

FUNDAMENTALS OF STRUCTURAL ANALYSIS

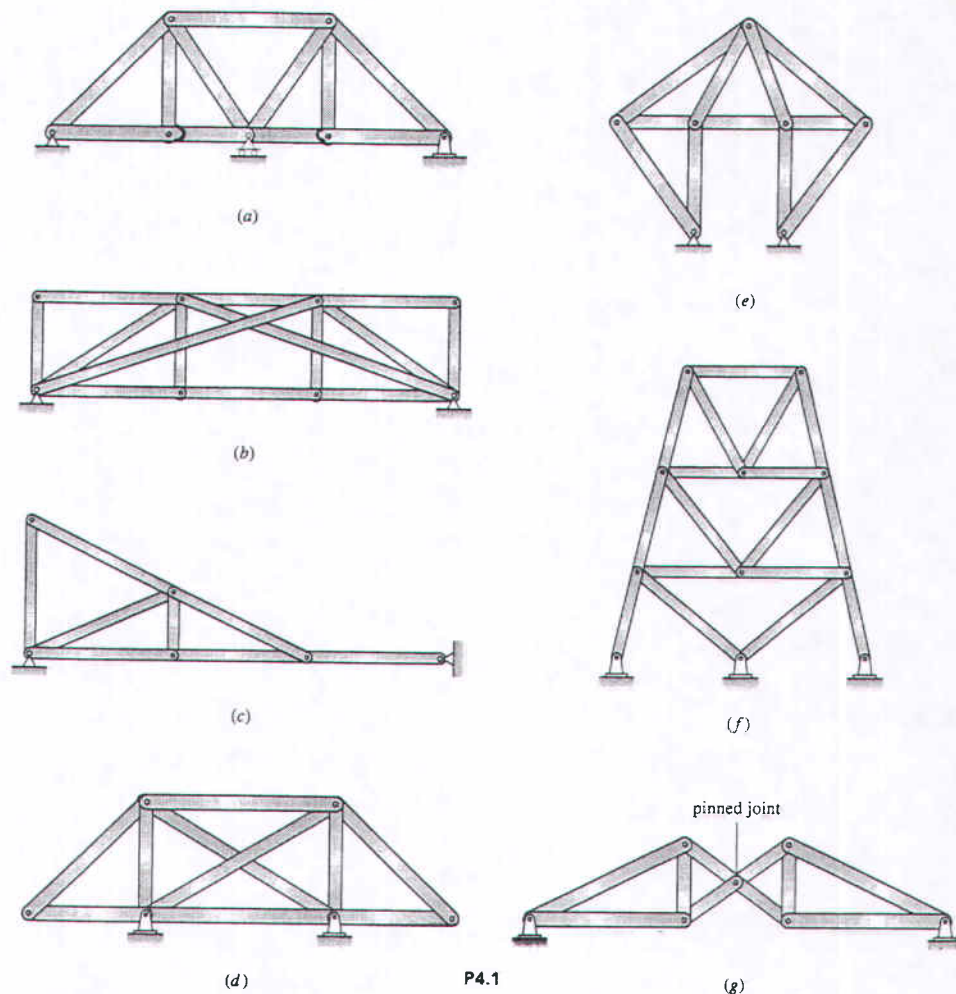
3rd Edition

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SOLUTIONS MANUAL

CHAPTER 4: TRUSSES

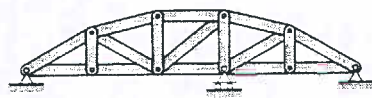
P4.1. Classify the trusses in Figure P4.1 as stable or unstable. If stable, indicate if determinate or indeterminate. If indeterminate, indicate the degree of indeterminacy.



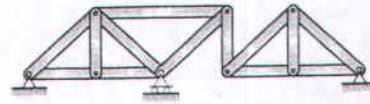
P4.1

a.)	Number of bars Number of reactions Number of Joints $(b + r = 16) > (2n = 14)$ <u>2° Indeterminate</u>	b=11 r=5 n=7
b.)	Number of bars Number of reactions Number of Joints $(b + r = 18) > (2n = 16)$ <u>2° Indeterminate</u>	b=14 r=4 n=8
c.)	Number of bars Number of reactions Number of Joints $(b + r = 10) = (2n = 10)$ <u>Geometrically Unstable. Reactions constitute a concurrent force system.</u>	b=7 r=3 n=5
d.)	Number of bars Number of reactions Number of Joints $(b + r = 14) > (2n = 12)$ <u>2° Indeterminate</u>	b=10 r=4 n=6
e.)	Number of bars Number of reactions Number of Joints $(b + r = 15) > (2n = 14)$ <u>1° Indeterminate</u>	b=11 r=4 n=7
f.)	Number of bars Number of reactions Number of Joints $(b + r = 23) > (2n = 22)$ <u>1° Indeterminate</u>	b=17 r=6 n=11
g.)	Number of bars Number of reactions Number of Joints $(b + r = 14) = (2n = 14)$ <u>Determinate</u>	b=10 r=4 n=7

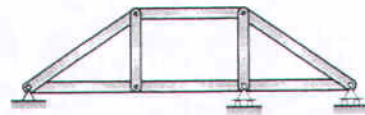
P4.2. Classify the trusses in Figure P4.2 as stable or unstable. If stable, indicate if determinate or indeterminate. If indeterminate, indicate the degree.



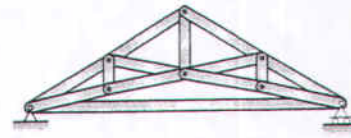
(a)



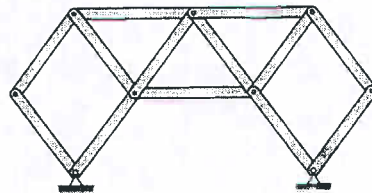
(b)



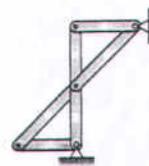
(c)



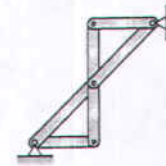
(d)



(e)



(f)



(g)

P4.2

a.) $5R + 17 F_{BAR} = 22 \text{ UNKNOWN}$
 $2 ESE \times 10 JTS = 20 ESE$
STABLE, INDET. 2°

d.) $3R + 14 F_{BAR} = 17 \text{ UNKNOWN}$
 $2 ESE \times 8 JTS = 16 ESE$
STABLE, INDET. 1°

b.) $5R + 13 F_{BAR} = 18 \text{ UNKNOWN}$
 $2 ESE \times 9 JTS = 18 ESE$
STABLE, DET.

e.) $4R + 13 F_{BAR} = 17 \text{ UNKNOWN}$
 $2 ESE \times 9 JTS = 18 ESE$
UNSTABLE

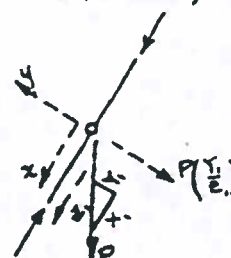
c.) $4R + 8 F_{BAR} = 12 \text{ UNKNOWN}$
 $2 ESE \times 6 JTS = 12 ESE$
STABLE, DET.

f.) $4R + 6 F_{BAR} = 10 \text{ UNKNOWN}$
 $2 ESE \times 5 JTS = 10 ESE$
STABLE, DET.

g.) $4R + 6 F_{BAR} = 10 \text{ UNKNOWN}$
 $2 ESE \times 5 JTS = 10 ESE$

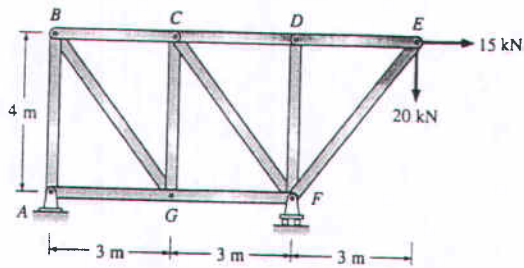
APPARENTLY DET., BUT IS UNSTABLE

12. DETERMINE ZERO BAR FORCES, THEN APPLY LOAD "P" @ JN.

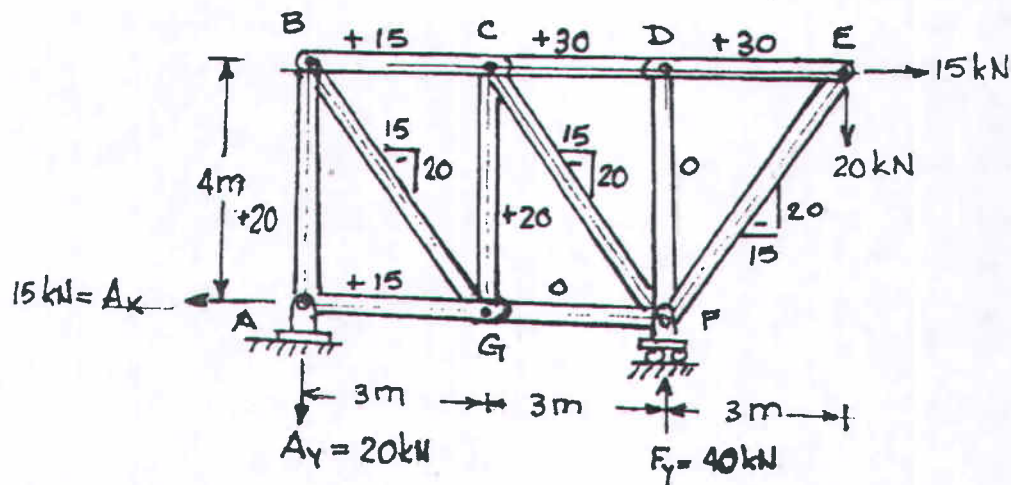


$\sum F_y = 0; P(\frac{1}{2}) \neq 0$

P4.3 Determine the forces in all bars of the truss. Indicate tension or compression.



