

FUNDAMENTALS OF STRUCTURAL ANALYSIS

3rd Edition

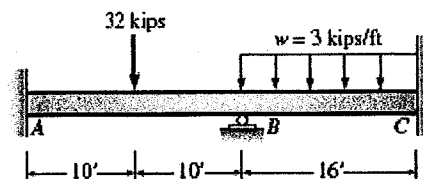
Kenneth M. Leet, Chia-Ming Uang, and Anne M. Gilbert

SOLUTIONS MANUAL

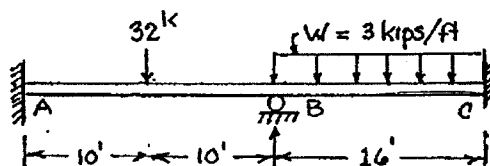
CHAPTER 13:

MOMENT DISTRIBUTION

P13.1 Analyze each structure by moment distribution.
Determine all reactions and draw the shear and moment diagrams, locating points of inflection and labeling values of maximum shear and moment in each span.
Unless otherwise noted, EI is constant.



P13.1



JOINT B DISTRIBUTION FACTORS

$$K_{AB} = \frac{I}{20} = \frac{4I}{80} \quad DF_{AB} = \frac{4}{9}$$

$$K_{BC} = \frac{I}{16} = \frac{5I}{80} \quad DF_{BC} = \frac{5}{9}$$

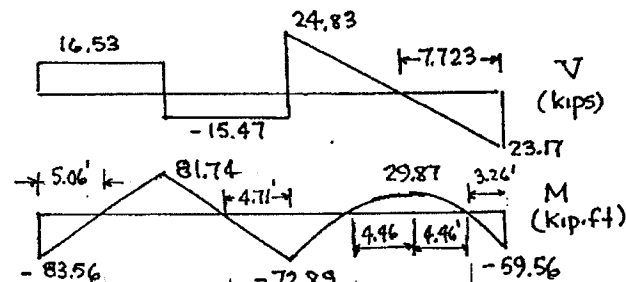
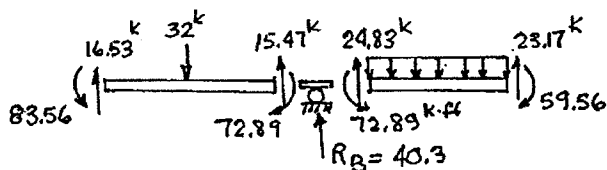
$$\Sigma K's = \frac{9I}{80}$$

FEMS

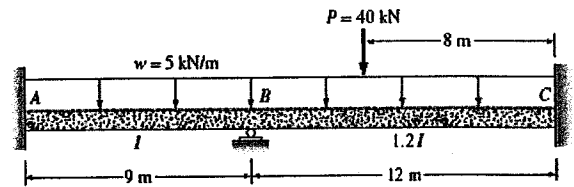
$$FEM_{AB} = \pm \frac{PL}{8} = \frac{32 \times 20}{8} = \pm 80 \text{ kip}\cdot\text{ft}$$

$$FEM_{BC} = \pm \frac{wL^2}{12} = \frac{3(16)^2}{12} = \pm 64 \text{ kip}\cdot\text{ft}$$

A		B		C	
-80		+80		-64	
-3.56		7.11		-8.89	
-83.56		+72.89		-72.89	
				+59.56	



P13.2 Analyze each structure by moment distribution. Determine all reactions and draw the shear and moment diagrams, locating points of inflection and labeling values of maximum shear and moment in each span. Unless otherwise noted, EI is constant.



P13.2

DISTR. FACTORS

$$K_{AB} = \frac{I}{9} = \frac{10I}{90} \quad DF_{BA} = \frac{10}{19}$$

$$K_{BC} = \frac{1.2I}{12} = \frac{9I}{90} \quad DF_{BC} = \frac{9}{19}$$

$$\Sigma K = \frac{19I}{90}$$

FIXED END MOMENTS (SEE TABLE 2) FRONT OVERLEAF

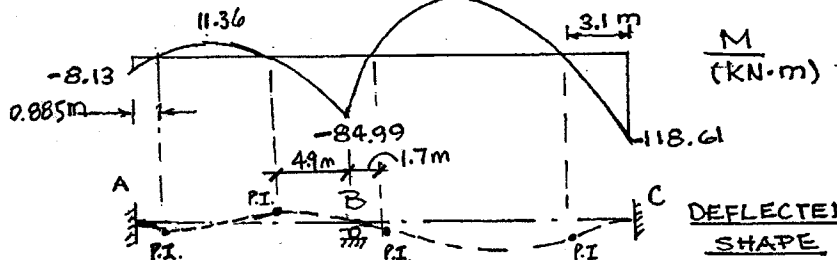
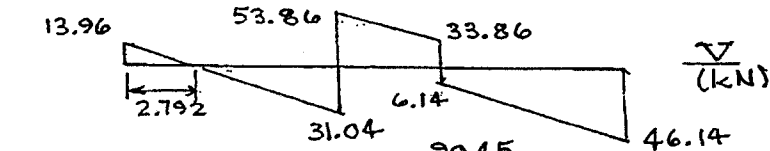
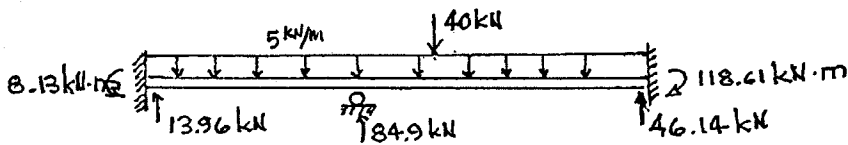
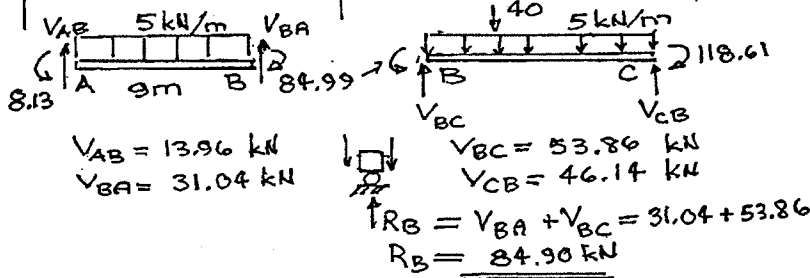
$$FEM_{BA} = FEM_{AB} = \pm \frac{wL^2}{12} = \pm \frac{5(9)^2}{12} = \pm 33.75 \text{ kN}\cdot\text{m}$$

$$FEM_{BC} = -\frac{wL^2}{12} - \frac{Pb^2a}{L^2} = -\frac{5(12)^2}{12} - \frac{40(64)4}{(12)^2} = -131.11 \text{ kN}\cdot\text{m}$$

$$FEM_{CB} = \frac{wL^2}{12} + \frac{Pba^2}{L^2} = \frac{5(12)^2}{12} + \frac{40(8)16}{(12)^2} = 95.55 \text{ kN}\cdot\text{m}$$

MOMENT DISTR.

A $\left(\frac{10}{19}\right)$		B $\left(\frac{9}{19}\right)$		C	FEM'S DEM'S END MOMENTS (kN·m)
-33.75	+33.75	-131.11	+95.55		
+25.62	+51.24	+46.12	23.06		
-8.13	84.99	84.99	118.61		



POINTS OF INFLECTION

The diagram shows a continuous beam with four supports labeled A, B, C, and D. The beam is subjected to the following loads and dimensions:

- A uniformly distributed load of 9 kips/ft is applied over a 10' span starting from support A.
- A point load of 20 kips is applied 6' to the right of support B.
- A uniformly distributed load of 9 kips/ft is applied over a 10' span ending at support D.
- The distance between supports A and B is 10'.
- The distance between supports B and C is 6'.
- The distance between supports C and D is 10'.

$$K_{BA} = \frac{I}{10} = \frac{4I}{60} = \frac{6}{11}$$

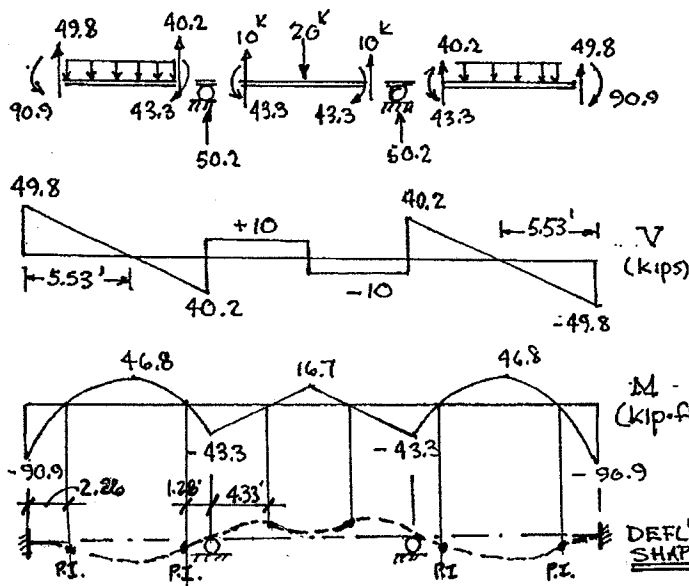
$$K_{BC} = \frac{I}{12} = \frac{5I}{60} = \frac{5}{11}$$

$$\Sigma K'_3 = \frac{111}{60}$$

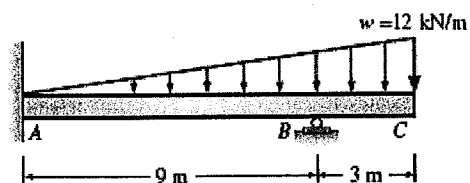
$$FEM_{AB} = FEM_{CD} = \frac{WL^2}{12} = \pm \frac{9(10)^2}{12} = \pm 75 \text{ kip}\cdot\text{ft}$$

$$FEM_{GD} = FEM_{DC} = \frac{PL}{8} = \frac{20 \times 12}{8} = \pm 30 \text{ Kip.ft}$$

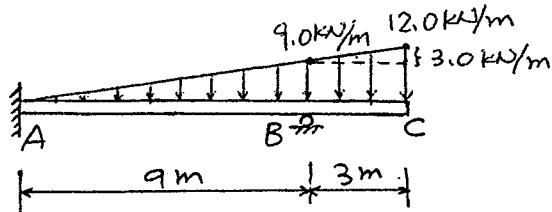
A	$\begin{matrix} 6 \\ 11 \end{matrix}$	$\begin{matrix} 5 \\ 11 \end{matrix}$	$\begin{matrix} 5 \\ 11 \end{matrix}$	$\begin{matrix} 6 \\ 11 \end{matrix}$	D	
-75	+75	-30	+30	-75	+75	FEM
-12.27	$\xleftarrow{-24.55}$	$\xrightarrow{-20.45}$	-10.23			DEM
			$\xleftarrow{+25.1}$	$\xrightarrow{+30.13}$		DEM (JOINT C)
		$\xleftarrow{12.55}$		$\xrightarrow{+15.07}$		COM
	$\xleftarrow{-6.85}$	$\xrightarrow{-5.7}$				DEM
-3.42			-2.85			COM
			$\xleftarrow{1.3}$	$\xrightarrow{1.55}$		DEM
	0.65			0.78		COM
	$\xleftarrow{-0.35}$	$\xrightarrow{-0.3}$				DEM
-0.175			-0.15			COM
			$\xleftarrow{+0.07}$	$\xrightarrow{+0.08}$	$\rightarrow 0.04$	DEM
-90.9	43.3	-43.3	43.3	-43.3	90.9	FINAL VALUES



P13.4 Analyze each structure by moment distribution. Determine all reactions and draw the shear and moment diagrams, locating points of inflection and labeling values of maximum shear and moment in each span. Unless otherwise noted, EI is constant.



P13.4

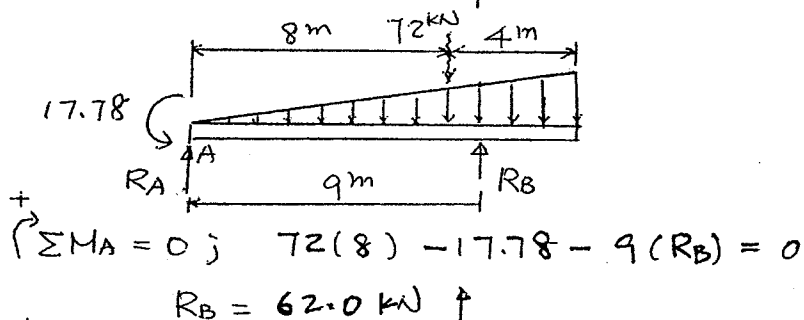


$$FEM_{AB} = -\frac{9.0(9)^2}{30} = -24.3 \text{ kN}\cdot\text{m}$$

$$FEM_{BA} = \frac{9.0(9)^2}{20} = 36.45 \text{ kN}\cdot\text{m}$$

$$FEM_{BC} = -(9.0 \times 3)1.5 - \frac{3.0 \times 3}{2}(2.0) = -49.5 \text{ kN}\cdot\text{m}$$

-24.3	+36.45	-49.5	FEM
	+13.05		D.M.
+6.53			C.O.M.
-17.78	+49.5	-49.5	FINAL M.



$$\sum M_A = 0; \quad 72(8) - 17.78 - 9(R_B) = 0$$

$$R_B = 62.0 \text{ kN} \uparrow$$

$$\sum F_y = 0; \quad R_A + R_B - 72 = 0$$

$$R_A = 10.0 \text{ kN} \uparrow$$

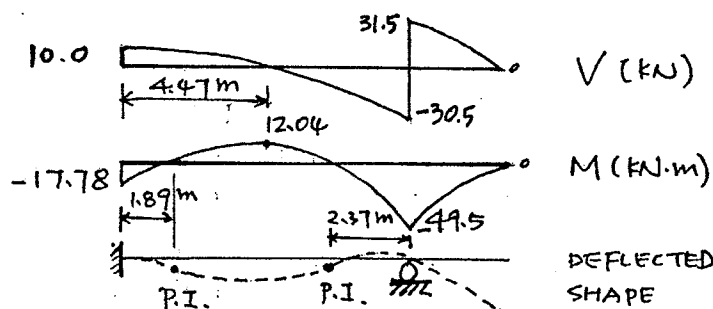
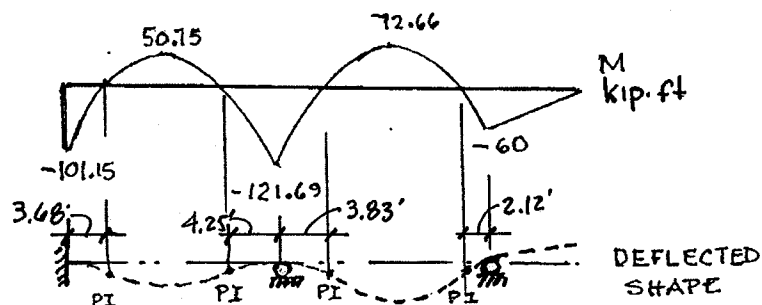
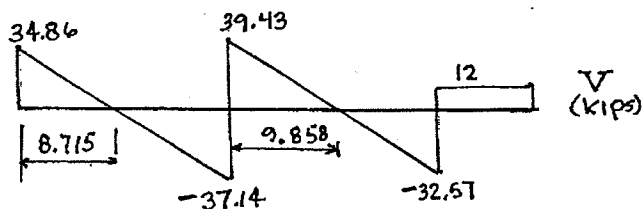
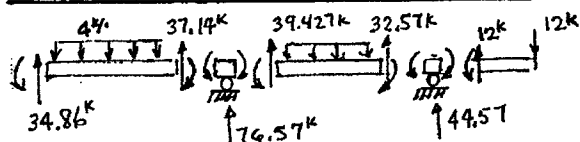
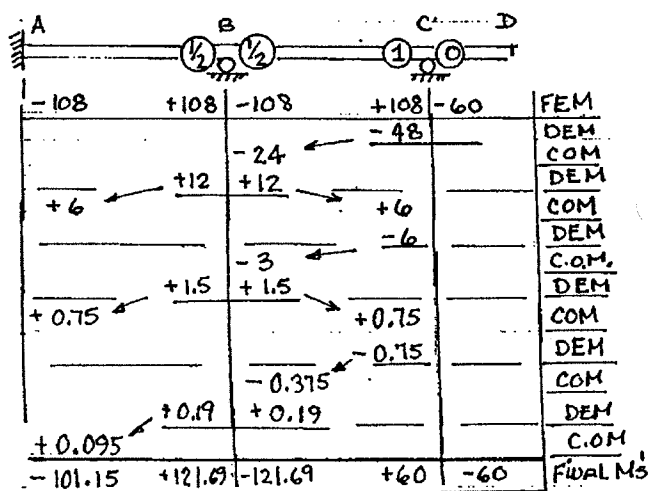


Diagram of a continuous beam with three supports (A, B, C) and a free end (D). The beam is subjected to a uniformly distributed load $w = 4$ kips/ft from A to B and a point load of 12 kips at D. The distances are 18' from A to B, 18' from B to C, and 5' from C to D.

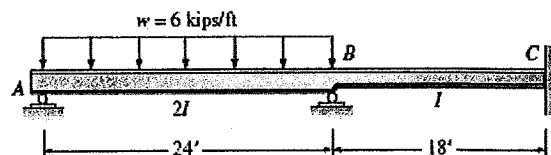
Diagram of a beam with a uniformly distributed load ($W = 4 \text{ kips/ft}$) and a point load (12 k) at the right end. The beam is supported by a pin at A and rollers at B and C. The distances are $18'$ from A to B, $18'$ from B to C, and $5'$ from C to D.

$$K_{CD} = 0 \quad \text{CANTILEVER}$$

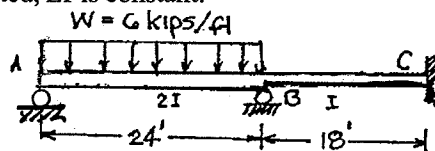
$$FEM = \frac{WL^2}{12} = \frac{4(18)^2}{12} = 108 \text{ kip}\cdot\text{ft}$$



P13.6 Analyze each structure by moment distribution.
Determine all reactions and draw the shear and moment diagrams, locating points of inflection and labeling values of maximum shear and moment in each span.
Unless otherwise noted, EI is constant.



