI} = \frac{0.6(20)^3}{3 \times 30,000 \times 400} = 0.23$$

CLOSING OF GAP DUE TO UNIT LOAD AT C

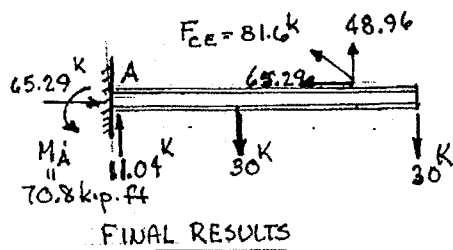
$$\Delta_{C0} + S_{CC} X_C = 0$$

$$19.45 + (0.23 + 0.00833) X_C = 0$$

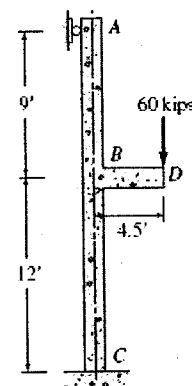
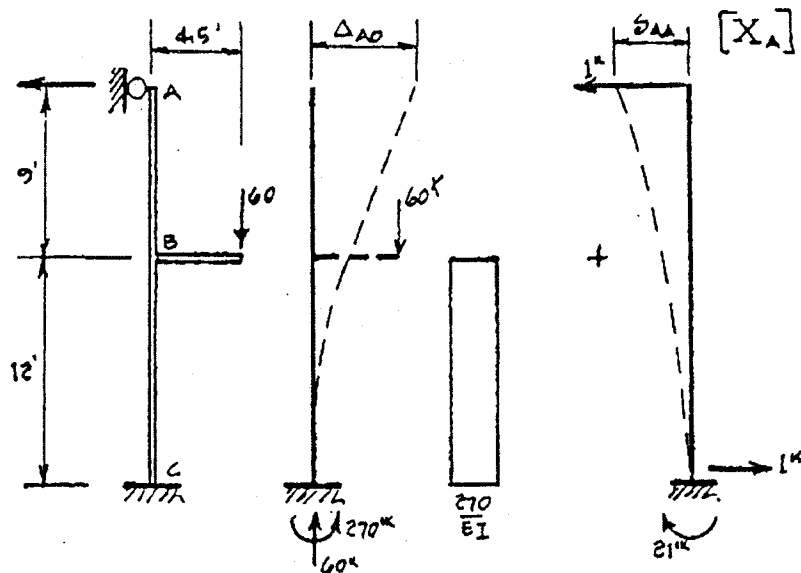
CABLE FORCE: $X_C = 81.6$ kips

AXIAL FORCE IN BEAM = $0.8 X_C = 65.29$ kips

$M_A = 30 \times 10 + 30 \times 25 - 48.96 \times 20 = 70.8$ kip-ft



P11.32. Determine the reactions at A and C in Figure
P11.32. EI is constant for all members.



P11.32

$$\Delta_A = 0$$

$$\Delta_{A0} + S_{AA} X_A = 0 \quad (1)$$

COMPUTE Δ_{A0} (USE MOMENT-AREA)

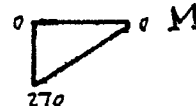
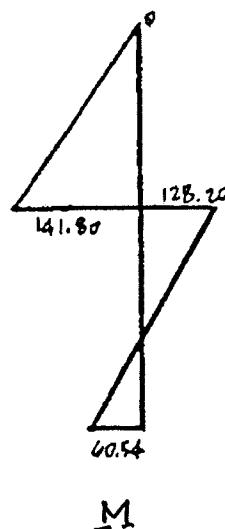
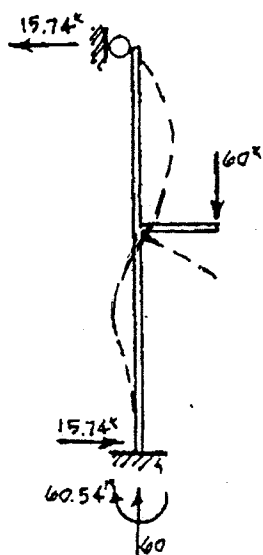
$$\Delta_{A0} = t_{AC} = \frac{270}{EI} (12) (6+9) = -48600/EI \quad (2)$$

COMPUTE S_{AA} (USE TABLE 10.1)

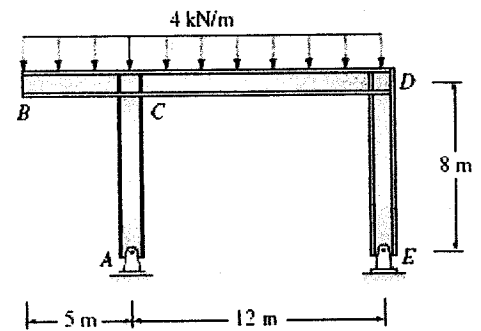
$$S_{AA} = PL^3/3EI = 1(21)^3/3EI = 3087/EI \quad (3)$$

SUBSTITUTE (2) & (3) INTO (1)

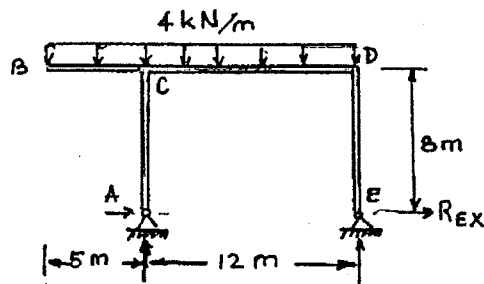
$$-48600/EI + 3087/EI [X_A] = 0 \quad \underline{X_A = 15.74''}$$



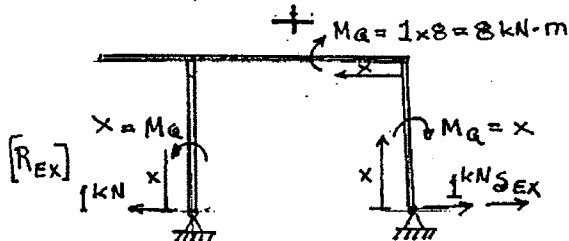
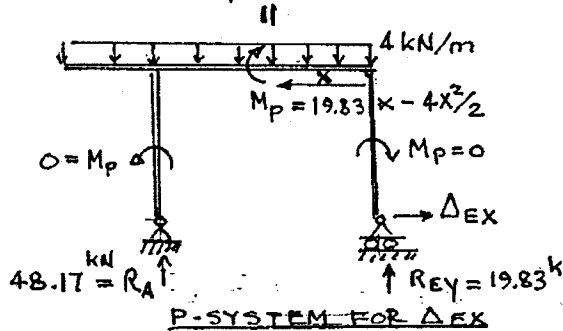
P11.33. Determine the reactions at supports A and E in Figure P11.33; EI is constant for all members.



P11.33



INDETERMINATE 1° ; SELECT R_{EX} AS REDUNDANT.