P4.3



$$\rightarrow + \quad \Sigma F_x = 0; \quad 15 \text{ kN} - A_x = 0$$

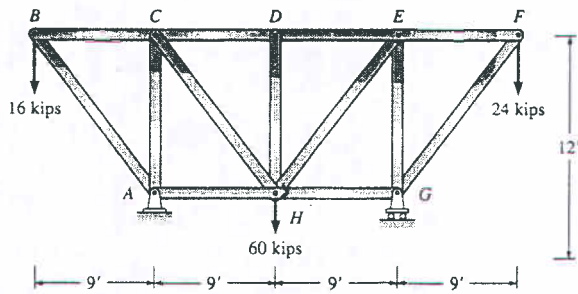
$$A_x = 15 \text{ kN} \leftarrow$$

$$\curvearrow + \quad \Sigma M_A = 0; \quad (15 \text{ kN})(4 \text{ m}) + (20 \text{ kN})(9 \text{ m}) - F_y(6 \text{ m}) = 0$$

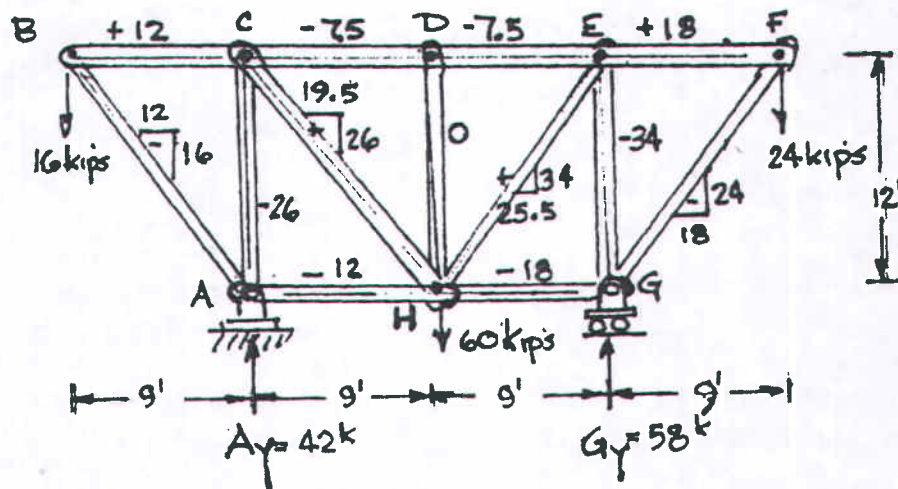
$$+ \quad \Sigma F_y = 0; \quad A_y - 40 \text{ kN} + 20 \text{ kN} = 0$$

$$A_y = 20 \text{ kN} \downarrow$$

P4.4 Determine the forces in all bars of the truss.
Indicate tension or compression.



P4.4



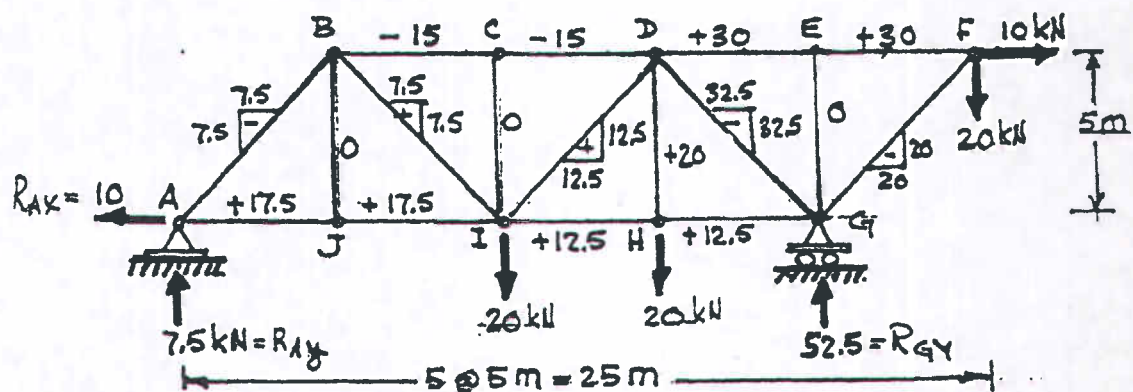
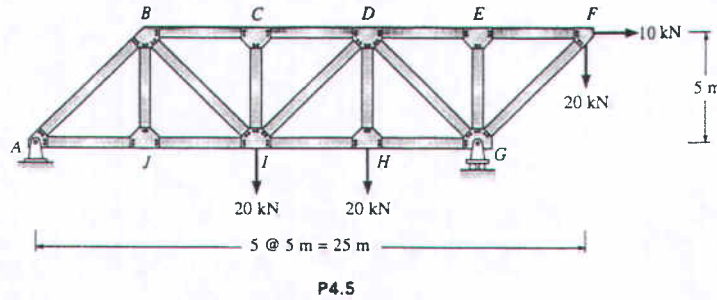
$$\sum M_A = 0; -(16^k)(9') + (60^k)(9') - G_Y(18') = 0$$

$$\underline{G_Y = 58 \text{ kips} \uparrow}$$

$$\sum F_Y = 0; A_Y - 16^k - 60^k - 24^k + 58^k = 0$$

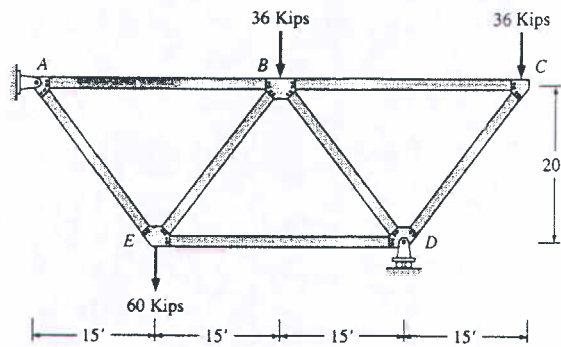
$$\underline{A_Y = 42 \text{ kips} \uparrow}$$

P4.5 Determine the forces in all bars of the truss.
Indicate tension or compression.

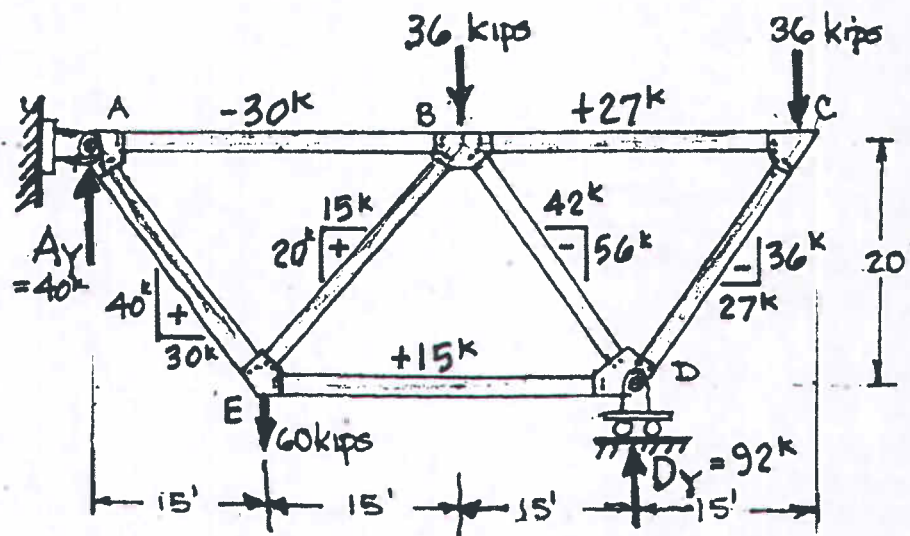


$$\begin{aligned} \sum M_A &= 0; & 20 \times 10 + 20 \times 15 + 20 \times 25 + 10 \times 5 - R_{Gy} \times 20 &= 0 \\ & & R_{Gy} &= 52.5 \text{ kN} \uparrow \\ \sum F_y &= 0; & -20 \times 3 + 52.5 + R_{Ay} &= 0 \\ & & R_{Ay} &= 7.5 \text{ kN} \uparrow \\ \sum F_x &= 0; & R_{Ax} &= 10 \text{ kN} \leftarrow \end{aligned}$$

P4.6 Determine the forces in all bars of the truss. Indicate tension or compression.



