

FUNDAMENTALS OF STRUCTURAL ANALYSIS

3rd Edition

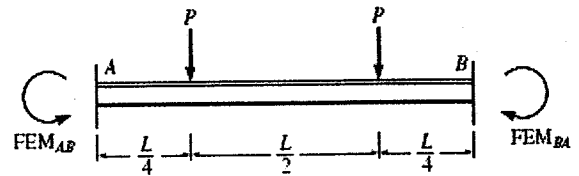
Kenneth M. Leet, Chia-Ming Uang, and Anne M. Gilbert

SOLUTIONS MANUAL

CHAPTER 12:

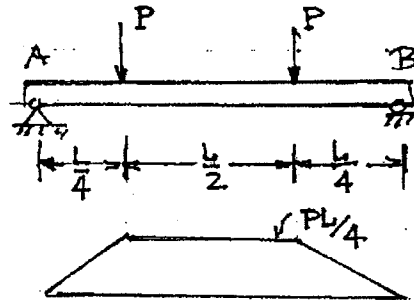
ANALYSIS OF INDETERMINATE BEAMS AND FRAMES BY THE SLOPE-DEFLECTION METHOD

P12.1. Using Equations 12.12 and 12.13, compute the fixed-end moments for the fixed-ended beam. See Figure P12.1.



P12.1

COMPUTE FEM'S



$$A_M = \frac{1}{2} \frac{PL}{4} \times \frac{L}{4} (2) + \frac{PL}{4} \times \frac{L}{2}$$

$$= \frac{3}{16} PL^2$$

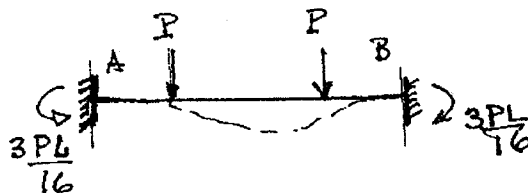
$$FEM_{AB} = \frac{2(A_M \bar{x})_A}{L^2} - \frac{4(A_M \bar{x})_B}{L^2}$$

$$= \frac{2}{L^2} \left(\frac{3}{16} PL^2 \times \frac{L}{2} \right) - \frac{4}{L^2} \left(\frac{3}{16} PL^2 \right) \frac{L}{2}$$

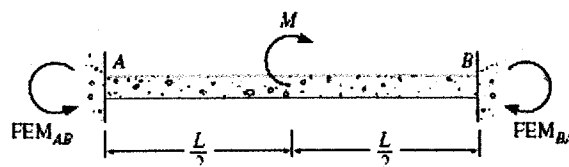
$$FEM_{AB} = \frac{3PL}{16} - \frac{3PL}{8} = -\frac{3PL}{16}$$

SIMILARLY FROM SYMMETRY

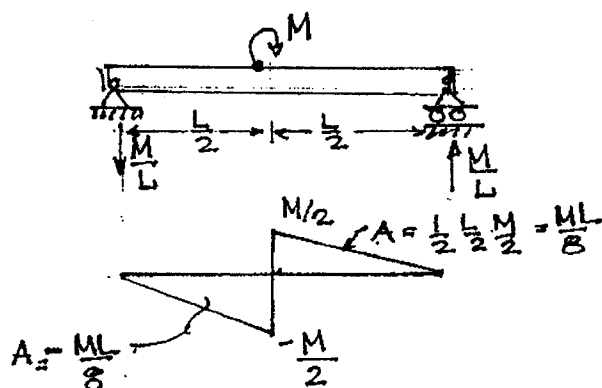
$$FEM_{BA} = +\frac{3PL}{16}$$



P12.2. Using Equations 12.12 and 12.13, compute the fixed-end moments for the fixed-ended beam. See Figure P12.2.



P12.2



$$\begin{aligned} (A_m \bar{x})_A &= \frac{1}{2} \times \frac{L}{2} \times \left(-\frac{M}{2}\right) \times \frac{2}{3} \times \frac{L}{2} \\ &\quad + \frac{1}{2} \times \frac{L}{2} \times \frac{M}{2} \left(\frac{L}{2} + \frac{1}{3} \times \frac{L}{2}\right) \\ &= -\frac{ML}{8} \times \frac{L}{3} + \frac{ML}{8} \left(\frac{2}{3}L\right) \\ &= \frac{ML^2}{24} \end{aligned}$$

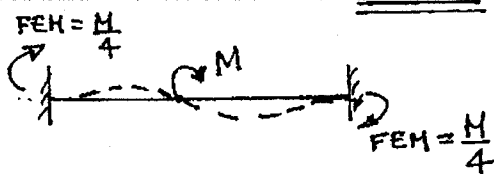
$$\text{SIMILARLY } (A_m \bar{x})_B = -\frac{ML^2}{24}$$

$$\begin{aligned} FEM_{AB} &= \frac{2}{L^2} (A_m \bar{x})_A - \frac{4}{L^2} (A_m \bar{x})_B \\ &= \frac{2}{L^2} \left(\frac{ML^2}{24}\right) - \frac{4}{L^2} \left(-\frac{ML^2}{24}\right) \end{aligned}$$

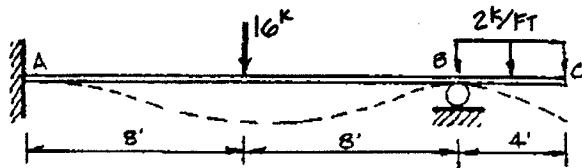
$$FEM_{AB} = \frac{M}{12} + \frac{M}{6} = \frac{3}{12}M = \frac{M}{4} \curvearrowright$$

$$\begin{aligned} FEM_{BA} &= \frac{4(A_m \bar{x})_A}{L^2} - \frac{2(A_m \bar{x})_B}{L^2} \\ &= \frac{4}{L^2} \frac{ML^2}{24} - \frac{2}{L^2} \left(-\frac{ML^2}{24}\right) \end{aligned}$$

$$FEM_{BA} = \frac{M}{6} + \frac{M}{12} = \frac{M}{4} \curvearrowleft$$



P12.3. Analyze by slope-deflection and draw the shear and moment curves for the beam in Figure P12.3. Given: $EI = \text{constant}$.



EI CONSTANT

$\theta_A = 0$; $\theta_B = \text{ONLY UNKNOWN DISPLACEMENT}$

$$\left. \begin{aligned} M_{AB} &= \frac{2EI}{16} [\theta_B] - \frac{PL}{8} \\ M_{BA} &= \frac{2EI}{16} [2\theta_B] + \frac{PL}{8} \end{aligned} \right\} \text{EQNS 1}$$

EQUILIBRIUM EQN @ JOINT B

$$\sum M_B = 0; \quad 16 - M_{BA} = 0 \rightarrow \text{EQN 2}$$

SUBSTITUTE EQN 1 INTO EQN 2

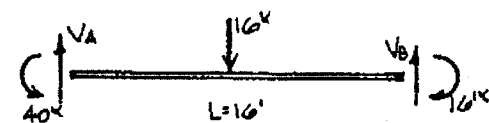
$$16 - \left[\frac{4EI}{16} \theta_B + 16 \left(\frac{16}{8} \right) \right] = 0 \quad \theta_B = \frac{-64}{EI}$$

SUBSTITUTE θ_B INTO EQN 1

$$M_{AB} = \frac{2EI}{16} \left[\frac{-64}{EI} \right] - \frac{16(16)}{8} \quad M_{AB} = -40 \text{ k}$$

$$M_{BA} = \frac{4EI}{16} \left[\frac{-64}{EI} \right] + \frac{16(16)}{8} \quad M_{BA} = 16 \text{ k}$$

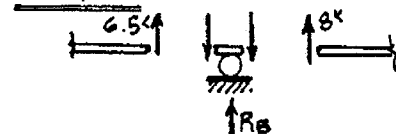
COMPLETE ANALYSIS USING STATICS



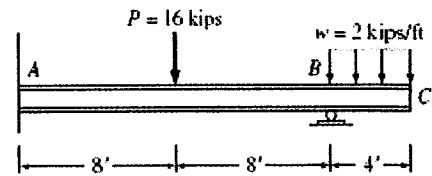
$$\sum M_A = 0; \quad 16(8) + 16 - 40 - V_B(16) = 0 \quad V_B = 6.5 \text{ k}$$

$$\sum F_y = 0; \quad V_A - 16 + 6.5 = 0 \quad V_A = 9.5 \text{ k}$$

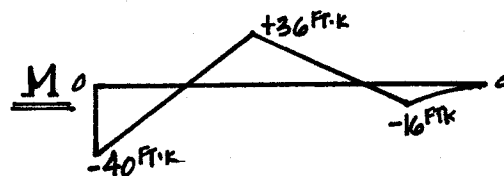
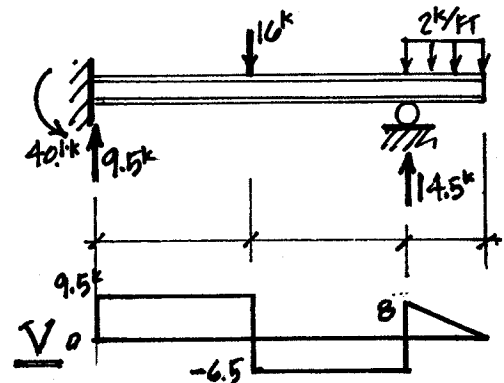
JOINT B



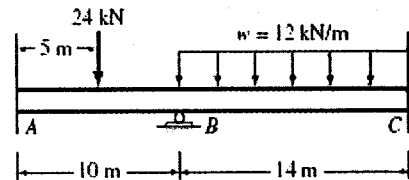
$$\sum F_y = 0; \quad -V_A - V_B + R_B = 0 \quad R_B = 14.5 \text{ k}$$



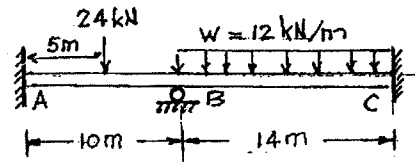
P12.3



P12.4. Analyze the beam in Figure P12.4 by slope-deflection and draw the shear and moment diagrams for the beam. EI is constant.



P12.4



$$\begin{aligned} \theta_A = \theta_C &= 0 \\ M_{AB} &= \frac{2EI}{10} (0 + \theta_B) - \frac{24 \times 10}{8} \\ M_{BA} &= \frac{2EI}{10} (2\theta_B + 0) + \frac{24 \times 10}{8} \\ M_{BC} &= \frac{2EI}{14} (2\theta_B + 0) - \frac{12(14)^2}{12} \\ M_{CB} &= \frac{2EI}{14} (0 + \theta_B) + \frac{12(14)^2}{12} \end{aligned} \quad (1)$$

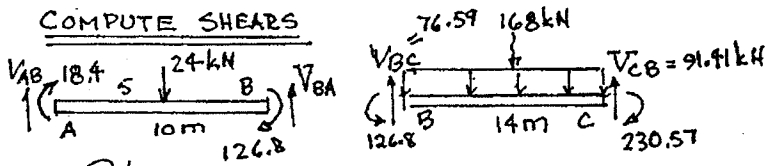
EQUIL. AT JOINT B

$$\begin{aligned} M_{BA} + M_{BC} &= 0 \\ \frac{4EI}{10} \theta_B + 30 + \frac{4EI}{14} (2\theta_B) - 196 &= 0 \\ \theta_B &= \frac{242}{EI} \quad (2) \end{aligned}$$

SUBSTI EQ (2) INTO EQ 1

$$\begin{aligned} M_{AB} &= \frac{2EI}{10} \left(\frac{242}{EI} \right) - 30 = 18.4 \text{ kN}\cdot\text{m} \\ M_{BA} &= \frac{4EI}{10} \left(\frac{242}{EI} \right) + 30 = 126.8 \text{ kN}\cdot\text{m} \\ M_{CB} &= \frac{2EI}{14} \left(\frac{242}{EI} \right) + 196 = 230.57 \text{ kN}\cdot\text{m} \end{aligned}$$

COMPUTE SHEARS



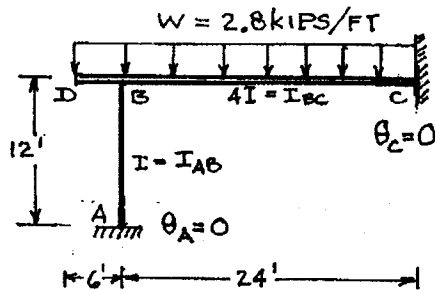
AB

$$\begin{aligned} \sum M_A = 0 &= 18.4 + 126.8 + 24 \times 5 - V_{BA} \times 10 \\ V_{BA} &= 26.52 \text{ kN} \\ \uparrow \sum F_y = 0 &= V_{AB} - 24 + 26.52 \\ V_{AB} &= -2.52 \text{ kN} \end{aligned}$$

BC

$$\begin{aligned} \sum M_B = 0 &= 168 \times 7 + 230.57 - 126.8 - V_{CB} \times 14 \\ V_{CB} &= 91.41 \text{ kN} \uparrow \\ \uparrow \sum F_y = 0 &= V_{BC} - 168 + 91.41 \\ V_{BC} &= 76.59 \text{ kN} \uparrow \end{aligned}$$

P12.5. Compute the reactions at A and C in Figure P12.5.
 Draw the shear and moment diagram for member BC.
 Given: $I = 2000 \text{ in}^4$ and $E = 3,000 \text{ kips/in}^2$.



$$FEM_{BC} = -\frac{wL^2}{12} = -\frac{2.8(24)^2}{12} = -134.4 \text{ kip}\cdot\text{ft}$$

$$FEM_{CB} = 134.4 \text{ kip}\cdot\text{ft}$$

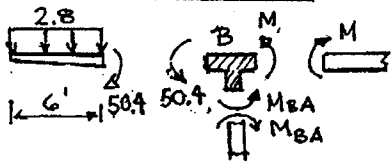
$$M_{BC} = \frac{2EI}{L}(2\theta_B) + FEM_{BC} = \frac{4EI}{L}\theta_B - 134.4$$

$$M_{CB} = \frac{2EI}{L}\theta_B + 134.4$$

$$M_{BA} = \frac{2EI}{L}\theta_B = \frac{4EI}{12}\theta_B$$

$$M_{AB} = \frac{2EI}{L}\theta_B = \frac{2EI}{12}\theta_B$$

EQUIL. EQ AT JOINT B



$$\sum M_B = 0; M_{BC} + M_{BA} + 16.8 = 0 \quad (2)$$

SUBSTI EQ'S (1) INTO (2)

$$\frac{4E(4I)}{24}\theta_B - 134.4 + \frac{4EI}{12}\theta_B + 50.4 = 0$$

$$EI\theta_B = 84$$

$$\theta_B = \frac{84}{EI} \quad (3)$$

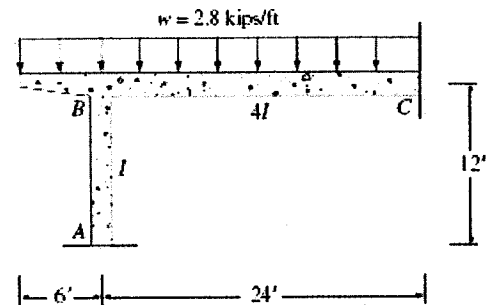
SUBSTITUTE (3) INTO EQS (1)

$$M_{BC} = \frac{4E(4I)}{24}\left(\frac{84}{EI}\right) - 134.4 = -78.4 \text{ K}\cdot\text{ft}$$

$$M_{BA} = \frac{4EI}{12}\left(\frac{84}{EI}\right) = 28 \text{ ft}\cdot\text{kips}$$

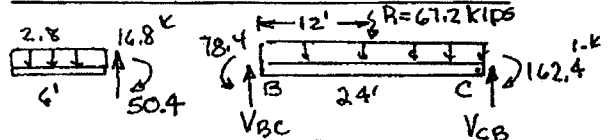
$$M_{CB} = \frac{2E(4I)}{24}\left(\frac{84}{EI}\right) + 134.4 = 162.4 \text{ K}\cdot\text{ft}$$

$$M_{AB} = \frac{2EI}{12}\left(\frac{84}{EI}\right) = 14 \text{ KIP}\cdot\text{FT}$$



P12.5

COMPUTE REACTIONS AT A AND C



COMPUTE V_{CB} ; $\sum M_B = 0$

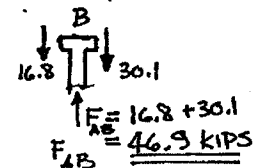
$$0 = 67.2(12) + 162.4 - 78.4 - V_{CB} 24$$

$$V_{CB} = 37.1$$

$$\uparrow \sum F_y = 0; V_{BC} - 67.2 + 37.1 = 0$$

$$V_{BC} = 30.1 \text{ kips}$$

AXIAL FORCE IN BA

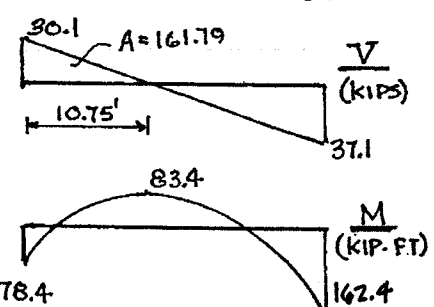
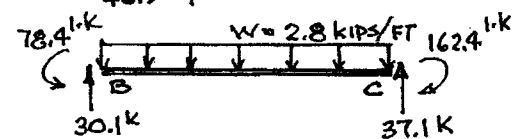
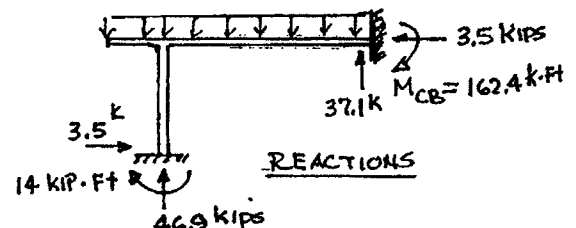
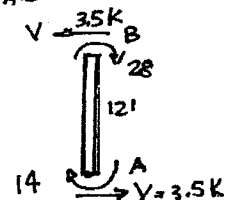


SHEAR IN COLUMN AB

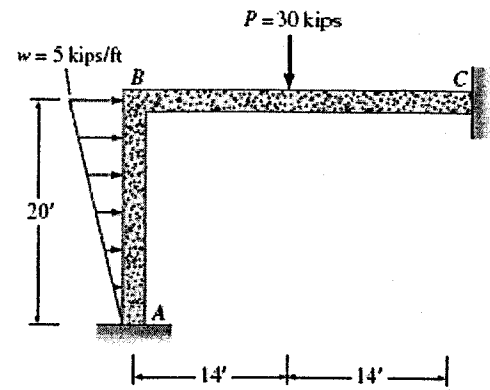
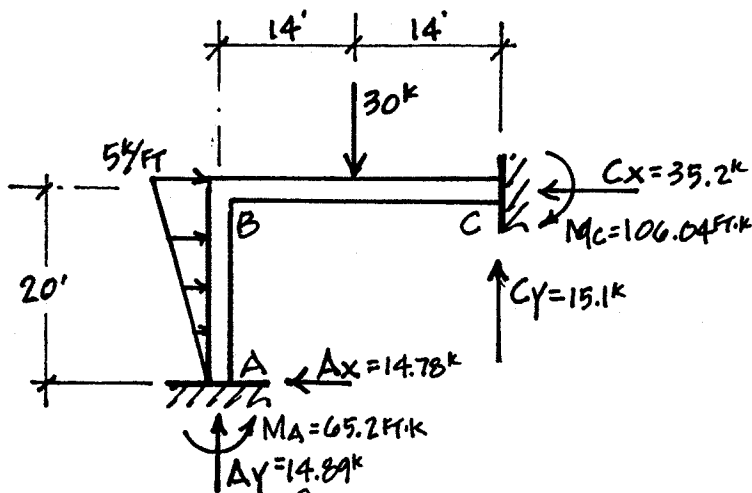
$$\sum M_A = 0$$

$$28 + 14 - V 12 = 0$$

$$V = 3.5 \text{ kips}$$



P12.6. Draw the shear and moment curves for the frame in Figure P12.6. Given: EI is constant.