P13.6



$$FEM_{AB} = -FEM_{BA} = \frac{wL^2}{12} = \frac{6 \times 24^2}{12} = 288$$

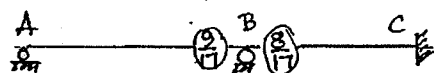
$$K_{AB} = \frac{3}{4} \left(\frac{2I}{L} \right) = \left(\frac{3}{4} \right) \frac{2I}{24} = \frac{I}{16} = \frac{9I}{144}$$

$$K_{BC} = \frac{I}{18} = \frac{8I}{144}$$

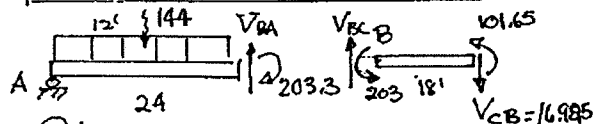
$$\Sigma K's = \frac{17I}{144}$$

$$DF_{AB} = \frac{K_{AB}}{\Sigma K's} = \frac{9I/144}{17I/144} = \frac{9}{17}$$

$$DF_{BC} = \frac{K_{BC}}{\Sigma K's} = \frac{8I/144}{17I/144} = \frac{8}{17}$$



-288	+288		FEM
+288	→ +144		
	-228.7	263.3	DEM
		→ 101.65	
0	+203.3	-203.3	FINAL M



$$\sum M_A = 0 = 144 \times 12 + 203.3 - V_{BA} \times 24$$

$$V_{BA} = 80.47 \text{ K} \uparrow$$

$$\sum M_B = 0 = -203.3 - 101.65 - V_{CB} \times 18$$

$$V_{CB} = 16.925 \text{ K} \downarrow$$

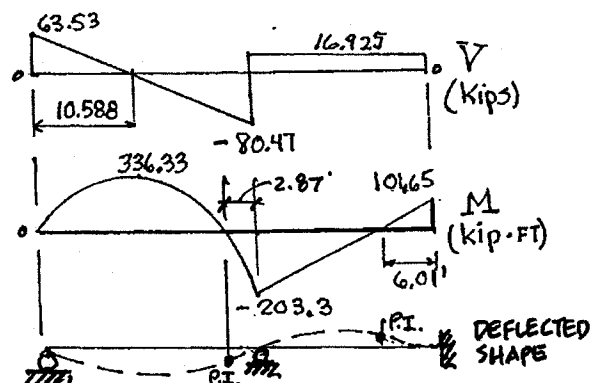
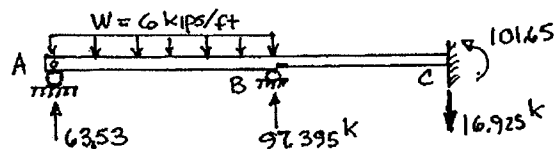
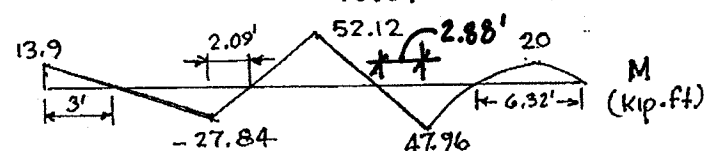
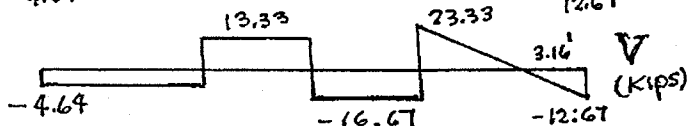
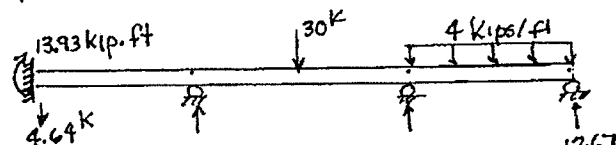
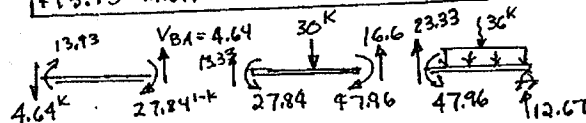


Diagram of a beam with a fixed support at A, a roller support at B, and a roller support at D. A 30 k point load is applied at C, and a 4 kips/ft distributed load is applied from C to D. Distances are 9' from A to B, 6' from B to C, and 9' from C to D.

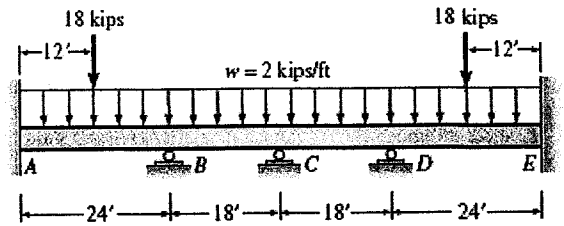
$$\begin{array}{l|l} K_{AB} = \frac{EI}{9} = \frac{4EI}{36} & DF_{BA} = 4/7 \\ K_{BC} = \frac{EI}{12} = \frac{3EI}{36} & DF_{BC} = 3/7 \\ \hline \Sigma K_s = \frac{7EI}{36} \end{array}$$

A number line from 0 to 1. The segments are labeled with fractions: $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{4}$.

0	0	-45	+45	-27	+27	FEM
						DEM
+ 12.86	$\uparrow 25.71$	$\uparrow 19.29$	$\searrow 9.64$	$\swarrow 13.5$	$\swarrow 27$	COM
			-7.07	-7.07		DEM
		-3.54				COM
	$+ 202$	$+ 1.52$	$\searrow 1.76$			DEM
$+ 1.01$			-38	-38		COM
		-0.19				DEM
	$+ .11$	$+ .08$	$\rightarrow .04$			COM
$+ .055$			$- .02$	$.02$		DEM
$+ 13.93$	$+ 27.84$	$+ 27.84$	$+ 47.96$	$- 47.96$	0	FINAL M

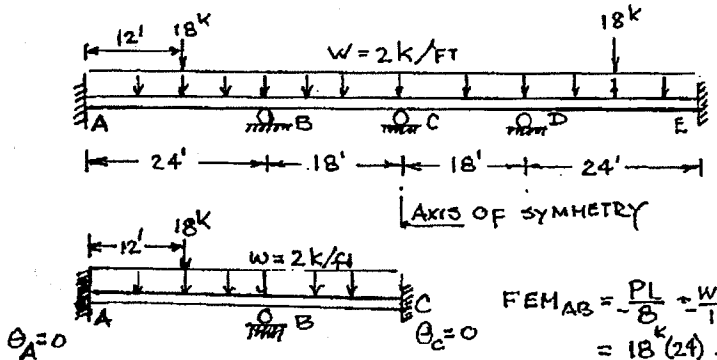


P13.8 Analyze by moment distribution.
 Modify stiffness as discussed in Section 13.5. EI is constant.
 Draw the shear and moment diagrams.



P13.8

SINCE STRUCTURE AND LOADS ARE SYMMETRICAL,
 $\theta_C = 0 \therefore$ TREAT JOINT C AS A FIXED SUPPORT.
 ANALYSE HALF STRUCTURE



$$FEM_{AB} = -\frac{PL}{8} - \frac{wL^2}{12}$$

$$= -\frac{18(24)}{8} - \frac{2(24)^2}{12} = -150 \text{ kip}\cdot\text{ft}$$

$$FEM_{BA} = +150$$

$$FEM_{BC} = -\frac{wL^2}{12} = -\frac{2(18)^2}{12} = -54 \text{ kip}\cdot\text{ft}$$

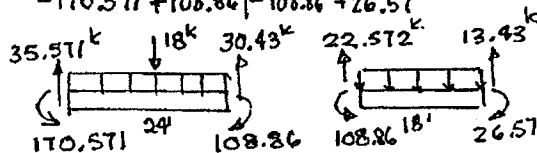
DISTR. FACTOR AT "B"

$$K_{AB} = \frac{I}{24} = \frac{3I}{12} = \frac{3}{1}$$

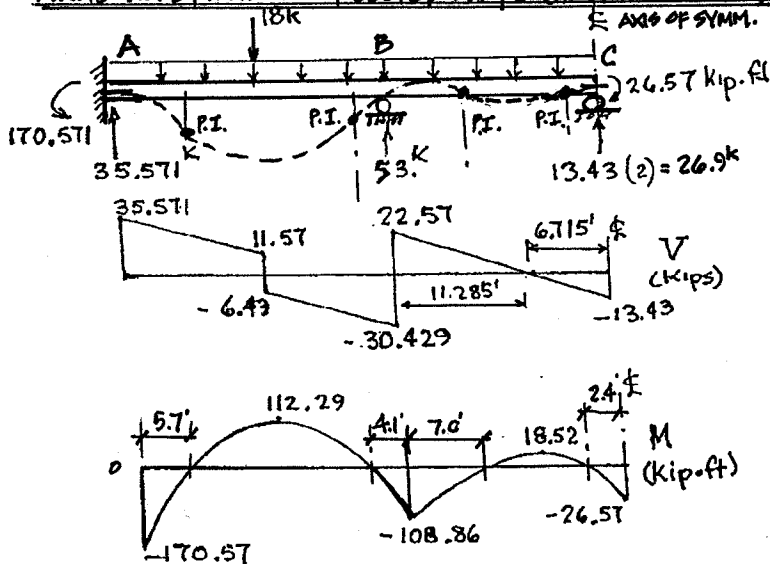
$$K_{BC} = \frac{I}{18} = \frac{4I}{12} = \frac{4}{3}$$

$$\Sigma K'_s = \frac{7I}{12}$$

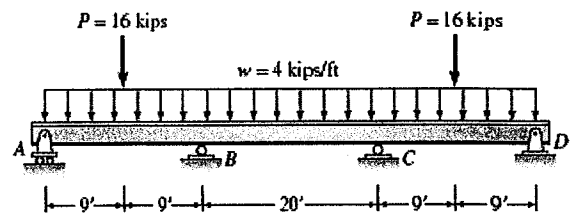
A	B	C	
-150	+150	-54	+54
	-41.143	-54.857	
-20.57		-27.43	
-170.571	+108.86	-108.86	+26.57



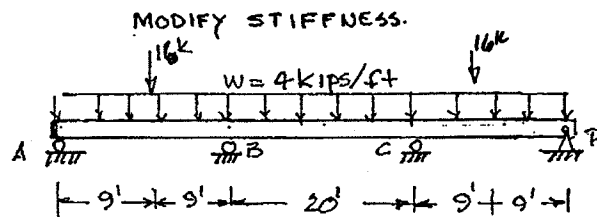
FINAL REACTIONS & DEFLECTED SHAPE (HALF STRUCTURE):



P13.9 Analyze by moment distribution.
 Modify stiffness as discussed in Section 13.5. EI is constant.
 Draw the shear and moment diagrams.



P13.9



$$K_{AB} = \frac{3}{4} \frac{I}{18} = \frac{I}{24} = \frac{5}{120} \quad \text{D.F. } \frac{5}{8}$$

$$K_{BC} = \frac{1}{2} \frac{I}{20} = \frac{I}{40} = \frac{3}{120} \quad \text{D.F. } \frac{3}{8}$$

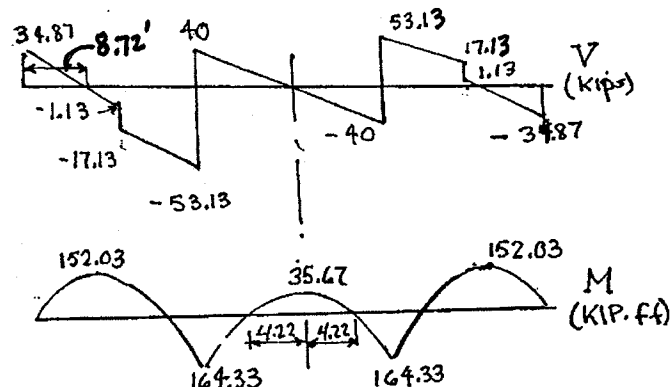
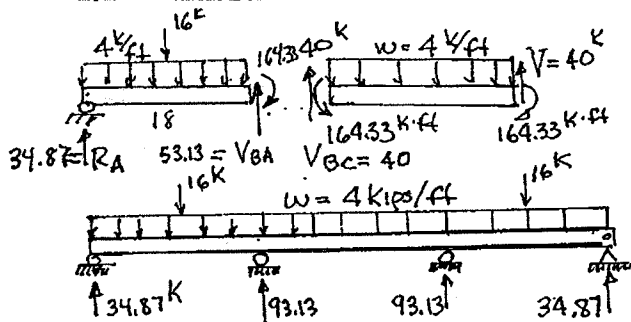
$$\Sigma K_s = \frac{8}{120}$$

$$FEM_{AB} = -\left(\frac{wL^2}{12} + \frac{PL}{8}\right)$$

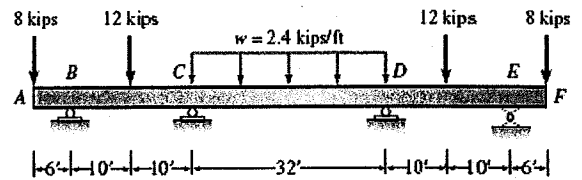
$$= -\frac{4(18)^2}{12} + \frac{16(18)}{8} = -144 \text{ kip}\cdot\text{ft}$$

$$-FEM_{BC} = \frac{wL^2}{12} = \frac{4(20)^2}{12} = +133.33$$

5	8								
-144	+144	-133.33	+133.33	-144	+144				FEM
+144									DEM
	+72								COM
	-56.67	-31							DEM
0	+164.33	-164.33	+164.33	-164.33	0				FINAL M



P13.10 Analyze by moment distribution.
Modify stiffness as discussed in Section 13.5. EI is constant.
Draw the shear and moment diagrams.



P13.10

COMPUTE MODIFIED STIFFNESSES

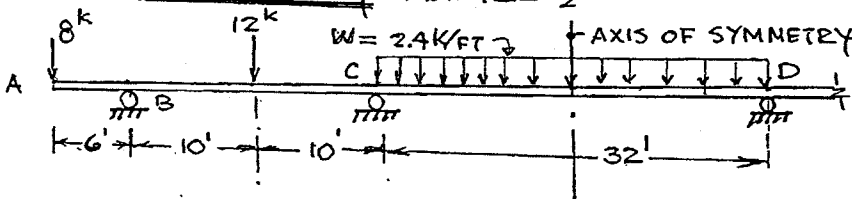
$$K_{BC} = K_{DE} = \frac{3}{4} \frac{I}{20} = 0.0375 I \quad 0.706$$

$$K_{CD} = K_{DC} = \frac{1}{2} \frac{I}{32} = 0.015625 I \quad 0.294$$

$$\Sigma K = 0.53125 I$$

$K/\Sigma K$

CONSIDER SYMMETRY: ANALYZE $\frac{1}{2}$ structure



MOMENT DISTR.

	A	B	C	D	E
FEM		+48	-30	+30	-204.8
		-18	-9	+204.8	-30
DEM			+129.76	+54.04	
FINAL END M'S	+48	-48	+150.76	-150.76	+150.76

MODIFIED STIFFNESSES

$$K_{BC} = K_{DE} = \frac{3}{4} \frac{I}{20}$$

$$K_{CD} = K_{DC} = \frac{1}{2} \frac{I}{32}$$

$$\Sigma K_s = 0.053125 I$$

DISTR. FACTORS
AT JOINT C and D

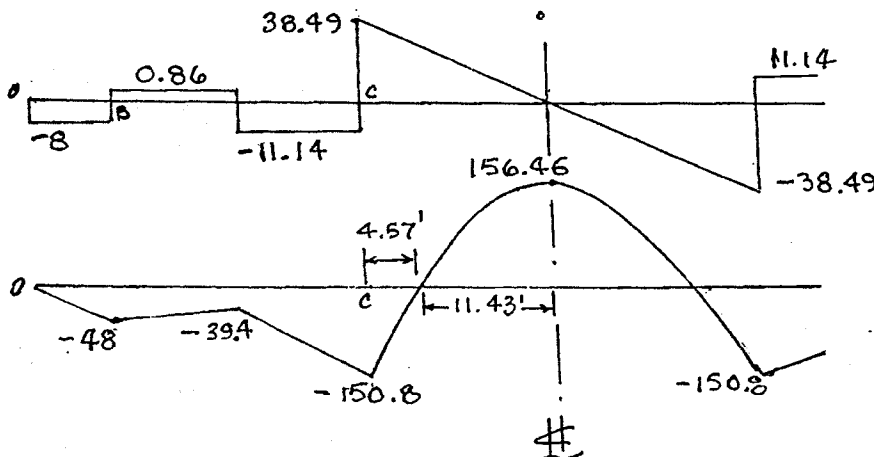
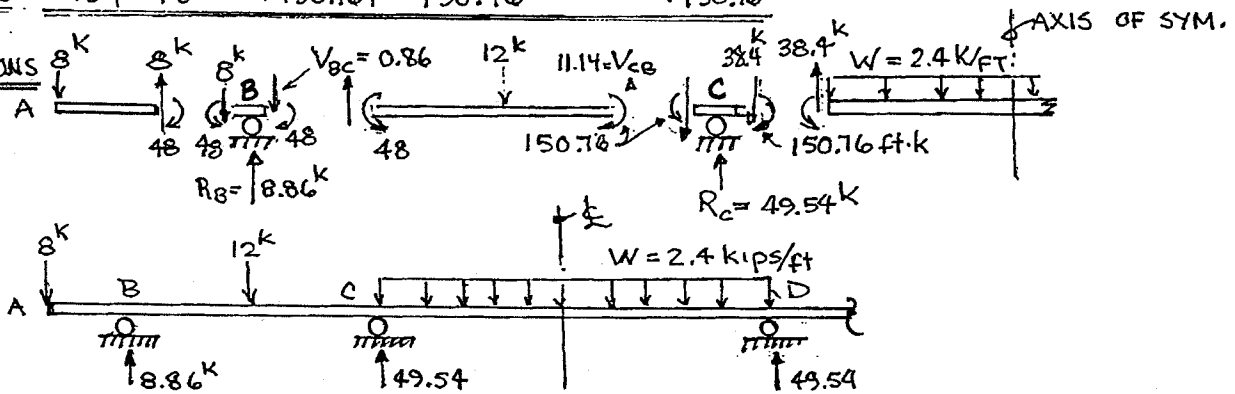
$$K_{CB} = 0.706$$

$$K_{CD} = 0.294$$

$$FEM_{BC} = FEM_{CB} = \frac{PL}{8} = 30 \text{ k-ft}$$

$$FEM_{CD} = FEM_{DC} = \frac{wL^2}{12} = 204.8$$

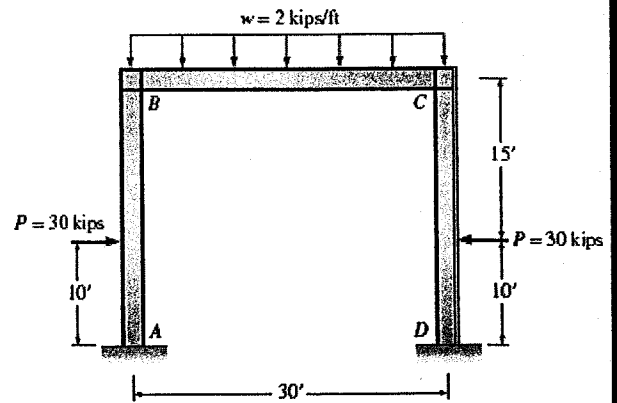
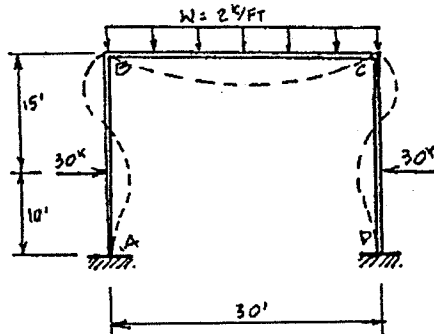
REACTIONS



SHEAR DIAGRAM
(kips)

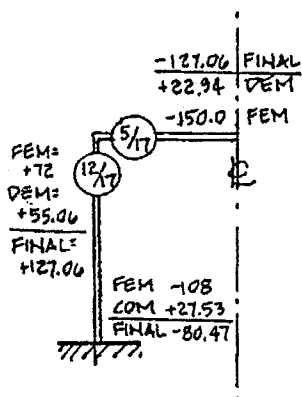
MOMENT DIAGRAM
KIP-FT

P13.11. Analyze the frame in Figure P13.11 by moment distribution. Determine all reactions and draw the shear and moment diagrams, locating points of inflection and labeling values of maximum shear and moment in each span. Given: EI is constant.