COMPATIBILITY EQUATION

$$\Delta_E = 0$$

$$\Delta_{EX} + R_{EX} \delta_{EX} = 0$$

$$\frac{2206.08}{EI} + R_{EX} \frac{1109.33}{EI} = 0$$

$$R_{EX} = -1.99 \text{ kN} \leftarrow$$

$$\frac{\Delta_{EX}}{1 \text{ kN}} = \int \frac{M_Q \cdot M_P dx}{EI}$$

$$\Delta_{EX} = \int_0^{12} (8)(19.83x - 2x^2) \frac{dx}{EI}$$

$$= \int_0^{12} (158.64x - 16x^2) \frac{dx}{EI}$$

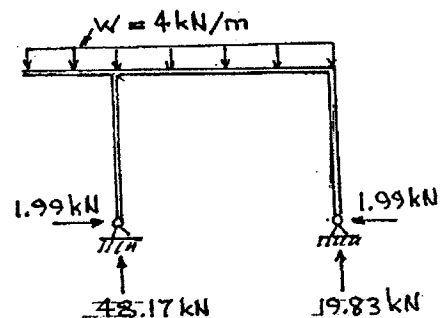
$$= \frac{1}{EI} \left[\frac{158.64x^2}{2} - \frac{16x^3}{3} \right]_0^{12}$$

$$\Delta_{EX} = \frac{11,422.08 - 9216}{EI} = \frac{2206.08}{EI}$$

$$\frac{\delta_{EX}}{1 \text{ kN}} = \int \frac{x^2 dx}{EI} + \int \frac{8^2 dx}{EI} + \int \frac{x^2 dx}{EI}$$

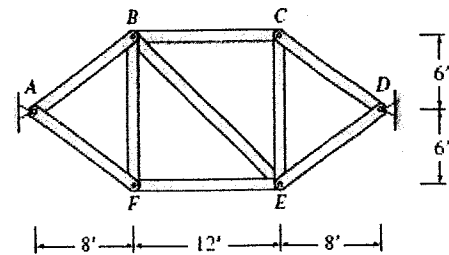
$$= \left[\frac{x^3}{3EI} \right]_0^8 + \left[\frac{64x}{EI} \right]_0^{12} + \left[\frac{x^3}{3EI} \right]_0^8$$

$$\delta_{EX} = \frac{1109.33}{EI}$$

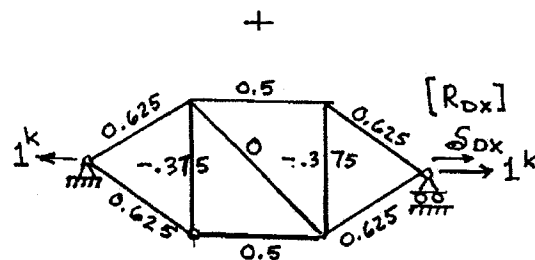
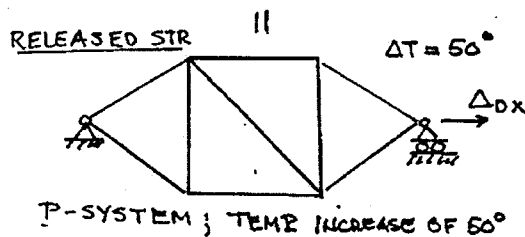
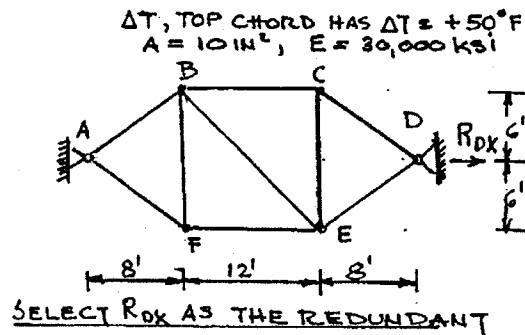


FINAL RESULTS

P11.34. Determine the reactions and bar forces that are created in the truss in Figure P11.34 when the top chords (ABCD) are subjected to a 50°F temperature change. Given: AE is constant for all bars, $A = 10 \text{ in}^2$, $E = 30,000 \text{ kips/in}^2$, $\alpha = 6.5 \times 10^{-6} \text{ (in/in)/}^\circ\text{F}$.



P11.34



Q-SYSTEM FOR Δ_{Dx}
 P AND Q SYSTEM FOR S_{Dx}

COMPUTE Δ_{Dx}

$$\Delta L_{AB} \text{ \& } \Delta L_{CD} = \Delta T \alpha L$$

$$= 50 \times 6.5 \times 10^{-6} \times (10 \times 12)$$

$$= 0.039 \text{ in}$$

$$\Delta_{BC} = \Delta T \alpha L = 50 \times 6.5 \times 10^{-6} (12 \times 12)$$

$$= 0.0468$$

$$\Delta_{Dx} = \sum F_Q \cdot \Delta L$$

$$\Delta_{Dx} = 0.625(0.039)2 + 0.5(0.0468)$$

$$\Delta_{Dx} = 0.07215 \text{ in}$$

COMPUTE S_{Dx} $F_P = F_Q$

$$S_{Dx} = \sum \frac{F_Q^2 L}{AE}$$

$$S_{Dx} = \frac{0.625^2 (10 \times 12)}{AE} [4] + \frac{0.375^2 (12 \times 12)}{AE} 2$$

$$+ \frac{(0.5)^2 (12 \times 12)}{AE} 2 = \frac{300}{AE}$$

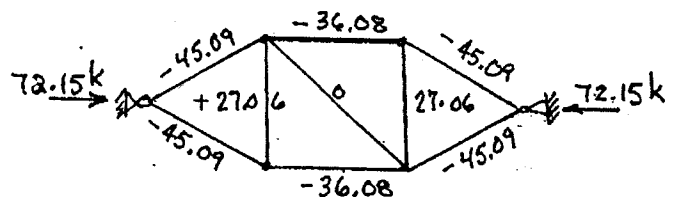
$$S_{Dx} = \frac{300}{10 \times 30,000} = 0.001 \text{ in}$$

COMPATIBILITY EQ

$$\Delta_{Dx} + R_{Dx} S_{Dx} = 0$$

$$0.07215 + R_{Dx} 0.001 = 0$$

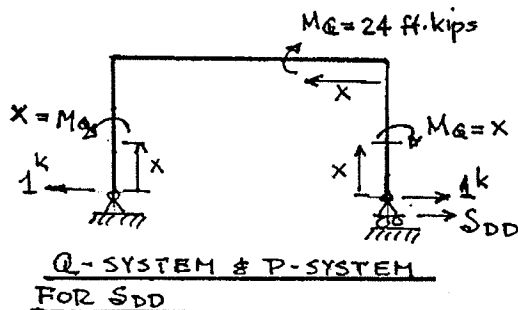
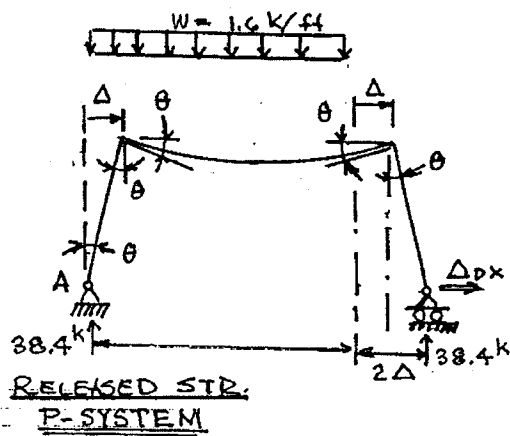
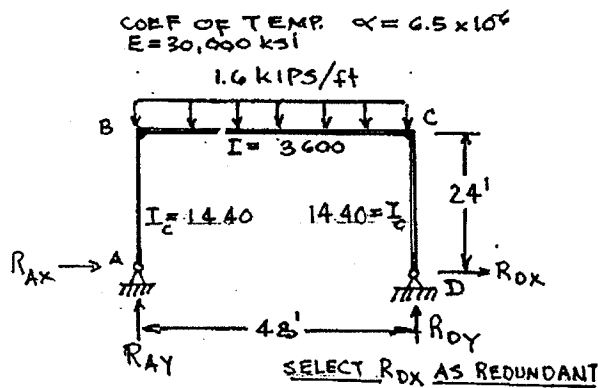
$$R_{Dx} = -72.15 \text{ kips} \leftarrow$$



ALL FORCES IN KIPS

FINAL RESULTS

P11.35. Determine the reactions that are created in the rigid frame in Figure P11.35 when the temperature of the top chord increases 60°F . Given: $I_{BC} = 3600 \text{ in}^4$, $I_{AB} = I_{CD} = 1440 \text{ in}^4$, $\alpha = 6.5 \times 10^{-6} (\text{in/in})/^\circ\text{F}$, and $E = 30,000 \text{ kips/in}^2$.