P12.6

UNKNOWN: θ_B

KNOWN: $\theta_A = \theta_C = 0$; $\psi_{AB} = \psi_{BC} = 0$

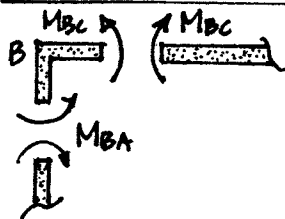
$$M_{AB} = \frac{2EI}{20'} (2\theta_A + \theta_B) - \frac{5 \text{ k/ft} (20')^2}{30} = 0.1 \theta_B EI - 66.7 = \underline{\underline{-65.2 \text{ ft-k}}}$$

$$M_{BA} = \frac{2EI}{20'} (2\theta_B) + \frac{5 \text{ k/ft} (20')^2}{20} = 0.2 \theta_B EI + 100 = \underline{\underline{+102.92 \text{ ft-k}}}$$

$$M_{BC} = \frac{2EI}{28'} (2\theta_B) - \frac{30 \text{ k} (28')}{8} = 0.143 \theta_B EI - 105 = \underline{\underline{-102.92 \text{ ft-k}}}$$

$$M_{CB} = \frac{2EI}{28'} (\theta_B) + \frac{30 \text{ k} (28')}{8} = 0.0714 \theta_B EI + 105 = \underline{\underline{+106.04 \text{ ft-k}}}$$

JOINT "B" EQUILIBRIUM:



$$\begin{aligned} \sum M_B &= 0; \quad M_{BA} + M_{BC} = 0 \\ (0.2 \theta_B EI + 100) + (0.143 \theta_B EI - 105) &= 0 \\ \theta_B &= \underline{\underline{14.578 / EI}} \end{aligned}$$

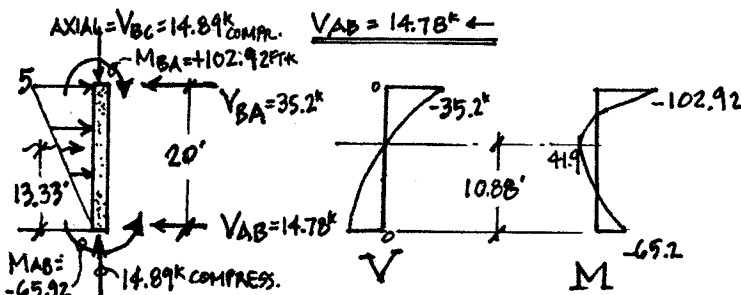
SEGMENT "AB": $\sum M_A = 0$;

$$+102.92 - 65.2 + \frac{5 \text{ k} (20') (13.33')}{2} - V_{BA} (20') = 0$$

$$V_{BA} = \underline{\underline{35.2 \text{ k}}}$$

$$\sum F_x = 0; \quad 50 \text{ k} - V_{BA} - V_{AB} = 0$$

$$V_{AB} = \underline{\underline{14.78 \text{ k}}}$$



SEGMENT "BC":

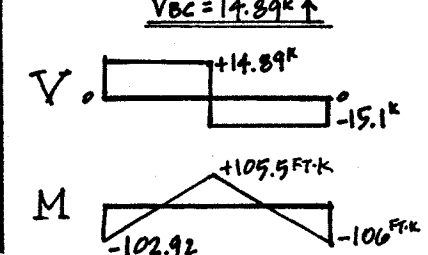
$$M_{BC} = -102.92 \quad M_{CB} = 106.04$$

$$\sum M_B = 0; \quad -102.92 + 106 + 30(14) - V_{CB}(28) = 0$$

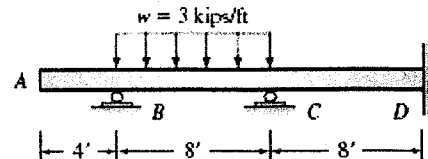
$$V_{CB} = \underline{\underline{15.1 \text{ k}}}$$

$$\sum F_y = 0; \quad -30 + 15.1 + V_{BC} = 0$$

$$V_{BC} = \underline{\underline{14.89 \text{ k}}}$$



P12.7. Analyze the beam in Figure P12.7. Draw the shear and moment curves. Given: $E = 29,000$ ksi and $I = 100 \text{ in}^4$.



P12.7

2 UNKNOWN θ_B, θ_C

$\theta_A = \theta_B$ UNLOADED CANTILEVER
AND $M_{AB} = M_{BA} = 0$

$$M_{BC} = \frac{2EI}{8'}(2\theta_B + \theta_C) - \frac{3k(8')^2}{12} = 0.5\theta_B EI + 0.25\theta_C EI - 16$$

$$M_{CB} = \frac{2EI}{8'}(2\theta_C + \theta_B) + 16 \text{ FT}\cdot\text{K} = 0.5\theta_C EI + 0.25\theta_B EI + 16$$

$$M_{CD} = \frac{2EI}{8'}(2\theta_C) = 0.5\theta_C EI$$

$$M_{DC} = \frac{2EI}{8'}(\theta_C) = 0.25\theta_C EI$$

JT. B $\sum M_B = 0$; $M_{BA} + M_{BC} = 0$
 $0.5\theta_B EI + 0.25\theta_C EI - 16 = 0$
 $\theta_B = \frac{32}{EI} - 0.5\theta_C$

JT. C $\sum M_C = 0$; $M_{CB} + M_{CD} = 0$
 $0.5\theta_C EI + 0.25\theta_B EI + 16 + 0.5\theta_C EI = 0$
 $(0.875\theta_C)EI + 24 = 0$

$$\theta_C = -27.43/EI$$

$$\theta_B = 45.72/EI$$

MEMBER END MOMENTS:

$$M_{BC} = 0$$

$$M_{CB} = 13.71 \text{ FT}\cdot\text{K}$$

$$M_{CD} = -13.71 \text{ FT}\cdot\text{K}$$

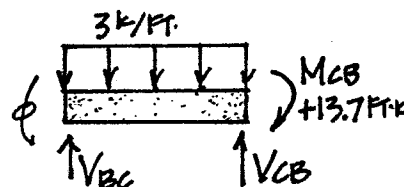
$$M_{DC} = -6.86 \text{ FT}\cdot\text{K}$$

SEGMENT BC

$\sum M_B = 0$;
 $\frac{3k(8')^2}{2} + 13.71 - V_{CB}(8') = 0$

$$V_{CB} = 13.72 \text{ k}\uparrow$$

$\sum F_y = 0$
 $-3k(8') + 13.72 \text{ k} + V_{BC} = 0$
 $V_{BC} = B_y = 10.29 \text{ k}\uparrow$



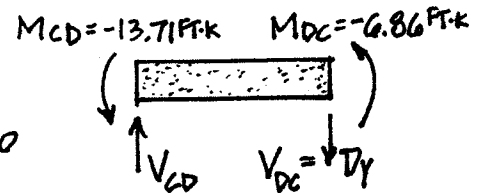
P12.7 Continued

P12.7 Continued

SEGMENT CD

$$\sum \overset{+}{\curvearrowleft} M_C = 0; \quad -13.7 \text{ FT}\cdot\text{K} - 6.86 \text{ FT}\cdot\text{K} + V_{DC} \downarrow (8') = 0$$

$$\underline{V_{DC} = D_Y = 2.6 \text{ K} \downarrow}$$



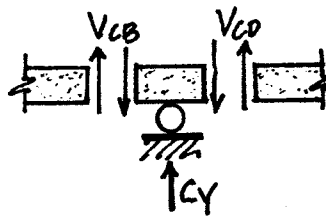
$$\uparrow \sum F_Y = 0; \quad -2.6 \text{ K} + V_{CD} = 0$$

$$\underline{V_{CD} = 2.6 \text{ K} \uparrow}$$

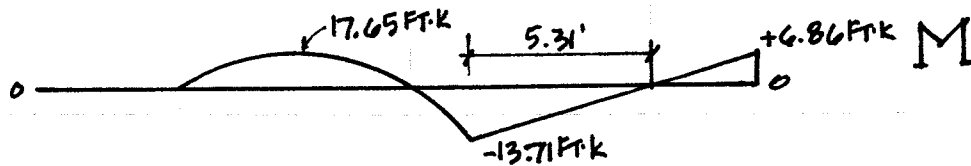
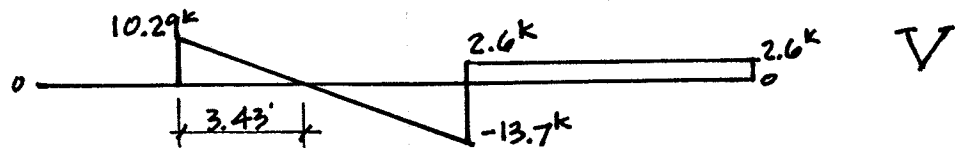
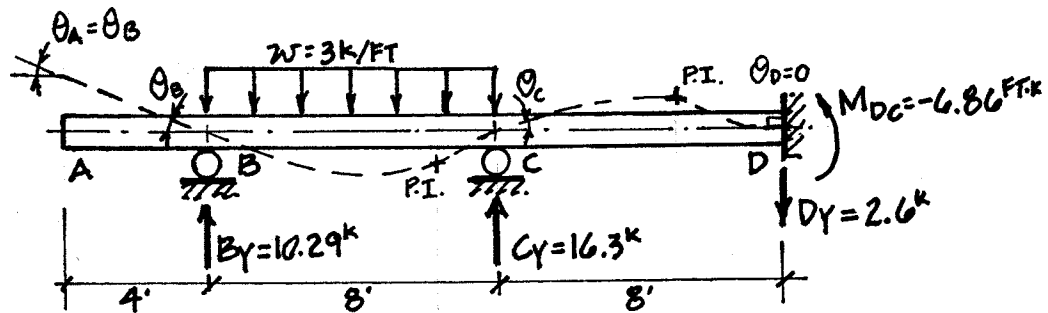
JT.C

$$\uparrow \sum F_Y = 0 \quad C_Y - V_{CB} - V_{CD} = 0$$

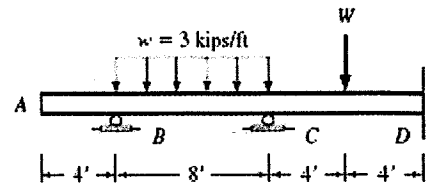
$$\underline{C_Y = 16.3 \text{ K} \uparrow}$$



ENTIRE STRUCTURE



P12.8. If no vertical deflection is permitted at end A for the beam in Figure P12.8, compute the required weight W that needs to be placed at midspan of CD. Given: $E = 29,000$ ksi and $I = 100$ in⁴.



P12.8

UNKNOWN: θ_c

$\theta_A = \theta_B$ UNLOADED CANTILEVER

THUS $M_{AB} = M_{BA} = 0$

$\theta_D = 0$

NO VERTICAL DISPLACEMENT @ A THUS $\theta_B = 0$

$$FEM_{BC/CB} = \pm \frac{wL^2}{12} = \frac{(3 \text{ K/FT})(8')^2}{12} = \pm 16 \text{ FT}\cdot\text{K}$$

$$FEM_{CD/DC} = \pm \frac{WL}{8} = \frac{W(8')}{8} = \pm W$$

$$M_{BC} = \frac{2EI}{8} (\theta_c) - 16 \text{ FT}\cdot\text{K} = 0.25\theta_c EI - 16$$

$$M_{CB} = \frac{2EI}{8} (2\theta_c) + 16 \text{ FT}\cdot\text{K} = 0.5\theta_c EI + 16$$

$$M_{CD} = \frac{2EI}{8} (2\theta_c) - W = 0.5\theta_c EI - W$$

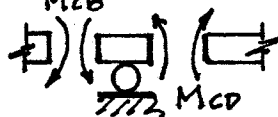
$$M_{DC} = \frac{2EI}{8} (\theta_c) + W = 0.25\theta_c EI + W$$

JT. B $\sum M_B = 0$; $M_{BC} = 0$
 $0.25\theta_c EI - 16 = 0$



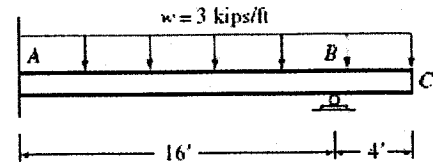
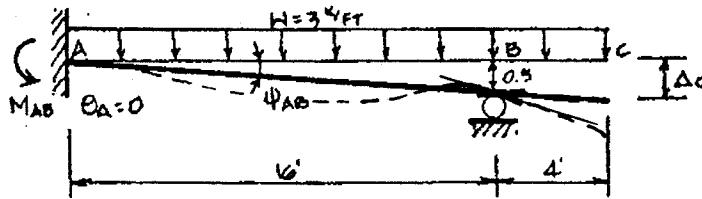
$$\theta_c = 64/EI$$

JT. C $\sum M_C = 0$ $M_{CB} + M_{CD} = 0$
 $(0.5\theta_c EI + 16) + (0.5\theta_c EI - W) = 0$
 $64 + 16 - W = 0$



$$W = 80 \text{ K}$$

P12.9. (a) Under the applied loads support B in Figure P12.9 settles 0.5 in. Determine all reactions. Given: $E = 30,000 \text{ kips/in}^2$, $I = 240 \text{ in}^4$. (b) Compute the deflection of point C.



P12.9

$$\psi_{AB} = +\frac{\Delta}{L} = \frac{0.5}{16(12)} = \frac{1}{384} \text{ RAD.}$$

$$E = 30,000 \text{ KSI}$$

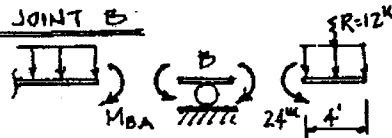
$$I = 240 \text{ IN}^4$$

A.) ANALYZE BY SLOPE DEFLECTION

$$M_{AB} = \frac{2EI}{16(12)} [\theta_B - 3\psi_{AB}] - \frac{wL^2}{12} \quad (1.)$$

$$M_{BA} = \frac{2EI}{16(12)} [2\theta_B - 3\psi_{AB}] + \frac{wL^2}{12} \quad (2.)$$

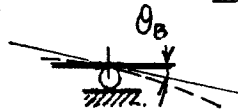
EQUILIBRIUM @ JOINT B



$$\sum M_B = 0; \quad 24 - M_{BA} = 0 \quad \underline{M_{BA} = 24(12) = 288 \text{ ft-k}} \quad (3.)$$

SUBSTITUTE (2) INTO (3.)

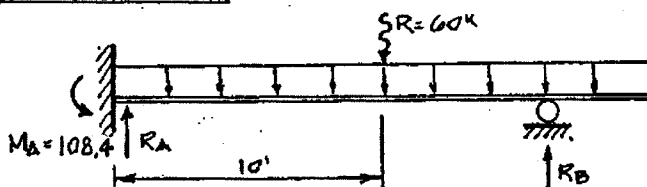
$$288 \text{ ft-k} = \frac{EI}{96} \left[2\theta_B - \frac{3}{384} \right] + 3(16)^2 \quad \underline{\theta_B = +0.00071 \text{ RAD.}}$$



$$M_{AB} = \left[\frac{2(30000)240}{192} \left[0.00071 - 0.00713 \right] - 768 \right] \frac{1}{12}$$

$$\underline{M_{AB} = -108.4 \text{ ft-k}}$$

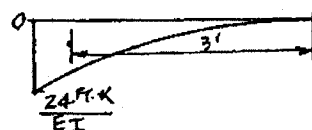
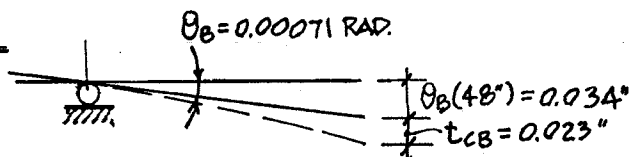
B.) REACTIONS



$$\sum M_A = 0; \quad 60(10) - 16R_B - 108.4 = 0 \quad \underline{R_B = 30.73 \text{ K}}$$

$$\sum F_y = 0; \quad 60 - R_B - R_A = 0 \quad \underline{R_A = 29.27 \text{ K}}$$

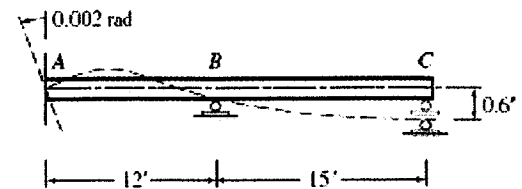
C.) Δ_c



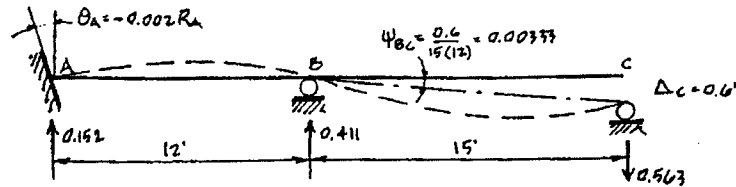
$$\Delta_{CB} = \frac{1}{3}(4) \frac{24(3)(1728)}{EI} = 0.023''$$

$$\Delta_c = 0.5'' + 0.034'' + 0.023'' = \underline{0.557''}$$

P12.10. In Figure P12.10, support *A* rotates 0.002 rad and support *C* settles 0.6 in. Draw the shear and moment curves. Given: $I = 144 \text{ in}^4$ and $E = 29,000 \text{ kips/in}^2$.