P13.11

MOMENT DISTRIBUTION



CONSIDER SYMMETRY, NO JOINT TRANSLATION

JOINT B

DISTRIBUTION FACTORS

$$K_{BA} = I/25$$

$$D.F._{BA} = K_{BA}/\Sigma K_s = 12/17$$

$$K_{BC} = 1/2(I/30)$$

$$D.F._{BC} = K_{BC}/\Sigma K_s = 5/17$$

$$\Sigma K_s = 17I/300$$

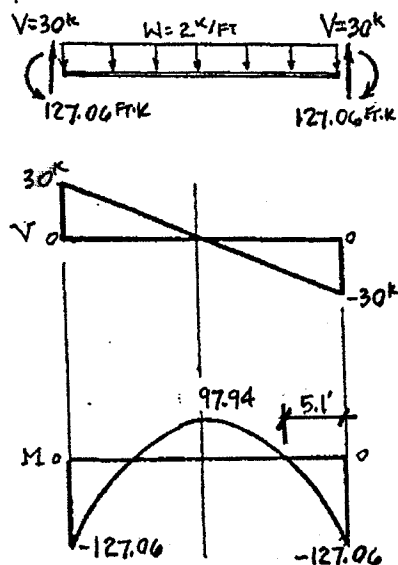
FIXED END MOMENTS

$$FEM_{BC} = \pm WL^2/12 = 2(30)^2/12 = \pm 150 \text{ k}$$

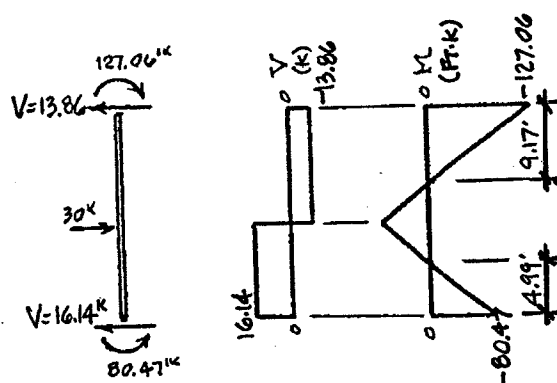
$$FEM_{AB} = Pab^2/L^2 = 30(10)(15)^2/(25)^2 = -108 \text{ k}$$

$$FEM_{DA} = Pab^2/L^2 = 30(15)(10)^2/(25)^2 = +72 \text{ k}$$

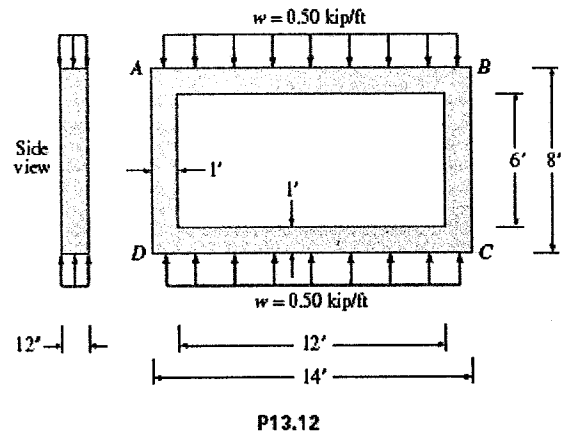
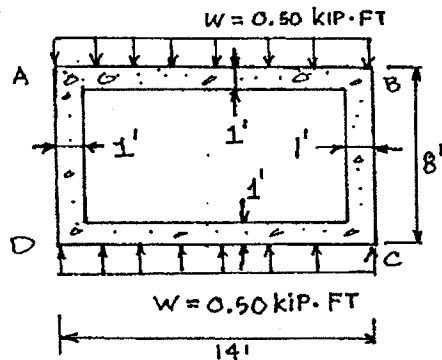
GIRDER



LEG AB (LEG CD SIMILAR)



P13.12. Analyze the reinforced concrete box in Figure P13.12 by moment distribution. Modify stiffnesses as discussed in Section 13.5. Draw the shear and moment diagrams for the top slab AB. Given: EI is constant.



BASE ANALYSIS ON $\frac{1}{2}$ DIMENSIONS.
 SINCE BOX SYMMETRICAL ABOUT HORIZ
 AND VERTIC $\frac{1}{2}$ S. CAN ADJUST STIFFNESS
 OF ALL SIDES BY $\frac{1}{2}$ & ELIMINATE C.O.M.

$$FEM = \frac{wL^2}{12} = \pm \frac{0.50(12)^2}{12} = \pm 7.04 \text{ K-FT}$$

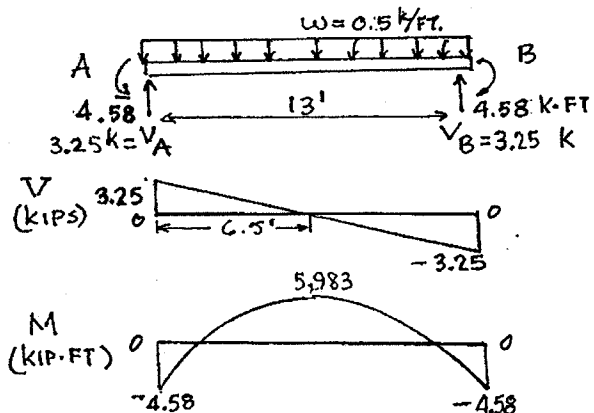
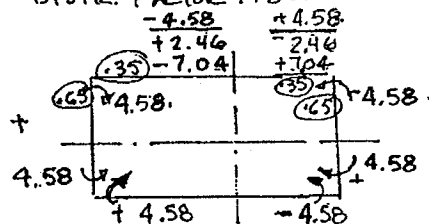
$$K_{AB} = \frac{I}{13} \times \frac{1}{2} = 0.0385 I$$

$$K_{AD} = \frac{I}{7} \times \frac{1}{2} = 0.0714 I$$

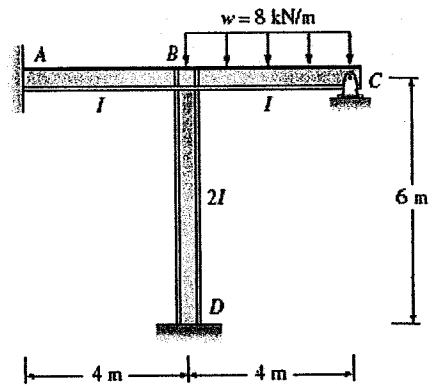
$$\sum K's = .1099 I$$

$$\text{DISTR FACTORS } AB = \frac{.0385 I}{.1099 I} = 0.35$$

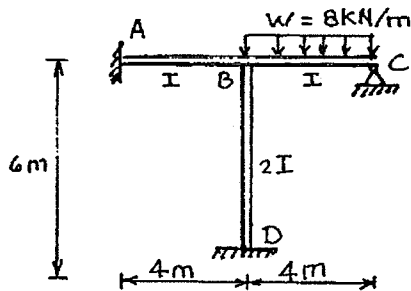
$$\text{DISTR. FACTOR } AD = 0.65$$



P13.13. Analyze the frame in Figure P13.13 by the moment distribution method. Determine all reactions and draw the moment and shear curves. Given: E is constant. Fixed supports at A and D .



P13.13



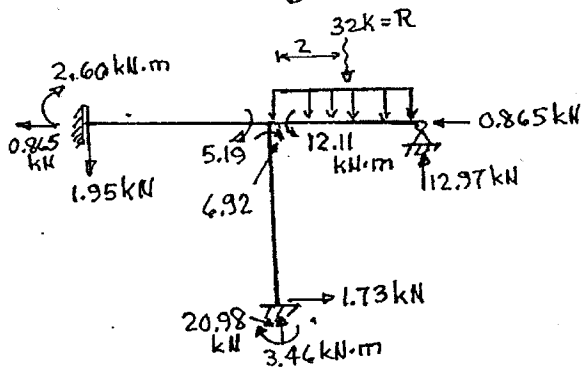
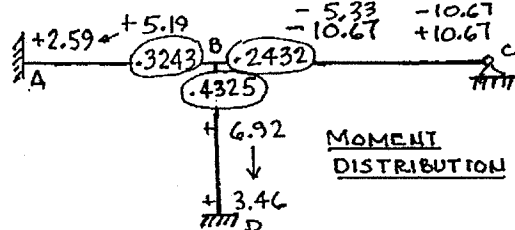
$$K_{AB} = \frac{I}{4}, K_{BC} = \frac{3I}{4}, K_{BD} = \frac{2I}{6}$$

$$\Sigma K = \frac{37I}{48}$$

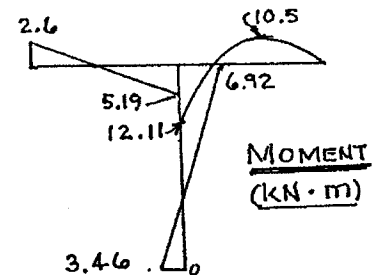
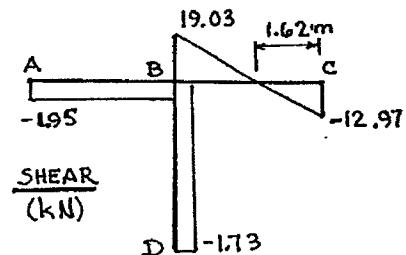
$$DF_{AB} = 0.3243, DF_{BC} = 0.2432, DF_{BD} = 0.4325$$

$$FEM_{BC} = -\frac{wL^2}{12} = -\frac{8(4)^2}{12} = -10.67 \text{ kN}\cdot\text{m}$$

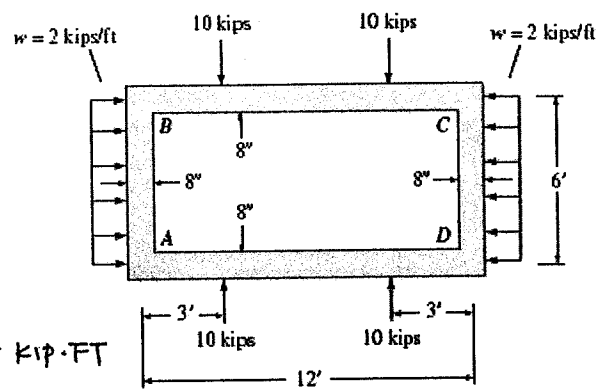
$$FEM_{CB} = 10.67 \text{ kN}\cdot\text{m}$$



REACTIONS AND MEMBER END MOMENTS



P13.14. The cross section of the rectangular ring in Figure P13.14 is 12 in $\times$ 8 in. Draw the moment and shear curves for the ring; $E = 3000 \text{ kips/in}^2$.



P13.14

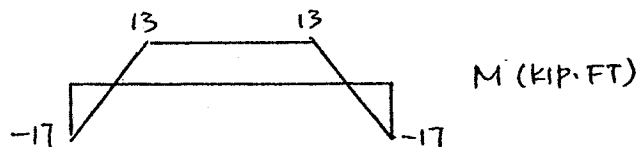
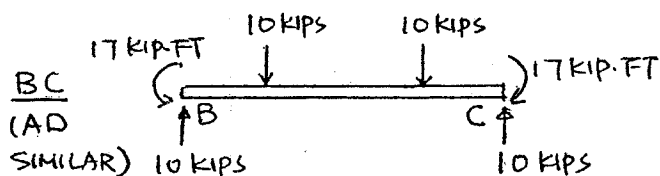
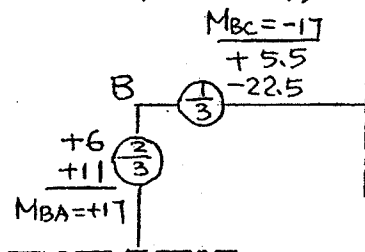
$$FEM_{BC} = -\frac{10(9)^2(3)}{12^2} - \frac{10(3)^2(9)}{12^2} = -22.5 \text{ kip}\cdot\text{ft}$$

$$FEM_{BA} = \frac{2(6)^2}{12} = +6 \text{ kip}\cdot\text{ft}$$

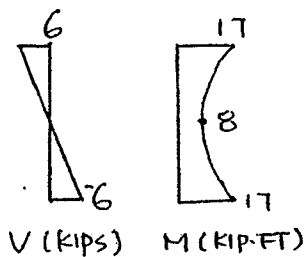
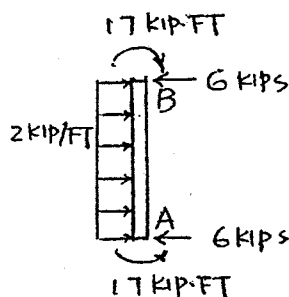
$$K_{BC} = \frac{1}{2} \frac{I}{12} = \frac{I}{24}, \quad DF_{BC} = \frac{1}{3}$$

$$K_{BA} = \frac{1}{2} \frac{I}{6} = \frac{I}{12}, \quad DF_{BA} = \frac{2}{3}$$

USING SYMMETRY, ANALYZE $\frac{1}{4}$ OF STRUCTURE.



AB
(CD SIMILAR)



$$\begin{aligned} K_{AB} &= \frac{3EI}{4 \times 21} = \frac{3EI}{28} = \frac{9I}{84} & \text{D.F. } \frac{9}{16} \\ K_{BD} &= \frac{EI}{12} = \frac{EI}{12} = \frac{7I}{84} & \frac{7}{16} \\ \Sigma K_S &= \frac{16I}{84} \end{aligned}$$

$$FEM_{AB} = \frac{WL^2}{12} = \frac{3(2)^2}{12} = \underline{\underline{110.25 \text{ kip} \cdot \text{ft}}}$$

$$M_{BC} = 36 \times 6 = -216 \text{ Kip. ft}$$

$$\begin{array}{r}
 0 \\
 + 110.25 \rightarrow \\
 - 110.25 \\
 \hline
 \end{array}$$

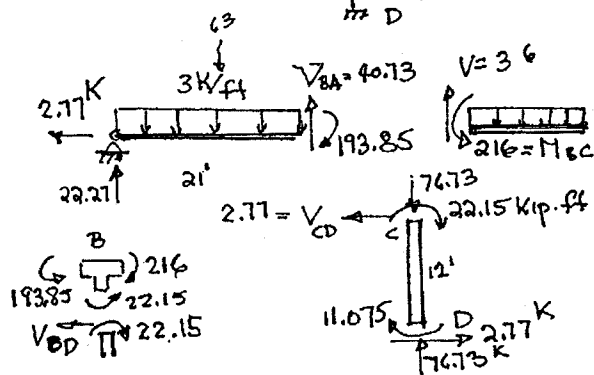
$$\begin{array}{r}
 + 197.85 \\
 + 28.48 \\
 + 55.125 \\
 + 110.25 \\
 \hline
 \end{array}$$

A ————— B ————— C

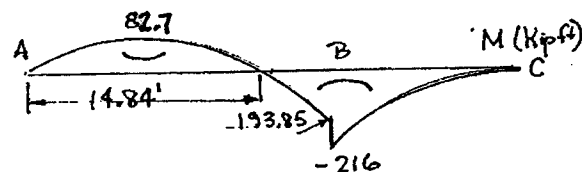
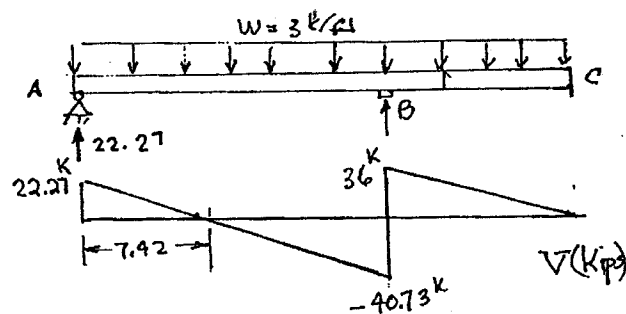
+ 22.15

+ 11.07

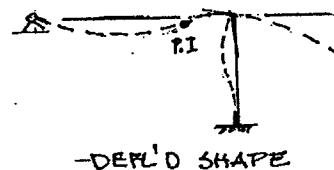
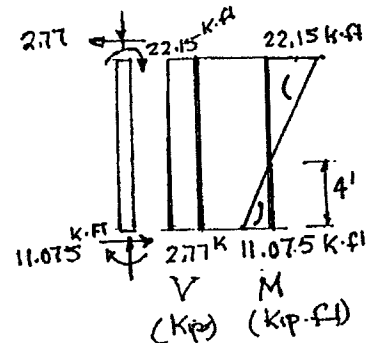
D



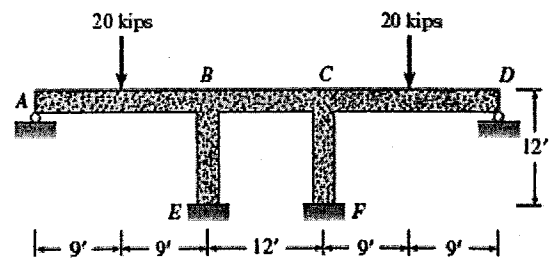
$$\sum M_D = 0 = 22.15 + 11.075 - V_{CD} \cdot 12$$
$$V_{CD} = 2.77 \text{ kips}$$



GIRDET

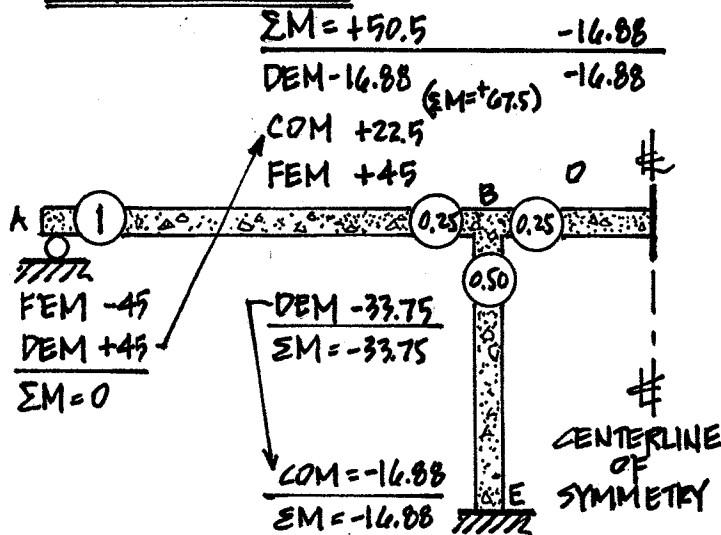


P13.16. Analyze the frame in Figure P13.16 by moment distribution. Determine all reactions and draw the shear and moment diagrams, locating points of inflection and labeling values of maximum shear and moment in each span. Given: EI is constant.