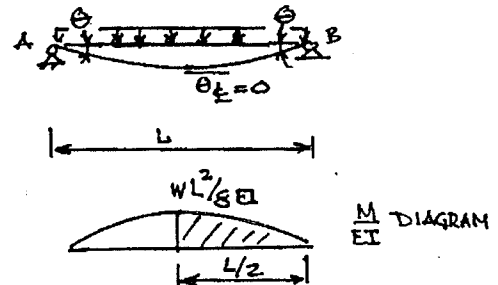
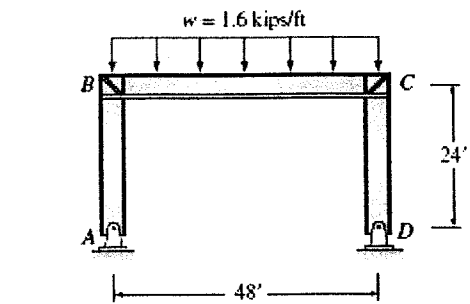


$$1 \times S_{DD} = 2 \int_0^{24} M_Q \frac{dQ}{EI_c} + \int_0^{48} (24) \frac{dX}{EI_{BC}}$$

$$S_{DD} = 2 \left(\frac{X^3}{3EI_{CD}} \right) \Big|_0^{24} + \frac{576X}{EI_{BC}} \Big|_0^{48}$$

$$= \left[\frac{2(13,824)}{3EI_{CD}} + \frac{27,648}{EI_{BC}} \right] 1728$$

$$S_{DD} = 0.369 + 0.442 = \underline{0.811 \text{ in.}}$$



USE MOMENT-AREA TO COMPUTE θ

$$\theta = \theta_D + \Delta\theta_{D/B}$$

$$\theta = 0 + \frac{2}{3} \left(\frac{L}{2} \times \frac{WL^2}{8EI} \right) = \frac{WL^3}{24EI}$$

$$\theta = \frac{1.6 \text{ k/ft} (48 \text{ ft})^3 (144) \text{ in}^2}{24 \times 30,000 \text{ k} \times 3600 \text{ in}^4 \text{ ft}^2}$$

$$\theta = 0.00983 \text{ RADIANS}$$

$$\Delta = \theta L = 0.00983 (24 \times 12) = 2.831''$$

$$\Delta_{DX} = 2\Delta = \underline{5.662 \text{ in.}}$$

$$\Delta_{TEMP} = 60^\circ (6.5 \times 10^{-6}) (48 \times 12) = 0.2246$$

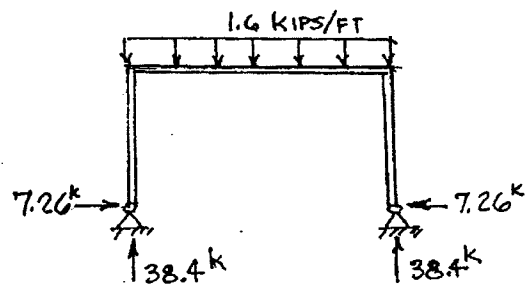
$$\Delta_{BX, \text{TOTAL}} = 5.662'' + 0.23'' = \underline{5.887 \text{ in}}$$

COMPATIBILITY EQ.

$$\Delta_{BX, \text{TOTAL}} + R_{Dx} S_{BB} = 0$$

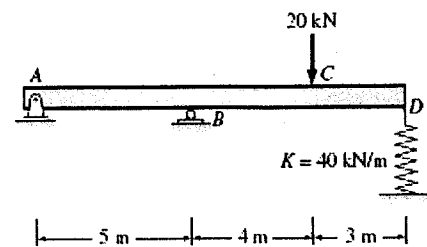
$$5.887 + R_{Dx} (0.811) = 0$$

$$R_{Dx} = \underline{-7.26 \text{ k}}$$

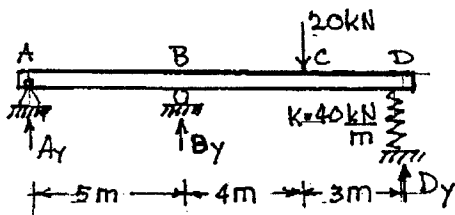


FINAL REACTIONS ALL EFFECTS

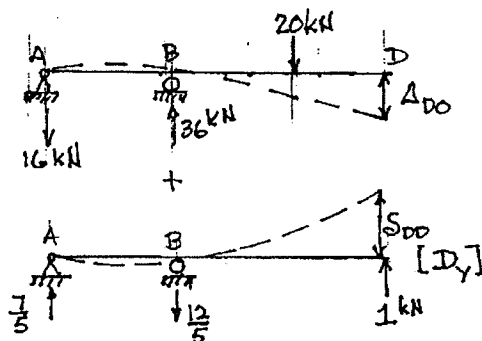
P11.36. Compute the reactions and draw the shear and moment curves for the beam in Figure P11.36. Given: EI is constant for the beam. $E = 200 \text{ GPa}$, $I = 40 \times 10^6 \text{ mm}^4$.



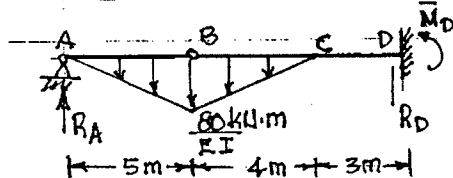
P11.36



SELECT D_y AS THE REDUNDANT



CALCULATE Δ_{D0} BY "CONJUGATE BEAM"



$$R_A = \frac{5 \times 80}{2} \left(\frac{5}{3} \right) \left(\frac{1}{5} \right) = \frac{200}{3EI}$$

$$-M_D = R_A \times 12 - \frac{5 \times 80}{2EI} \left(\frac{5}{3} \times 7 \right) - \frac{4 \times 80}{2EI} \left(\frac{2}{3} \times 4 + 3 \right) = -\frac{1840}{EI}$$

$$\Delta_D = \frac{1840}{EI} \downarrow$$

$$S_{DD} = \frac{Pa^2}{3EI} (L+a) \quad (\text{SEE BACK OF FRONT COVER})$$

$$S_{DD} = \frac{1(7)^2(12)}{3EI} = \frac{196}{EI}$$

COMPATIBILITY EQ

$$\Delta_D(\downarrow) = \Delta_{D0}(\downarrow) - S_{DD} D_y$$

$$\Delta_D = \frac{D_y}{K} = \frac{1840}{EI} - \frac{196 D_y}{EI}$$

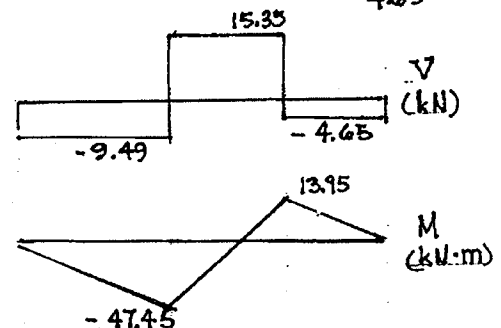
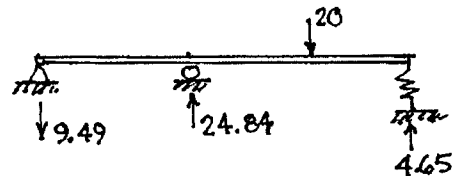
$$D_y = \frac{1840}{\frac{EI}{K} + 196}$$