P12.10



$$I = 144 \text{ in}^4 ; E = 29,000 \text{ ksi}; \text{ ALL FEM} = 0$$

$$M_{AB} = 2EI/12(12) [2(-0.002) + \theta_B] = -0.008E + 2E\theta_B$$

$$M_{BA} = 2EI/12(12) [2\theta_B - 0.002] = 4\theta_B - 0.004E$$

$$M_{BC} = 2EI/15(12) [2\theta_B + \theta_C - 3(0.00333)] = 3.2\theta_B E + 1.6\theta_C E - 0.016E$$

$$M_{CB} = 2EI/15(12) [2\theta_C + \theta_B - 3(0.00333)] = 3.2\theta_C E + 1.6\theta_B E - 0.016E$$



$$(1) M_{BA} + M_{BC} = 0$$

$$(2) M_{CB} = 0$$

$$4E\theta_B - 0.004E + 3.2\theta_B E + 1.6\theta_C E - 0.016E = 0$$

$$(3) 7.2\theta_B E + 1.6\theta_C E = 0.02E$$

$$(4) -1.6\theta_B E - 3.2\theta_C E = -0.016E$$

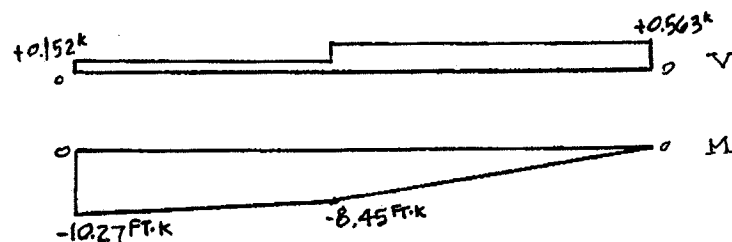
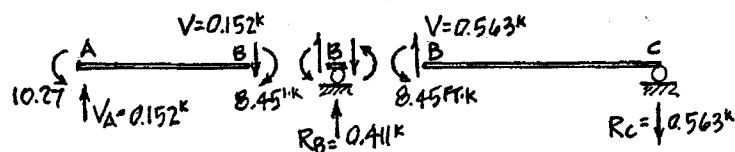
$$\theta_B = 0.001875 \text{ RADIAN}$$

SUBSTITUTE θ_B INTO EQUATION (4), SOLVING FOR θ_C

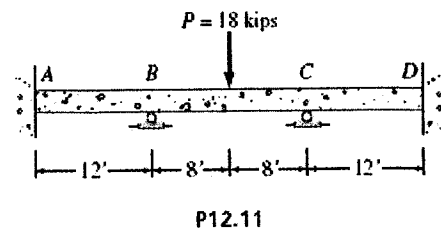
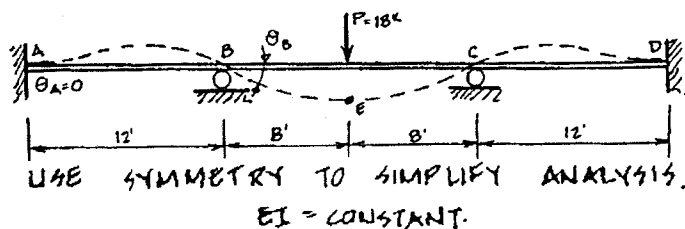
$$\theta_C = 0.004163 \text{ RADIAN}$$

$$M_{AB} = -0.008(29,000) + 2(29,000)(0.001875) = -123.25 \text{ FT-K} = -10.27 \text{ FT-K}$$

$$M_{BA} = 4(0.001875)(29,000) - 0.004(29,000) = 121.5 \text{ FT-K} = 8.45 \text{ FT-K}$$



P12.11. (a) Compute all reactions and draw the shear and moment curves for the beam in Figure P12.11. Given: EI is constant. (b) Compute the deflection under the load.



(A) $FEM = PL/8 = 18(16)/8 = 36 \text{ Ft}\cdot\text{K}$

BY SYMMETRY $\theta_B = -\theta_C$

$M_{AB} = (2EI/12) \theta_B$

$M_{AB} = EI \theta_B / 6$

$M_{BA} = (2EI/12) 2\theta_B$

$M_{BA} = EI \theta_B / 3$

$M_{BC} = (2EI/16) (2\theta_B + \theta_C) - 36$

SETTING $\theta_C = -\theta_B$: $M_{BC} = (EI/8) \theta_B - 36$

$M_{BC} = \frac{EI \theta_B}{8} - 36$

JOINT B: EQUATION OF EQUILIBRIUM : $\Sigma M_B = 0$

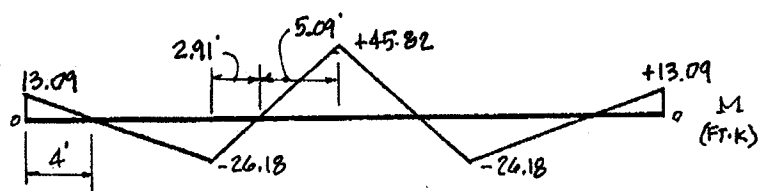
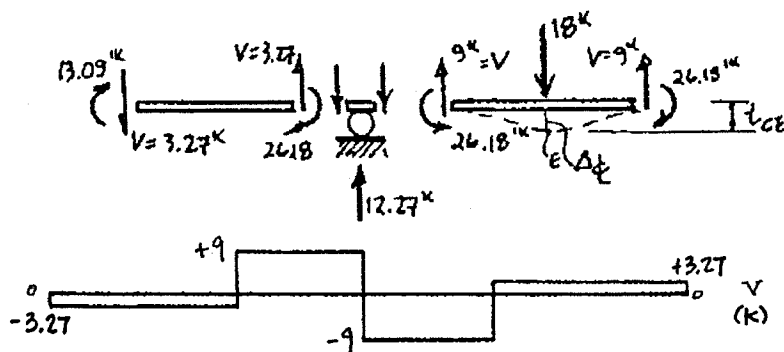


$M_{BA} + M_{BC} = 0$; $EI \theta_B / 3 + EI \theta_B / 8 - 36 = 0$

$\theta_B = 864 / 11EI$

$M_{AB} = 13.09 \text{ Ft}\cdot\text{K}$

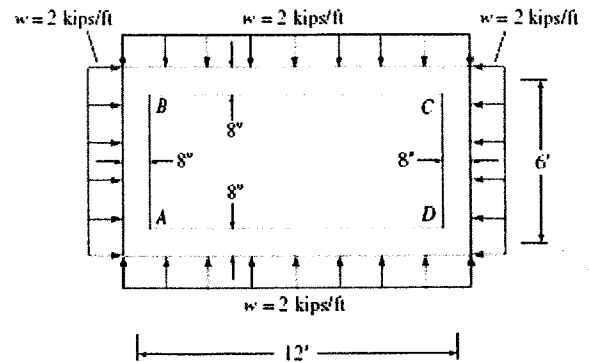
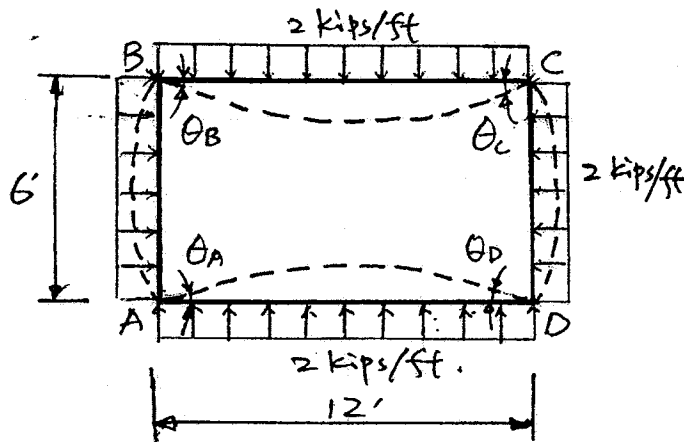
$M_{BA} = 26.18 \text{ Ft}\cdot\text{K}$



(B) $\Delta \delta = t_{CE} = \frac{1}{2} (5.09) (45.82 / EI) \left[2.91 + \frac{2}{3} (5.09) \right] - \frac{1}{2} (2.91) 26.18 / EI (2.91 / 3)$

$\Delta \delta = \underline{\underline{698.1 / EI}}$

P12.12. (a) Determine the member end moments for the rectangular ring in Figure P12.12, and draw the shear and moment curves for members AB and AD. The cross section of the rectangular ring is 12 in $\times$ 8 in and $E = 3000$ kips/in². (b) What is the axial force in member AD and in member AB?



P12.12
BY SYMMETRY,

$$\theta_A = -\theta_B = -\theta_D = \theta_C$$

THUS ONLY ONE UNKNOWN

$$FEM_{AD} = \frac{+WL^2}{12} = \frac{+(2)(12)^2}{12} = +24 \text{ kip}\cdot\text{ft}$$

$$FEM_{AB} = \frac{-WL^2}{12} = \frac{-2(6)^2}{12} = -6 \text{ kip}\cdot\text{ft}$$

$$\begin{aligned} (a) \quad M_{AD} &= \frac{2EI}{L} [2\theta_A + \theta_D] + 24 \\ &= \frac{2EI}{12} [2\theta_A - \theta_A] + 24 = \frac{EI}{6} \theta_A + 24 \\ M_{AB} &= \frac{2EI}{L} [2\theta_A + \theta_B] - 6 \\ &= \frac{2EI}{6} [2\theta_A - \theta_A] - 6 = \frac{EI}{3} \theta_A - 6 \end{aligned}$$

JOINT A :

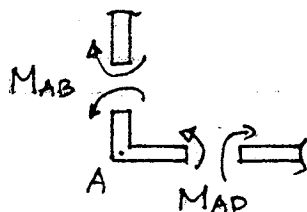
$$\sum M_A = 0; M_{AD} + M_{AB} = 0$$

$$\frac{EI}{6} \theta_A + 24 + \frac{EI}{3} \theta_A - 6 = 0$$

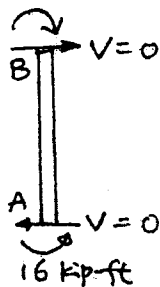
$$\theta_A = \frac{-36}{EI}$$

$$M_{AB} = \frac{EI}{3} \left(\frac{-36}{EI} \right) - 6 = -18 \text{ kip}\cdot\text{ft}$$

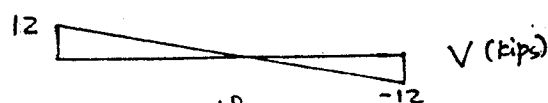
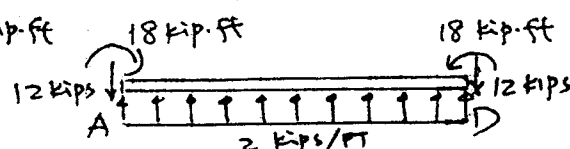
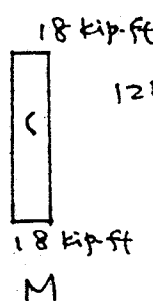
$$M_{AD} = \frac{EI}{6} \left(\frac{-36}{EI} \right) + 24 = +18 \text{ kip}\cdot\text{ft}$$



18 kip·ft



0
V

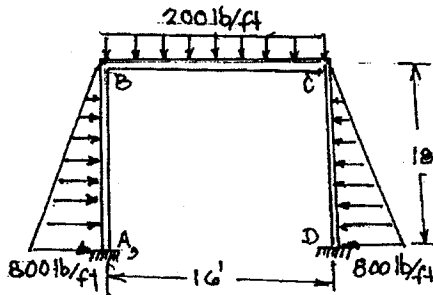


(b)

AXIAL FORCE IN AD = 0 KIP

AXIAL FORCE IN AB = 12 KIPS (COMPRESSION)

P12.13. Figure P12.13 shows the forces exerted by the soil pressure on a typical 1-ft length of a concrete tunnel as well as the design load acting on the top slab. Assume that a fixed-end condition at the bottom of the walls at A and D is produced by the connection to the foundation mat. Determine the member end moments and draw the shear and moment curves. Also draw the deflected shape. EI is constant.



FROM SYMMETRY: $\theta_B = -\theta_C$

FIXED END AT A & D: $\theta_A = \theta_D = 0$

SEE P. 442 TEXT FOR FEM'S

FIXED END MOMENTS MEMBER BC

$$FEM_{BC} = \frac{wL^2}{12} = \frac{0.2(16)^2}{12} = 4.266 \text{ kip}\cdot\text{ft}$$

$$FEM_{CB} = -\frac{wL^2}{12} = -\frac{0.2(16)^2}{12} = -4.266 \text{ kip}\cdot\text{ft}$$

$$FEM_{AB} = -\frac{wL^2}{20} = -\frac{0.8(18)^2}{20} = -12.96 \text{ kip}\cdot\text{ft}$$

$$FEM_{BA} = \frac{wL^2}{30} = \frac{0.8(18)^2}{30} = 8.64 \text{ kip}\cdot\text{ft}$$

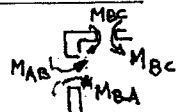
$$M_{BC} = \frac{2EI}{16} (2\theta_B + \theta_C) - 4.266 \text{ AND } \theta_C = -\theta_B$$

$$M_{BC} = \frac{EI}{8} (\theta_B) - 4.266$$

$$M_{BA} = \frac{2EI}{18} (2\theta_B + 0) + 8.64 = \frac{2EI}{9} \theta_B + 8.64$$

$$M_{AB} = \frac{2EI}{18} (\theta_B) - 12.96$$

JOINT Equil Eq



$$M_{BA} + M_{BC} = 0$$

$$\frac{4EI}{18} \theta_B + 8.64 + \frac{EI}{8} \theta_B - 4.266 = 0$$

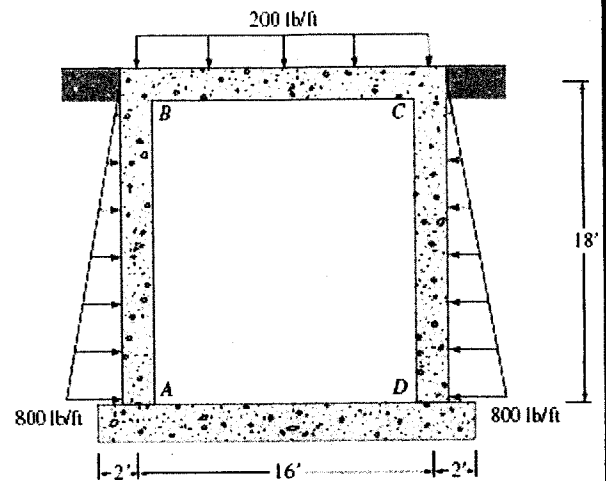
$$\theta_B = -\frac{12.61}{EI}$$

SUBSTITUTE θ_B INTO EQS FOR MOMENT

$$M_{BC} = \frac{EI}{8} \left(\frac{-12.61}{EI} \right) - 4.266 = 5.84 \text{ kip}\cdot\text{ft}$$

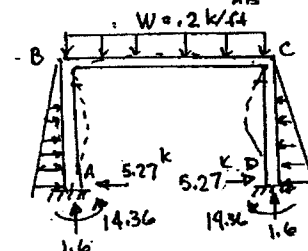
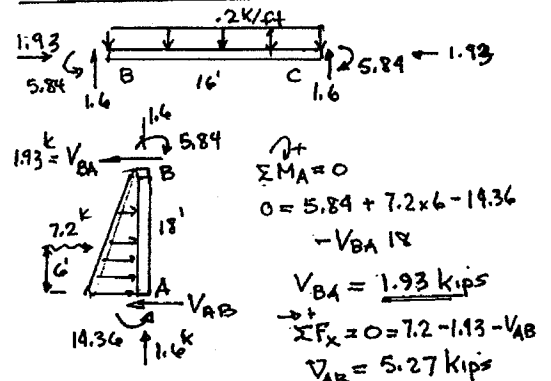
$$M_{AB} = \frac{EI}{9} \left(\frac{-12.61}{EI} \right) - 12.96 = -14.36 \text{ kip}\cdot\text{ft}$$

$$M_{BA} = \frac{2EI}{9} \left(\frac{-12.61}{EI} \right) + 8.64 = 5.84 \text{ kip}\cdot\text{ft}$$

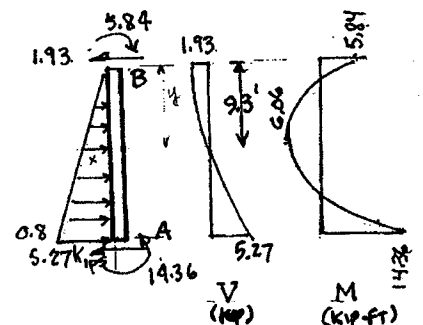
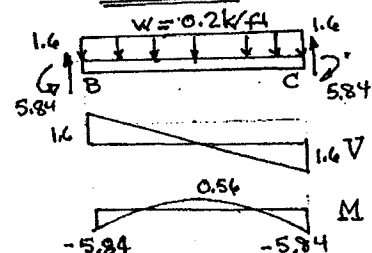


P12.13

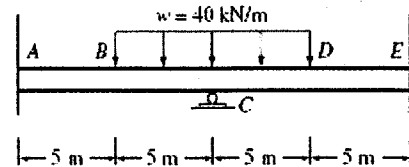
MEMBER FORCES



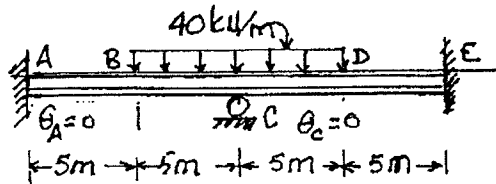
REACTIONS



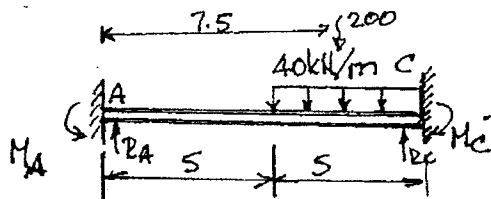
P12.14. Compute the reactions and draw the shear and moment curves for the beam in Figure P12.14. Given: $E = 200 \text{ GPa}$ and $I = 120 \times 10^6 \text{ mm}^4$.