P4.6



$$\sum M_A = 0;$$

$$60k(15') + 36k(30') + 36k(60') - D_Y(45) = 0$$

$$\underline{\underline{D_Y = 92k \uparrow}}$$

$$\sum F_Y = 0;$$

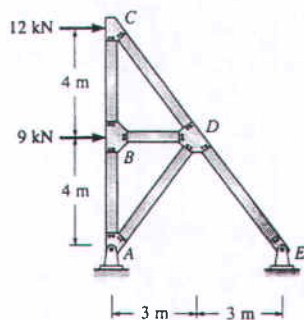
$$A_Y - 60k - 36k - 36k + 92k = 0$$

$$\underline{\underline{A_Y = 40k \uparrow}}$$

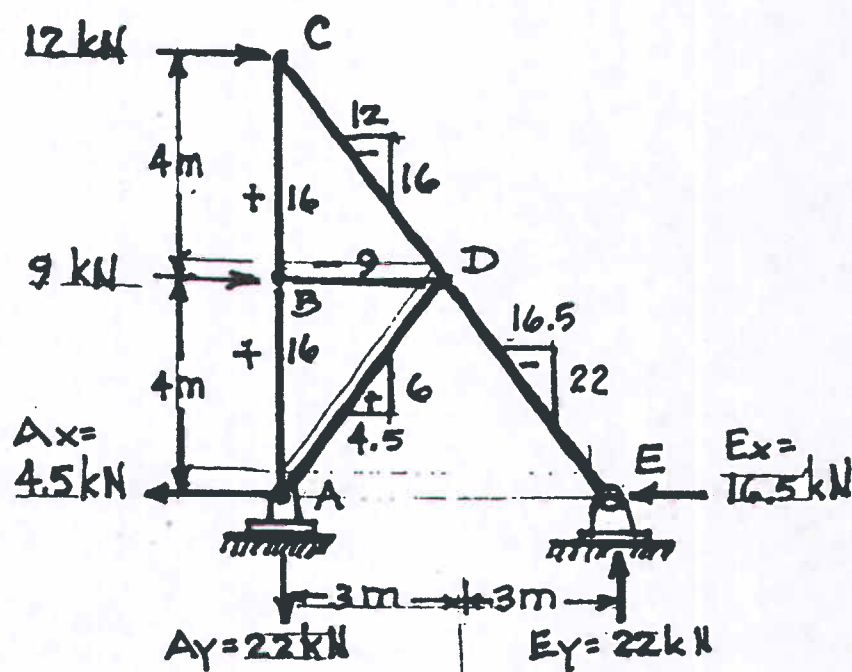
$$\sum F_X = 0;$$

$$\underline{\underline{A_X = 0}}$$

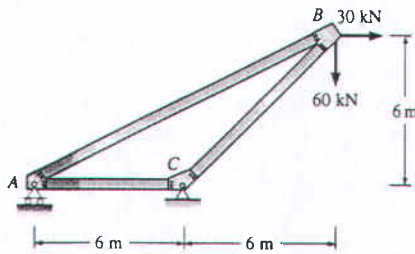
P4.7 Determine the forces in all bars of the truss. Indicate tension or compression.



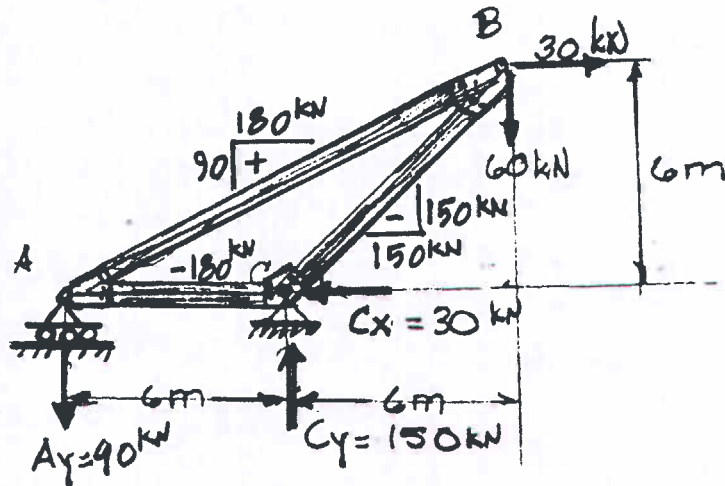
P4.7



P4.8 Determine the forces in all bars of the truss. Indicate tension or compression.



P4.8



$$\sum M_C = 0;$$

$$60 \text{ kN}(6 \text{ m}) + 30 \text{ kN}(6 \text{ m}) - A_y(6 \text{ m}) = 0$$

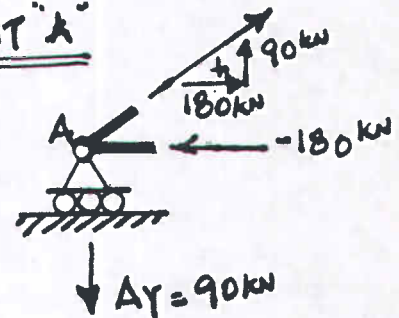
$$\underline{A_y = 90 \text{ kN} \downarrow}$$

$$\sum F_y = 0;$$

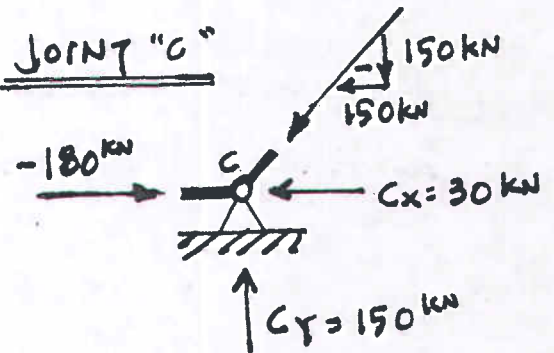
$$-90 \text{ kN} + C_y - 60 \text{ kN} = 0$$

$$\underline{C_y = 150 \text{ kN} \uparrow}$$

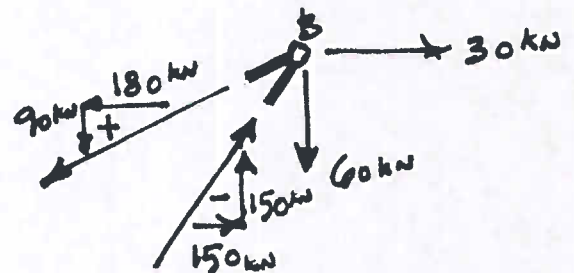
JOINT "A"



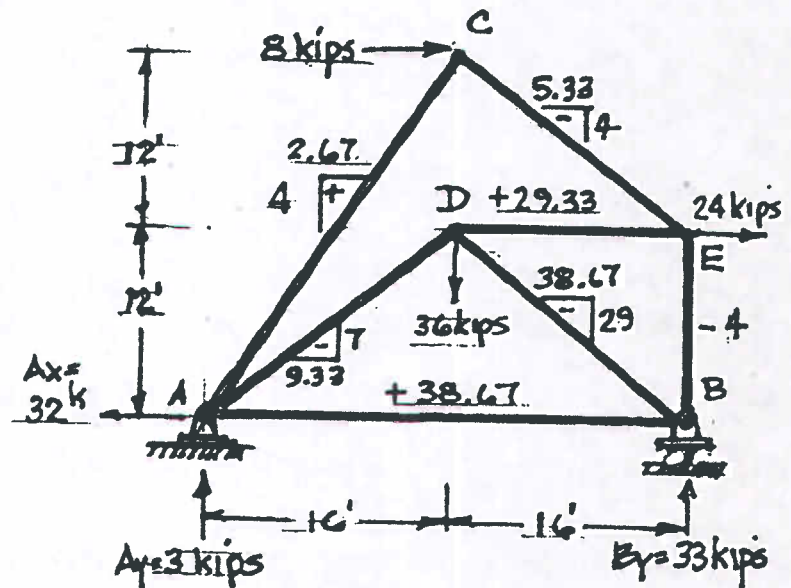
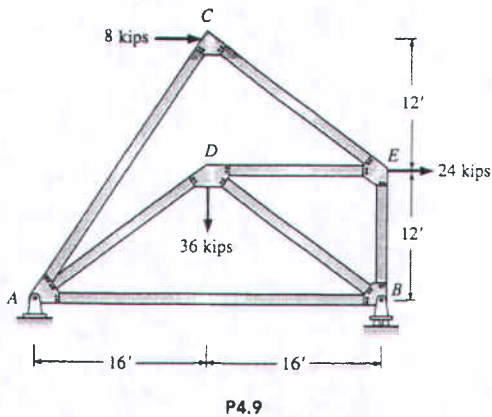
JOINT "C"



CHECK JOINT "B"



P4.9 Determine the forces in all bars of the truss. Indicate tension or compression.



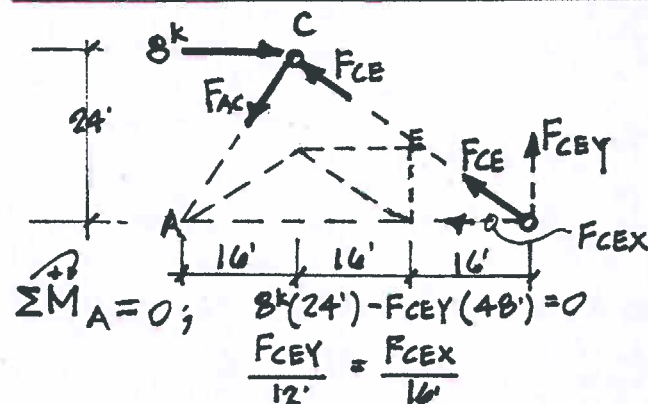
$$\sum M_A = 0; \quad 8k(24') + 36k(16') + 24k(12') - B_y(32') = 0$$

$$\underline{B_y = 33k \uparrow}$$

$$\sum F_y = 0; \quad A_y - 36k + 33k = 0$$

$$\underline{A_y = 3k \uparrow}$$

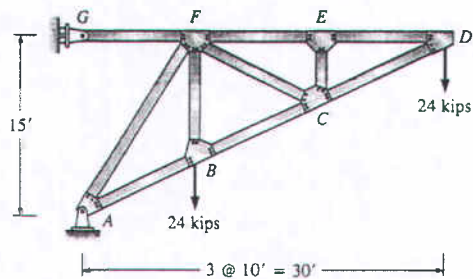
FORCE IN BAR CE : CONSIDER JOINT C



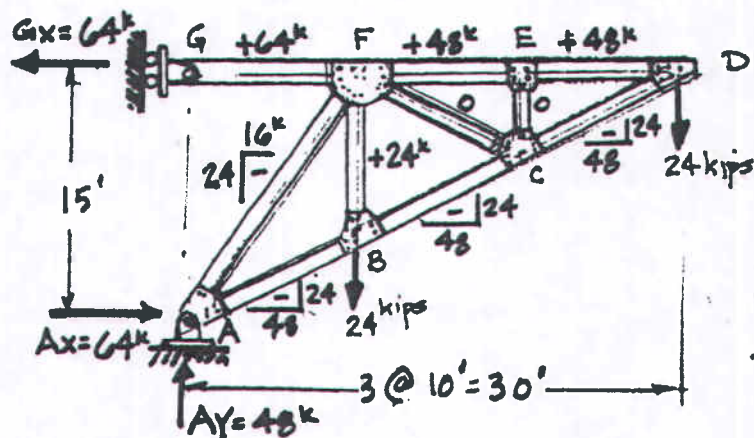
$$\underline{F_{CEY} = 4k}$$

$$\underline{F_{CEX} = 5.33k}$$

P4.10 Determine the forces in all bars of the truss.
Indicate tension or compression.