P13.16

HALF STRUCTURE :

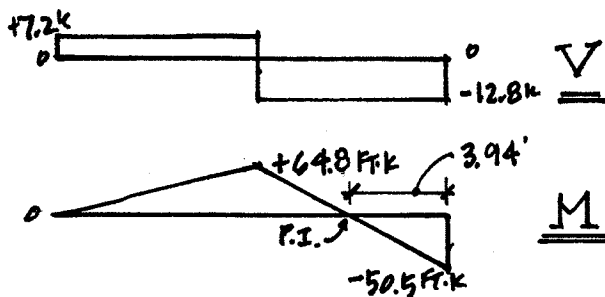
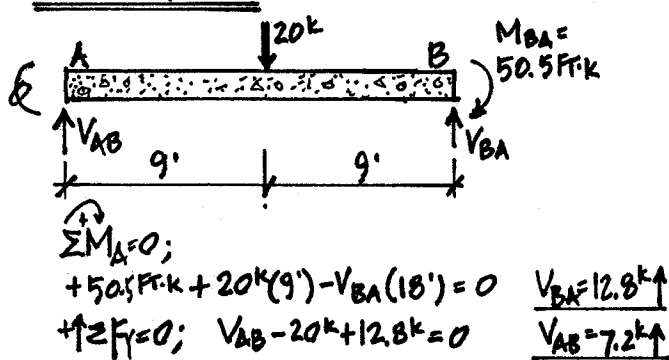


DUE TO SYMMETRIC STRUCTURE & SYMMETRIC LOADS; EVALUATE HALF STRUCTURE : θ_B BETWEEN BC = 0

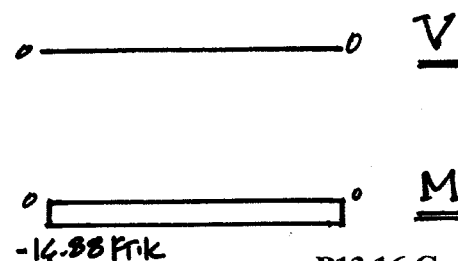
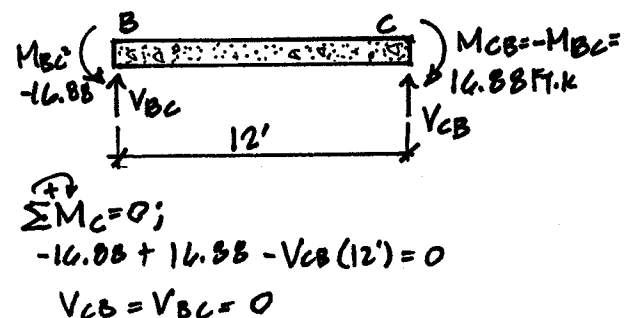
JOINT B	DISTRIBUTION FACTORS
$K_{BA} = \frac{3}{4} \frac{EI}{18} = \frac{3EI}{72}$	$DF_{BA} = 0.25$
$K_{BC} = \frac{1}{2} \frac{EI}{12} = \frac{3EI}{72}$	$DF_{BC} = 0.25$
$K_{BE} = \frac{EI}{12} = \frac{6EI}{72}$	$DF_{BE} = 0.50$
$\Sigma K_{OB} = \frac{12EI}{72}$	1.0 ✓

$$FEM_{AB,BA} = \pm PL/8 = \frac{20k(18)}{8} = \pm 45 \text{ ft.k}$$

SEGMENT "AB":



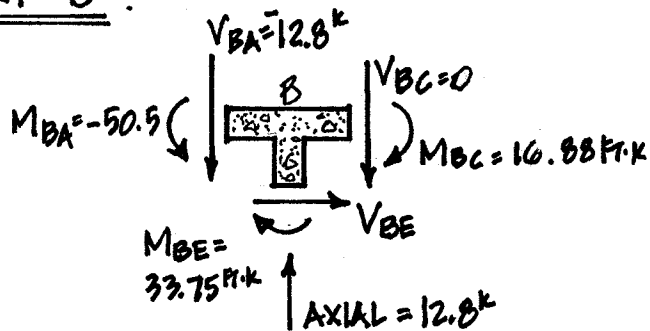
SEGMENT "BC":



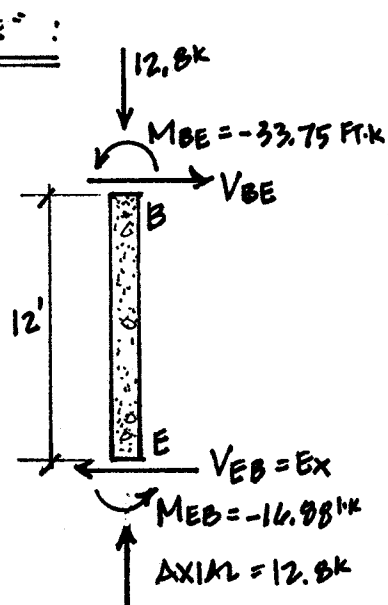
P13.16 Continued

P13.16 Continued

JOINT 'B':



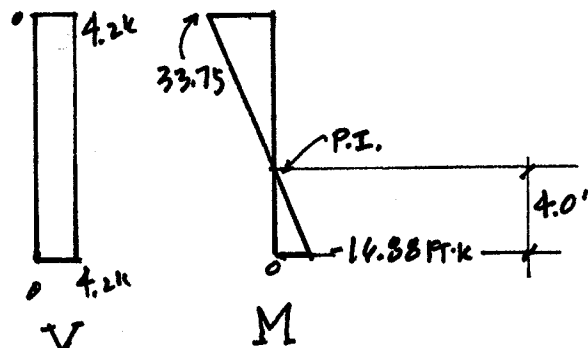
SEGMENT "BE":



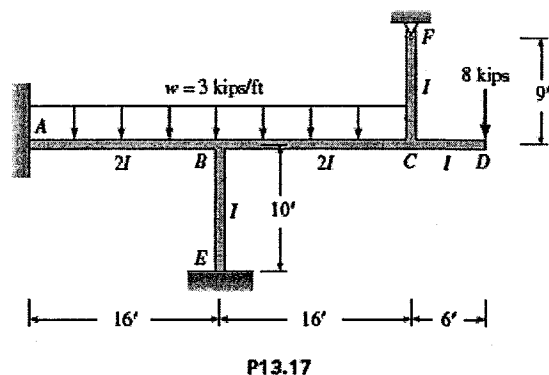
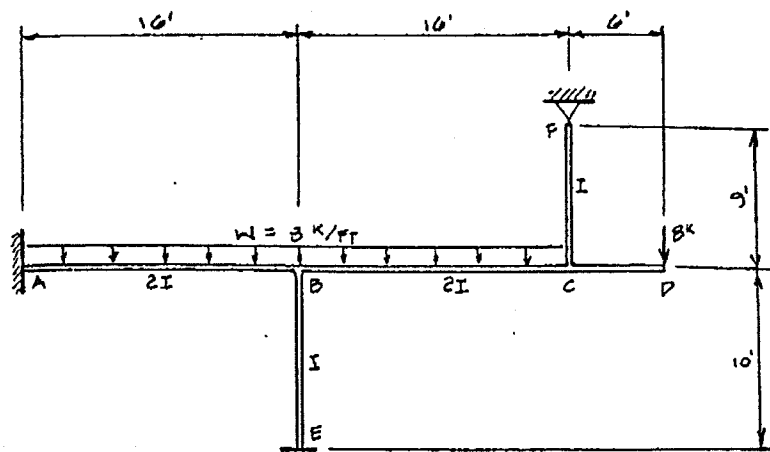
$$\sum M_E = 0; -33.75 \text{ ft}\cdot k - 16.88 \text{ ft}\cdot k + V_{BE}(12') = 0$$

$$\underline{V_{BE} = 4.2k \rightarrow}$$

$$\sum F_x = 0; V_{EB} = 4.2k \leftarrow = E_x$$



P13.17. Analyze the frame in Figure P13.17 by moment distribution. Determine all reactions and draw the shear and moment diagrams, locating points of inflection and labeling values of maximum shear and moment in each span. E is constant, but I varies as noted.



JOINT B	D.F.'s	JOINT C	D.F.'s
$K_{BA} = \frac{2I}{16}$	D.F. _{BA} = 0.36	$K_{CB} = \frac{2I}{16}$	D.F. _{CB} = 0.6
$K_{BC} = \frac{2I}{16}$	D.F. _{BC} = 0.36	$K_{CD} = \frac{3}{4}(\frac{I}{6})$	D.F. _{CD} = 0.4
$K_{BE} = \frac{I}{10}$	D.F. _{BE} = 0.28	$\Sigma K's = 5I/24$	
$\Sigma K's = 0.35I$			

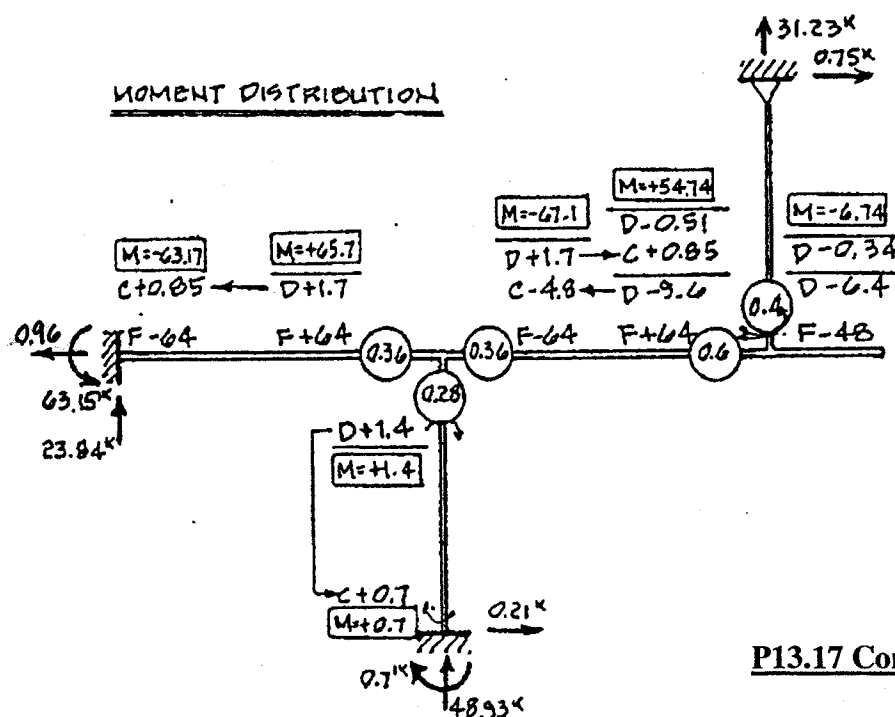
FIXED END MOMENTS

$$F_{AB} = F_{BC} = -3(16)^2/12 = -64 \text{ k}$$

$$F_{BA} = F_{CB} = 3(16)^2/12 = +64 \text{ k}$$

$$F_{CD} = -48 \text{ k}$$

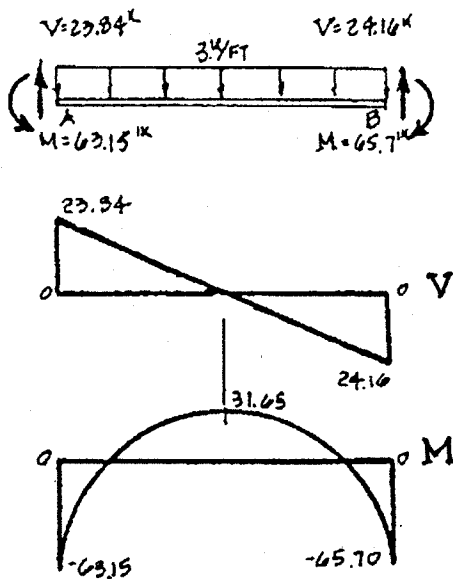
MOMENT DISTRIBUTION



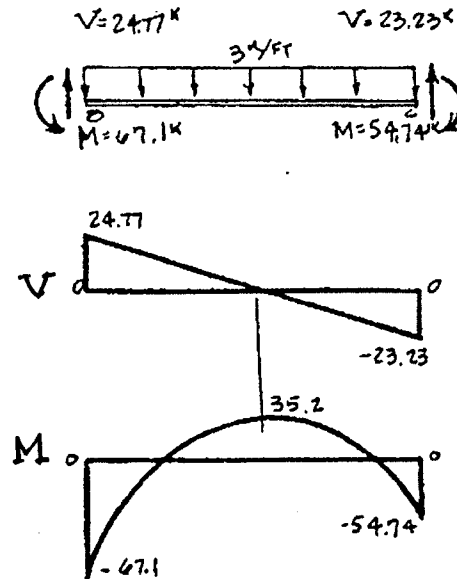
P13.17 Continued

P13.17 Continued

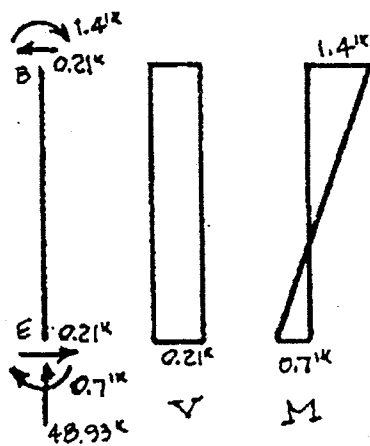
MEMBER AD



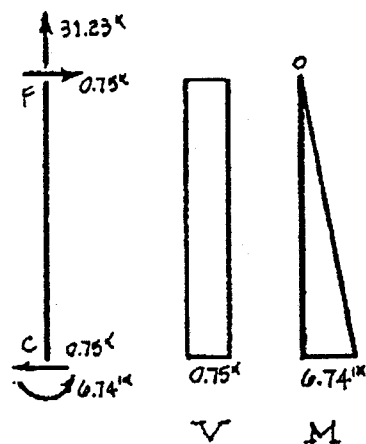
MEMBER BC



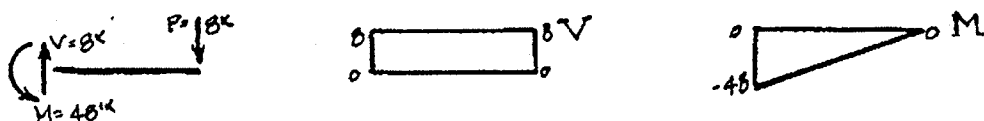
MEMBER DE



MEMBER FC



MEMBER CD



SYMMETRIC STRUCTURE WITH SYMMETRIC LOADS; EVALUATE HALF THE STRUCTURE: $\theta_{A-C-D} = 0$

$$K_{BA} = \frac{EI}{20} \quad DF_{BA} = 0.5$$

$$K_{BC} = \frac{1.5EI}{30'} \quad DF_{BC} = 0.5$$

$$\Sigma K @ B = 0.1 EI \quad 1.0$$

$$K_{CB} = \frac{1.5 EI}{30'} \quad DF_{CB} = 0.40$$

$$K_{CH} = \frac{EI}{20'} \quad DF_{CH} = 0.40$$

$$K_{CD} = \frac{1}{2} \left(\frac{1.5EI}{30} \right) \quad DF_{CD} = 0.20$$

$$\Sigma K @ C = 0.125 EI \quad 1.0$$

$$FEM = \frac{WL^2}{12} = \frac{3 \text{ k/ft} (30)^2}{12} = \pm 225 \text{ ft.k}$$

$$\Sigma M = -118.4 \text{ Ft}\cdot\text{k}$$

DEM 4028

COM - 056

DEM + 5 143

COM -11.25

DEM +112 E

FEM = 225.0

222.0

$$\Sigma M = +260.5$$

DEM - 0.06

COM + 0 14

DEM -1.12

COM +2.81

DEM - 22 E

DEM - 22.9
COM 154.2

FFM +22F

12M 1229.0

$$\Sigma M = -236.8$$

DEM - Q. Q3

DEM-REG

REF 0.70

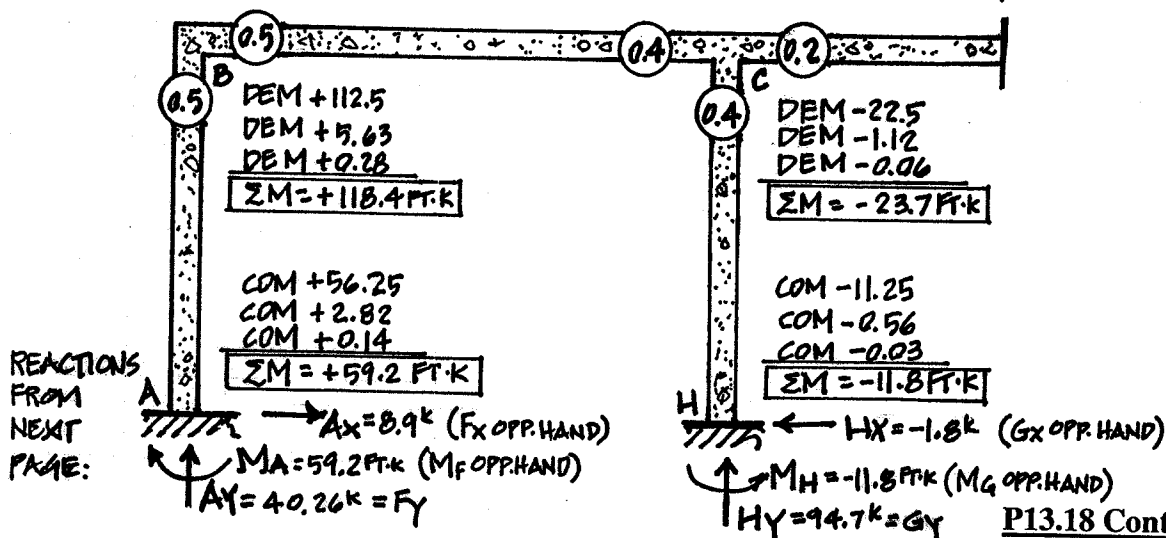
DEM - 11.25

DEM-11.25

EFM-225

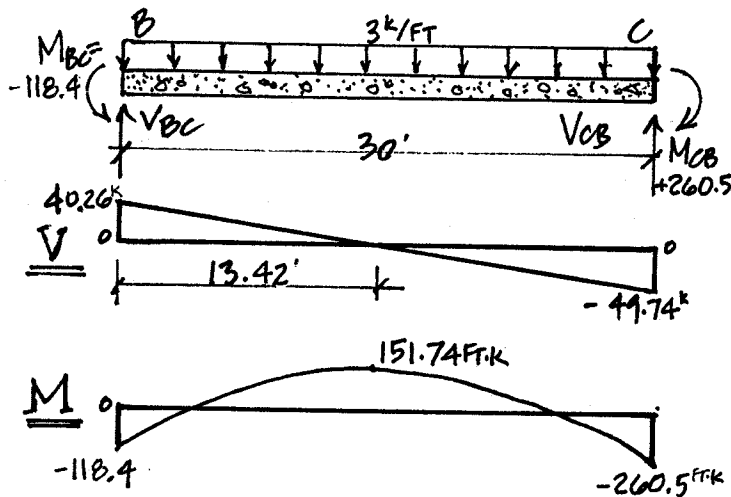
FLM-225

SYMMETRY
 $\theta_k = 0$.