P12.14



since the structure and the applied load are symmetrical with respect to a vertical axis thru support C, $\theta_C = 0$ and the joint at C can be treated as a fixed end.



$$M_A = FEM_A = -\frac{5wL^2}{192} \quad M_C = FEM_C = \frac{11wL^2}{192}$$

$$M_A = -\frac{5(40)(10)^2}{192} \quad M_C = \frac{11(40)(10)^2}{192}$$

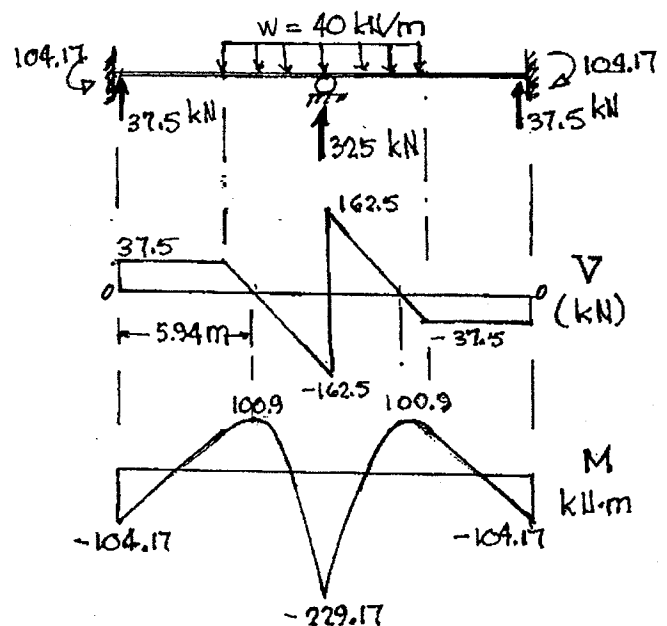
$$M_A = -104.17 \text{ kN}\cdot\text{m}; \quad M_C = 229.17 \text{ kN}\cdot\text{m}$$

$$\sum M_A = 0 = 200 \times 7.5 + 229.17 - 104.17 - R_C \times 10$$

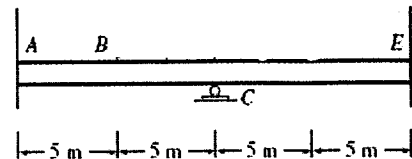
$$R_C = V_C = 162.5 \text{ kN}$$

$$\sum F_y = 0 = R_A - 200 + R_C$$

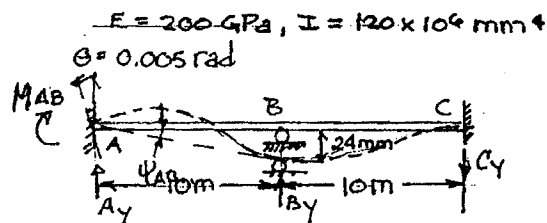
$$R_A = 200 - R_C = 37.5 \text{ kN}$$



P12.15. Consider the beam in Figure P12.14 without the applied load. Compute the reactions and draw the shear and moment curves for the beam if support C settles 24 mm and support A rotates counterclockwise 0.005 rad.



P12.14



KINEMATIC unknown: θ_B

$$\theta_A = -0.005, \theta_C = 0$$

$$\psi_{AB} = \psi_{BA} = \frac{\Delta_B}{L} = \frac{24}{10 \times 1000} = 0.0024 \text{ rad}$$

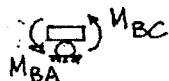
$$\psi_{BC} = \psi_{CB} = -0.0024 \text{ rad}$$

$$\frac{2EI}{L} = \frac{2 \times 200 \times 10^6 (120 \times 10^6)}{10} = 4800 \text{ kN}\cdot\text{m}$$

$$M_{BA} = \frac{2EI}{L} (2\theta_B + \theta_A - 3\psi_{BA}) \quad (1)$$

$$M_{BC} = \frac{2EI}{L} (2\theta_B - 3\psi_{BC}) \quad (2)$$

EQUILIBRIUM AT B



$$M_{BA} + M_{BC} = 0 \quad (3)$$

SUBSTI. EQ'S 1 & 2 INTO 3

$$4\theta_B + \theta_A - 3\psi_{BA} - 3\psi_{BC} = 0$$

$$\theta_B = -\frac{1}{4}\theta_A = -\frac{1}{4}(-0.005)$$

$$\theta_B = +0.00125 \text{ RADIANS}$$

$$M_{AB} = \frac{2EI}{L} (2\theta_A + \theta_B - 3\psi_{AB})$$

$$= 4800 [2 \times -0.005 + (0.00125) - (3 \times 0.0024)]$$

$$M_{AB} = -76.56 \text{ kN}\cdot\text{m}$$

M_{CB}

$$M_{CB} = \frac{2EI}{L} (2\theta_C + \theta_B - 3\psi_{CB})$$

$$= 4800 (0 + 0.00125 + 3 \times 0.0024)$$

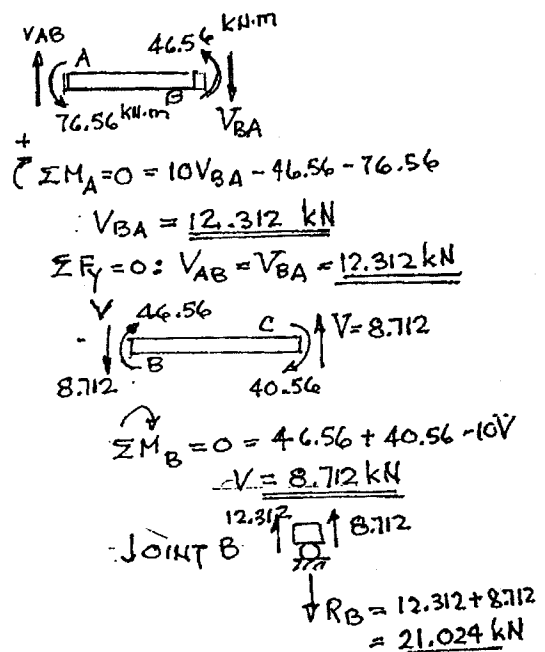
$$= 40.56 \text{ kN}\cdot\text{m}$$

M_{BC}

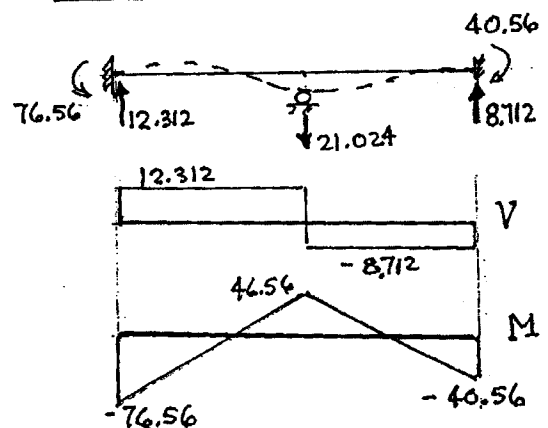
$$M_{BC} = 4800 (2\theta_B + \theta_C - 3\psi_{CB})$$

$$= 4800 (2 \times 0.00125 + 0 - 3 \times (-0.0024))$$

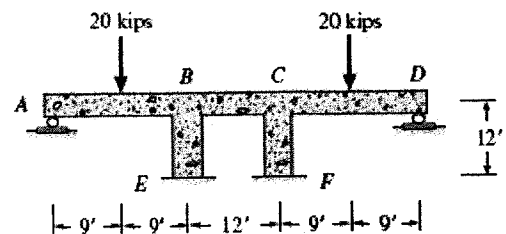
$$= 46.56 \text{ kN}\cdot\text{m}$$



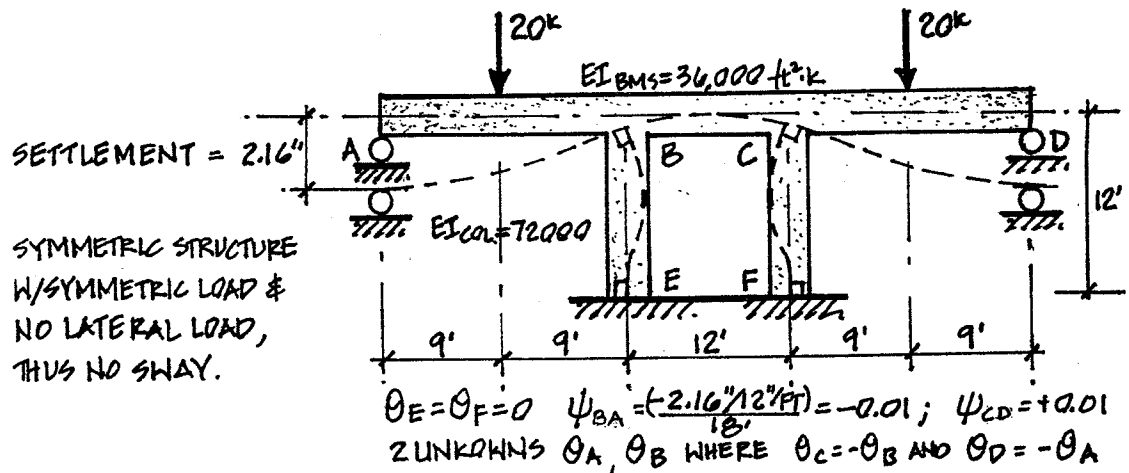
SHEAR & MOMENT DIAGRAMS



P12.16. Analyze the frame in Figure P12.16. In addition to the applied loads, supports *A* and *D* settle by 2.16 in. $EI = 36,000 \text{ kip-ft}^2$ for beams and $EI = 72,000 \text{ kip-ft}^2$ for columns. Use symmetry to simplify the analysis.



P12.16



$$M_{AB} = \frac{2(36,000 \text{ ft}^2 \cdot \text{k})}{18'} (2\theta_A + \theta_B - 3(-0.01)) - \frac{20 \text{ k}(18')}{8} = 8000\theta_A + 4000\theta_B + 75$$

$$M_{BA} = \frac{2(36,000 \text{ ft}^2 \cdot \text{k})}{18'} (2\theta_B + \theta_A - 3(-0.01)) + 45 \text{ ft} \cdot \text{k} = 8000\theta_B + 4000\theta_A + 165$$

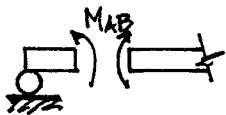
$$M_{BC} = \frac{2(36,000 \text{ ft}^2 \cdot \text{k})}{12'} (2\theta_B + \theta_C) = 6000\theta_B$$

WHERE $\theta_C = -\theta_B$

$$M_{BE} = \frac{2(72,000 \text{ ft}^2 \cdot \text{k})}{12'} (2\theta_B) = 24,000\theta_B$$

$$M_{EB} = \frac{2(72,000 \text{ ft}^2 \cdot \text{k})}{12'} (\theta_B) = 12,000\theta_B$$

JT. A $\sum M_A = 0$:

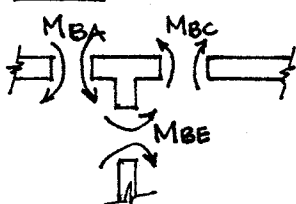


$$M_{AB} = 0$$

$$8000\theta_A + 4000\theta_B + 75 = 0$$

$$\theta_A = -0.009375 - 0.5\theta_B$$

JT. B $\sum M_B = 0$:



$$M_{BA} + M_{BC} + M_{BE} = 0$$

$$38000\theta_B + 4000\theta_A + 165 = 0$$

SUBSTITUTE θ_A FROM ABOVE, GIVES:

$$36,000\theta_B + 127.5 = 0$$

$$\theta_B = -0.003542$$

$$\theta_A = -0.0076$$

P12.16 Continued

P12.16 Continued

MEMBER END MOMENTS:

$$M_{AB} = 0$$

$$M_{BA} = +106.1 \text{ FT}\cdot\text{K}$$

$$M_{BC} = -21.3 \text{ FT}\cdot\text{K}$$

$$M_{BE} = -85 \text{ FT}\cdot\text{K}$$

$$M_{EB} = -42.2 \text{ FT}\cdot\text{K}$$

$$M_{DC} = 0$$

$$M_{CD} = -106.1 \text{ FT}\cdot\text{K}$$

$$M_{CB} = +21.3 \text{ FT}\cdot\text{K}$$

$$M_{CF} = +85 \text{ FT}\cdot\text{K}$$

$$M_{FC} = +42.2 \text{ FT}\cdot\text{K}$$

SEGMENT AB [CD IS OPPOSITE HAND]

$$\sum M_B = 0;$$

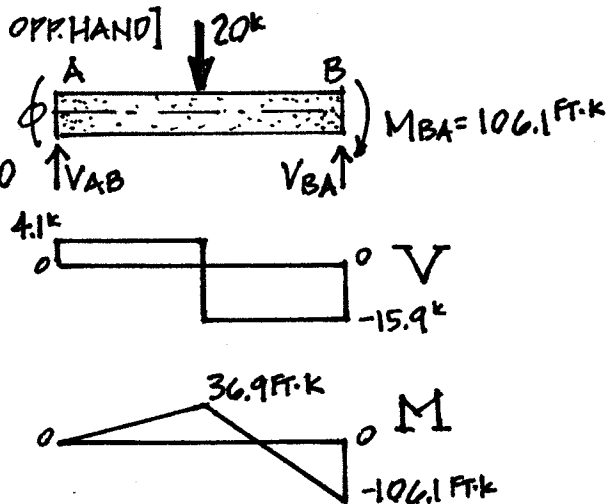
$$106.1 - 20^k(9') + V_{AB}(18') = 0$$

$$V_{AB} = 4.1^k \uparrow$$

$$\sum F_y = 0$$

$$V_{AB} - 20^k + V_{BA} = 0$$

$$V_{BA} = 15.9^k \uparrow$$

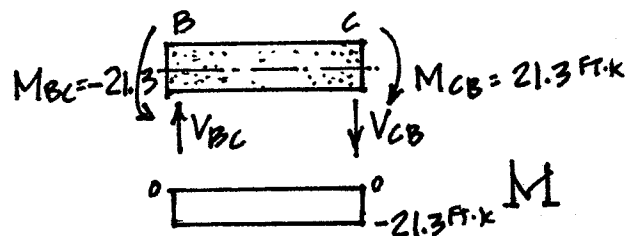


SEGMENT BC

$$\sum M_B = 0;$$

$$-21.3 + 21.3 - V_{BC}(12') = 0$$

$$V_{BC} = 0; \quad V_{CB} = 0$$



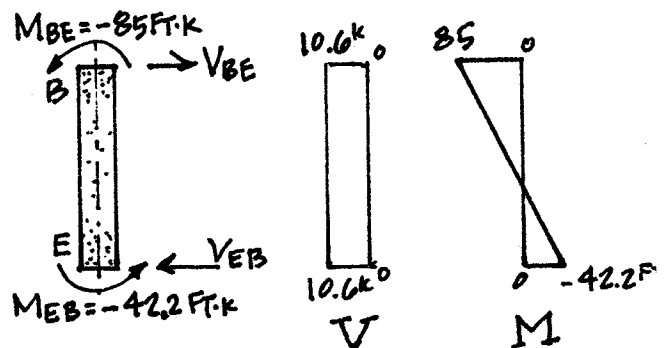
SEGMENT BE [CF IS OPPOSITE HAND]

$$\sum M_B = 0;$$

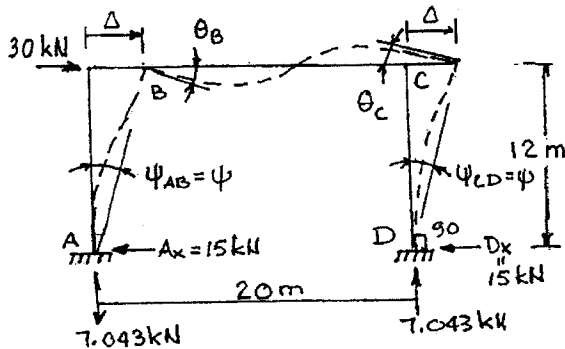
$$-42.2 \text{ FT}\cdot\text{K} - 85 \text{ FT}\cdot\text{K} + V_{EB}(12') = 0$$

$$V_{EB} = 10.6^k \leftarrow$$

$$V_{BE} = 10.6^k \rightarrow$$



P12.17. Analyze the frame in Figure P12.17. Given: EI is constant.