$$\text{AND } \frac{EI}{K} = \frac{200 \times 40}{40} = 200$$

$$D_y = \frac{1840}{200 + 196} = 4.65 \text{ kN} \uparrow$$

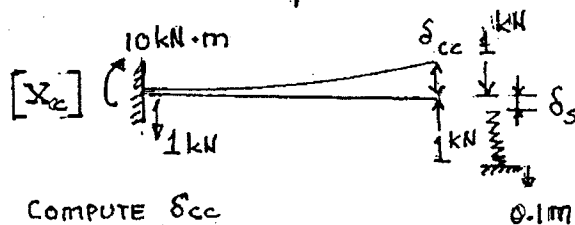
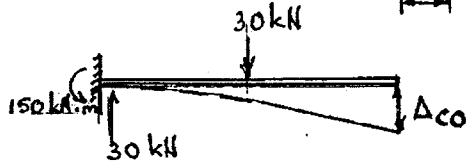
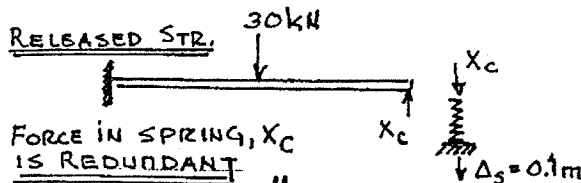
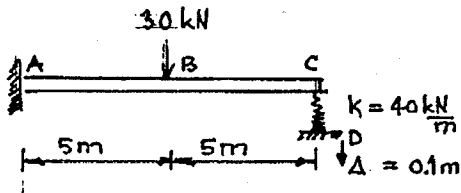
$$\Sigma M_A = 0: R_B = 24.84 \text{ kN} \uparrow$$

$$\Sigma F_y = 0: R_A = 9.49 \text{ kN} \downarrow$$

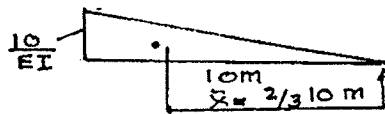


P11.37. Compute the reactions and draw the shear and moment curves for the beam in Figure P11.37. In addition to the applied load, the support at D settles by 0.1 m. EI is constant for the beam. $E = 200$ GPa, $I = 60 \times 10^6 \text{ mm}^4$.

$$E = 200 \text{ GPa}, I = 60 \times 10^6 \text{ mm}^4$$



COMPUTE δ_{cc}



$$\delta_{cc} = \frac{1}{2} \times 10 \times \frac{20}{EI} = \frac{333.33}{EI}$$

$$\delta_{cc} = \frac{333.33}{200 \times 60} = 0.0278 \text{ m}$$

$$F = K \delta_s$$

$$1 = 40 \delta_s$$

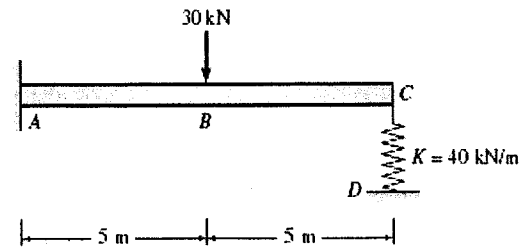
$$0.025 \text{ m} = \frac{1}{40} \text{ m} = \delta_s$$

COMPATIBILITY EQUATION

$$-\Delta_{co} + \delta_{cc} X_c = -0.025 X_c - 0.1$$

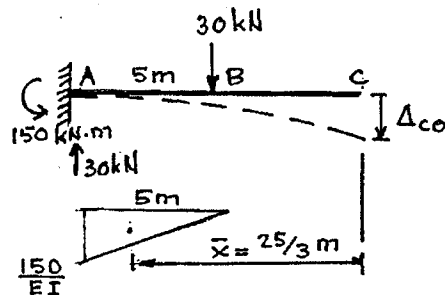
$$-0.2604 + 0.0278 X_c = -0.025 X_c - 0.1$$

$$X_c = 3.038$$



P11.37

COMPUTE Δ_{co} USING MOMENT-AREA

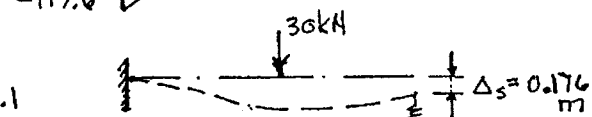
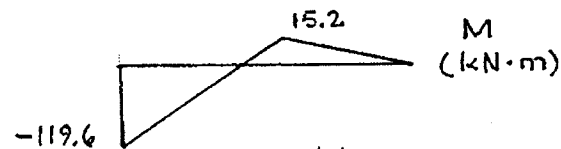
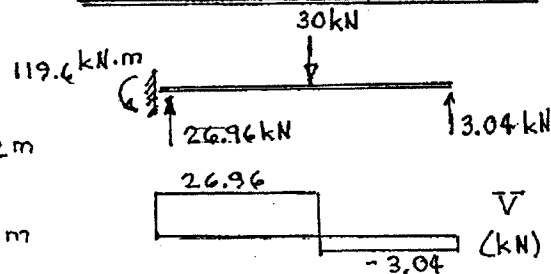


$$\Delta_{co} = \frac{5(150)(25)}{2EI} = \frac{3125}{EI} \text{ kN}\cdot\text{m}^3$$

$$\Delta_{co} = \frac{3125 \text{ kN}\cdot\text{m}^3}{(200 \frac{\text{kN}}{\text{m}^2} \times 10^6)(60 \times 10^6 \frac{\text{m}^4}{10^{12} \text{ m}^4})} = -0.2604 \text{ m}$$

$$\Delta_{co} = -0.2604 \text{ m}$$

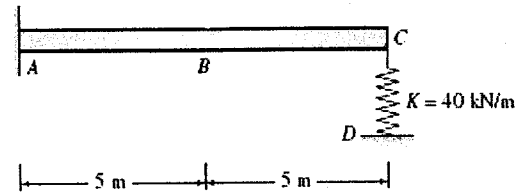
SHEAR AND MOMENT DIAGRAMS



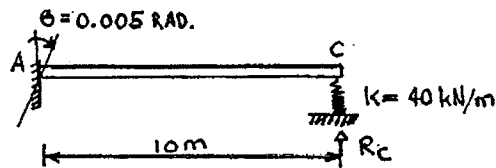
DEFLECTED SHAPE

$$\Delta_s = 0.1 + \frac{3.048}{40} = 0.176 \text{ m}$$

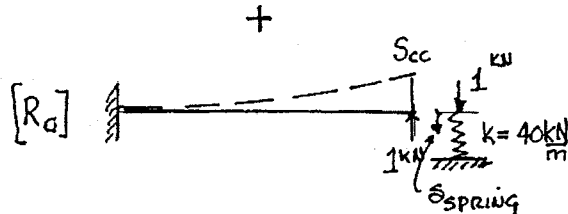
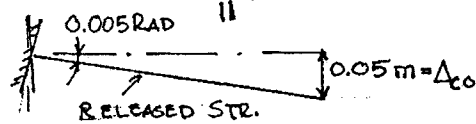
P11.38. Consider the beam in Figure P11.37 without the applied load and support settlement. Compute the reactions and draw the shear and moment curves for the beam if support *A* rotates clockwise by 0.005 rad.