P13.18 Continued

P13.18 Continued



SEGMENT "BC" :

$$\sum M_B = 0;$$

$$-118.4 + 3 \text{ k/ft} (30')^2 / 2 - V_{CB} (30') + 260.5 = 0$$

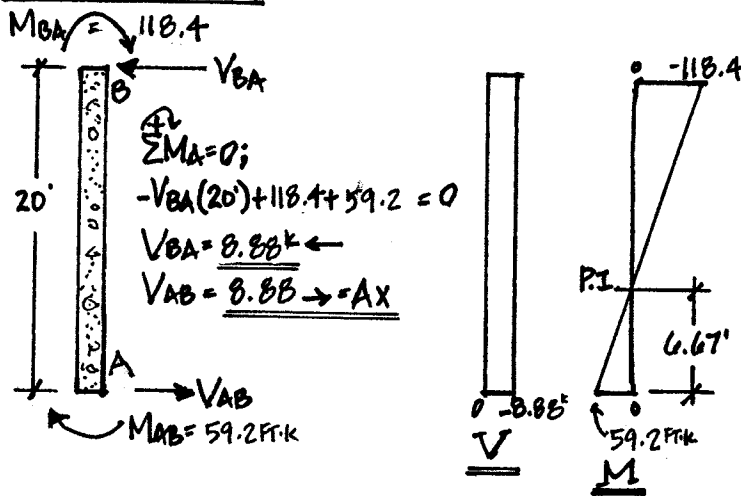
$$V_{CB} = 49.74 \text{ k} \uparrow$$

$$+\uparrow \sum F_y = 0;$$

$$-3 \text{ k/ft} (30') + 49.74 + V_{BC} = 0$$

$$V_{BC} = 40.26 \text{ k} \uparrow$$

SEGMENT "AB" :



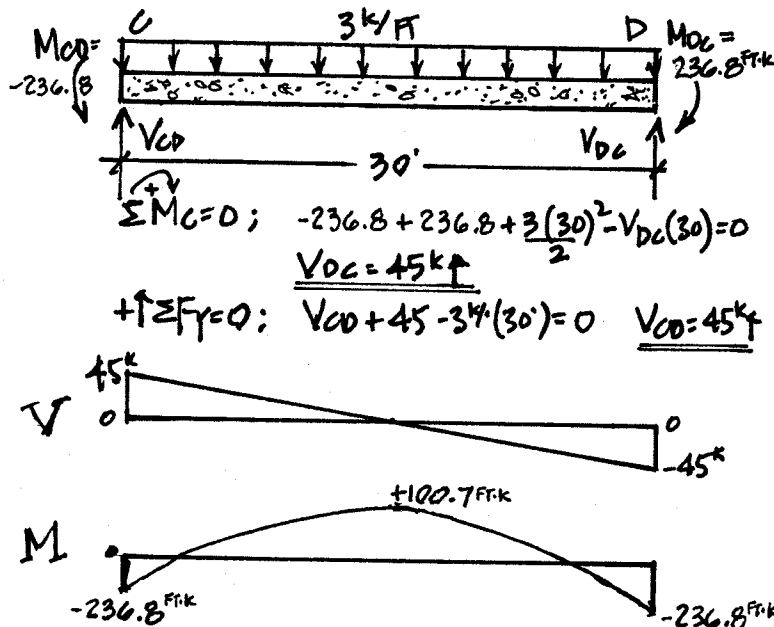
$$\sum M_A = 0;$$

$$-V_{BA} (20') + 118.4 + 59.2 = 0$$

$$V_{BA} = 8.88 \text{ k} \leftarrow$$

$$V_{AB} = 8.88 \text{ k} \rightarrow = A_x$$

SEGMENT "CD" :



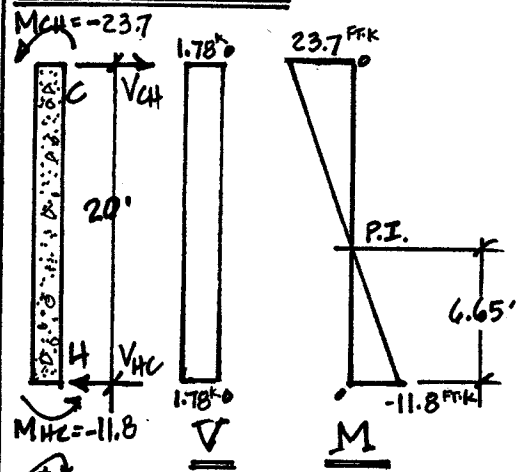
$$\sum M_C = 0;$$

$$-236.8 + 236.8 + 3 \frac{(30')^2}{2} - V_{DC} (30') = 0$$

$$V_{DC} = 45 \text{ k} \uparrow$$

$$+\uparrow \sum F_y = 0; V_{CD} + 45 - 3 \text{ k/ft} (30') = 0 \quad V_{CD} = 45 \text{ k} \uparrow$$

SEGMENT "CH" :



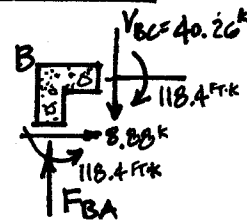
$$\sum M_H = 0;$$

$$-23.7 - 11.8 + V_{CH} (20') = 0$$

$$V_{CH} = 1.775 \text{ k} \rightarrow$$

$$V_{HC} = 1.775 \text{ k} \leftarrow = H_x$$

JOINT "B" :

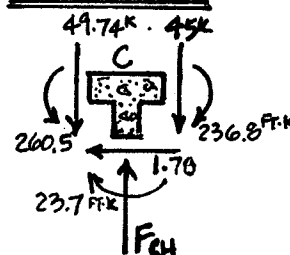


$$+\uparrow \sum F_y = 0;$$

$$F_{BA} = 40.26 \text{ k} \uparrow$$

$$A_y = 40.26 \text{ k} \uparrow$$

JOINT "C" :

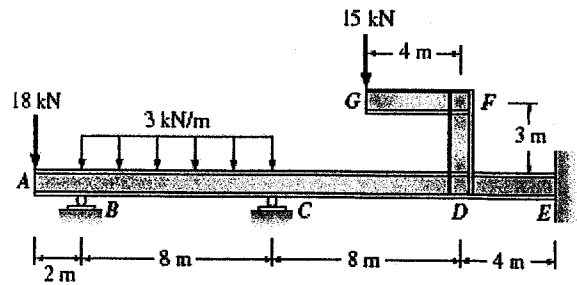


$$+\uparrow \sum F_y = 0;$$

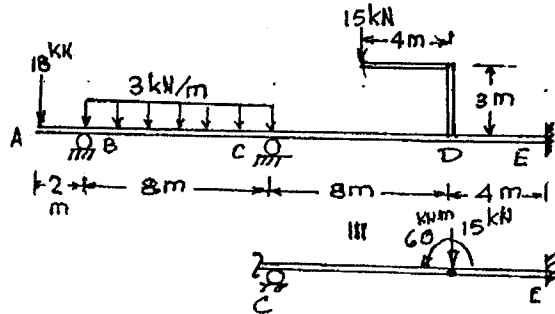
$$F_{CH} = 94.7 \text{ k}$$

$$H_y = 94.7 \text{ k}$$

P13.19. Analyze the beam in Figure P13.19 by the moment distribution method. Determine all reactions and draw the moment and shear curves for beam *ABCDE*; *EI* is constant.



P13.19



$$K_{BC} = \frac{I}{8} \times \frac{3}{4} = \frac{3I}{32} = \frac{9I}{96}$$

$$K_{CE} = \frac{I}{12} = \frac{8I}{96}$$

$$\sum K's = \frac{17I}{96}$$

D.F.
 $\frac{9/17}{8/17}$

$$FEM_{BC} = \frac{wL^2}{12} = \frac{3(8)^2}{12} = \pm 16 \text{ kN}\cdot\text{m}$$

$$FEM_{CE} = -\frac{Pb^2a}{L^2} - \frac{Mb(2a-b)}{L^2}$$

$$= -\frac{15(4)^2 \cdot 8}{144} - \frac{60 \times 4(16-4)}{12^2}$$

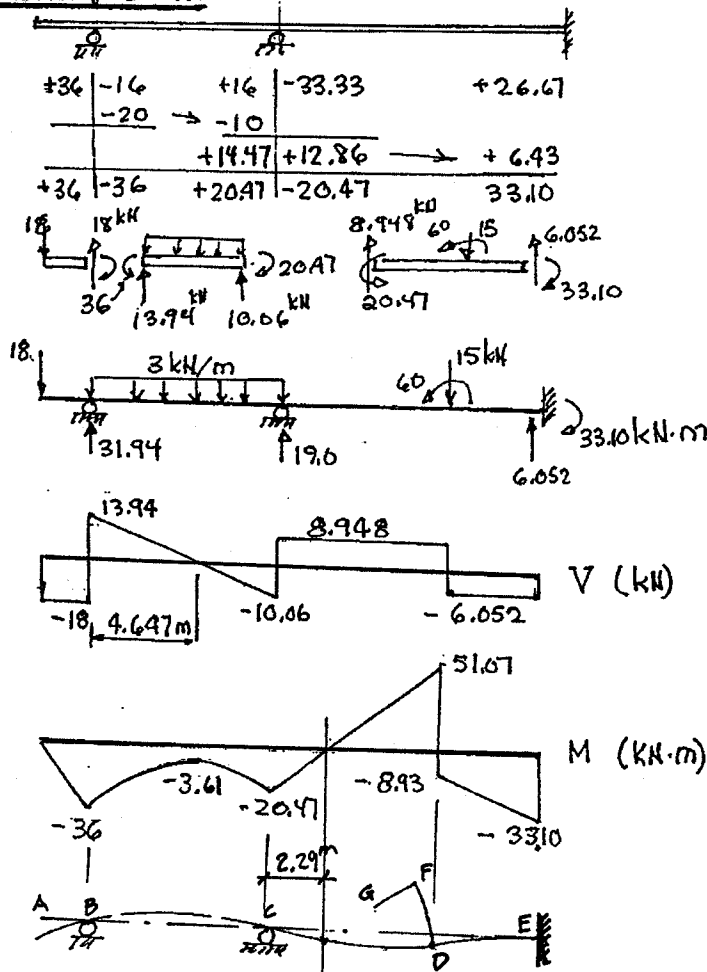
$$= -33.33 \text{ kN}\cdot\text{m}$$

$$FEM_{EC} = +\frac{Pba^2}{L^2} - \frac{Ma(2b-a)}{L^2}$$

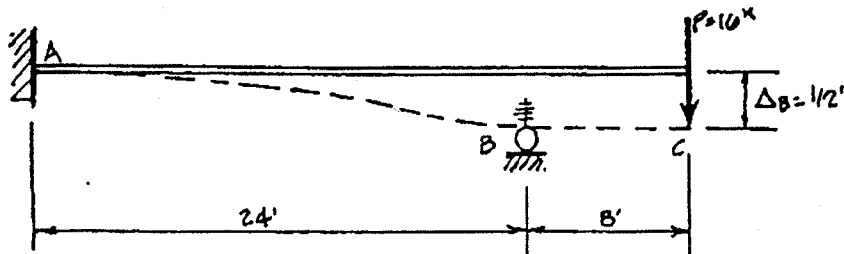
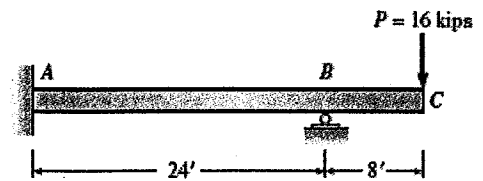
$$= \frac{15(4)^2 \cdot 8}{144} - \frac{60 \times 8(8-8)}{144}$$

$$= +26.67 \text{ kN}\cdot\text{m}$$

MOMENT DISTR.



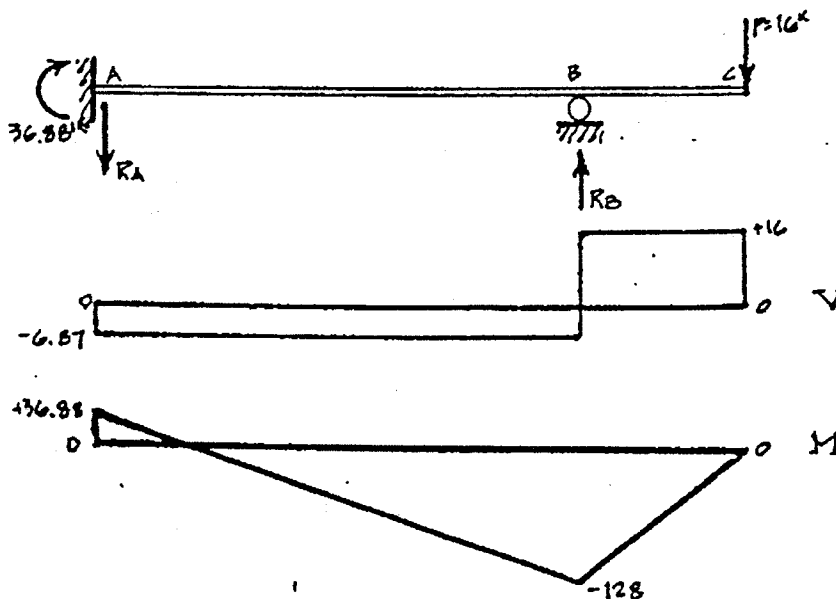
P13.20. If support B in Figure P13.20 settles $\frac{1}{2}$ in. under the 16-kip load, determine the reactions and draw the shear and moment curves for the beam. Given: $E = 30,000 \text{ kips/in}^2$, $I = 600 \text{ in}^4$.



USING EQU. 12.22

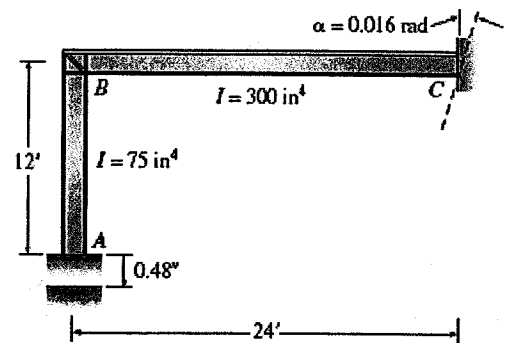
$$\begin{aligned} FEM_{BA} = FEM_{AB} &= -6EI\Delta / L^2 = -6(30000)(6000)(\frac{1}{2}) / (24(12))^2 \\ &= -651.04 \text{ IN}\cdot\text{K} = \underline{-94.25 \text{ FT}\cdot\text{K}} \end{aligned}$$

-94.25	-94.25	-128	FEM
	+182.25		DEM
+36.88			COM
+36.88	+128	-128	FINAL



$$\begin{aligned} \sum M_A = 0; \quad 36.88 + 16(32) - R_B(24) &= 0 & R_B &= 22.87 \text{ k}\uparrow \\ \sum F_y = 0; \quad R_A + 22.87 - 16 &= 0 & R_A &= -6.87 \text{ k}\downarrow \end{aligned}$$

P13.21. If support *A* in Figure P13.21 is constructed 0.48 in too low and the support at *C* is accidentally constructed at a slope of 0.016 rad clockwise from a vertical axis through *C*, determine the moment and reactions created when the structure is connected to its supports.
Given: $E = 29,000$ kips/in².