THE ANALYSIS CAN BE SIMPLIFIED CONSIDERING ANTISYMMETRY:

$$\begin{aligned}\theta_A &= \theta_D = 0 \\ \psi_{AB} &= \psi_{CD} = \psi \\ \theta_B &= \theta_C = \theta\end{aligned}$$

SINCE MEMBER AB AND CD HAVE THE SAME BOUNDARY CONDITIONS AND SAME PROPERTIES: E, I , ETC

THE HORIZONTAL REACTIONS ARE EQUAL
I.E., $A_x = D_x = 15 \text{ kN}$

$$\begin{aligned}M_{AB} &= \frac{2EI}{12}(\theta_B - 3\psi) = \frac{EI}{6}(\theta - 3\psi) \\ M_{BA} &= \frac{2EI}{12}(2\theta_B - 3\psi) = \frac{EI}{6}(2\theta - 3\psi) \\ M_{BC} &= \frac{2EI}{20}(2\theta_B + \theta_C) = \frac{EI}{10}(3\theta)\end{aligned} \quad (1)$$

WRITE EQUIL. EQ'S

1. JOINT B: $M_{BA} + M_{BC} = 0$

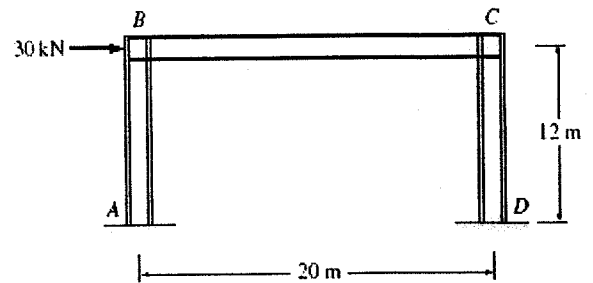
$$\begin{aligned}M_{BC} &= \frac{EI}{10}(3\theta) \\ M_{BA} &= \frac{EI}{6}(2\theta - 3\psi) \\ \frac{EI}{6}(2\theta - 3\psi) + \frac{EI}{10}(3\theta) &= 0 \\ \theta &= \frac{15}{19}\psi \quad (2)\end{aligned}$$

2. SHEAR EQ FOR AB: $\sum M_A = 0$

$$\begin{aligned}M_{BA} + M_{AB} - 15^k(12\text{m}) &= 0 \\ \frac{EI}{6}(2\theta - 3\psi) + \frac{EI}{6}(\theta - 3\psi) &= 180 \\ \text{SUBSTI } \psi &= 19/15\theta\end{aligned}$$

$$\begin{aligned}(2\theta - 3(\frac{19\theta}{15}) + (\theta - 3(\frac{19\theta}{15})) &= 180(\frac{6}{EI}) \\ \theta &= \frac{234.783}{EI} \quad (3)\end{aligned}$$

$$\begin{aligned}\psi &= \frac{19}{15}\theta = \frac{234.783}{15} \quad (19) \\ \psi &= \frac{297.391}{EI} \quad (4)\end{aligned}$$



P12.17

SUBSTI. EQ'S 2 AND 3 INTO (1)

$$M_{AB} = \frac{EI}{6}(\frac{234.783}{EI} - 3[\frac{297.783}{EI}])$$

$$M_{AB} = -109.565 \text{ kN}\cdot\text{m}$$

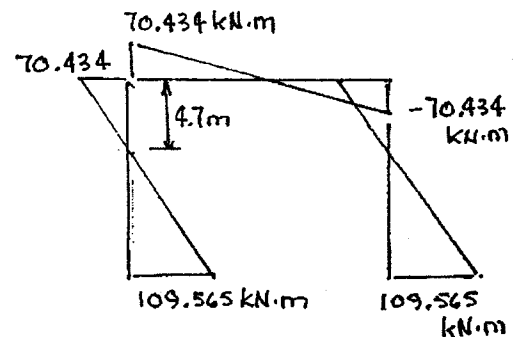
$$M_{BA} = \frac{EI}{6}(2 \times \frac{234.783}{EI} - 3 \times \frac{297.783}{EI})$$

$$M_{BA} = -70.434 \text{ kN}\cdot\text{m}$$

$$M_{BC} = \frac{EI}{10}(3 \times \frac{234.783}{EI}) = 70.434 \text{ kN}\cdot\text{m}$$

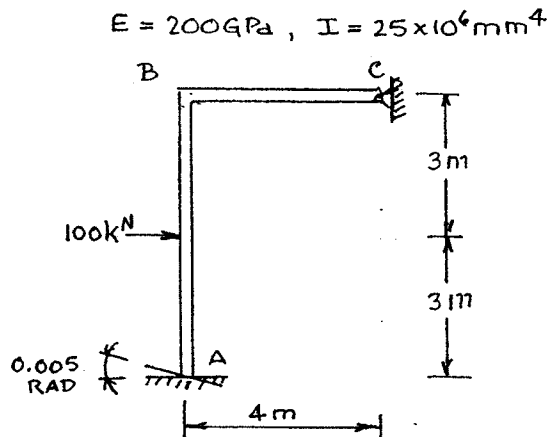
SHEARS IN BC

$$\begin{aligned}\sum M_B &= 0 \\ 0 &= 70.434 + 70.434 - V(20) \\ V &= 7.043 \text{ kN}\end{aligned}$$



MOMENT DIAGRAM

P12.18. Analyze the structure in Figure P12.18. In addition to the applied load, support *A* rotates clockwise by 0.005 rad. Also $E = 200 \text{ GPa}$ and $I = 25 \times 10^6 \text{ mm}^4$ for all members.



$$FEM_{AB} = -\frac{PL}{8} = -\frac{100(6)}{8} = -75, FEM_{BA} = 75 \text{ kN}\cdot\text{m}$$

MEMBER END MOMENTS

$$\left. \begin{aligned} M_{AB} &= \frac{2EI}{6} (2 \times 0.005 + \theta_B) - 75 \text{ kN}\cdot\text{m} \\ M_{BA} &= \frac{2EI}{6} (2\theta_B + 0.005) + 75 \\ M_{BC} &= \frac{2EI}{4} (2\theta_B + \theta_C) \\ M_{CB} &= \frac{2EI}{4} (2\theta_C + \theta_B) \end{aligned} \right\} \text{EQ. 1.}$$

JOINT EQUIL. EQS.

$$\begin{aligned} \text{JOINT C} \\ M_{CB} = 0 &= \frac{2EI}{4} (2\theta_C + \theta_B) \\ \theta_C &= -\frac{\theta_B}{2} \quad (2) \end{aligned}$$

JOINT B

$$\begin{aligned} \text{JOINT B} \\ M_{BC} + M_{BA} &= 0 \\ \frac{2EI}{4} (2\theta_B - \frac{\theta_B}{2}) + \frac{EI}{3} (2\theta_B + 0.005) + 75 &= 0 \\ \frac{EI}{2} (\frac{3\theta_B}{2}) + \frac{EI}{3} (2\theta_B + 0.005) &= -75 \end{aligned}$$

$$\theta_B = -0.011767 \text{ RADIANS} \quad (3)$$

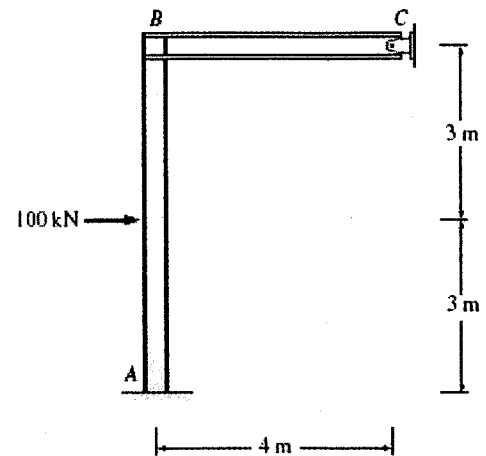
$$\theta_C = -\frac{\theta_B}{2} = 0.005884 \text{ RADIANS} \quad (4)$$

SUBSTI. (3) & (4) INTO EQ'S 1

$$\begin{aligned} M_{BC} &= \frac{EI}{2} (-2 \times 0.011767 + 0.005884) \\ &= \frac{200 \times 25}{2} (-0.019456) = -44.125 \text{ kN}\cdot\text{m} \end{aligned}$$

$$M_{BA} = 44.125 \text{ kN}\cdot\text{m}$$

$$M_{AB} = -77.94 \text{ kN}\cdot\text{m}$$

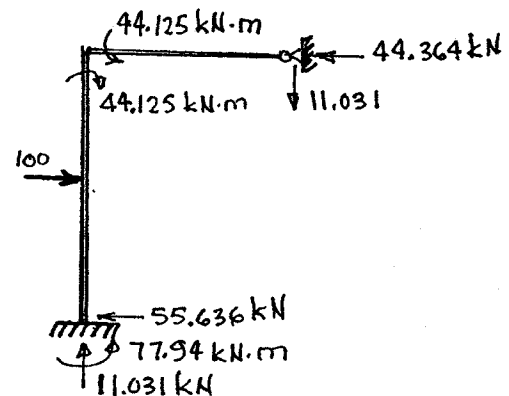


P12.18

ANALYSIS OF MEMBERS

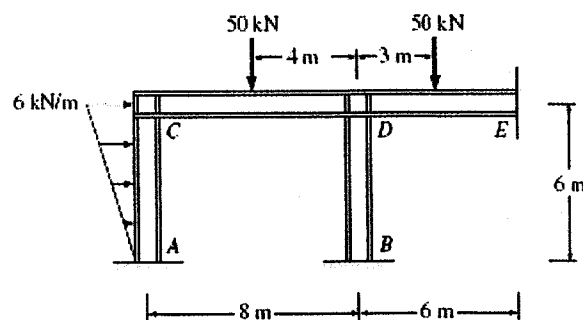
$$\begin{aligned} \text{MEMBER BC} \\ V_{BC} = 44.125 \text{ kN} \\ \sum M_B = 0 = C_y \cdot 4 - 44.125 \\ C_y = 11.031 \text{ kN} \downarrow \\ V_{BC} = 11.031 \text{ kN} \uparrow \end{aligned}$$

$$\begin{aligned} \text{MEMBER AB} \\ 44.364 = V_{BA} \\ M_{BA} = 44.125 \\ \sum M_A = 0 \\ 44.125 + 100 \times 3 - 77.94 - V_{BA} \cdot 6 = 0 \\ V_{BA} = 44.364 \text{ kN} \leftarrow \\ V_{AB} = 100 - 44.364 = 55.636 \text{ kN} \leftarrow \end{aligned}$$



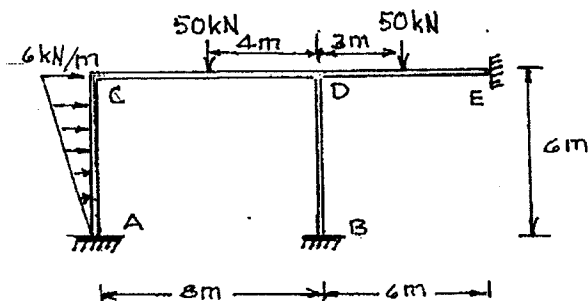
REACTIONS AND MEMBER END MOMENTS

P12.19. Analyze the frame in Figure P12.19. Given: EI is constant.



P12.19

EI IS CONSTANT



$$FEM_{CA} = \frac{wL^2}{20} = \frac{6(6)^2}{20} = 10.8 \text{ kN}\cdot\text{m}$$

$$FEM_{AC} = \frac{wL^2}{30} = \frac{6(6)^2}{30} = -7.2 \text{ kN}\cdot\text{m}$$

$$FEM_{CD} = \frac{PL}{8} = -\frac{50(8)}{8} = -50 \text{ kN}\cdot\text{m}$$

$$FEM_{DC} = \frac{PL}{8} = 50 \text{ kN}\cdot\text{m}$$

$$FEM_{DE} = -\frac{PL}{8} = -\frac{50 \times 6}{8} = -37.5 \text{ kN}\cdot\text{m}$$

$$FEM_{ED} = +\frac{PL}{8} = +37.5 \text{ kN}\cdot\text{m}$$

MEMBER END-MOMENTS

$$\theta_A = \theta_B = \theta_E = 0$$

$$M_{AC} = \frac{2EI}{6}(\theta_C) - 7.2$$

$$M_{CA} = \frac{2EI}{6}(2\theta_C) + 10.8$$

$$M_{CD} = \frac{2EI}{8}(2\theta_C + \theta_D) - 50$$

$$M_{DC} = \frac{2EI}{8}(2\theta_D + \theta_C) + 50$$

$$M_{DB} = \frac{2EI}{6}(2\theta_D)$$

$$M_{BD} = \frac{2EI}{6}(\theta_D)$$

$$M_{DE} = \frac{2EI}{6}(2\theta_D) - 37.5$$

$$M_{ED} = \frac{2EI}{6}(\theta_D) + 37.5$$

JOINT EQUIL EQ'S

$$(1) \sum M_C = 0; \frac{2EI}{6}(2\theta_D) + 10.8 + \frac{2EI}{8}(2\theta_C + \theta_D) - 50 = 0 \quad (1)$$

$$(2) \sum M_D = 0; M_{DC} + M_{DB} + M_{DE} = 0$$

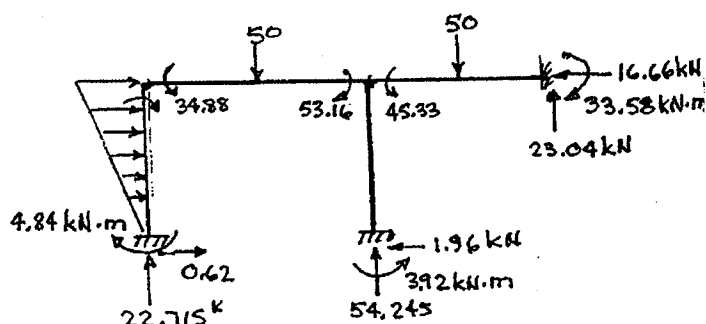
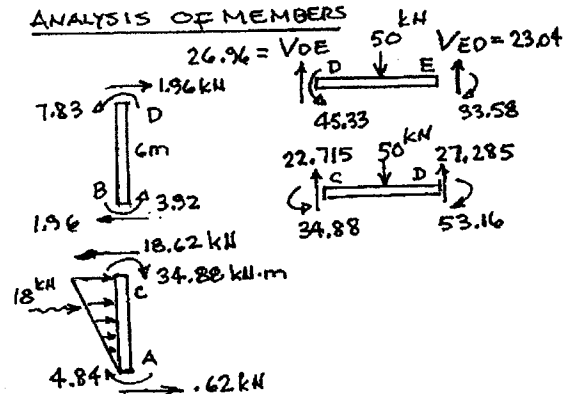
$$\frac{2EI}{8}(2\theta_D + \theta_C) + 50 + \frac{2EI}{6}(2\theta_D) + \frac{2EI}{6}(2\theta_D) - 37.5 = 0 \quad (2)$$

$$\text{SOLVING EQ'S (1) AND (2)} \quad \theta_C = \frac{36.116}{EI} \quad \theta_D = -\frac{11.74}{EI}$$

SUBSTITUTE VALUES OF θ_C AND θ_D INTO EQ'S FOR MEMBER END MOMENTS

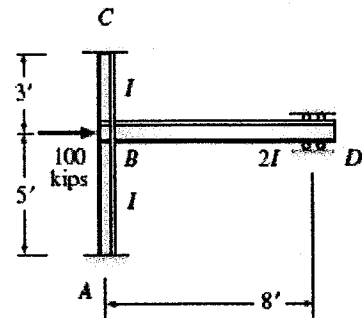
$$\begin{aligned} M_{AC} &= 4.84 \text{ kN}\cdot\text{m} & M_{DE} &= -45.33 \text{ kN}\cdot\text{m} \\ M_{CA} &= 34.88 \text{ kN}\cdot\text{m} & M_{ED} &= 33.58 \text{ kN}\cdot\text{m} \\ M_{CD} &= 34.88 \text{ kN}\cdot\text{m} & M_{DC} &= 53.16 \text{ kN}\cdot\text{m} \\ M_{DB} &= -7.83 \text{ kN}\cdot\text{m} & & \\ M_{BD} &= -3.92 \text{ kN}\cdot\text{m} & & \end{aligned}$$

ANALYSIS OF MEMBERS



REACTIONS AND MEMBER END MOMENTS

P12.20. Analyze the frame in Figure P12.20. Note that support D can translate in the horizontal direction only. Compute all reactions and draw the shear and moment curves. Given: $E = 29,000$ ksi and $I = 100$ in⁴.