P4.10



$$\overset{+}{\curvearrowleft} \sum M_A = 0; \quad 24^k(10') + 24^k(30') - G_x(15') = 0$$

$$G_x = 64^k \leftarrow$$

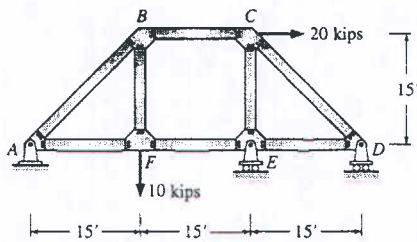
$$\uparrow \sum F_y = 0; \quad A_y - 24^k - 24^k = 0$$

$$A_y = 48^k \uparrow$$

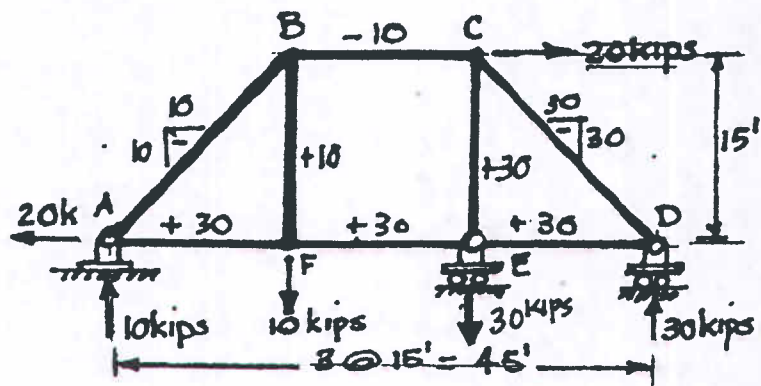
$$\overset{+}{\rightarrow} \sum F_x = 0; \quad G_x - A_x = 0$$

$$A_x = 64^k \rightarrow$$

P4.11 Determine the forces in all bars of the truss.
Indicate tension or compression.



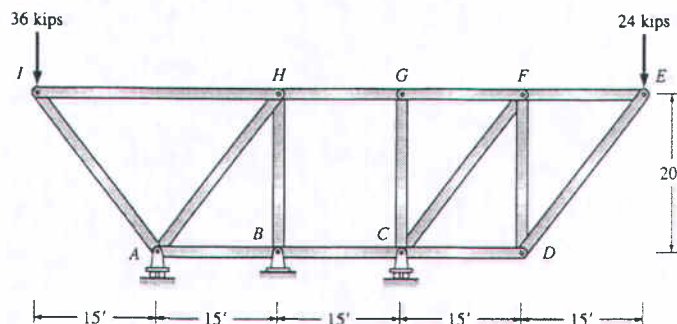
P4.11



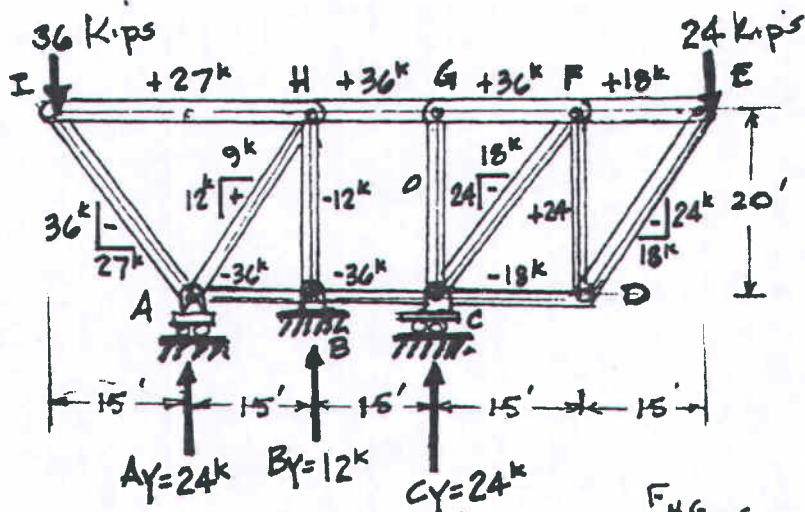
SOLVE BY METHOD OF JOINTS

START AT JOINT F

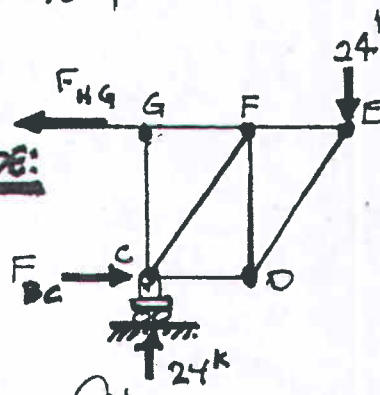
P4.12 Determine the forces in all bars of the truss.
Indicate tension or compression.



P4.12



FBD RIGHT SIDE:



$$\sum M_C = 0; (24^k)(30') - F_{HG}(20') = 0$$

$$F_{HG} = 36^k$$

$$+\uparrow \sum F_y = 0; C_y - 24 = 0$$

$$C_y = 24^k \uparrow$$

ENTIRE STRUCTURE:

$$(+\sum M_A = 0; -B_y(15') - 24^k(30') + 24^k(60') - 36^k(15') = 0$$

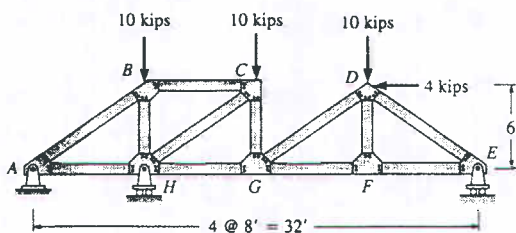
$$B_y = 12^k \uparrow$$

$$+\uparrow \sum F_y = 0;$$

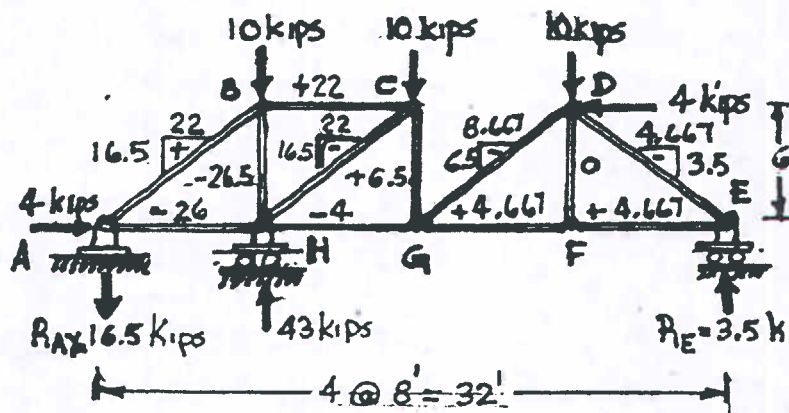
$$A_y - 36^k + 12^k + 24^k - 24^k = 0$$

$$A_y = 24^k \uparrow$$

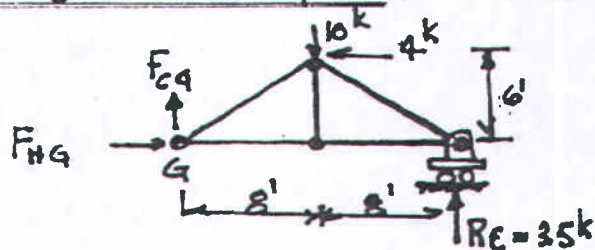
P4.13 Determine the forces in all bars of the truss.
Indicate tension or compression.