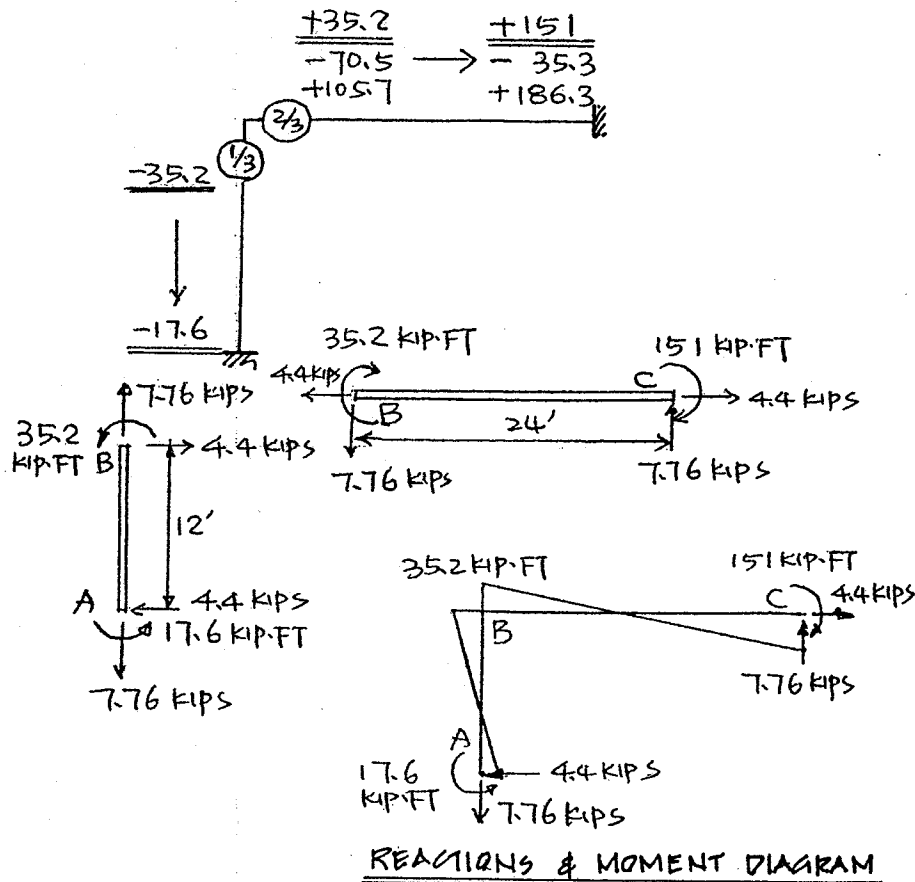


P13.21

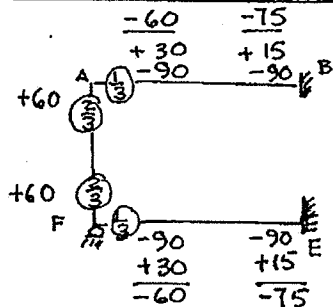
$$\begin{aligned}
 FEM_{BC} &= \frac{6EI\Delta}{L^2} + \frac{2EI\alpha}{L} \\
 &= \frac{6(29000)(300)(0.48)}{(24 \times 12)^2} + \frac{2(29000)(300)(0.016)}{24 \times 12} \\
 &= 1268.8 \text{ kip-in} = +105.7 \text{ kip-ft} \\
 FEM_{CB} &= \frac{6EI\Delta}{L^2} + \frac{4EI\alpha}{L} \\
 &= \frac{6(29000)(300)(0.48)}{(24 \times 12)^2} + \frac{4(29000)(300)(0.016)}{24 \times 12} \\
 &= 2235.4 \text{ kip-in} = +186.3 \text{ kip-ft}
 \end{aligned}$$

$$K_{AB} = \frac{I}{12} = \frac{75}{12}, \quad DF_{AB} = \frac{1}{3}$$

$$K_{BC} = \frac{I}{24} = \frac{300}{24}, \quad DF_{BC} = \frac{2}{3}$$



MOMENT DISTRIBUTION



1. MEMBER EB IS STRAIGHT $\therefore \theta_B = \theta_E = 0$
 $\therefore$ JOINTS B AND E CAN BE TREATED AS FIXED
 SUPPORTS. ALSO $\Delta_B = \Delta_E = \Delta$

3. THE VALUES OF $V \neq M$ ARE IDENTICAL IN MEMBERS AB, FE, DE AND CB, AND $V = 25 \text{ KIPS}$.

$$FEM_{BA} = FEM_{EF} = FEM_{AB} = FEM_{FE} = \frac{2EI}{4} (0 + 0 - 3\psi) = \frac{3EI}{2} \psi$$

ARBITRARILY LET $\frac{3EI}{2}\psi = 90 \text{ kN}\cdot\text{m}$

$$K_{AB} = K_{FE} = \frac{I}{L} = \frac{I}{4}$$

$$K_{AF} = K_{AF} = \frac{3I}{2L} = \frac{3I}{2 \times 3}$$

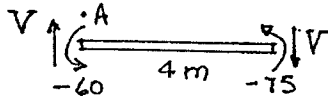
$$= K_5 = \frac{3I}{4}$$

$$\begin{aligned} \text{D.F.} \quad \frac{I/4}{3/4 I} &= \frac{1}{3} \\ \frac{2I/4}{3/4 I} &= \frac{2}{3} \end{aligned}$$

Page 13-22A

P13.22 Continued

COMPUTE V IN AB & FE



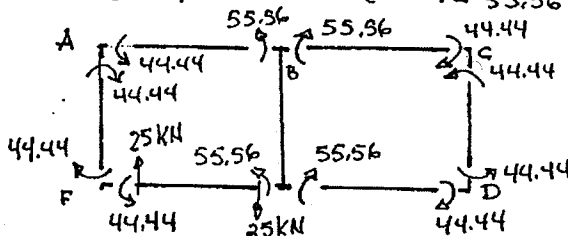
$$\sum M_A = 0 = 4V - 60 - 75$$

$$V = \frac{135}{4} = 33.75 \text{ kN}$$

SINCE $V = 25 \text{ kN}$ REDUCE ALL FORCES

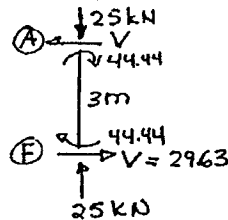
BY RATIO OF $25/33.75 = 0.740741$

$$60(0.7407) = 44.44 \quad 75(0.7407) = 55.56 \text{ kN}\cdot\text{m}$$



MEMBER END MOMENTS
(kN·m)

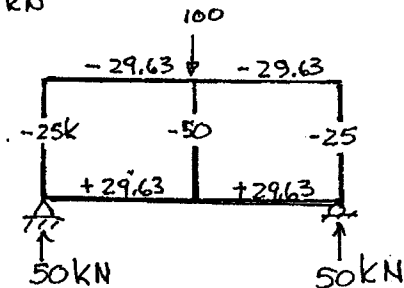
FORCES IN MEMBER AF



$$\sum M_F = 0$$

$$0 = 44.44 + 44.44 - V(3)$$

$$V = 29.63 \text{ kN}$$

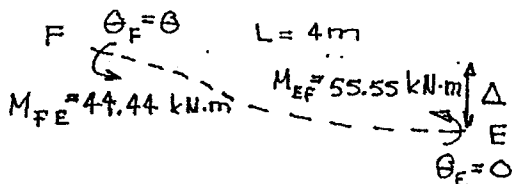


AXIAL FORCES IN ALL
MEMBERS IN kN.

BOTTOM CHORD IN TENSION
TOP CHORD IN COMPRESSION
SOME TRUSS ACTION,
VERTICAL FORCE TRANSMITTED BY V.

DEFLECTION @ MIDSPAN

SINCE MEMBER END MOMENTS ESTABLISHED,
USE SLOPE-DEFL EQ TO COMPUTE
DISPLACEMENT AT MIDPOINT



$$M_{FE} = -44.44 = \frac{2EI}{L} (2\theta - 3\psi)$$

$$M_{EF} = 55.55 = \frac{2EI}{L} (\theta - 3\psi)$$

$$\text{SOLVING FOR } \theta = \frac{11.11L}{2EI}$$

$$\psi = \frac{-66.66L}{3 \times 2EI}$$

$$\psi = \frac{\Delta}{L} = \frac{-66L}{3 \times 2EI}$$

$$\Delta = \frac{-66L^2}{6EI} = \frac{-66(4)^2}{6 \times 200 \times 250}$$

$$\Delta = 0.00356 \text{ m} = 3.56 \text{ mm}$$

ALTERNATE COMPUTATION BY VIRTUAL
WORK:

USE Q-SYSTEM = $\frac{1}{100}$ P-SYSTEM

$$1 \text{ kN} \times \delta = \sum \int M_Q M_P \frac{dx}{EI} = \frac{1}{100EI} \int M_P^2 dx$$

$$\delta = \frac{1}{100EI} \left[4 \int_0^4 M_P^2 dx + 2 \int_{-1.5}^{1.5} M_P^2 dx \right]$$

MEMBER AB MEMBER AF

$$= \frac{1}{100EI} \left[4 \int_0^4 (-44.5 + 25x)^2 dx + 2 \int_{-1.5}^{1.5} (29.63x)^2 dx \right]$$

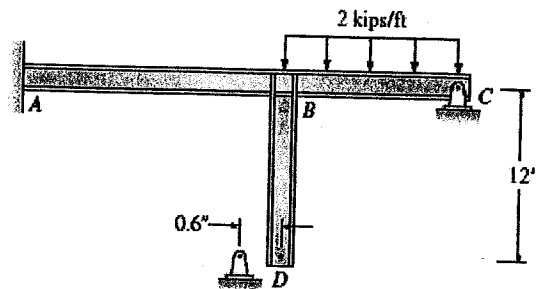
$$= \frac{1}{100EI} [4(3454.3) + 2(1380.7)]$$

$$= \frac{17,779}{100EI} = \frac{17,779}{100(200 \times 10^6)(250 \times 10^{-6})}$$

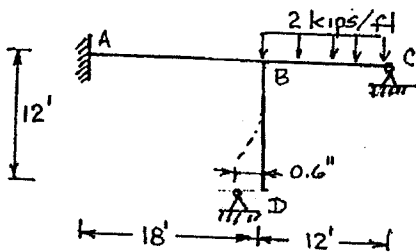
$$\delta = 0.00356 = 3.56 \text{ mm}$$

P13.23. Due to a construction error, the support at D has been constructed 0.6 in to the left of column BD . Using moment distribution, determine the reactions that are created when the frame is connected to the support and the uniform load is applied to member BC . Draw the shear and moment diagrams and sketch the deflected shape. $E = 29,000 \text{ kips/in}^2$, $I = 240 \text{ in}^4$ for all members.

$$I = 240 \text{ in}^4, E = 29,000 \text{ kips/in}^2$$



P13.23



$$FEM_{BC} = -\frac{WL^2}{12} = -\frac{2(12)^2}{12} = -24 \text{ kip}\cdot\text{ft}$$

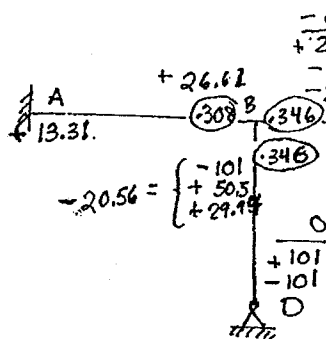
$$FEM_{CB} = 24 \text{ kip}\cdot\text{ft}$$

$$FEM_{BD} = -\frac{6EI\Delta}{L^2} = -\frac{6(29,000)(240)(0.6)}{(12 \times 12)^2} = -1208.14 \text{ kips} = -101.41 \text{ k}$$

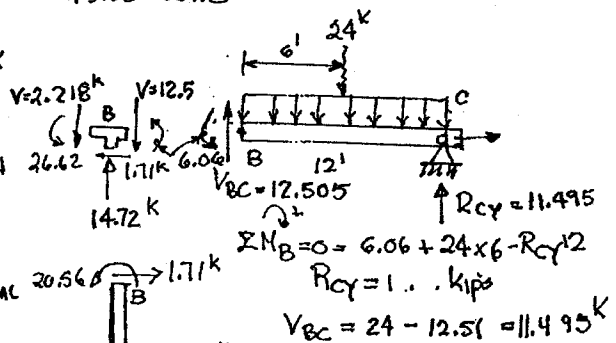
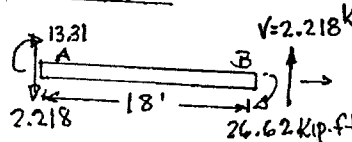
$$K_{AB} = \frac{I}{L} = \frac{240}{18} = 13.33 = \frac{D.F.}{.308}$$

$$K_{BC} = \frac{3I}{4L} = \frac{3(240)}{4(12)} = 15.0 = .344$$

$$K_{BD} = \frac{3I}{4L} = \frac{3(240)}{4(12)} = 15.00 = \frac{.346}{43.33 = \Sigma K's}$$



FREE BODIES



AXIAL LOAD IN GIRDERS

DEPENDS ON AXIAL STIFFNESS.

$$\Delta L = \frac{FL}{AE} \text{ INVERSELY PROPORTIONAL TO } L$$

$$F = AE(\Delta L)$$

$$\frac{F_{AB}}{F_{BC}} = \frac{12}{18}$$

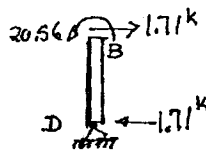
$$\therefore F_{AB} = \frac{12}{18} F_{BC}$$

$$F_{AB} + F_{BC} = 1.71$$

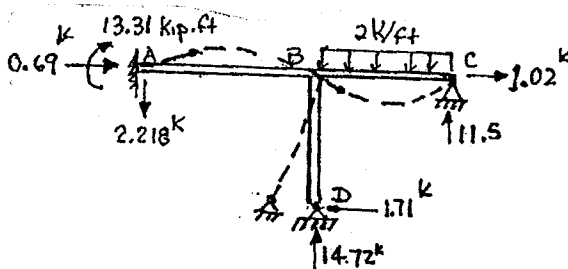
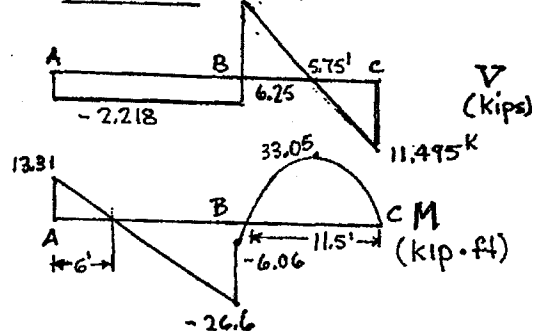
$$\frac{2}{3} F_{BC} + F_{BC} = 1.71$$

$$F_{BC} = 1.02 \text{ kips}$$

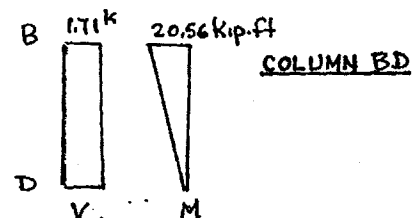
$$F_{AB} = .69$$



GIRDER ABC

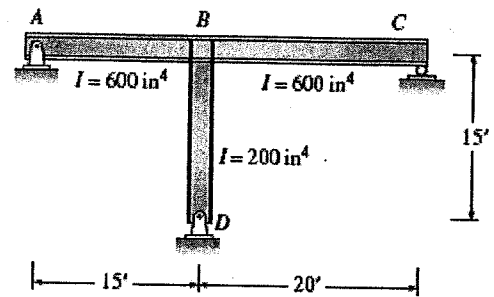


REACTIONS

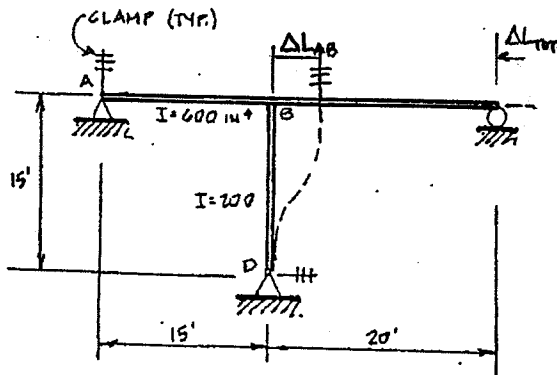


COLUMN BD

P13.24. What moments are created in the frame in Figure P13.24 by a temperature change of $+80^{\circ}\text{F}$ in girder ABC? The coefficient of temperature expansion $\alpha_t = 6.6 \times 10^{-6} \text{ (in/in)/}^{\circ}\text{F}$ and $E = 29,000 \text{ kips/in}^2$.



P13.24



ANALYSIS FOR TEMPERATURE FORCES.

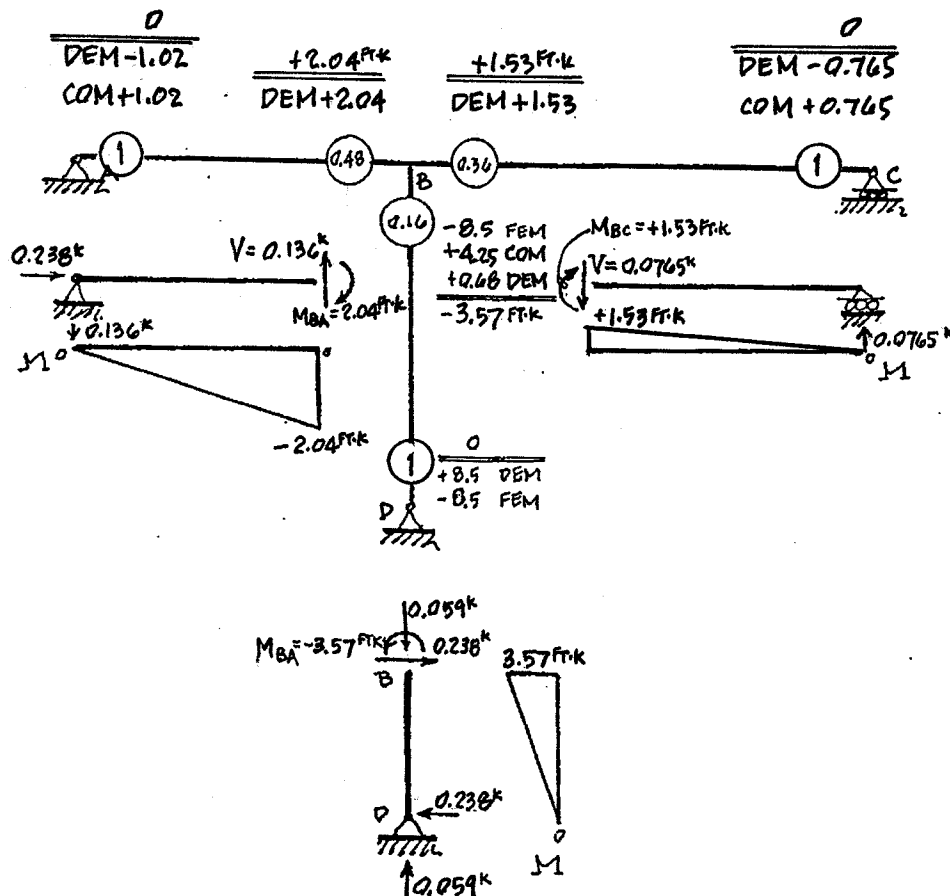
FRAME IS CLAMPED IN DEFLECTED POSITION.

$$\begin{aligned} K_{AB} &= \frac{3}{4} \left(\frac{600}{15} \right) = 40 & D.F._{AB} &= 0.48 \\ K_{BC} &= \frac{3}{4} \left(\frac{600}{20} \right) = 30 & D.F._{BC} &= 0.36 \\ K_{BD} &= \frac{3}{4} \left(\frac{200}{15} \right) = 13.33 & D.F._{BD} &= 0.16 \\ \Sigma K's &= & & 83.33 \end{aligned}$$

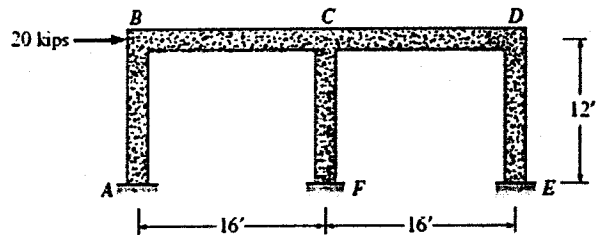
$$\Delta L_{TF} = \alpha_T (L) \Delta T = 6.6 (10^{-6}) (15' + 20') (12'') (80^{\circ}) = 0.2218''$$

$$FEM_{DB} = \frac{6EI\Delta/L^2} = \frac{6(29,000)(200)(\Delta L_{AB})}{(15 \times 12'')^2} = 8.5 \text{ FEM}$$

$$\text{WHERE } \Delta L_{AB} = 6.6 (10^{-6}) (15 \times 12'') (80) = 0.09504''$$



P13.25. Determine the reactions and the moments induced in the members of the frame in Figure P13.25. Also determine the horizontal displacement of joint B. Given: EI is constant for all members, $I = 1,500 \text{ in}^4$, and $E = 3,000 \text{ kips/in}^2$.