P12.20

2 UNKNOWN $\theta_B, \Delta_x @ D$
 $\theta_A = \theta_C = \theta_D = 0$

$$\psi_{AB} = \frac{\Delta_x / 12' \text{ FT}}{5'} = 0.01667 \Delta_x$$

$$\psi_{CB} = \frac{-\Delta_x / 12' \text{ FT}}{3'} = -0.02778 \Delta_x$$

COUNTERCLOCKWISE

$$EI = \frac{29,000 \text{ KSI} (100 \text{ in}^4)}{144 \text{ in}^2 / \text{ft}^2} = 20,138.9 \text{ ft}^2 \cdot \text{k}$$

$$M_{AB} = \frac{2EI}{5'} (2\theta_A^0 + \theta_B - 3(0.01667 \Delta_x)) = (0.4\theta_B - 0.02 \Delta_x) EI$$

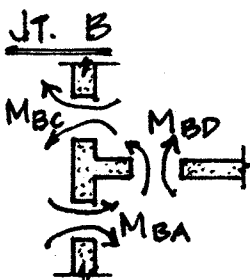
$$M_{BA} = \frac{2EI}{5'} (2\theta_B + \theta_A^0 - 3(0.01667 \Delta_x)) = (0.8\theta_B - 0.02 \Delta_x) EI$$

$$M_{BC} = \frac{2EI}{3'} (2\theta_B + \theta_C^0 - 3(-0.02778 \Delta_x)) = (1.33\theta_B + 0.0556 \Delta_x) EI$$

$$M_{CB} = \frac{2EI}{3'} (2\theta_C^0 + \theta_B - 3(-0.02778 \Delta_x)) = (0.667\theta_B + 0.0556 \Delta_x) EI$$

$$M_{BD} = \frac{2E(2I)}{8'} (2\theta_B + \theta_D^0 - 0) = \theta_B EI$$

$$M_{DB} = \frac{2E(2I)}{8'} (2\theta_D^0 + \theta_B - 0) = 0.5\theta_B EI$$

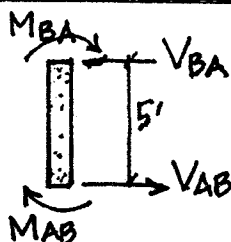


$$\sum M_B = 0; \quad M_{BA} + M_{BC} + M_{BD} = 0$$

$$(3.133\theta_B + 0.03556 \Delta_x) EI = 0$$

$$\theta_B = -0.011349 \Delta_x$$

SEGMENT AB



$$\sum M_A = 0; \quad M_{AB} + M_{BA} - V_{BA}(5') = 0$$

$$V_{BA} = \frac{M_{AB} + M_{BA}}{5'} = \frac{(1.2\theta_B - 0.04 \Delta_x) EI}{5'}$$

$$V_{BA} = (0.24\theta_B - 0.008 \Delta_x) EI$$

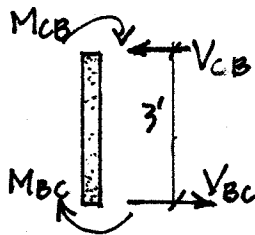
WHERE $\theta_B = -0.011349 \Delta_x$

$$V_{BA} = -0.0107 \Delta_x EI \rightarrow$$

P12.20 Continued

P12.20 Continued

SEGMENT BC



$$\sum \overset{+}{\curvearrowright} M_C = 0;$$

$$M_{BC} + M_{CB} - V_{BC}(3') = 0$$

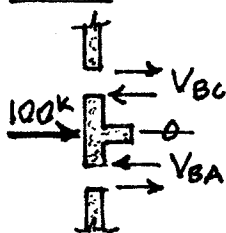
$$V_{BC} = \frac{M_{BC} + M_{CB}}{3'} = \frac{(2\theta_B + 0.111\Delta x)EI}{3'}$$

$$V_{BC} = (0.667\theta_B + 0.03703\Delta x)EI$$

$$\text{WHERE } \theta_B = -0.011349\Delta x$$

$$V_{BC} = 0.0294\Delta x EI \rightarrow$$

JT. B



$$\sum \overset{+}{\rightarrow} F_x = 0;$$

$$100k - V_{BA} - V_{BC} = 0$$

$$100k - 0.0107\Delta x EI - 0.0294\Delta x EI = 0$$

$$\Delta x = 0.124''$$

$$\theta_B = -0.011349\Delta x = -0.001407 \text{ RAD.}$$

MEMBER END FORCES:

$$M_{AB} = -61.2 \text{ Ft.k}$$

$$M_{BA} = -72.6 \text{ Ft.k}$$

$$M_{BC} = 101.1 \text{ Ft.k}$$

$$M_{CB} = 119.9 \text{ Ft.k}$$

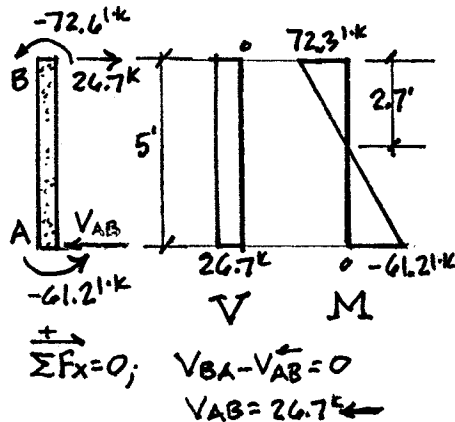
$$M_{BD} = -28.3 \text{ Ft.k}$$

$$M_{DB} = -14.2 \text{ Ft.k}$$

$$V_{BA} = 26.72 \text{ k}$$

$$V_{BC} = 73.4 \text{ k}$$

SEGMENT AB

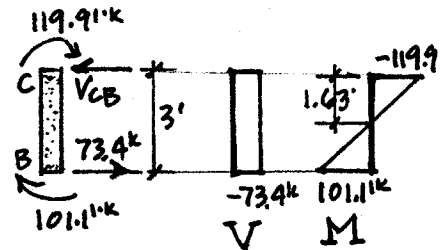


$$\sum \overset{+}{\rightarrow} F_x = 0;$$

$$V_{BA} - V_{AB} = 0$$

$$V_{AB} = 26.7 \text{ k}$$

SEGMENT BC



$$\sum \overset{+}{\rightarrow} F_x = 0;$$

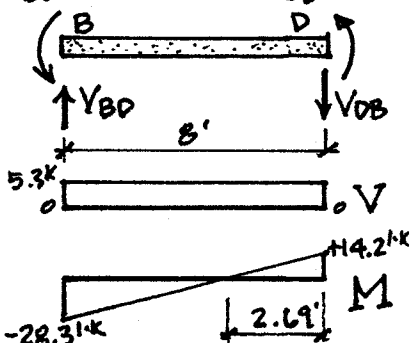
$$V_{BC} - V_{CB} = 0$$

$$V_{CB} = 73.4 \text{ k}$$

SEGMENT BD

$$M_{BD} = -28.3 \text{ k}$$

$$M_{DB} = -14.2 \text{ k}$$



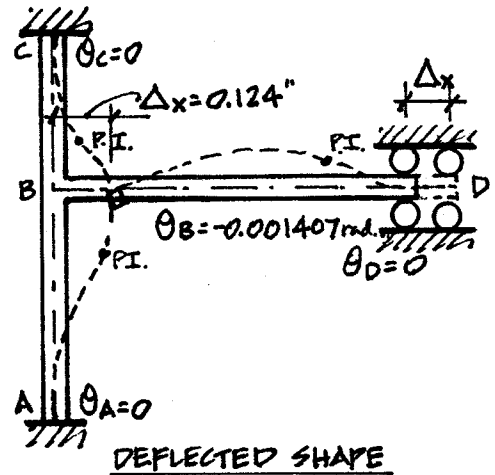
$$\sum \overset{+}{\curvearrowright} M_B = 0$$

$$-28.3 - 14.2 + V_{DB}(8') = 0$$

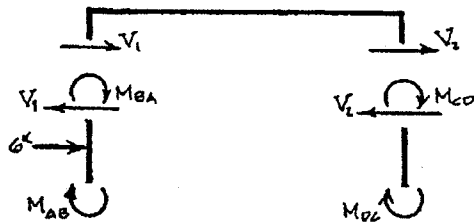
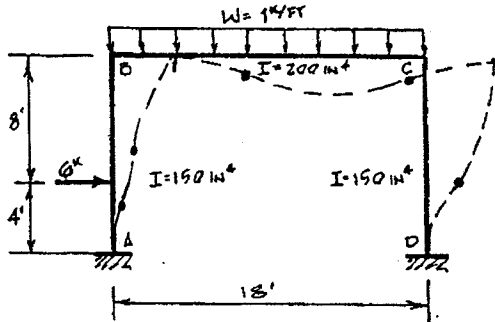
$$V_{DB} = 5.3 \text{ k}$$

$$\sum \overset{+}{\uparrow} F_y = 0;$$

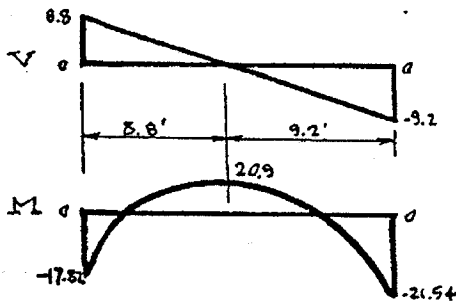
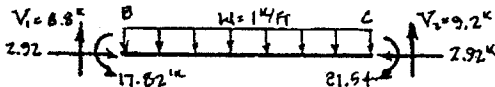
$$V_{BD} = -V_{DB} = 5.3 \text{ k}$$



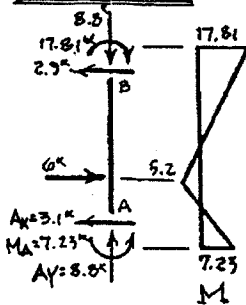
P12.21. Analyze the frame in Figure P12.21. Compute all reactions. Also $I_{BC} = 200 \text{ in}^4$ and $I_{AB} = I_{CD} = 150 \text{ in}^4$. E is constant.



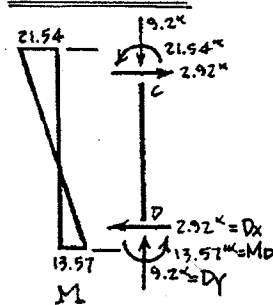
MEMBER "BC":



MEMBER "AB":



MEMBER "CD":



UNKNOWN θ_B , θ_C AND Δ

LET $K_{COL} = K = EI/12$

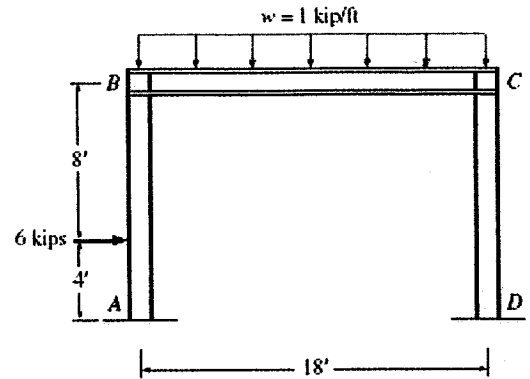
THEN $K_{BH} = 4EI/3(18) = 8K/9$

$FEM_{AB} = Pab^2/L^2 = 6(4)(64)/(12)^2 = -10.67 \text{ FT-K}$

$FEM_{BA} = Pba^2/L^2 = 6(8)(16)/(12)^2 = 5.33 \text{ FT-K}$

$FEM_{BC} = WL^2/12 = -1(18)^2/12 = -27 \text{ FT-K}$

$FEM_{CB} = -FEM_{BC} = +27 \text{ FT-K}$



P12.21

EXPRESS MEMBER END MOMENTS W/ SLOPE DEFN EQN

$$M_{AB} = \frac{2EI}{12} [\theta_B - 3\psi] - 10.67 = 2K\theta_B - 6K\psi - 10.67$$

$$M_{BA} = \frac{2EI}{12} [2\theta_B - 3\psi] + 5.33 = 4K\theta_B - 6K\psi + 5.33$$

$$M_{BC} = \frac{2(8K)}{5} [2\theta_B + \theta_C] - 27 = 3.56K\theta_B + 1.78K\theta_C - 27$$

$$M_{CB} = \frac{2(8K)}{5} [2\theta_C + \theta_B] + 27 = 1.78K\theta_B + 3.56K\theta_C + 27$$

$$M_{CD} = 2K [2\theta_C - 3\psi] = 4K\theta_C - 6K\psi$$

$$M_{DC} = 2K [\theta_C - 3\psi] = 2K\theta_C - 6K\psi$$

EQUILIBRIUM EQUATIONS

$$\text{JOINT B: } M_{BA} + M_{BC} = 0 \quad (1)$$

$$\text{JOINT C: } M_{CB} + M_{CD} = 0 \quad (2)$$

$$\sum F_x = 0; V_1 + V_2 = 0 \quad (3)$$

$$\sum M_A = 0; M_{AB} + M_{BA} + 4(4) - V_1(12) = 0$$

$$V_1 = \frac{M_{AB} + M_{BA} + 24}{12}$$

$$V_2 = \frac{M_{CD} + M_{DC}}{12}$$

SUBSTITUTE EQN (4) INTO (3)

$$\frac{M_{AB} + M_{BA} + 24}{12} + \frac{M_{CD} + M_{DC}}{12} = 0 \quad (5)$$

EXPRESS EQNS (1), (2) AND (5) IN TERMS OF DISPLMTS

$$3.56K\theta_B + 1.78K\theta_C - 6K\psi = 21.67$$

$$1.78K\theta_B + 3.56K\theta_C - 6K\psi = -27.00$$

$$6K\theta_B + 6K\theta_C - 24K\psi = -18.66$$

SOLVING GIVES:

$$\psi = 0.935/K; \theta_B = 4.523/K; \theta_C = -3.893/K$$

SUBSTITUTING VALUES OF DISPLACMTS INTO M1 EQNS

$$M_{AB} = -7.23 \text{ FT-K}$$

$$M_{BA} = 17.82 \text{ FT-K}$$

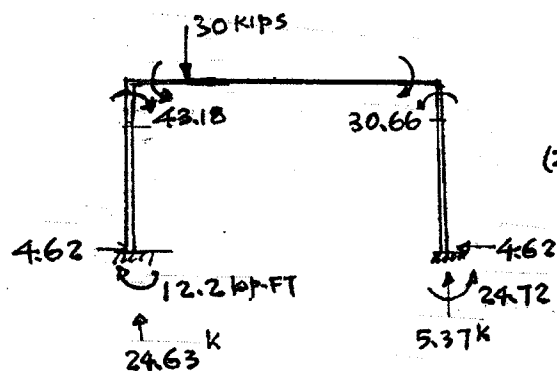
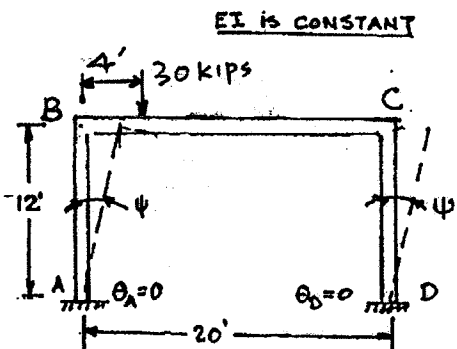
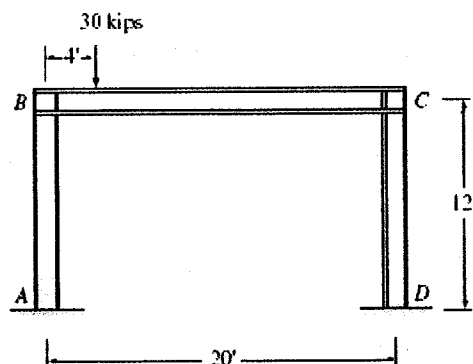
$$M_{CD} = -21.54 \text{ FT-K}$$

$$M_{DC} = 13.57 \text{ FT-K}$$

$$M_{BC} = -17.82 \text{ FT-K}$$

$$M_{CB} = 21.54 \text{ FT-K}$$

P12.22. Analyze the frame in Figure P12.22. Notice that sidesway is possible because the load is unsymmetric. Compute the horizontal displacement of joint B. Given: $E = 29,000 \text{ kips/in}^2$ and $I = 240 \text{ in}^4$ for all members.

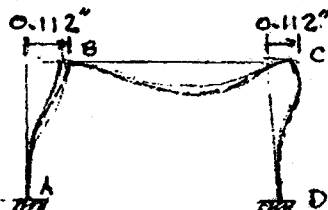


$$M_{AB} = \frac{2K(15.49)}{K} - 6K\left(\frac{3.13}{K}\right) = -12.2 \text{ kip}\cdot\text{ft}$$

$$M_{BA} = \frac{4K(15.49)}{K} - 6K\left(\frac{3.13}{K}\right) = 43.18 \text{ kip}\cdot\text{ft}$$

$$M_{CD} = \frac{4K(-2.97)}{K} - 6K\left(\frac{3.13}{K}\right) = -30.66 \text{ kip}\cdot\text{ft}$$

$$M_{DC} = \frac{2K(-2.97)}{K} - 6K\left(\frac{3.13}{K}\right) = -24.72 \text{ kip}\cdot\text{ft}$$



UNKNOWN: θ_B, θ_C, ψ

FOR COLUMNS $K_c = \frac{EI}{12} = K$

P12.22

FOR GIRDERS $K_g = \frac{EI}{20} = .6K$

$$FEM_{BC} = -\frac{Pba^2}{L^2} = -\frac{30(16)^2(4)}{20^3} = -76.8 \text{ kip}\cdot\text{ft}$$

$$FEM_{CB} = \frac{Pba^2}{L^2} = \frac{30(16)(4)^2}{20^3} = 19.2 \text{ kip}\cdot\text{ft}$$

(1) USE SLOPE-DEFL. EQ.

$$M_{AB} = 2K(\theta_B - 3\psi) = 2K\theta_B - 6K\psi$$

$$M_{BA} = 2K(2\theta_B - 3\psi) = 4K\theta_B - 6K\psi$$

$$M_{BC} = 2(.6K)(2\theta_B + \theta_C) - 76.8 = 2.4K\theta_B + 1.2K\theta_C - 76.8$$

$$M_{CB} = 2(.6K)(2\theta_C + \theta_B) + 19.2 = 2.4K\theta_C + 1.2K\theta_B + 19.2$$

$$M_{CD} = 2K(2\theta_C - 3\psi) = 4K\theta_C - 6K\psi$$

$$M_{DC} = 2K(\theta_C - 3\psi) = 2K\theta_C - 6K\psi$$

(2) EQUILIBRIUM EQUATIONS

$$\text{JOINT B: } M_{BA} + M_{BC} = 0 \quad (a)$$

$$\text{JOINT C: } M_{CB} + M_{CD} = 0 \quad (b)$$

$$\text{SHEAR EQ: } \frac{M_{BA} + M_{AB}}{12} + \frac{M_{CD} + M_{DC}}{12} = 0 \quad (c)$$

EXPRESS EQUILIBRIUM EQS in terms of displacements

$$6.4K\theta_B + 1.2K\theta_C - 6K\psi = 76.8 \quad (a')$$

$$1.2K\theta_B + 6.4K\theta_C - 6K\psi = -19.2 \quad (b')$$

$$.45K\theta_B + .5K\theta_C - 2K\psi = 0 \quad (c')$$

SOLVE EQ'S FOR θ_B, θ_C & ψ AND

SUBSTITUTE INTO EQ in (1)

$$\theta_B = 15.49/K$$

$$\theta_C = -2.97/K$$

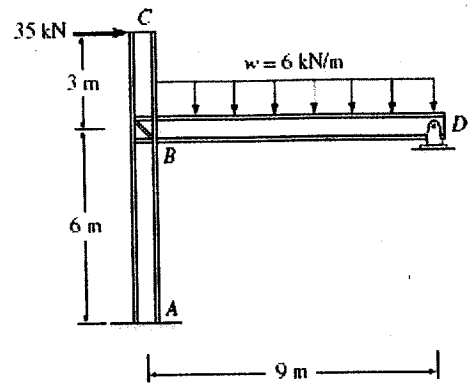
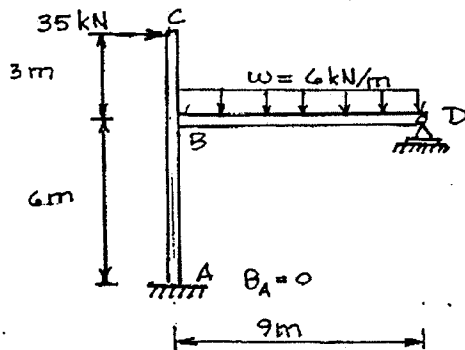
$$\psi = 3.13/K = 3.13 \left(\frac{12}{EI} \right)$$

$$= \frac{3.13 \times 12}{29,000 \times 240} (12)^2$$

$$\psi = 7.771 \times 10^{-4} \text{ RADIANS}$$

$$\Delta_B = \psi L = \psi(12 \times 12) = 0.112''$$

P12.23. Compute the reactions and draw the shear and moment diagrams for beam BD in Figure P12.23. Also EI is constant.