P4.13



COMPUTE R_E : ΣM ABOUT G



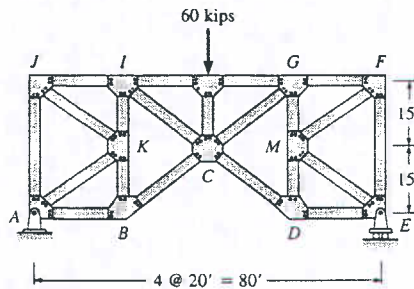
$$\sum M_G = 0; 10 \times 8' - 4 \times 6' - R_E \times 16' = 0$$

$$R_E = 3.5 \text{ kips} \uparrow$$

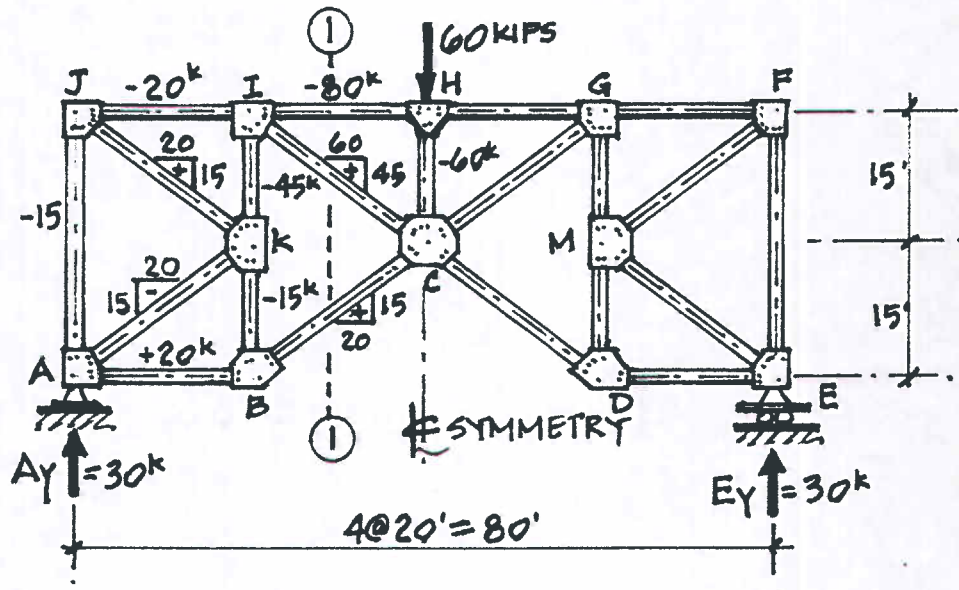
$$\sum F_x = 0; F_{HG} - 4 = 0 \therefore F_{HG} = 4 \text{ k} \rightarrow$$

COMPLETE BY METHOD OF JOINTS

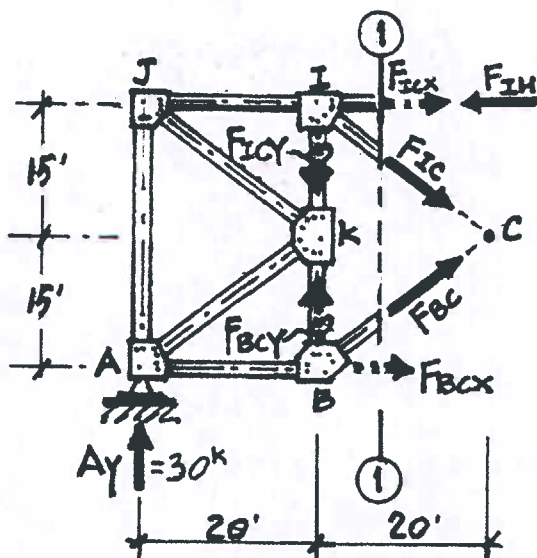
P4.14 Determine the forces in all bars of the truss.
Indicate tension or compression.



P4.14



FREEBODY LEFT OF SECTION ①:



$$\begin{aligned} \sum M_C = 0; & 30k(40') - F_{IH}(15') = 0 \\ & F_{IH} = 80k \text{ COMPRESSION} \end{aligned}$$

$$\begin{aligned} \sum M_I = 0; & 30k(20') + F_{BCx}(30') = 0 \\ & F_{BCx} = 20k \text{ TENSION} \end{aligned}$$

$$F_{BCx}/20' = F_{BCy}/15'$$

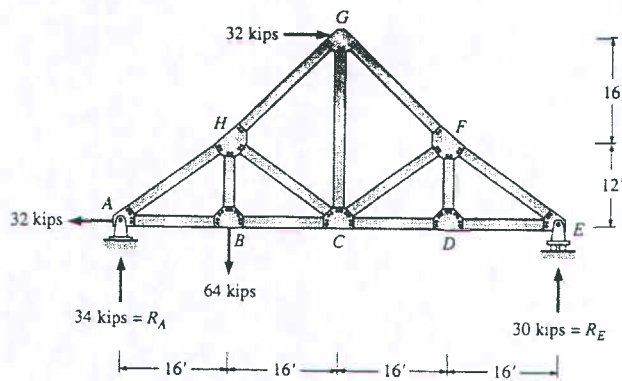
$$F_{BCy} = 15k$$

$$F_{BC} = \sqrt{(20k)^2 + (15k)^2} = 25k \text{ TENS.}$$

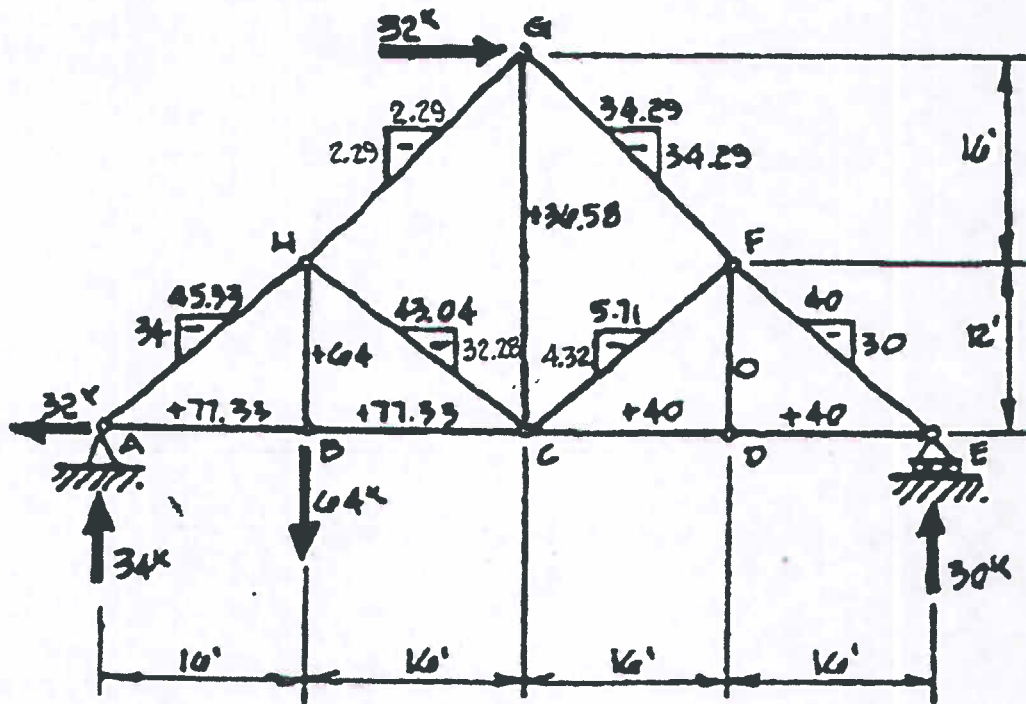
$$\begin{aligned} \sum M_B = 0; & 30k(20') - 80k(30') + F_{ICx}(30') = 0 \\ & F_{ICx} = +60k \text{ TENSION} \end{aligned}$$

COMPLETE ANALYSIS BY METHOD OF JOINTS

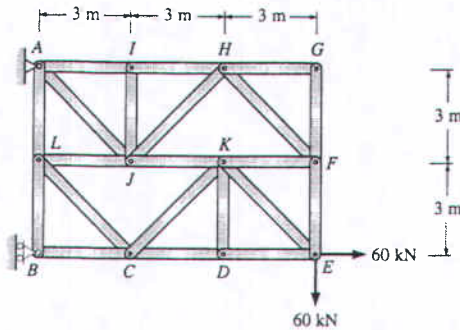
P4.15 Determine the forces in all bars of the truss.
Indicate tension or compression.



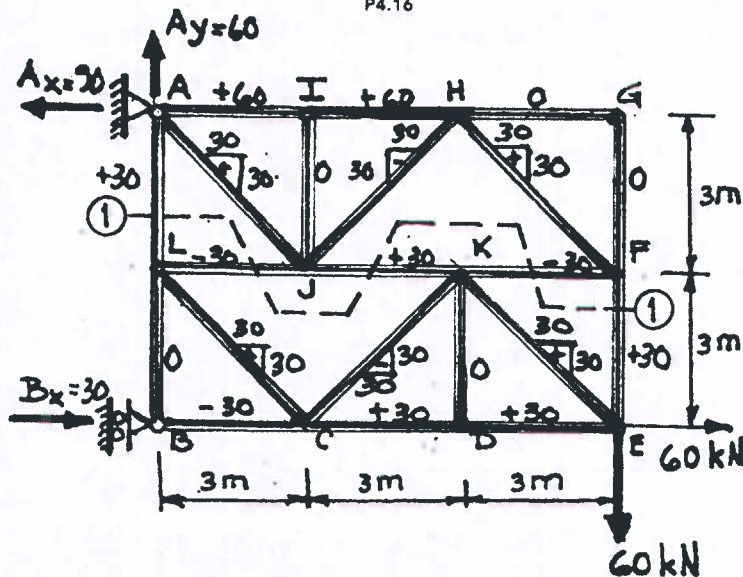
P4.15



P4.16. Determine the forces in all bars of the truss. *Hint:* If you have trouble computing bar forces, review *K* truss analysis in Example 4.6.