P11.37



SUPPORT A ROTATES 0.005 RADIANS
CLOCKWISE, DRAW V AND M DIAGRAM
 $E = 200 \text{ GPa}$, $I = 60 \times 10^6 \text{ mm}^4$
TAKE R_B AS REDUNDANT.



$$S_{CC} = 0.0278 \text{ m (SEE P.11.37)}$$

$$\delta_{\text{SPRING}} = \frac{1 \text{ kN}}{40} = 0.025 \text{ m}$$

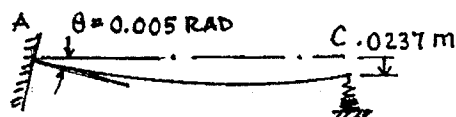
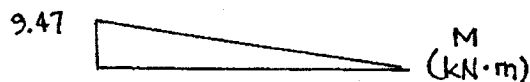
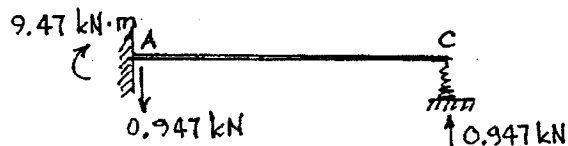
COMPATIBILITY EQ

$$-\Delta_{C0} + S_{CC} R_C = -\delta_{\text{SPRING}} \times R_C$$

$$-0.05 + 0.0278 R_B = -0.025 R_C$$

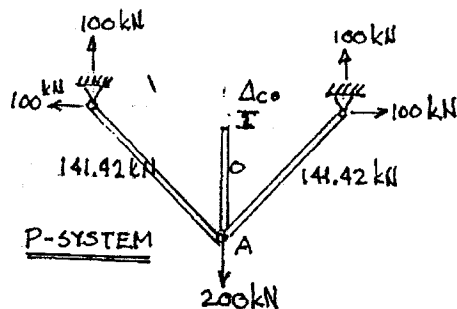
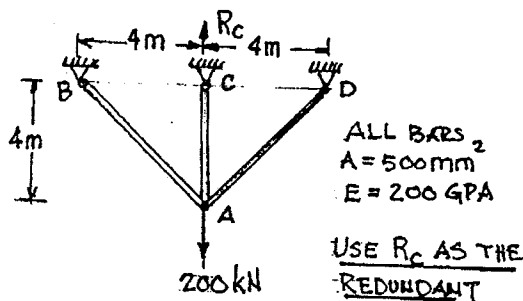
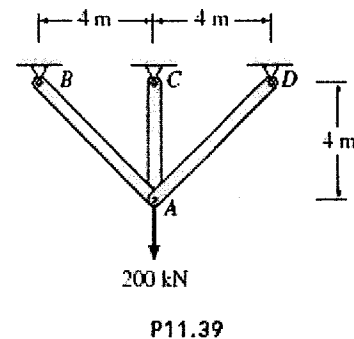
$$0.0528 R = .05$$

$$R_C = .947 \text{ kN}$$



DEF'L'D SHAPE

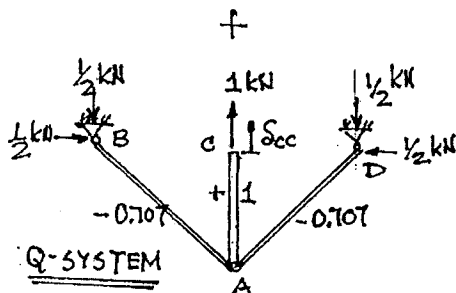
P11.39. Determine the vertical and horizontal displacements at A of the pin-connected structure in Figure
P11.39. Given: $E = 200 \text{ GPa}$ and $A = 500 \text{ mm}^2$ for all members.



$$\Delta_{CO}: 1 \text{ kN} \cdot \Delta_{CO} = \sum F_P F_P L / AE$$

$$\Delta_{CO} = \left[-0.707 (141.42) (4\sqrt{2}) \right] 2 / AE$$

$$\Delta_{CO} = -1131.82 / AE$$



$$S_{CC}: 1 \text{ kN} \cdot S_{CC} = \sum F_Q^2 L / AE$$

$$S_{CC} = \frac{1^2 (4)}{AE} + \frac{(-0.707)^2 (5.66) [2]}{AE}$$

$$S_{CC} = 9.66 / AE$$

COMPATIBILITY EQ

$$\Delta_{CO} + R_C S_{CC} = 0$$

$$-1131.82 / AE + R_C (9.66 / AE) = 0$$

$$R_C = 117.17 \text{ kips} \uparrow$$

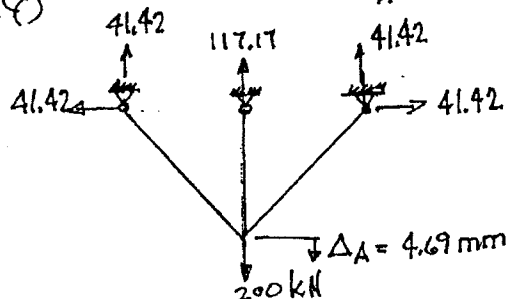
$$\Delta_{AX} = 0 \text{ (SYMMETRY)}$$

$$\Delta_A = \Delta L \text{ OF BAR AC}$$

$$\Delta_A = \frac{F_{AC} L}{AE}$$

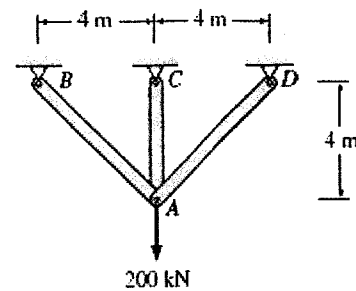
$$= \frac{117.17 (4 \text{ m})}{500 \text{ mm}^2 \times \frac{\text{m}^2}{10^6 \text{ mm}^2}} \frac{200 \text{ kN} \times 10^6}{\text{m}^2}$$

$$\Delta_A = 0.00469 \text{ m} = 4.69 \text{ mm} \downarrow$$

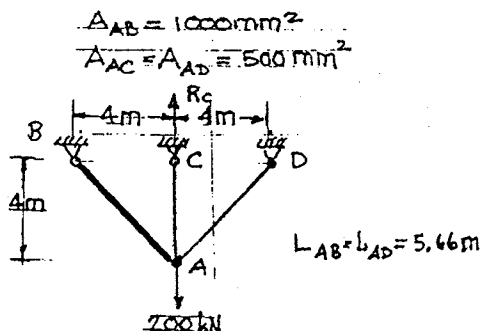


FINAL RESULTS

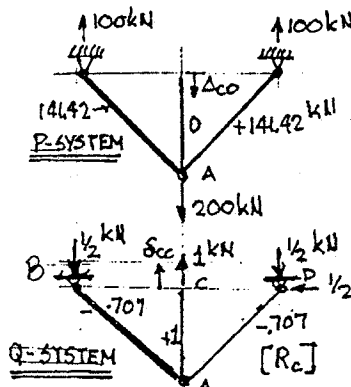
P11.40. Determine the vertical and horizontal displacements at A of the pin-connected structure in Figure P11.39. Given: $E = 200 \text{ GPa}$, $A_{AB} = 1000 \text{ mm}^2$, and $A_{AC} = A_{AD} = 500 \text{ mm}^2$.