P12.23

$$FEM_{BD} = -\frac{wL^2}{12} = -\frac{6(9)^2}{12} = -40.5 \text{ kN}\cdot\text{m}$$

$$FEM_{DB} = \frac{wL^2}{12} = 40.5 \text{ kN}\cdot\text{m}$$

$$M_{AB} = \frac{2EI}{6} \theta_B$$

$$M_{BA} = \frac{2EI}{6} (2\theta_B)$$

$$M_{BD} = \frac{2EI}{9} (2\theta_B + \theta_D) - 40.5$$

$$M_{DB} = \frac{2EI}{9} (2\theta_D + \theta_B) + 40.5$$

JOINT EQUIL. EQ'S

$$(1) \quad M_{DB} = 0 \quad \begin{matrix} \nearrow \\ \theta_D = 0 \end{matrix} \quad \begin{matrix} \nearrow \\ \theta_B = 0 \end{matrix} \quad \begin{matrix} \nearrow \\ \theta_A = 0 \end{matrix}$$

$$\frac{2EI}{9} (2\theta_D + \theta_B) + 40.5 = 0 \quad (1)$$

$$(2) \quad \sum M = 0 \quad \begin{matrix} \curvearrowright \\ \theta_B = 0 \end{matrix} \quad \begin{matrix} \curvearrowright \\ \theta_D = 0 \end{matrix} \quad \begin{matrix} \curvearrowright \\ \theta_A = 0 \end{matrix}$$

$$-105 + M_{BD} + M_{BA} = 0$$

$$-105 + \frac{2EI}{9} (2\theta_B + \theta_D) - 40.5 + \frac{2EI}{6} (2\theta_B) = 0 \quad (2)$$

$$(1a) \quad \frac{4EI}{9} \theta_D + \frac{2EI}{9} \theta_B = -40.5$$

$$(2a) \quad \frac{4EI}{9} \theta_B + \frac{2EI}{9} \theta_D + \frac{2EI}{3} \theta_B = 145.5$$

$$\text{FROM (1a) and (2a) } \theta_B = \frac{165.75}{EI}$$

$$\theta_D = -\frac{174}{EI}$$

SUBSTITUTING θ_B & θ_D INTO EQ'S FOR member end MOMENTS GIVES:

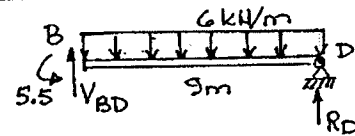
$$M_{AB} = \frac{2EI}{6} \left(\frac{165.75}{EI} \right) = 55.25 \text{ kN}\cdot\text{m}$$

$$M_{BA} = \frac{2EI}{6} \left(2 \left(\frac{165.75}{EI} \right) \right) = 110.5 \text{ kN}\cdot\text{m}$$

$$M_{BD} = \frac{2EI}{9} \left[2 \left(\frac{165.75}{EI} \right) - \frac{174}{EI} \right] - 40.5$$

$$M_{BD} = -5.5 \text{ kN}\cdot\text{m}$$

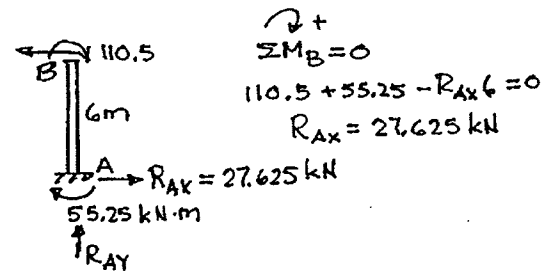
COMPUTE REACTIONS



$$\sum M_B = 0; \quad 54 \text{ kN} \times \frac{9}{2} - R_D 9 - 5.5 = 0$$

$$R_D = 26.39 \text{ kN}$$

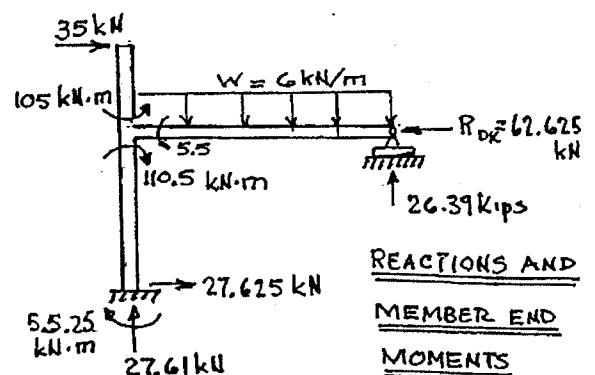
$$\sum F_y = 0; \quad V_{BD} = 54 - 26.39 = 27.61 \text{ kN}$$



$$\sum M_B = 0$$

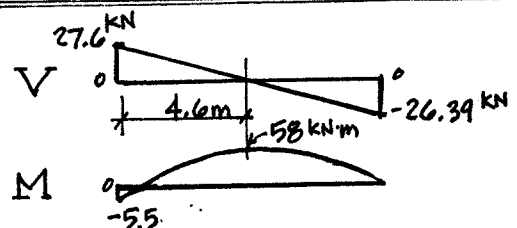
$$110.5 + 55.25 - R_{AX} 6 = 0$$

$$R_{AX} = 27.625 \text{ kN}$$

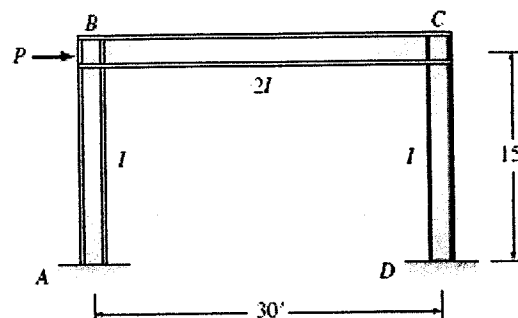


REACTIONS AND
MEMBER END
MOMENTS

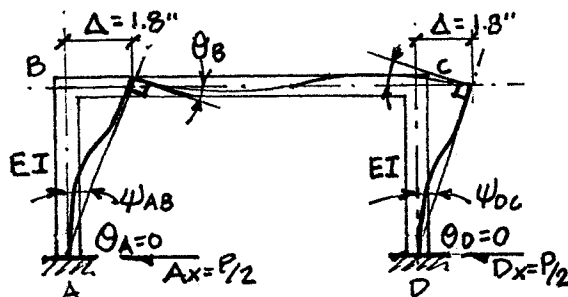
MEMBER BD: SHEAR & MOMENT DIAGRAMS



P12.24. If the horizontal displacement of the frame is limited to 1.8 in, compute the maximum lateral force P that can be applied to the frame. Given: $E = 29,000$ ksi and $I = 500$ in⁴.



P12.24



UNKNOWN θ_B , WHERE $\theta_C = \theta_B$
 ROTATIONS ARE CLOCKWISE AND
 $\psi_{AB} = \psi_{DC} = \frac{1.8''/12\text{ft}}{15'} = 0.01$
 $\theta_A = \theta_D = 0$

$$EI = \frac{29,000 \text{ KSI} (500 \text{ IN}^4)}{144 \text{ IN}^2/\text{FT}^2} = 100,694.4 \text{ FT}^2\text{-K}$$

BY SYMMETRY $A_x = D_x = P/2$

MEMBER END MOMENTS:

$$M_{AB} = \frac{2EI}{15'} (\theta_B - 3(0.01)) = (0.133\theta_B - 0.004)EI$$

$$= -322.2 \text{ FT-K} = M_{DC}$$

$$M_{BA} = \frac{2EI}{15'} (2\theta_B - 3(0.01)) = (0.2667\theta_B - 0.004)EI$$

$$= -241.6 \text{ FT-K} = M_{CD}$$

$$M_{BC} = \frac{2E(2I)}{30'} (2\theta_B + \theta_C) = 0.40\theta_B EI$$

↑ WHERE $\theta_C = \theta_B$

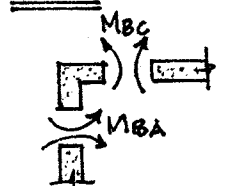
$$= 241.6 \text{ FT-K} = M_{CB}$$

JT. B $\sum M_B = 0;$

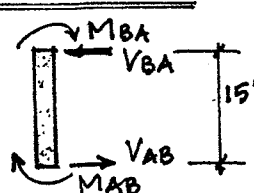
$$M_{BA} + M_{BC} = 0$$

$$(0.667\theta_B - 0.004)EI = 0$$

$$\theta_B = 0.006$$



SEGMENT AB



$$\sum M_A = 0;$$

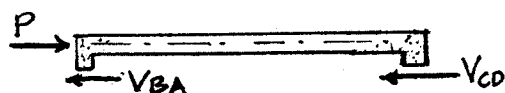
$$M_{AB} + M_{BA} - V_{BA}(15') = 0$$

$$V_{BA} = \frac{M_{AB} + M_{BA}}{15'} = -37.6 \text{ K} \rightarrow$$

SEGMENT CD, BY SYMMETRY

$$V_{CD} = \frac{M_{CD} + M_{DC}}{15'} = 37.6 \text{ K} \rightarrow$$

SEGMENT BC



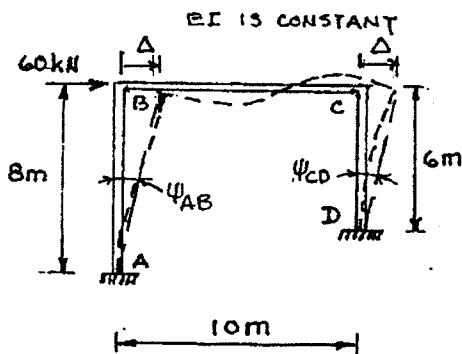
$$\sum F_x = 0;$$

$$P - V_{BA} - V_{CD} = 0$$

$$P = \frac{M_{AB} + M_{BA}}{15'} + \frac{M_{CD} + M_{DC}}{15'}$$

$$P = 75.2 \text{ K}$$

P12.25. Determine all reactions at points A and D in Figure P12.25. EI is constant.



$$\psi_{AB} = \frac{\Delta}{8}, \quad \psi_{CD} = \frac{\Delta}{6} \therefore \frac{\psi_{CD}}{\psi_{AB}} = \frac{\Delta}{6} \cdot \frac{8}{\Delta} = \frac{4}{3}$$

$$\text{and } \psi_{CD} = \frac{4}{3} \psi_{AB}$$

$$\begin{aligned} M_{AB} &= \frac{2EI}{8} (\theta_B - 3\psi_{AB}) = -93.72 \text{ kN}\cdot\text{m} \\ M_{BA} &= \frac{2EI}{8} (2\theta_B - 3\psi_{AB}) = -71.29 \text{ kN}\cdot\text{m} \\ M_{BC} &= \frac{2EI}{10} (2\theta_B + \theta_C) = 71.29 \text{ kN}\cdot\text{m} \\ M_{CB} &= \frac{2EI}{10} (2\theta_C + \theta_B) = 88.67 \text{ kN}\cdot\text{m} \\ M_{CD} &= \frac{2EI}{6} (2\theta_C - 4\psi_{AB}) = -88.67 \text{ kN}\cdot\text{m} \\ M_{DC} &= \frac{2EI}{6} (\theta_C - 4\psi_{AB}) = -147.58 \text{ kN}\cdot\text{m} \end{aligned}$$

EQUIL. EQS

JOINT B $\Sigma M_B = 0; M_{BA} + M_{BC} = 0$ (1)

JOINT C $\Sigma M_C = 0; M_{CB} + M_{CD} = 0$ (2)

SHEAR EQ. $\Sigma F_x = 0: \frac{M_{AB} + M_{BA}}{8} + \frac{M_{CD} + M_{DC}}{6} + 60 = 0$ (3)

$$\left(\frac{EI}{2} \theta_B - \frac{6}{8} \psi_{AB} \right) + \frac{EI}{5} (2\theta_B + \theta_C) = 0 \quad (1a)$$

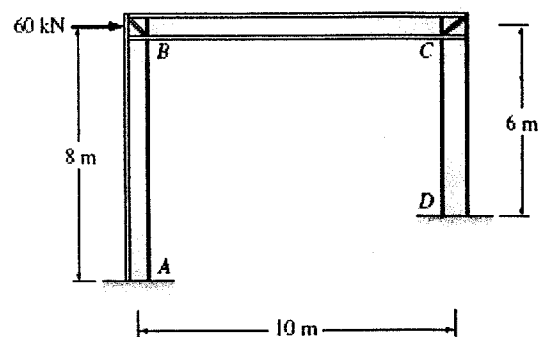
$$\frac{EI}{5} (2\theta_C + \theta_B) + \frac{EI}{3} (2\theta_C - 4\psi_{AB}) = 0 \quad (2a)$$

$$\frac{2EI}{8} (\theta_B - 3\psi_{AB}) + \frac{EI}{8} (2\theta_B - 3\psi_{AB}) + \frac{EI}{3} (2\theta_C - 4\psi_{AB}) + \frac{EI}{3} (\theta_C - 4\psi_{AB}) + 60 = 0$$

$$\frac{6\theta_B}{64} - \frac{12}{64} \psi_{AB} + \frac{\theta_C}{6} - \frac{8}{18} \psi_{AB} = -\frac{60}{EI} \quad (3a)$$

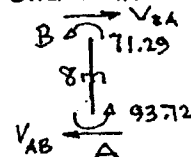
SOLVING (1a), (2a), and (3a) GIVES.

$$\theta_B = \frac{89.74}{EI} \quad \theta_C = \frac{176.77}{EI} \quad \psi_{AB} = \frac{154.88}{EI}$$

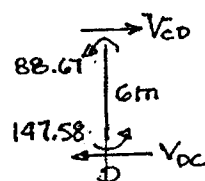


P12.25

SHEARS IN LEGS

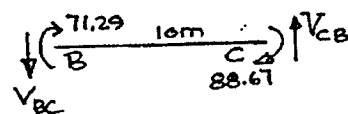


$$\begin{aligned} \Sigma M_A = 0 \\ -71.29 - 93.72 + V_{BA} 8 = 0 \\ V_{BA} = 20.62 \text{ kN} \\ V_{AB} = V_{BA} = 20.62 \text{ kN} \end{aligned}$$

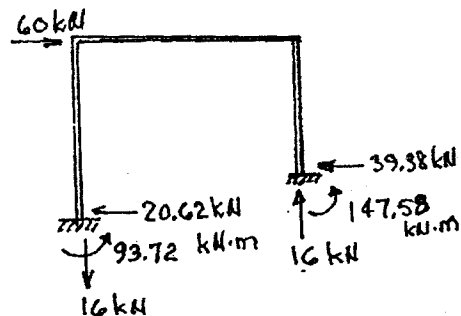


$$\begin{aligned} \Sigma M_D = 0 \\ -88.67 - 147.58 + V_{CD} 6 = 0 \\ V_{CD} = 39.38 \text{ kN} \\ V_{DC} = 39.38 \text{ kN} \end{aligned}$$

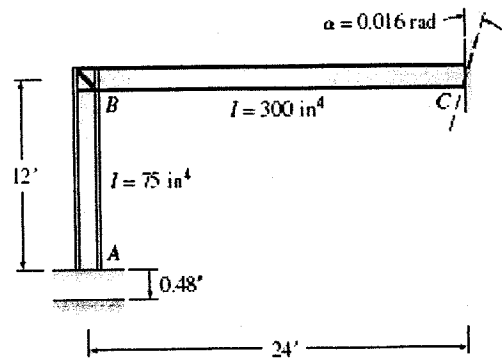
SHEAR IN GIRDER



$$\begin{aligned} \Sigma M_B = 0; 71.29 + 88.67 - V_{CB} 10 = 0 \\ V_{CB} = 16 \text{ kN} \uparrow \\ V_{BC} = -V_{CB} = 16 \text{ kN} \downarrow \end{aligned}$$



P12.26. If support *A* in Figure P12.26 is constructed 0.48 in too low and the support at *C* is accidentally constructed at a slope of 0.016 rad clockwise from a vertical axis through *C*, determine the moment and reactions created when the structure is connected to its supports.
Given: $E = 29,000 \text{ kips/in}^2$.