P13.25

JOINT "B" & "D" DISTRIBUTION FACTORS

$$\begin{aligned} K_{BA} &= EI/12 & D.F. BA &= D.F. DE = 0.571 \\ K_{BC} &= EI/16 & D.F. BC &= D.F. DC = 0.429 \\ \Sigma K &= 0.146EI & \Sigma &= 1.0 \end{aligned}$$

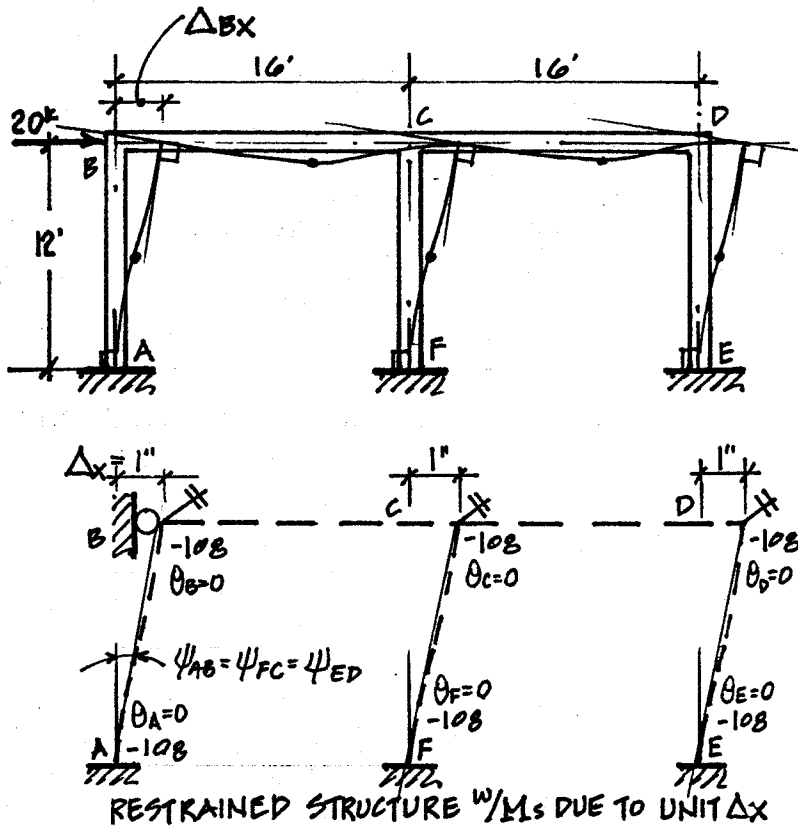
JOINT "C"

$$\begin{aligned} K_{CB} &= EI/16 & D.F. CB &= 0.30 \\ K_{CD} &= EI/16 & D.F. CD &= 0.30 \\ K_{CF} &= EI/12 & D.F. CF &= 0.40 \\ \Sigma K &= 0.2083EI & &= 1.0 \end{aligned}$$

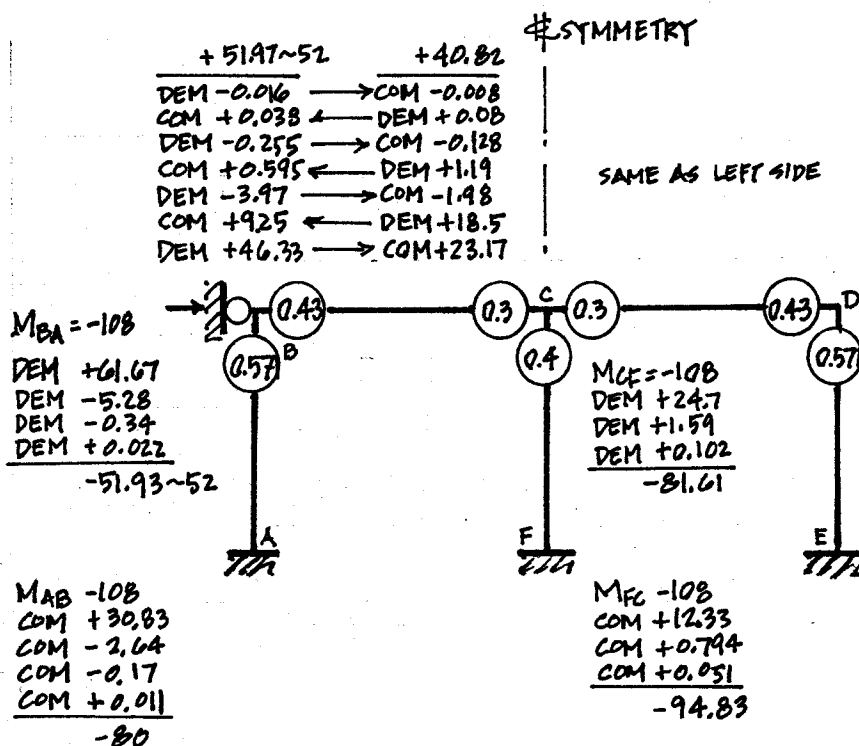
COLUMN MOMENTS IN 'RESTRAINED STR.

$$M_{AB} = M_{BA} = 6EI/L^2 = \frac{6(3000)(1500)}{(12 \times 12)^2 (12')} = -108 \text{ Ft.K}$$

$$M_{FC} = M_{CF} = M_{ED} = M_{DE} = -108 \text{ Ft.K}$$



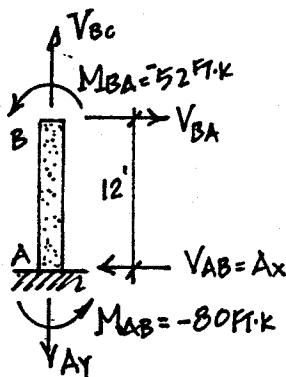
RESTRAINED STRUCTURE w/ Ms DUE TO UNIT ΔX



P13.25 Continued

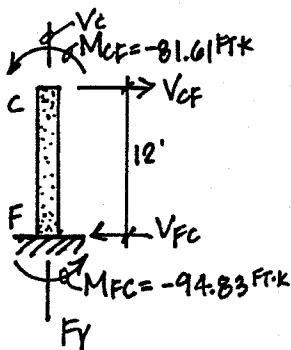
P13.25 Continued

COLUMN "AB" & "DE":



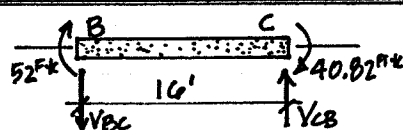
$$\begin{aligned} \sum M_A = 0; & \quad -52 - 80 + V_{BA}(12') = 0 \\ V_{BA} = 11^k & \rightarrow = V_{DE} \\ V_{AB} = 11^k & \leftarrow = V_{ED} \end{aligned}$$

COLUMN "CF":



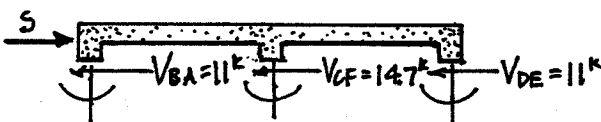
$$\begin{aligned} \sum M_F = 0; & \quad -81.61 - 94.83 + V_{cf}(12') = 0 \\ V_{cf} = 14.7^k & \rightarrow \\ V_{fc} = 14.7^k & \leftarrow \end{aligned}$$

GIRDER "BC" (CD' OPP. HAND):



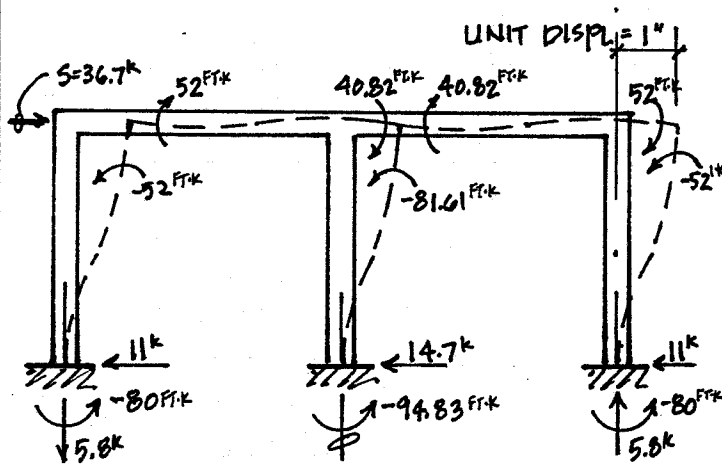
$$\begin{aligned} \sum M_B = 0; & \quad 52 + 40.82 - V_{cb}(16) = 0 \\ V_{cb} = 5.8^k & \uparrow = (-V_{cd}) \\ V_{bc} = 5.8^k & \downarrow = (-V_{dc}) \end{aligned}$$

GIRDER "BCD":

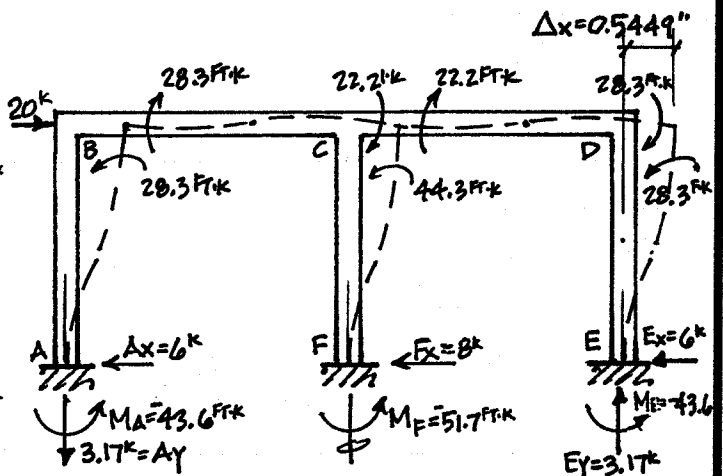


$$\begin{aligned} \sum F_x = 0; & \quad S - (2 \times 11^k) - 14.7^k = 0 \\ S = 36.7^k & \rightarrow \end{aligned}$$

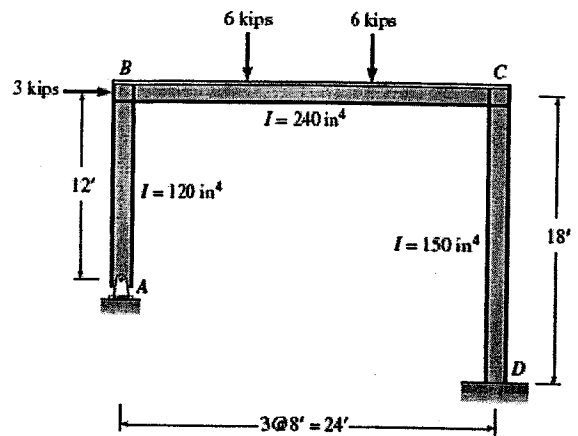
$$\therefore \text{SCALING FACTOR} = P/S = 20^k / 36.7 = 0.5449$$



REACTIONS & END MOMENTS DUE TO UNIT DISPLACEMENT



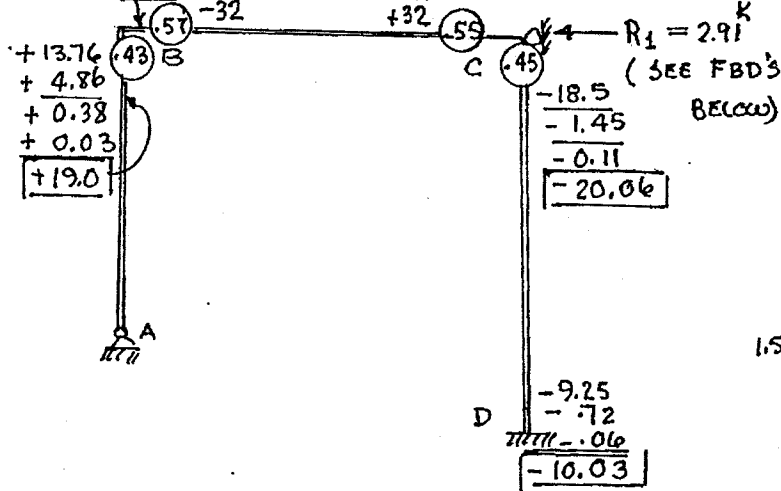
FINAL REACTIONS & MEMBER END MOMENTS



DISTRIBUTION FACTORS

CASE A : NO SIDESWAY

$$\begin{array}{r}
 M \\
 (\text{kip}\cdot\text{ft}) \\
 \hline
 +0.04 \\
 -0.07 \\
 \hline
 +0.5 \rightarrow \\
 -0.88 \leftarrow \\
 +6.45 \rightarrow \\
 -11.31 \leftarrow \\
 \hline
 -19.14 \\
 +18.24 \rightarrow
 \end{array}
 \quad
 \begin{array}{r}
 +20.06 \\
 -0.14 \\
 \hline
 0.25 \\
 -1.77 \\
 \hline
 +3.22 \\
 -22.62 \\
 \hline
 +9.12
 \end{array}$$



$$K_{AB} = \frac{3}{4} \frac{I}{L} = \frac{3}{4} \frac{(120)}{12} = 7.5$$

$$7.5/17.5 = 0.43$$

$$K_{BC} = \frac{240}{24} = 10$$

$$10/17.5 = \frac{0.57}{1.00}$$

JOINT C

$$K_{BC} = \frac{240}{24} = 10$$

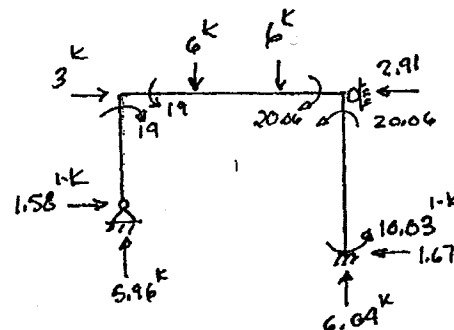
D.F.

$$10/18.33 = 0.55$$

$$K_{CD} = \frac{150}{18} = 8.33$$

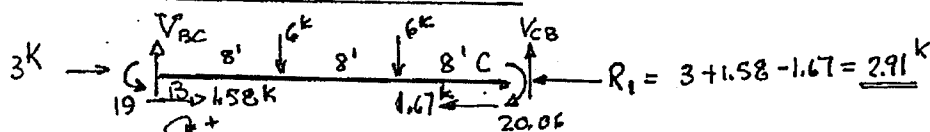
$$\Sigma K' = 18.33$$

$$8.33/18.33 = 0.45$$



REACTIONS: CASE A

COMPUTE SHEARS & FORCE R_1



$$\Sigma M_B = 0 = 6 \times 8 + 6 \times 16 - V_C \times 24 + 20.66$$

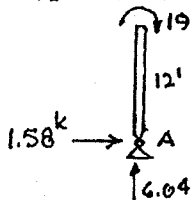
$$V_{CR} = 6.04^k$$

$$V_{BC} = 5.96^k$$

$$\sum M_A = 0 = 19 - Y_{BA} 12$$

$$V_{BA} = 1.58$$

$$1.58 = \frac{19}{12} = \underline{V_{BA}}$$



6.04
 $\downarrow$
 1.67
 20.06 K.p.ft

$$\sum M_D = 0 = -20.06 - 10.63 + V_{BA} \cdot 18$$

$$V_{BA} = \underline{1.67 \text{ kips}}$$

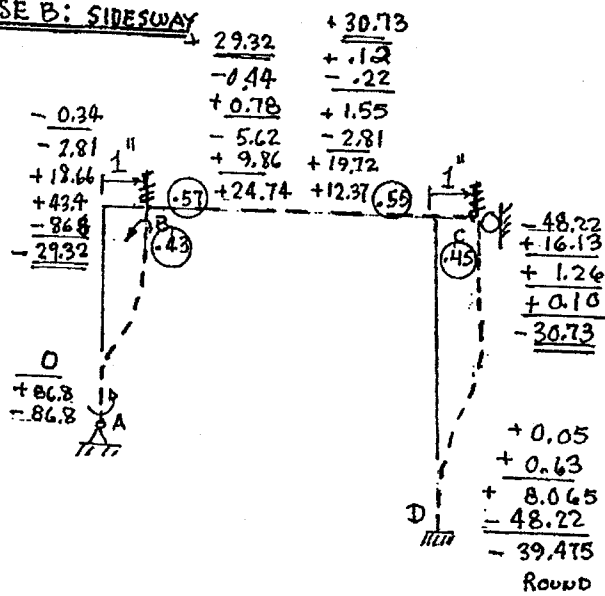
$$V_{BA} = \underline{1.67 \text{ kips}}$$

$D \downarrow 1003 \text{ kpf}$
 V_{AB}

P13.26 Continued

P13.26 Continued

CASE B: SIDESWAY



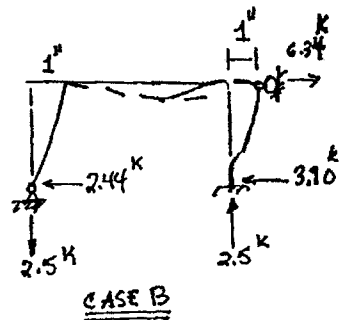
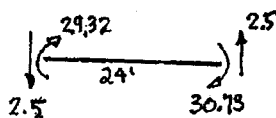
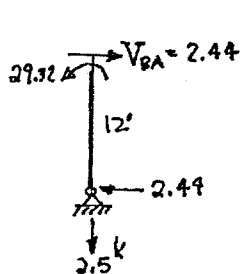
INTRODUCE A 1" DISPL. OF GIRDER

$$FEM_{AB} = \frac{2EI}{L}(-3\psi) \text{ where } \psi = \frac{1}{L}$$

$$FEM_{AB} = FEM_{BA} = \frac{2EI \cdot 20}{12 \times 12} \left(-3 \times \frac{1}{12 \times 12}\right) = -104.167 = -86.80 \text{ kip-ft}$$

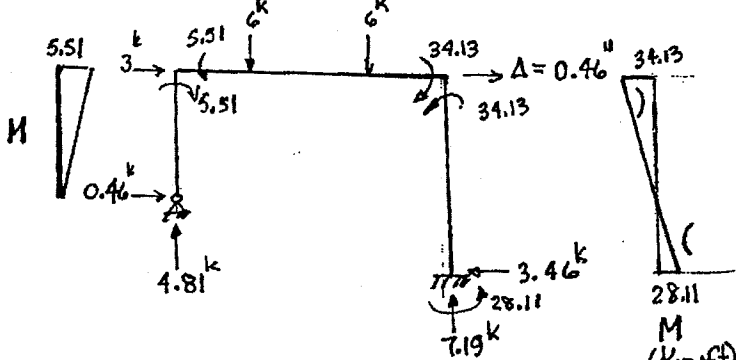
$$FEM_{CD} = FEM_{DC} = \frac{2EI \cdot 50}{18 \times 12} \left(-3 \times \frac{1}{18 \times 12}\right) = -578.70 \text{ kip-in} = -48.22 \text{ Kip-ft}$$