P11.39



SELECT R_C AS REDUNDANT



$$\Delta_{C0} = \frac{1}{AE} \sum F_Q F_P L$$

$$= \frac{1}{500(200)} \left[(0.707)(141.42)(5.66) + (-0.707)(141.42)(5.66) \right] + \frac{1}{1000(200)} (103.03)(4)$$

$$= -0.00566 + (-0.00283)$$

$$\Delta_{C0} = 0.00849$$

$$\Delta_{CC} = \frac{1}{AE} \sum F_Q^2 L$$

$$= \frac{(0.707)^2 (5.66)}{1000 \times 200} + \frac{(-0.707)^2 (5.66)}{500 \times 200} + \frac{1^2 (4)}{500(200)}$$

$$= 0.0000141 + 0.0000282 + 0.00004$$

$$= 0.0000824 = 0.0824 \times 10^{-3} \text{ m}$$

COMPATIBILITY Eq.

$$\Delta_{C0} + \Delta_{CC} R_C = 0$$

$$0.00849 + 0.0000834 R_C = 0$$

$$R_C = 103.03 \text{ kN}$$

$$\Delta_{Ay} = \frac{F_L}{AE} = \frac{103.03 \text{ kN}(4)}{500 \times 200} = 0.00412 \text{ m}$$

$$\text{OR } 4.12 \text{ mm}$$

* Since bar AC is vertical and the force is known, there is no need to consider other bars.

FINAL RESULTS:

$$R_{By} = 48.485 \text{ kN}$$

$$R_{Dy} = 48.485 \text{ kN}$$

$$R_{Dx} = 48.485 \text{ kN}$$

$$F_{AD} = \sqrt{2} (48.485) = 68.57 \text{ kN}$$

$$\sum M_B = 0 = -R_{Dy}(8) - 103.03(4) + 200(4)$$

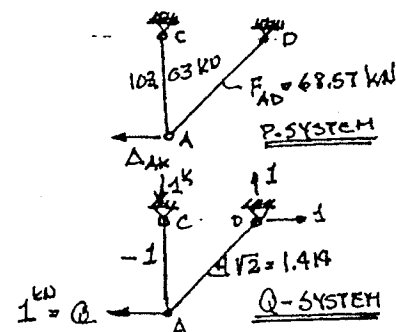
$$R_{Dy} = 48.485 \text{ kN}$$

$$R_{By} = 48.485 \text{ kN} \text{ (Symmetry)}$$

$$R_{Dx} = R_{Dy} = 48.485$$

COMPUTE Δ_{Ax}

CONSIDER BARS AC AND AD IN THE Q AND P-SYSTEMS.



$$\Delta_{Ax} = \frac{1}{AE} \sum F_Q F_P L = \sum F_Q \Delta_{LP}$$

$$= \frac{1}{500 \times 200} \left[(-0.00412) + \sqrt{2} (0.00566) \right]$$

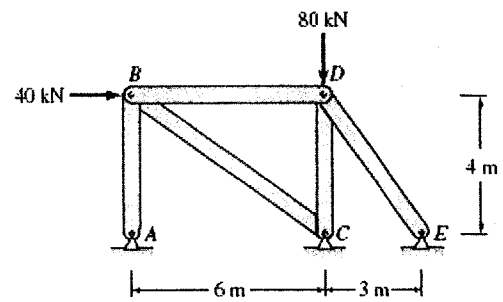
$$= -0.00412 + 0.00566$$

$$= 0.00154 \text{ mm}$$

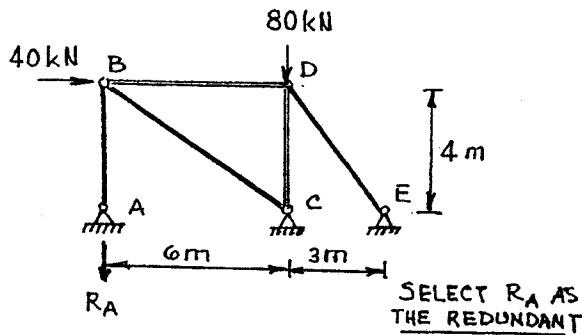
$$\Delta_{Ax} = 1.54 \text{ mm}$$

* SINCE WE KNOW THE FORCES IN TWO BARS, WE CAN COMPUTE Δ_L OF EACH BAR. THIS PERMITS US TO CONSIDER TWO BARS AC & AD RATHER THAN ALL 3 BARS.

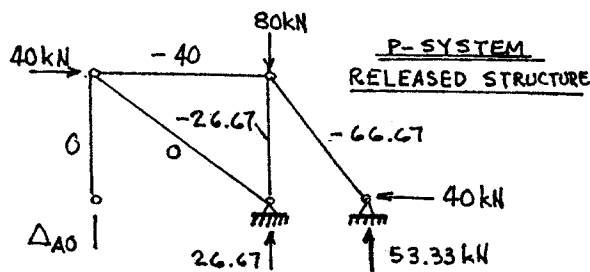
P11.41. Determine the reactions and all bar forces for the truss in Figure P11.41. Given: $E = 200 \text{ GPa}$ and $A = 1000 \text{ mm}^2$ for all bars.



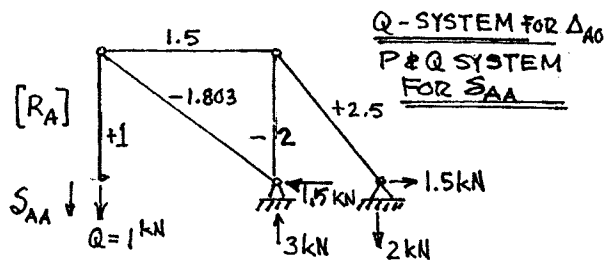
P11.41



||



+



$$\Delta_{A0}: 1 \cdot \Delta_{A0} = \sum F_Q \cdot \frac{F_{PL}}{AE}$$

$$\Delta_{A0} = \frac{1}{AE} [(1.5)(-40 \times 6) + (-2)(-26.67)(4) + (2.5)(-66.67)(5)]$$

$$\Delta_{A0} = -\frac{980}{AE}$$

$$S_{AA}: 1 \text{ kN} \cdot S_{AA} = \sum F_Q^2 \frac{L}{AE}$$

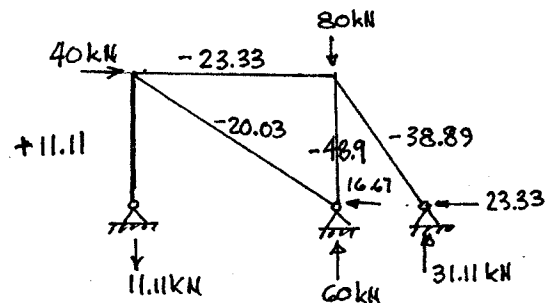
$$S_{AA} = \left[\frac{(1.5)^2 \times 6}{AE} + \frac{(2)^2 \times 4}{AE} + \frac{(-2)^2 \times 4}{AE} + \frac{(2.5)^2 \times 5}{AE} + \frac{(-1.803)^2 \times 7.21}{AE} \right] = \frac{88.2}{AE}$$

COMPATIBILITY EQ.