P4.16



ESTABLISH ZERO BARS (SHOW ON SKETCH)

IJ, HG, GF, KD, & LB

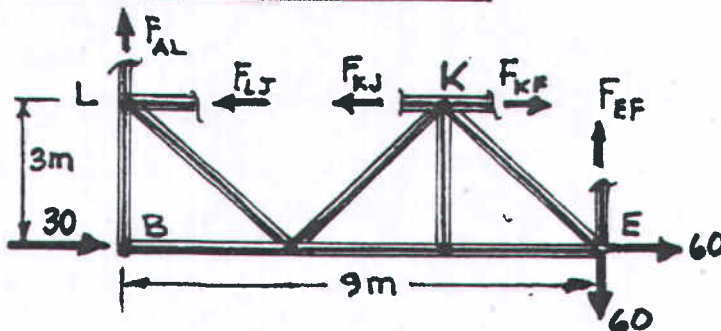
COMPUTE REACTIONS: $\sum M_A = 0$

$$60 \times 9 - 60 \times 6 - B_x \times 6 = 0$$

$$B_x = 30 \text{ kN} \rightarrow$$

$$\begin{aligned} \sum F_y = 0; & \quad A_y - 60 = 0 \quad \underline{A_y = 60 \text{ kN} \uparrow} \\ \sum F_x = 0; & \quad 30 + 60 - A_x = 0 \quad \underline{A_x = 90 \text{ kN} \leftarrow} \end{aligned}$$

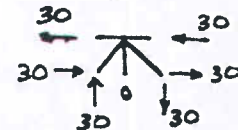
FREEBODY BELOW SECTION ①



$$\begin{aligned} \sum M_L = 0: & \quad -30 \times 3 - 60 \times 3 + 60 \times 9 - F_{EF} \times 3 = 0 \\ & \quad \underline{F_{EF} = 30 \text{ kN}} \end{aligned}$$

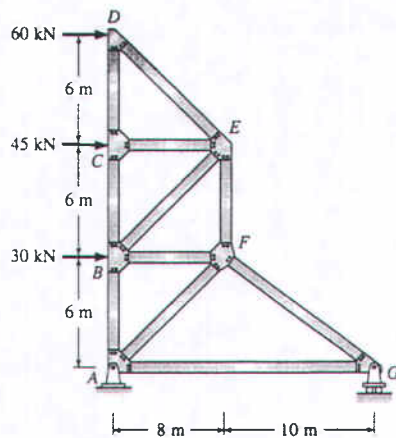
COMPLETE BY METHOD OF JOINTS

CHECK JOINT K

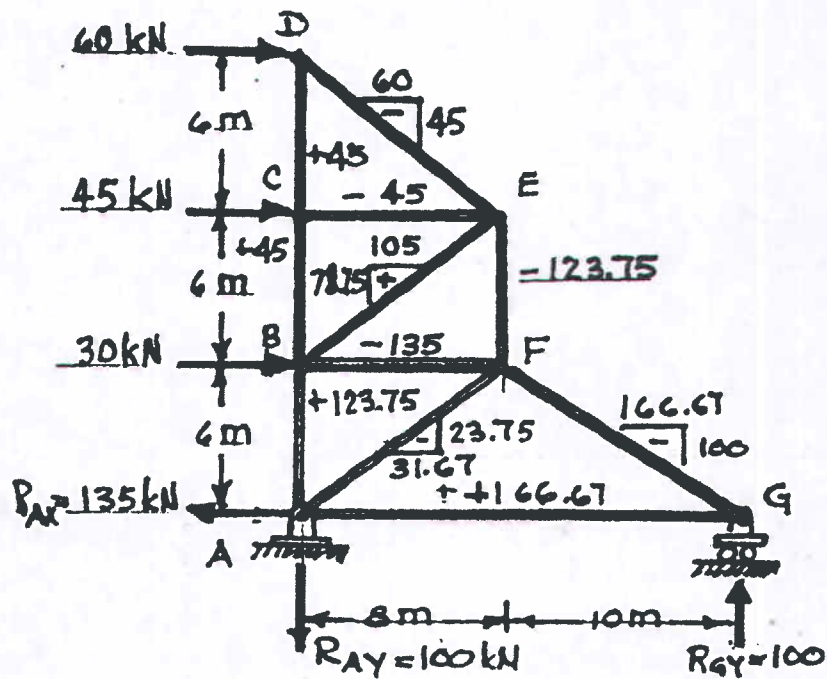


$$\begin{aligned} \sum F_x &= 0 \quad \therefore \text{JOINT IN EQUIL} \\ \sum F_y &= 0 \end{aligned}$$

P4.17 Determine the forces in all bars of the truss.
Indicate tension or compression.



P4.17



$$\sum M_A = 0; \quad 60 \times 18 + 45 \times 12 + 30 \times 6 - 18 R_{Gy} = 0$$

$$R_{Gy} = 100 \text{ kN} \uparrow$$

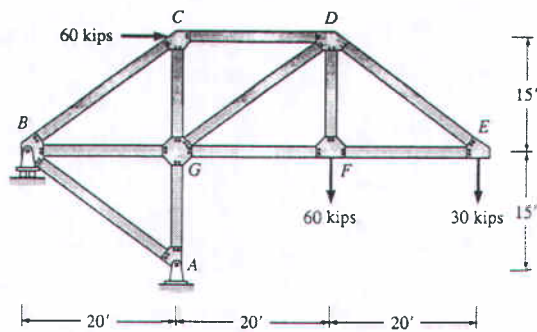
$$\sum F_x = 0; \quad 60 + 45 + 30 - R_{Ax} = 0$$

$$R_{Ax} = 135 \text{ kN} \leftarrow$$

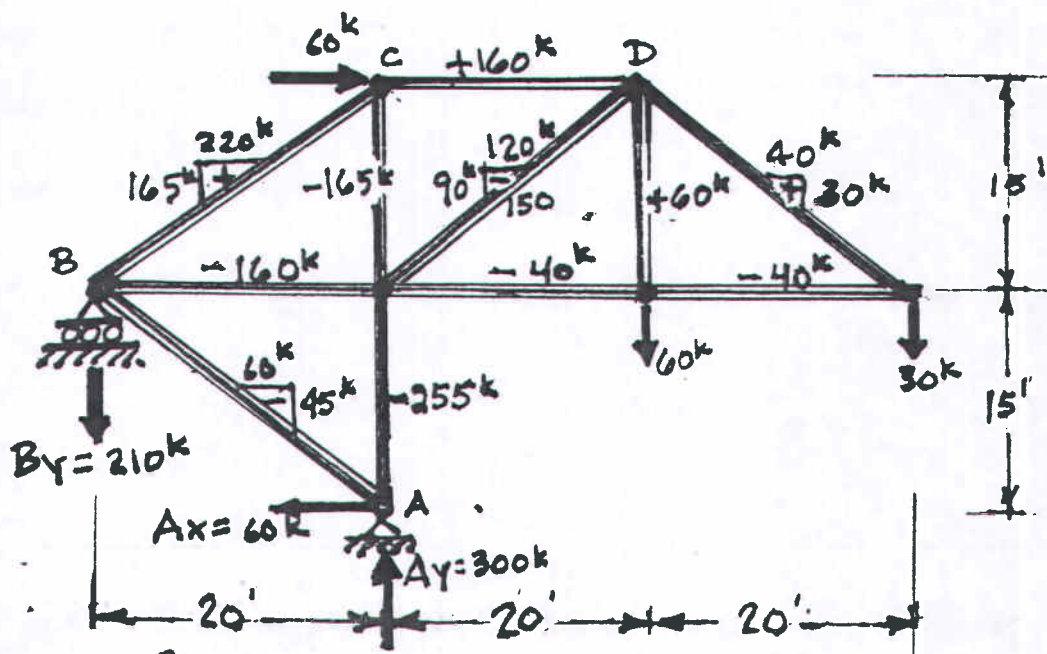
$$\sum F_y = 0; \quad 100 \text{ kN} - R_{Ay} = 0$$

$$R_{Ay} = 100 \text{ kN} \downarrow$$

P4.18 Determine the forces in all bars of the truss.
Indicate tension or compression.



P4.18



$$\sum M_A = 0; \quad -B_y(20') + 60k(30') + 60k(20') + 30k(40') = 0$$

$$\underline{B_y = 210k \downarrow}$$

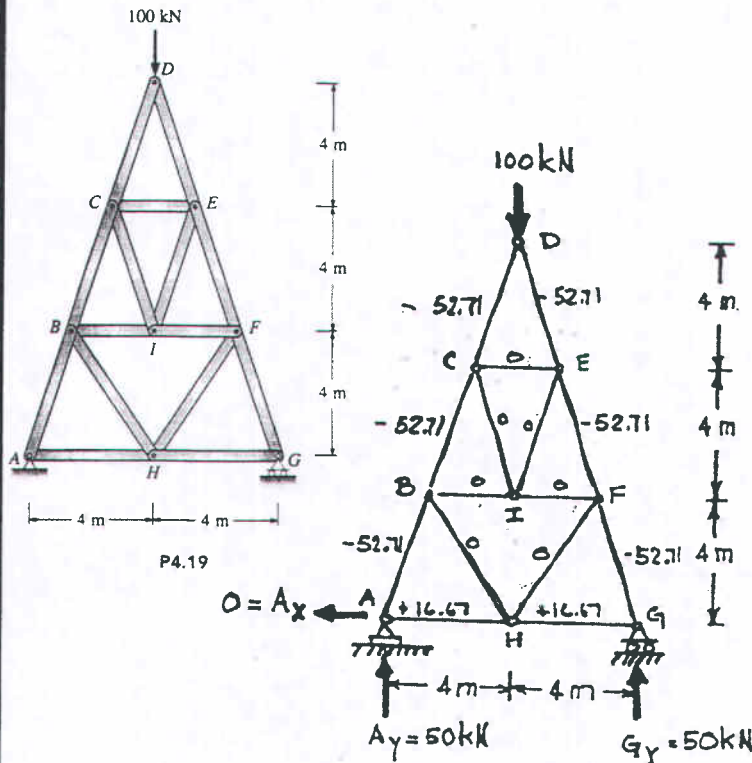
$$+\uparrow \sum F_y = 0; \quad -210k - 60k - 30k + A_y = 0$$

$$\underline{A_y = 300k \uparrow}$$

$$+\rightarrow \sum F_x = 0; \quad -A_x + 60k = 0$$

$$\underline{A_x = 60k \leftarrow}$$

P4.19 Determine the forces in all bars of the truss. Indicate tension or compression.