P12.26

$$I_B = 4I_{col} ; E = 29,000 \text{ ksi}$$

$$\theta_C = 0.016 \text{ RAD}$$

$$\psi_{BC} = \frac{-0.48}{24(12)} = -1/600 \text{ (COUNTERCLOCKWISE)}$$

$$\theta_B = \text{UNKNOWN}$$

$$(1) M_{AB} = [2EI_c/12(12)] [\theta_B]$$

$$(2) M_{BA} = [2EI_c/12(12)] [2\theta_B]$$

$$(3) M_{BC} = \frac{2E(4I_c)}{24(12)} [2\theta_B + 0.016 - 3(\frac{1}{600})]$$

$$(4) M_{CB} = \frac{2E(4I_c)}{24(12)} [2(0.016) + \theta_B - 3(\frac{1}{600})]$$

EQUILIBRIUM @ JOINT B

$$(5) M_{BA} + M_{BC} = 0$$

SUBSTITUTE EQUATIONS 2 AND 3 INTO EQUATION 5

$$\frac{2EI_c}{144} [2\theta_B] + \frac{8EI_c}{24(12)} [2\theta_B + 0.016 + \frac{1}{200}] = 0$$

$$\theta_B = -0.007 \text{ RAD}$$

SUBSTITUTE θ_B BACK INTO SLOPE DEFLECTION EQUATIONS

$$M_{AB} = \frac{2(29000)(75)}{144} [-0.007]$$

$$M_{AB} = -17.62 \text{ FT-K}$$

$$M_{BA} = 2M_{AB}$$

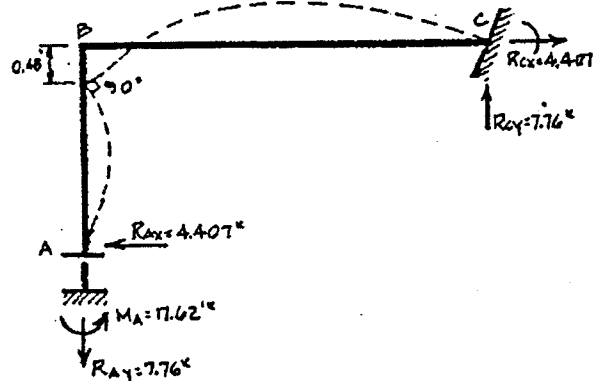
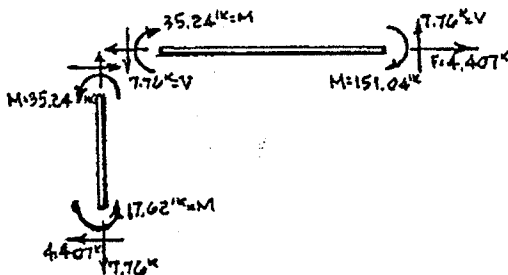
$$M_{BA} = -35.24 \text{ FT-K}$$

$$M_{BC} = \frac{29000(75)}{36} [2(-0.007) + 0.016 + 1/200]$$

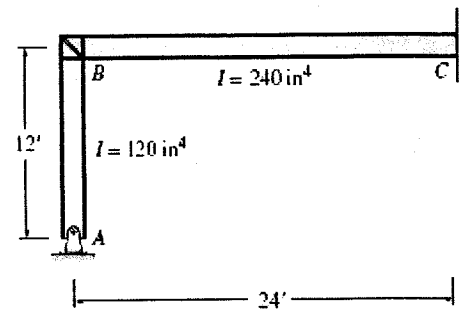
$$M_{BC} = 35.24 \text{ FT-K}$$

$$M_{CB} = \frac{29000(75)}{36} [0.032 + (-0.007) + 0.005]$$

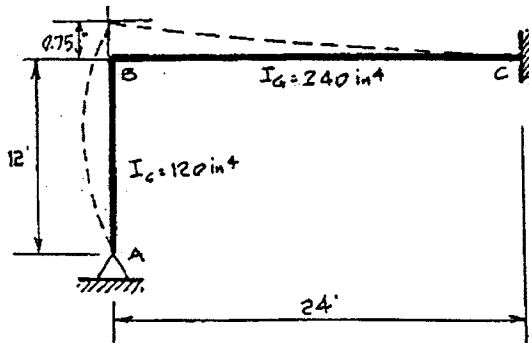
$$M_{CB} = 151.04 \text{ FT-K}$$



P12.27. If member AB in Figure P12.27 is fabricated $\frac{3}{4}$ in too long, determine the moments and reactions created in the frame when it is erected. Sketch the deflected shape. Given: $E = 29,000 \text{ kips/in}^2$.



P12.27



$$I_A = 2I_C; \quad E = 29,000 \text{ ksi}$$

$$\theta_C = 0$$

$$\psi_{BC} = 0.75/12(24) = 0.0026 \text{ RAD}$$

$$\theta_A \text{ AND } \theta_B = \text{UNKNOWN}$$

$$M_{AB} = \frac{2EI_C}{12(12)} [2\theta_A + \theta_B]$$

$$M_{BA} = \frac{2EI_C}{12(12)} [2\theta_B + \theta_A]$$

$$M_{BC} = \frac{2E2I_C}{24(12)} [2\theta_B - 3(0.0026)]$$

$$M_{CB} = \frac{2E2I_C}{24(12)} [\theta_B - 3(0.0026)]$$

EQUILIBRIUM EQUATIONS

$$(1) M_{AB} = 0$$

$$(2) M_{BA} + M_{BC} = 0$$

SUBSTITUTE MOMENT EQUATIONS INTO EQUILIBRIUM EQUATIONS

$$(1a) \frac{2EI_C}{144} [2\theta_A + \theta_B] = 0$$

$$\theta_A = -\theta_B/2$$

$$(2a) \frac{2EI_C}{144} [2\theta_B + \theta_A] + \frac{2EI_C}{144} [2\theta_B - 0.0078] = 0$$

SUBSTITUTE θ_A INTO 2a

$$\theta_B = 0.00223 \text{ RAD}$$

$$\theta_A = -0.001115 \text{ RAD}$$

SUBSTITUTE VALUES FOR θ_A AND θ_B INTO MOMENT EQUATIONS

$$M_{AB} = 0$$

$$M_{AB} = 0$$

$$M_{BA} = \frac{2(29000)(120)}{144} [2(0.00446) - 0.00112] \frac{1}{12}$$

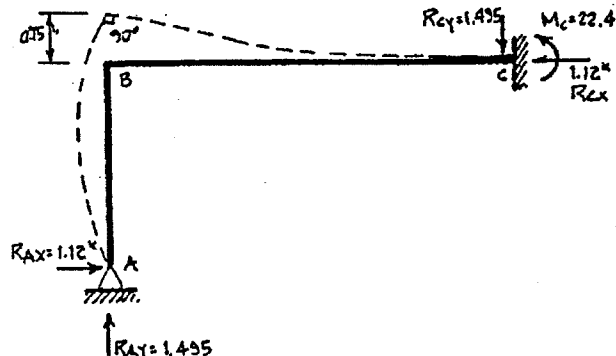
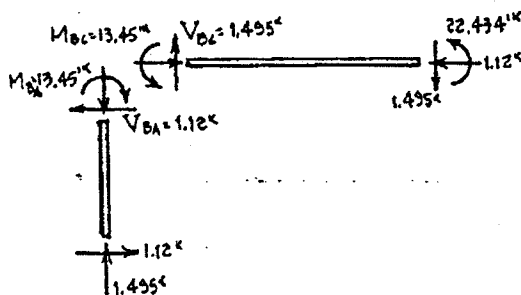
$$M_{BA} = 13.45 \text{ ft-k}$$

$$M_{BC} = \frac{29000}{72} [2(0.00223) - 3(0.0026)] [20] \frac{1}{12}$$

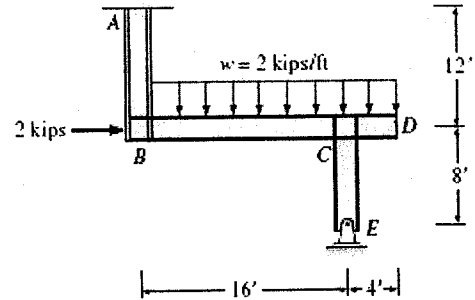
$$M_{BC} = -13.45 \text{ ft-k}$$

$$M_{CB} = \frac{29000}{72} (120) [0.00223 - 0.0078] \frac{1}{12}$$

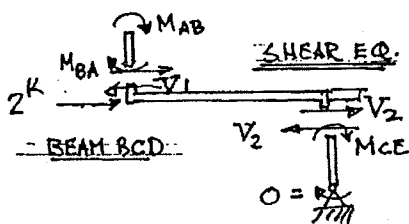
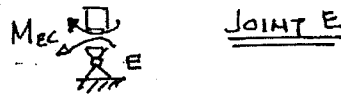
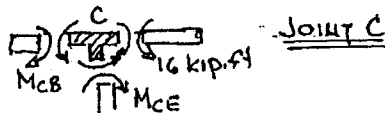
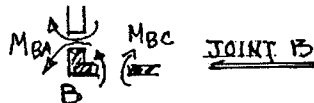
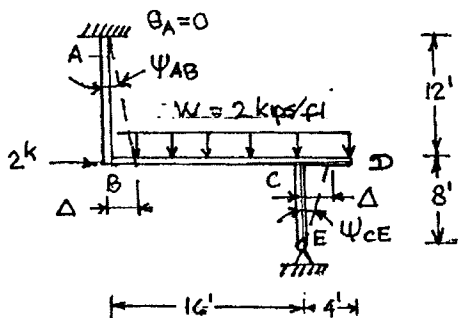
$$M_{CB} = -22.434 \text{ ft-k}$$



P12.28. Set up the equilibrium equations required to analyze the frame in Figure P12.28 by slope deflection. Express the equilibrium equations in terms of the appropriate displacements; EI is constant for all members.



P12.28



UNKNOWN: $\Delta, \theta_B, \theta_C, \theta_E$

$$\psi_{AB} = -\frac{\Delta}{12}, \psi_{CE} = \frac{\Delta}{8}$$

$$\psi_{AB}/\psi_{CE} = -\frac{\Delta/12}{\Delta/8}$$

$$\psi_{AB} = -\frac{2}{3}\psi_{CE}$$

$$FEM_{BC} = \frac{wL^2}{12} = \frac{2(16)^2}{12} = 42.66 \text{ kip-ft}$$

(1) EXPRESS END MOMENTS IN TERMS OF DISPLACEMENTS

$$M_{AB} = \frac{2EI}{12}(\theta_B - 3\psi_{AB})$$

$$M_{BA} = \frac{2EI}{12}(2\theta_B - 3\psi_{AB})$$

$$M_{BC} = \frac{2EI}{16}(2\theta_B + \theta_C) - 42.66$$

$$M_{CB} = \frac{2EI}{16}(2\theta_C + \theta_B) + 42.66$$

$$M_{CE} = \frac{2EI}{8}(2\theta_C + \theta_E - 3\psi_{CE})$$

$$M_{EC} = \frac{2EI}{8}(2\theta_E + \theta_C - 3\psi_{CE})$$

(2) EQUILIBRIUM EQUATIONS

(A) JOINT B: $\sum M_B = 0 \quad M_{BA} + M_{BC} = 0$

(B) JOINT C: $\sum M_C = 0 \quad M_{CB} + M_{CE} - 16 = 0$

(C) JOINT E: $\sum M_E = 0 \quad M_{EC} = 0$

SHEAR EQUATION $\sum F_x = 0$

$$2^k - V_1 + V_2 = 0$$

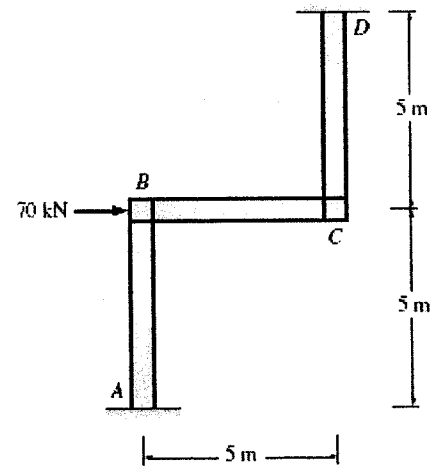
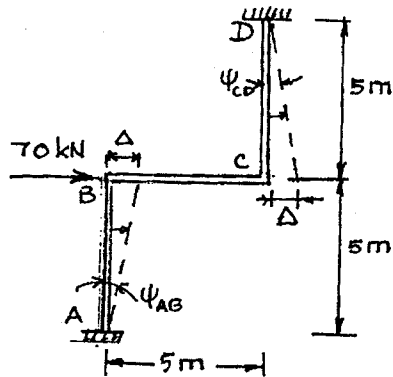
where $V_1 = \frac{M_{AB} + M_{BA}}{12}$ & $V_2 = \frac{M_{CE}}{8}$

(D) $\sum Z = \frac{M_{AB} + M_{BA}}{12} + \frac{M_{CE}}{8} = 0$

(3) COMPLETE SOLUTION BY SUBSTITUTING VALUES IN STEP 1 INTO EQUIL EQ'S. AND SOLVING FOR THE UNKNOWN DISPLACEMENTS.

P12.29. Analyze the frame in Figure P12.29. Also EI is constant.

EI CONSTANT ALL MEMBERS



P12.29

$$\psi_{AB} = \frac{\Delta}{5} ; \quad \psi_{CD} = -\frac{\Delta}{5}$$

$$\psi_{AD} = \psi ; \quad \psi_{CD} = -\psi$$

$$M_{AB} = \frac{2EI}{5} (\theta_B - 3\psi) = -116.66 \text{ kN}\cdot\text{m}$$

$$M_{BA} = \frac{2EI}{5} (2\theta_B - 3\psi) = -58.33 \text{ kN}\cdot\text{m}$$

$$M_{BC} = \frac{2EI}{5} (2\theta_B + \theta_C) = 58.33 \text{ kN}\cdot\text{m}$$

$$M_{CB} = \frac{2EI}{5} (2\theta_C + \theta_B) = -58.33 \text{ kN}\cdot\text{m}$$

$$M_{CD} = \frac{2EI}{5} (2\theta_C + 3\psi) = 58.33 \text{ kN}\cdot\text{m}$$

$$M_{DC} = \frac{2EI}{5} (\theta_C + 3\psi) = 116.66 \text{ kN}\cdot\text{m}$$

EQUIL EQ'S

JOINT B: $\sum M_B = 0$; $M_{BC} + M_{CB} = 0$

$$4\theta_B + \theta_C - 3\psi = 0 \quad (1)$$

JOINT C: $\sum M_C = 0$; $M_{CB} + M_{CD} = 0$

$$4\theta_C + \theta_B + 3\psi = 0 \quad (2)$$

SHEAR EQ: $\sum F_x = 0$; $-V_1 - V_2 + 70 = 0$

$$\frac{M_{AB} + M_{BA}}{5} - \frac{M_{CD} + M_{DC}}{5} + 70 = 0$$

BOTH TERMS OPPOSE 70kN

$$3\theta_B - 6\psi - 3\theta_C - 6\psi = -\frac{70 \times 5 \times 5}{EI}$$

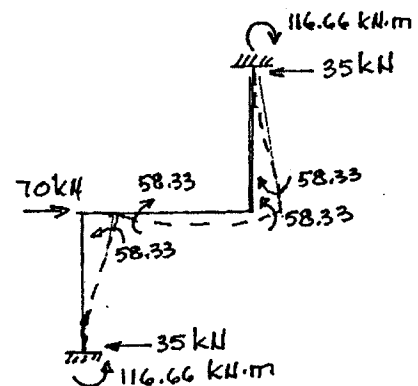
$$\theta_B - \theta_C - 4\psi = -\frac{350(5)}{2EI} \quad (3)$$

SOLVING EQ'S (1), (2), AND (3)

$$\theta_C = -\frac{145.83}{EI}, \quad \theta_B = \frac{145.83}{EI}, \quad \psi = \frac{145.83}{EI}$$

THESE VALUES SUBSTITUTE INTO MOMENT EQS. ABOVE.

* M_{AB} & M_{BA} ARE NEGATIVE MOMENTS
 M_{CD} & M_{DC} ARE POSITIVE MOMENTS



REACTIONS, END MOMENTS AND DEF'D SHAPE

P12.30. Determine the degree of kinematic indeterminacy for each structure in Figure P12.30. Neglect axial deformations.

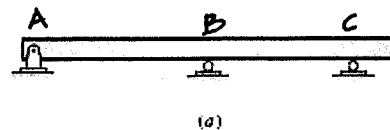
(a.) INDETERMINATE 3° : ROTATION @ A, B, & C

(b.) INDETERMINATE 3° : ROTATION @ B, C, & D

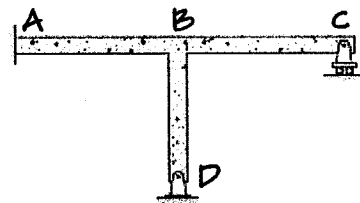
(c.) INDETERMINATE 6° : ROTATION @ A, B, C, D, E & F

(d.) INDETERMINATE 13° :

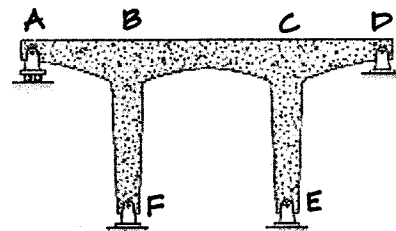
10 JOINT ROTATIONS &
3 DEGREES OF SIDESWAY



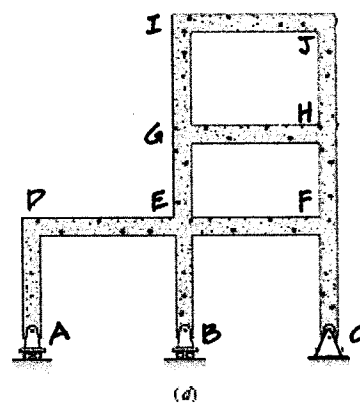
(a)



(b)



(c)



(d)

P12.30