ROUND TO 39.48

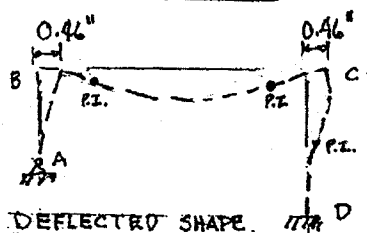


TO ACCOUNT FOR FORCE OF 2.91 k, MULTIPLY CASE B X $\frac{2.91}{6.34} = 0.458$

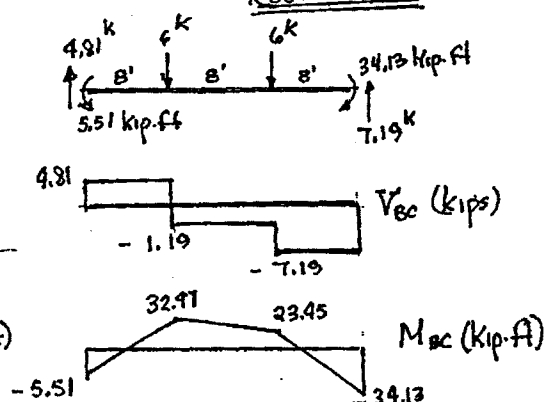
ROUND TO 0.46



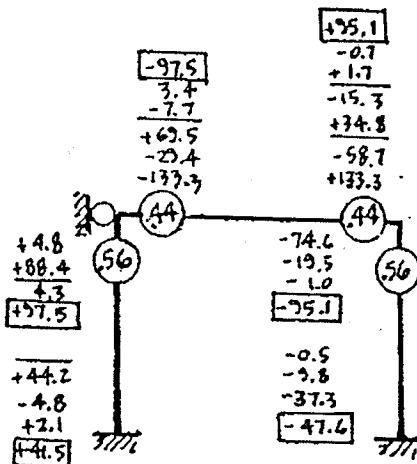
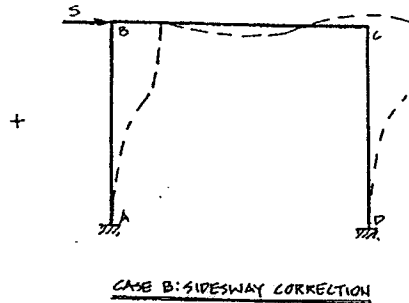
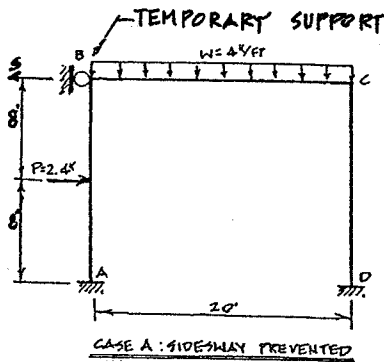
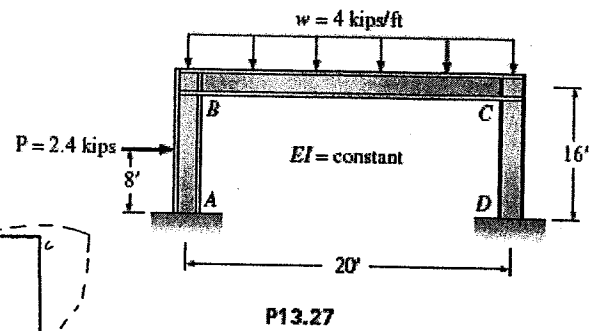
FINAL RESULTS



DEFLECTED SHAPE



P13.27. Analyze the frame in Figure P13.27 by moment distribution. Draw the shear and moment curves. Sketch the deflected shape.



CASE A: SIDESWAY PREVENTED

$$FEM_{AB} = \frac{P \cdot h}{8} = \frac{2.4(16)}{8} = 4.8^k$$

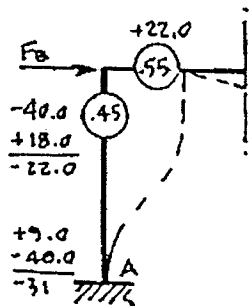
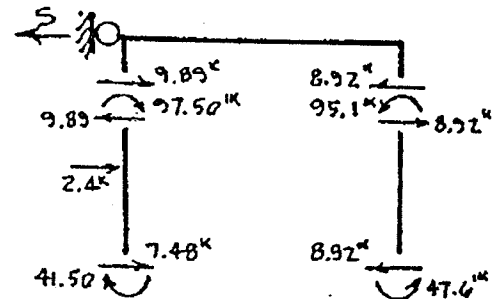
$$FEM_{BC} = \frac{wL^2}{12} = \frac{4(20)^2}{12} = 133.33^k$$

$$K_{AB} = \frac{I}{16} = \frac{5I}{80} \quad D.F. = 0.556$$

$$K_{BC} = \frac{I}{20} = \frac{4I}{80} \quad D.F. = 0.444$$

$$\Sigma K = \frac{9I}{80} \quad 1.000$$

COMPUTE HOLDING FORCE S
@ JOINT B. $S = 0.97^k$



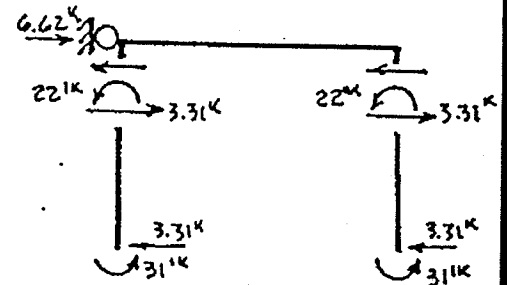
$$FEM_{AB} = FEM_{CD} = \frac{6EI\Delta}{L^2}$$

ASSUME Δ IS SELECTED
SO FEM'S = 40 FT-K

$$K_{AB} = \frac{I}{16} = \frac{5I}{80} \quad D.F. = 0.45$$

$$K_{BC} = \frac{3I}{20} = \frac{6I}{80} \quad D.F. = 0.55$$

$$\Sigma K = \frac{11I}{80}$$

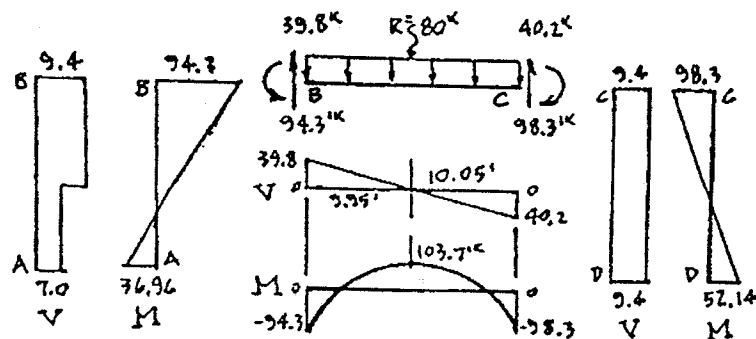
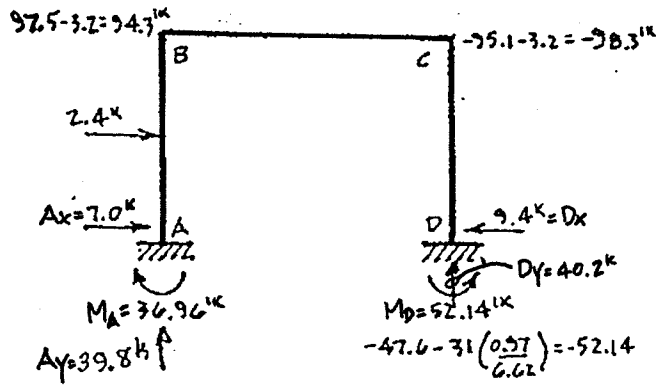


P13.27 Continued

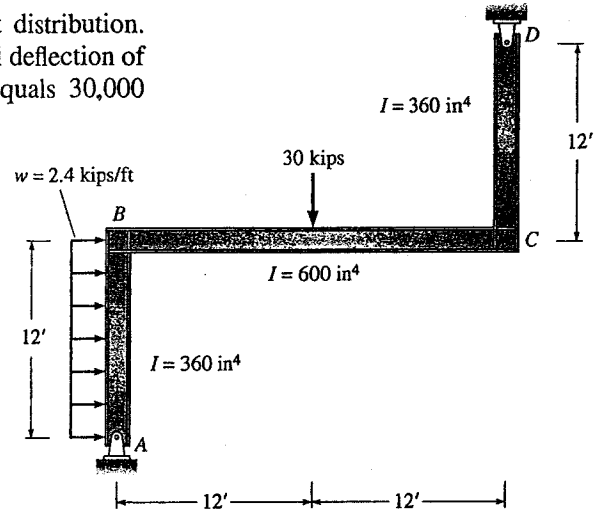
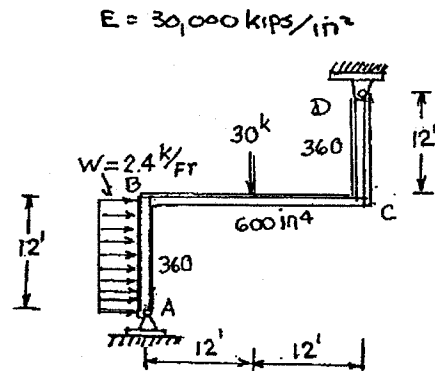
P13.27 Continued

CASE B: SIDESWAY CORRECTION
TO ELIMINATE 0.97^k HOLDING FORCE

MULTIPLY MOMENTS IN CASE B BY 0.97/6.62 AND ADD TO CASE A.



P13.28. Analyze the frame in Figure P13.28 by moment distribution. Draw the shear and moment curves. Compute the horizontal deflection of joint B. Sketch the deflected shape. E is constant and equals 30,000 kips/in².



P13.28

$$E = 30,000 \text{ kips/in}^2$$

$$FEM_{BA} = FEM_{AB} = \pm \frac{wL^2}{12} = \pm \frac{2.4(12)^2}{12} = \pm 28.8 \text{ kip}\cdot\text{ft}$$

$$FEM_{CB} = FEM_{BC} = \pm \frac{PL}{8} = \pm \frac{30(24)}{8} = \pm 90 \text{ kip}\cdot\text{ft}$$

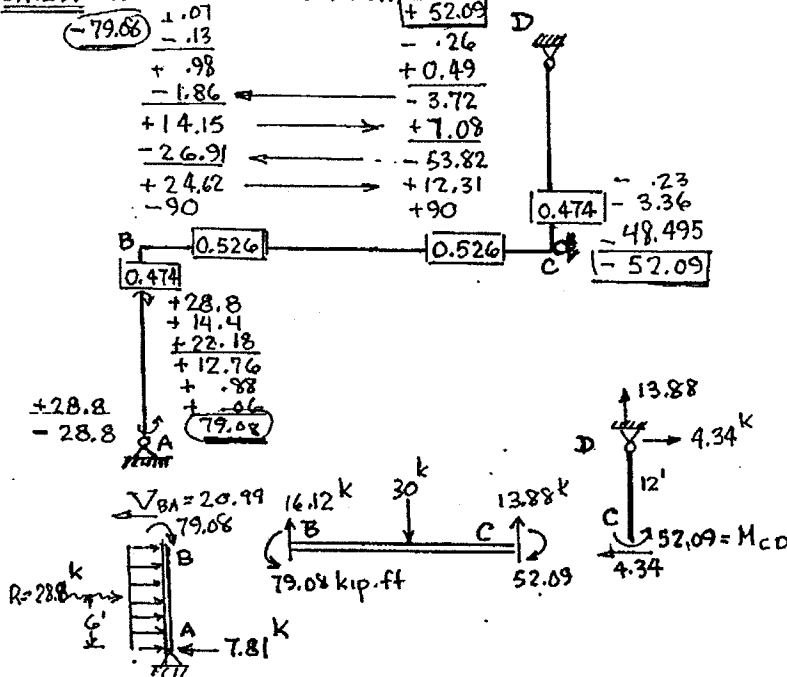
$$K_{AB} = K_{CD} = \frac{360}{12} \times \frac{3}{4} = 22.5$$

$$K_{BC} = \frac{600}{24} = 25$$

$$\Sigma K_S = 47.5$$

DISTR. FACTOR	
$22.5/47.5 = 0.474$	$25/47.5 = 0.526$

CASE A: NO SIDESWAY MOMENT DISTRIBUTION:



FORCE IN TEMPORARY SUPPORT AT C

$$\sum F_x = 0 = 20.99 + 4.34 - F_C = 0$$

$$F_C = 25.3 \text{ kips}$$

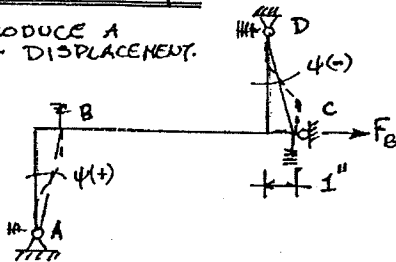
P13.28 Continued

P13.28 Continued

CASE B. SIDESWAY:

CLAMP ALL JOINTS

INTRODUCE A UNIT DISPLACEMENT.

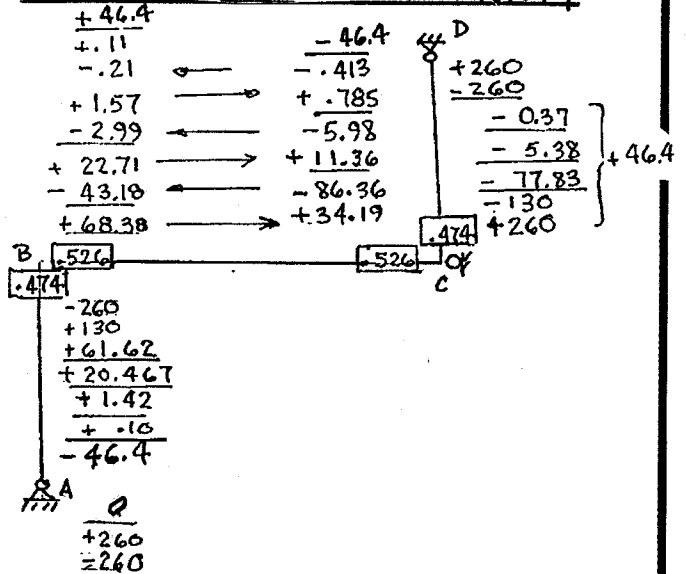


$$FEM = \frac{2EI}{L}(-3\psi) \quad \text{WHERE } \psi = \frac{1}{12 \times 12}$$

$$FEM_{AB} = FEM_{BA} = \frac{2(30,000)(360)(-1)}{(12 \times 12)(144)} = -3125 \text{ in. kips} = -260 \text{ ft. kips}$$

$$FEM_{CD} = FEM_{DC} = \frac{2(30,000)(360)(-1/144)}{144} = 260 \text{ ft. kips}$$

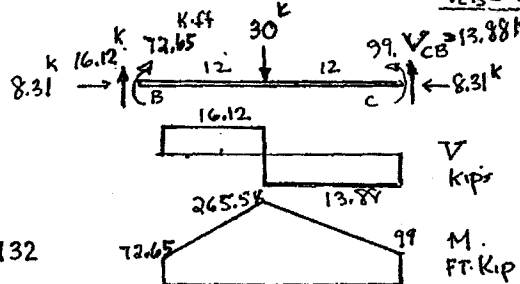
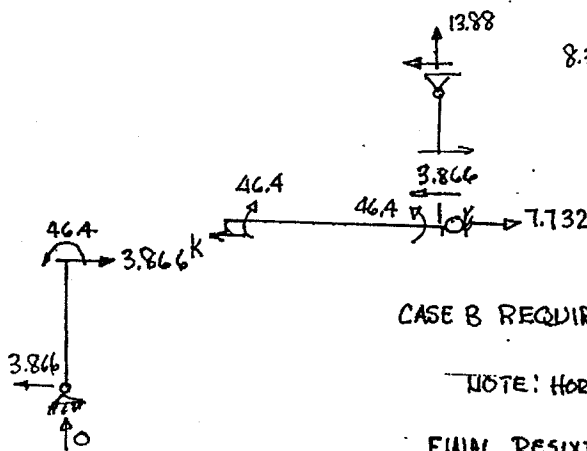
MOMENT DISTRIBUTION CASE B SIDESWAY:



$$\sum M_B = 0$$

$$0 = 72.65 + 30(12) - 99 - V_{CB} 24$$

$$V_{CB} = 13.88 \text{ kips}$$

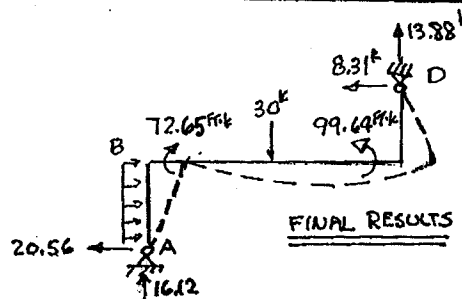


$$\text{CASE B REQUIRED} = \frac{25.3}{7.132} = 3.27$$

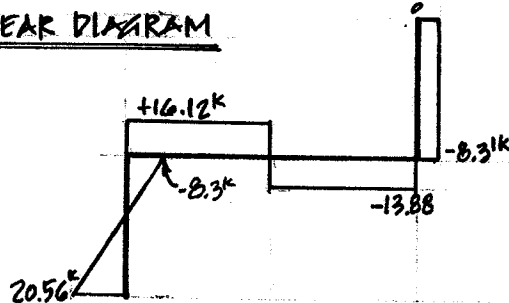
NOTE: HORIZ DISPL OF JOINT C = 3.27"

FINAL RESULTS CASE A + 3.72(CASE B)

$$\begin{aligned} \text{CASE A} + \text{CASE B} \\ M_{BA} &= 79.08 - (46.4)(3.27) = 72.65 \text{ kip-ft} \\ M_{CD} &= 52.09 + (46.4)(3.27) = 99.64 \text{ kip-ft} \\ R_{AX} &= 7.921 + 3.866(3.27) = 20.56 \text{ kips} \\ R_{DX} &= 4.33 - 3.866(3.27) = 8.31 \text{ kips} \\ M_{BC} &= -79.08 + 46.4(3.27) = 72.65 \text{ kip-ft} \\ M_{CB} &= +52.09 - 46.4(3.27) = -99.64 \text{ kip-ft} \\ V_{BC} &= 16.12 - 0(3.27) = 16.12 \text{ k} \end{aligned}$$



SHEAR DIAGRAM



MOMENT DIAGRAM