$$\Delta_{A0} + R_A S_{AA} = 0$$

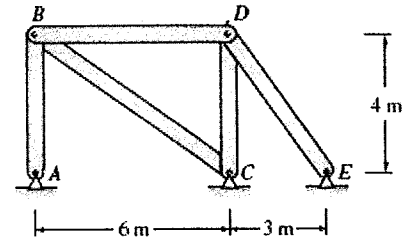
$$-\frac{980}{AE} + R_A \frac{88.2}{AE} = 0$$

$$R_A = 11.11 \text{ kN} \downarrow$$



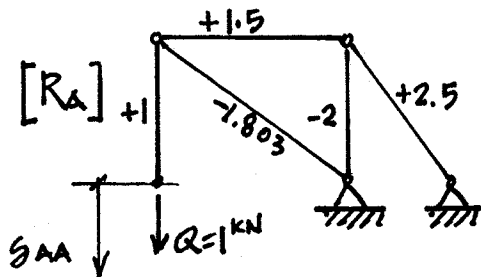
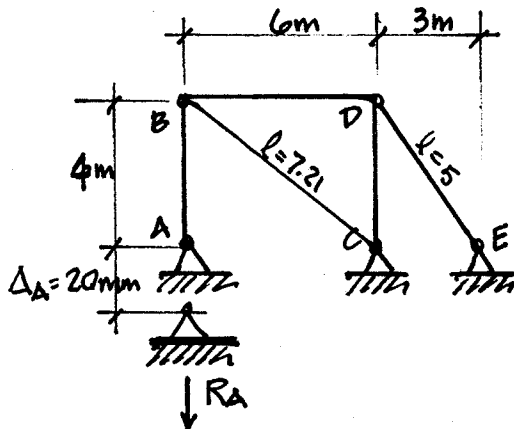
BAR FORCES AND REACTIONS

P11.42. Consider the truss in Figure P11.41 without the applied loads. Determine the reactions and all bar forces for the truss if support A settles vertically 20 mm.

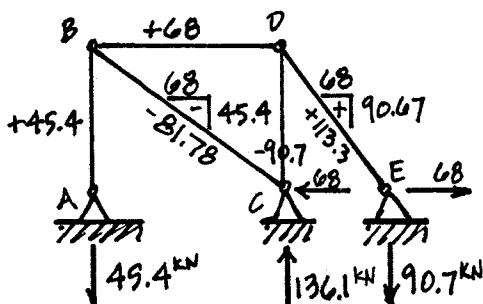


P11.41

ALL BARS $A = 1000 \text{ mm}^2$, $G = 200 \text{ GPa}$
SELECT R_A AS THE REDUNDANT



PR-SYSTEMS FOR S_{AA}



FINAL BAR FORCES & REACTIONS

$$\Delta_A = -20 \text{ mm}$$

$$\Delta_{A0} + S_{AA} X = -20 \text{ mm}$$

$$\text{WHERE } R_A = X$$

$$\Delta_{A0} = 0$$

$$\text{VIRTUAL WORK } Q \cdot \delta = \sum F_Q^2 L / AE$$

$$1 \text{ kN} \cdot S_{AA} = \sum F_Q^2 L / AE$$

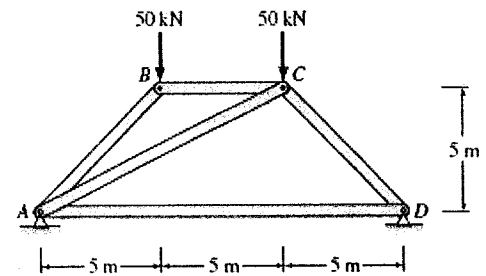
$$= (1)^2(4) + (1.5)^2(6) + (2.5)^2(5) + (-1.803)^2(7.21) + (-2)^2(4)$$

$$= 88.19 / AE = 0.4409 \text{ mm}$$

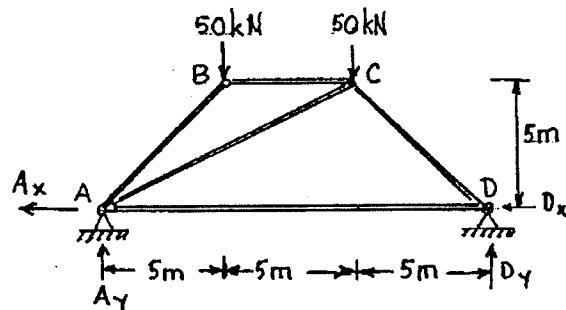
$$(\Delta_{A0} = 0) + 0.4409 X = -20 \text{ mm}$$

$$\underline{\underline{X = R_A = 45.4 \text{ kN}}}$$

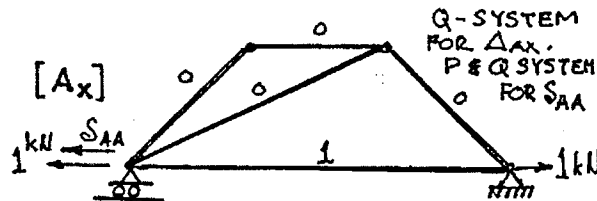
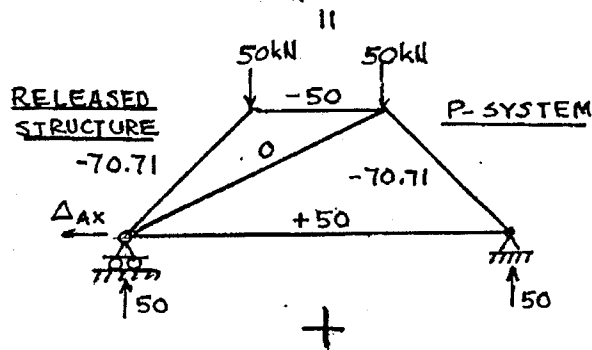
P11.43. Determine the reactions and all bar forces for the truss in Figure P11.43. $E = 200 \text{ GPa}$ and $A = 1000 \text{ mm}^2$ for all bars.



P11.43



SELECT A_x AS THE REDUNDANT



$$\Delta_{AH} \cdot 1^{\text{kN}} = \sum F_Q F_P \frac{L}{AE} = \frac{1^{\text{kN}} \times 50^{\text{kN}} \times 15}{AE} = \frac{750}{AE}$$

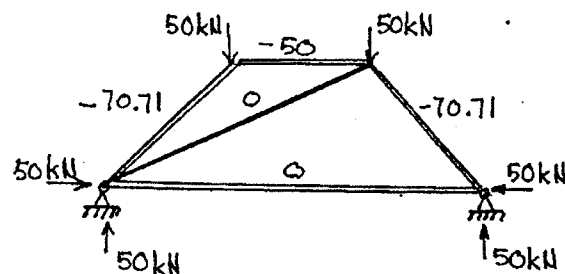
$$S_{AA} \cdot 1^{\text{kN}} = \sum \frac{F_Q^2 L}{AE} = \frac{1^2 \times 15}{AE} = \frac{15}{AE}$$

COMPATIBILITY EQ. $\Delta_A = 0$

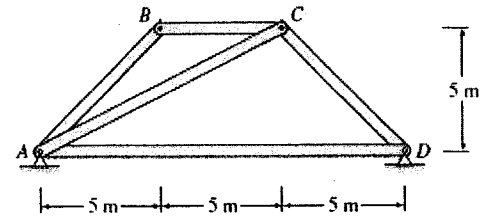
$$\Delta_{AH} + R_A S_{AA} = 0$$

$$750/AE + R_A 15/AE = 0$$

$$R_A = 50 \text{ kN} \rightarrow$$



P11.44. Consider the truss in Figure P11.43 without the applied loads. Determine the reactions and all bar forces for the truss if member AC is fabricated 10 mm too short.