P4.19

COMPUTE REACTIONS

$$\sum \mathcal{M}_A = 0; 100(4) - G_y(8) = 0 \therefore G_y = 50 \text{ kN}$$

$$\sum F_y = 0; -100 + A_y + G_y = 0$$

$$A_y = 100 - G_y = 50 \text{ kN}$$

$$\sum F_x = 0; A_x = 0$$

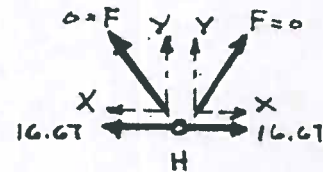
NOTE: SINCE STRUCTURE AND LOAD ARE SYMMETRICAL, THE FORCES ARE EQUAL IN CORRESPONDING MEMBERS ON EITHER SIDE OF THE CENTERLINE



JOINT A

$$\begin{aligned} \sum F_y = 0 \\ 50 - Y_{AB} &= 0 \\ Y_{AB} &= 50 \text{ kN} \\ Y_{AB} &= \frac{X_{AB}}{3} \\ X_{AB} &= \frac{50}{3} = 16.67 \text{ kN} \\ F_{AB} &= \frac{\sqrt{10}}{3} Y_{AB} = 52.71 \\ \sum F_x = 0: F_{AH} - X_{AB} &= 0 \\ F_{AH} &= X_{AB} = 16.67 \text{ kN} \end{aligned}$$

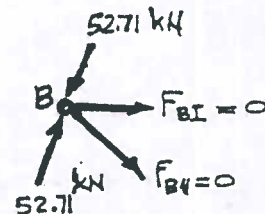
CONSIDER A FREEBODY OF JOINT H



BECAUSE OF SYMMETRY THE FORCES F IN BARS BH AND FH ARE EQUAL IN MAGNITUDE AND SENSE. ASSUME TENSION.

TO SATISFY $\sum F_x = 0$, THE X-COMPONENTS OF F MUST BE EQUAL IN MAGNITUDE. SINCE THE Y-COMPONENTS ARE EQUAL, THE ONLY WAY EQUILIBRIUM CAN BE SATISFIED I.E. $Y + Y = 0$ IS FOR Y 'S TO BE 0. IF Y 'S = ZERO, F 'S MUST EQUAL ZERO

JOINT B

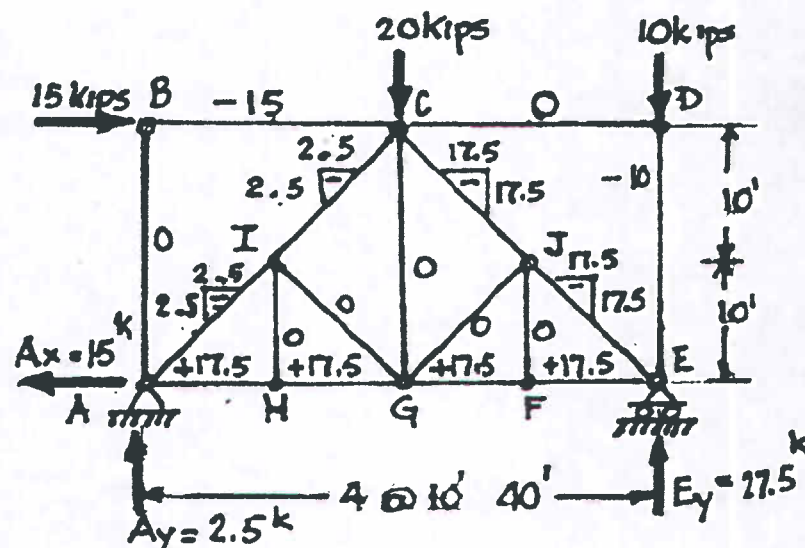
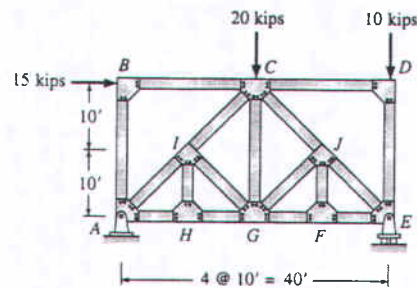


SINCE $F_{BH} = 0$, JOINT B CONSISTS OF THREE BARS, TWO OF WHICH ARE COLINEAR. THEREFORE FORCE F_{BI} IN THE NONCOLINEAR BAR IS ZERO, SEE CASE 2 IN SECTION 4.5 OF TEXT.

SIMILAR ANALYSIS OF OTHER JOINTS WILL SHOW ALL INTERIOR BARS EXCEPT AH AND HG ARE ZERO.

THIS ANALYSIS SHOWS THAT FORCES TAKE THE MOST DIRECT PATH TO THE SUPPORTS. THE INTERNAL BARS ARE REQUIRED FOR OTHER LOADS AND TO BRACE BARS ABCD AND DEFG.

P4.20 Determine the forces in all bars of the truss.
Indicate tension or compression.



COMPUTE REACTIONS:

$$\sum M_A^+ = 0; \quad 15 \times 20 + 20 \times 20 + 10 \times 40 - 40 E_y = 0$$

$$E_y = 27.5 \text{ kips } \uparrow$$

$$\sum F_y = 0; \quad A_y - 20 - 10 + 27.5 = 0$$

$$A_y = 2.5 \text{ kips } \uparrow$$

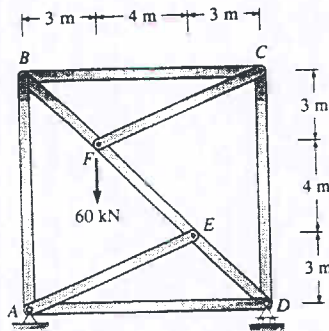
$$\sum F_x = 0; \quad A_x = 15 \text{ kips } \leftarrow$$

DETERMINE ZERO BARS:

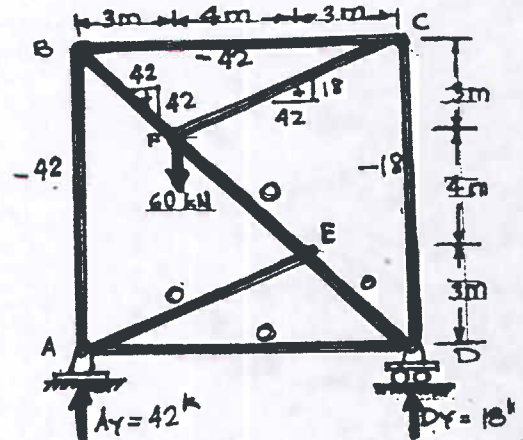
AB, CD, FJ, HI, IG, GJ, CG

SOLVE BY METHOD OF JOINTS

P4.21 Determine the forces in all bars of the truss. Indicate tension or compression.

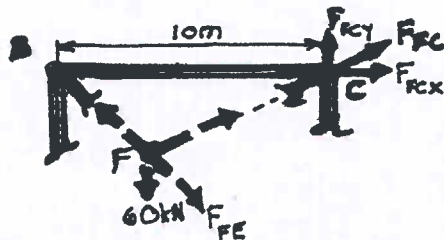


P4.21



REACTIONS: $\sum M_A = 0; 60 \times 3 + D_y \times 10 = 0$
 $D_y = 18 \text{ kN} \uparrow$
 $\sum F_y = 0; A_y - 60 + 18 = 0$
 $A_y = 42 \text{ kN} \uparrow$

FORCE IN BAR FC: CONSIDER JOINT F

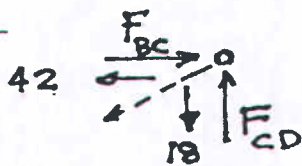


$\sum M_B = 0; 60 \text{ kN}(3 \text{ m}) - F_{cy} 10 = 0$
 $F_{cy} = 18 \text{ kN}$ TENSION

$\frac{F_{cy}}{3} = \frac{F_{cx}}{7}$

$F_{cx} = \frac{7}{3}(18) = 42 \text{ kN}$

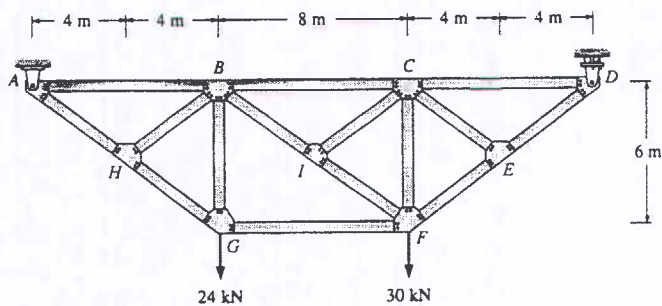
JOINT C



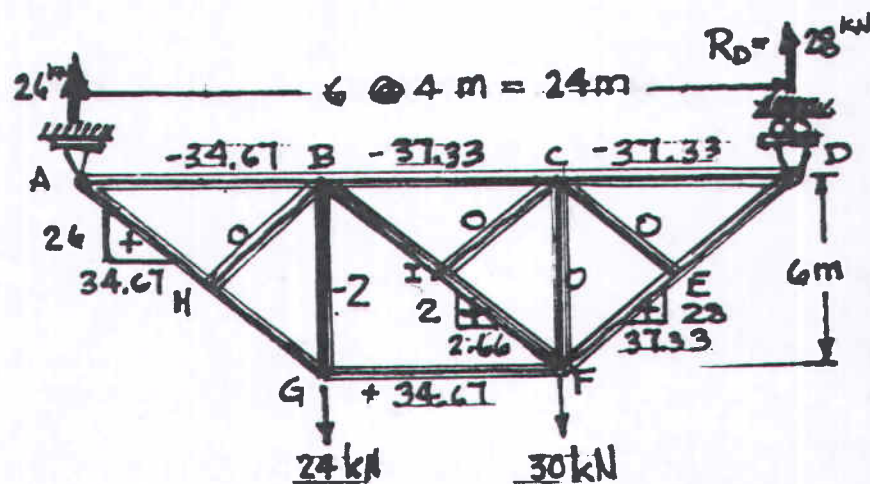
$\sum F_x = 0; F_{BC} = 42 \text{ kN COMP.}$
 $F_{CD} = 18 \text{ kN COMP.}$

COMPLETE THE ANALYSIS BY METHOD OF JOINTS. (SEE TOP FIGURE FOR RESULTS).