P11.43

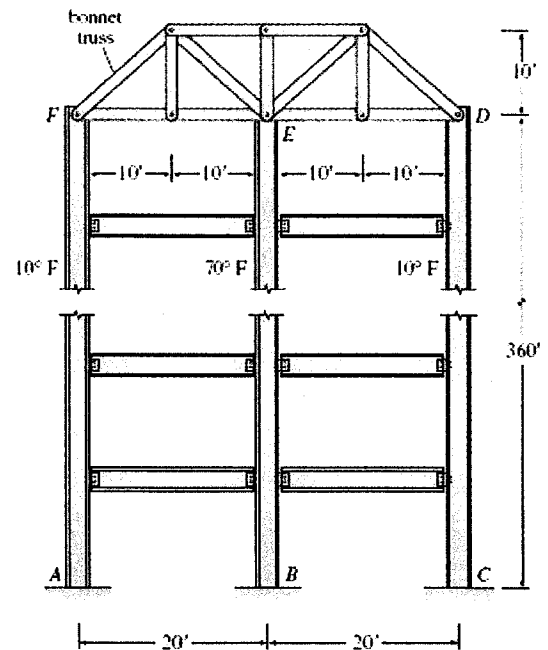
NO BARS ARE STRESSED. THE REDUNDANCY OCCURS BECAUSE BAR AD IS ONLY STRESSED IF SUPPORTS A AND D MOVE APART HORIZONTALLY. ABOVE THE JOINTS, THE STRUCTURE IS DETERMINATE, I.E. STARTING AT JOINT B , BAR FORCES CAN BE DETERMINED BY METHOD OF JOINTS. FOR BARS AB , BC , AC , AND CD , A CHANGE IN LENGTH WILL PRODUCE MOVEMENT OF JOINTS BUT NO STRESSES.

P11.45. Practical Design Example

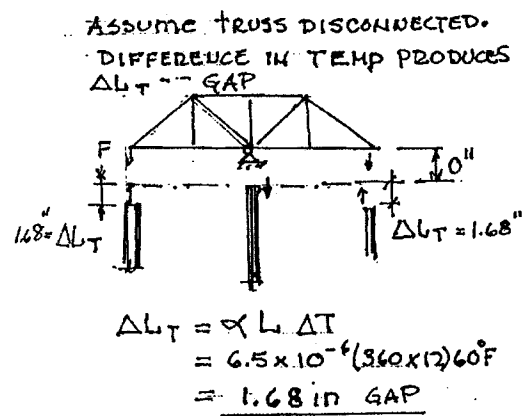
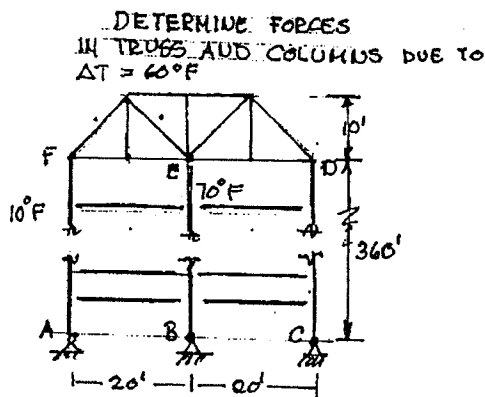
The tall building in Figure P11.45 is constructed of structural steel. The exterior columns, which are uninsulated, are exposed to the outside ambient temperature. To reduce the differential vertical displacements between the interior and exterior columns due to temperature differences between the interior and exterior of the building, a *bonnet truss* has been added at the top of the building. For example, if a bonnet truss was not used to restrain the outer columns from shortening in the winter due to a 60°F temperature difference between the interior and exterior columns, points *D* and *F* at the top of the exterior columns would move downward 1.68 inches relative to the top of the interior column at point *E*. Displacements of this magnitude in the upper stories would produce excessive slope of the floor and would damage the exterior facade.

If the temperature of interior column *BE* is 70°F at all times but the temperature of the exterior columns in winter drops to 10°F, determine (a) the forces created in the columns and the truss bars by the temperature differences and (b) the vertical displacements of the tops of the columns at points *D* and *E*. Slotted truss connections at *D* and *F* have been designed to act as rollers and transmit vertical force only, and the connection at *E* is designed to act as a pin. The shear connections between the beam webs and the columns may be assumed to act as hinges.

Given: $E = 29,000 \text{ kips/in}^2$. The average area of the interior column is 42 in^2 and 30 in^2 for the exterior columns. The areas of all members of the truss are 20 in^2 . The coefficient of temperature expansion $\alpha = 6.5 \times 10^{-6} (\text{in/in})/^\circ\text{F}$. Note: The interior columns must be designed for both the floor loads and the compression force created by the temperature differential.



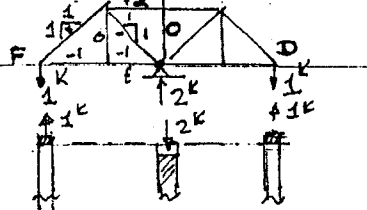
P11.45



P11.45 Continued

P11.45 Continued

DETERMINE THE AMOUNT THE GAP IS CLOSED BY A UNIT LOAD APPLIED TO EXT. COLUMNS.



δ OF EXTERIOR COLUMNS / 1 k

$$\delta = \frac{PL}{AE} = \frac{1(360)12}{30 \times 29,000} = 0.004966"$$

δ OF INTERIOR COLUMN

$$\delta = \frac{PL}{AE} = \frac{2(360)12}{42 \times 29,000} = 0.007094"$$

DISPLACEMENT OF POINTS D & F OF TRUSS RELATIVE TO SUPPORT E

$$1^k \delta_F = \sum \frac{F_a F_p L}{A E} \quad F_a = F_p$$

$$\delta_F = \frac{((\sqrt{2}/2)(10\sqrt{2}))^2}{20E} + \frac{(-1)(-1)(10)}{20E}$$

$$\frac{(2 \times 2 \times 10)}{20E} = \frac{5.8284(12)}{29,000}$$

$$\delta_F = \delta_D = 0.002412 \text{ in.}$$

TOTAL CLOSING OF GAP BY UNIT load

$$\delta = 0.004966$$

$$\delta_{BB} = 0.007094$$

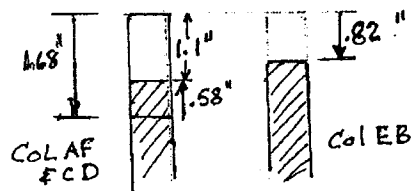
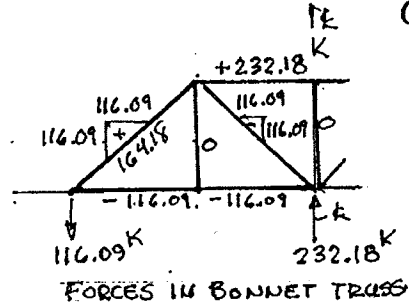
$$\delta_{TRUSS} = \frac{0.002412}{0.01472} \text{ in}$$

$$\times 0.01472 = 1.68 \text{ in}$$

$$X = 116.09 \text{ kips}$$

FORCE IN EXTERIOR COLUMNS 116.09 kips (TENSION)

FORCE IN INTERIOR COLUMN 232.18 kips (COMPRESSION)



FINAL DIFFERENCE IN LENGTH

$$1.1 - 0.82 = 0.28"$$