P4.22 Determine the forces in all bars of the truss.
Indicate tension or compression.



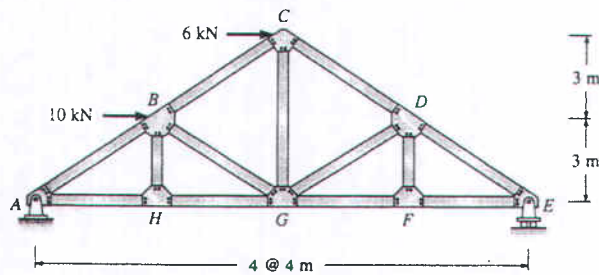
P4.22



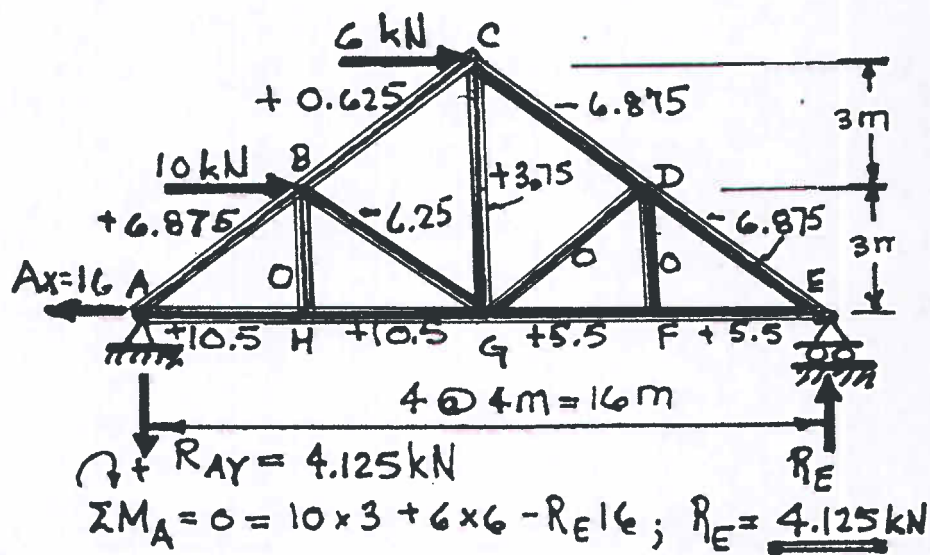
$$\begin{aligned} \sum M_A = 0; & \quad 24 \times 8 + 30 \times 16 - R_D \times 24 = 0 \\ & \quad \underline{R_D = 28 \text{ kN} \uparrow} \end{aligned}$$

ZERO BARS: BH, CI, CE, CF
ANALYSE BY METHOD OF JOINTS

P4.23 Determine the forces in all bars of the truss. Indicate tension or compression.



P4.23



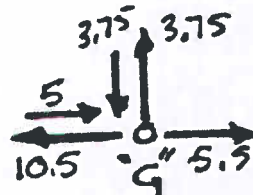
JOINT E

$$F_{DE} = \frac{4}{3}(4.125) = 5.5 \text{ kN TENSION}$$

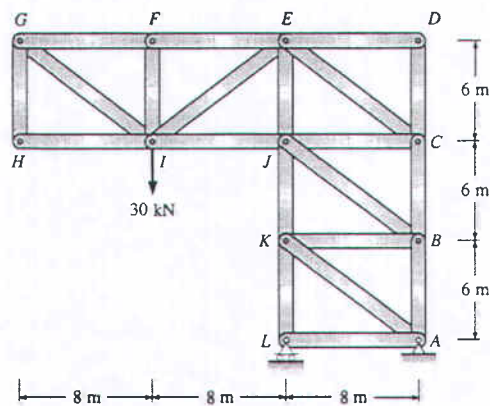
$$F_{FE} = \frac{5}{3}(4.125) = 6.875 \text{ kN COMPRESSION}$$

$$X_{AB} = \frac{4}{3}(4.125) = 5.5 \text{ kN}$$

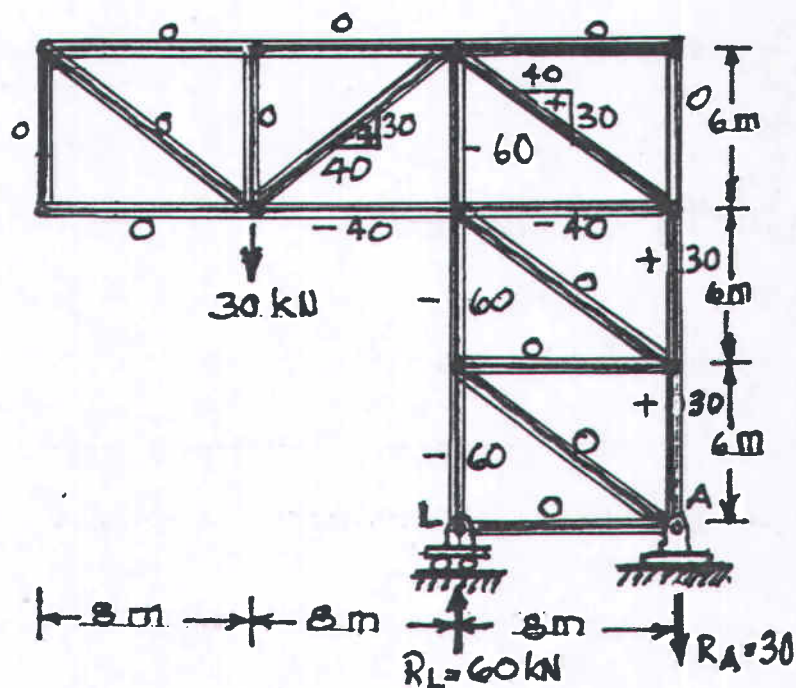
CHECK
J.T.A
 $\sum F_x = 0$
 $\sum F_y = 0$



P4.24 Determine the forces in all bars of the truss. Indicate tension or compression.



P4.24



COMPUTE REACTIONS

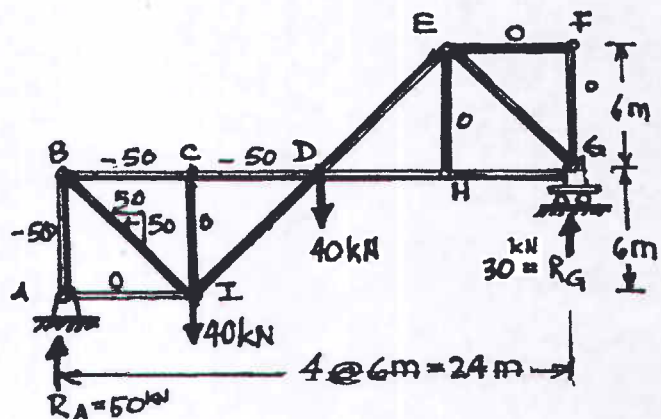
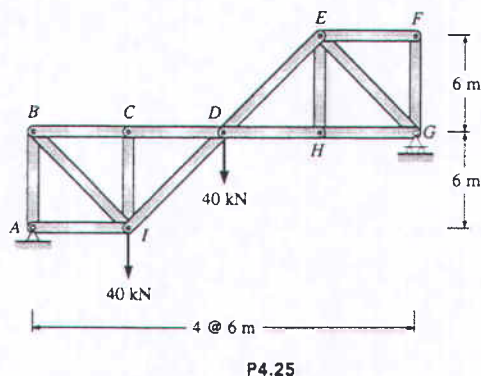
$$\uparrow \Sigma M_L = 0; R_A \cdot 8 - 30 \times 8 = 0$$

$$R_A = 30 \text{ kN} \downarrow$$

$$\sum F_y = 0; -30 - 30 + R_L = 0$$

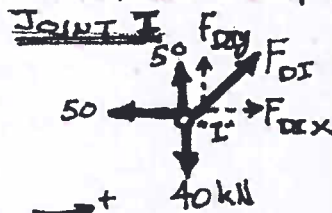
$R_L = 60 \uparrow$

P4.25. Determine the forces in all bars of the truss in Figure P4.25. If your solution is statically inconsistent, what conclusions can you draw about the truss? How might you modify the truss to improve its behavior? Try analyzing the truss with your computer program. Explain your results.



$$\begin{aligned} \sum M_A = 0; & \quad 40 \times 6 + 40 \times 12 = R_G \times 24 = 0 \\ & \quad R_G = 30 \text{ kN} \\ \uparrow \sum F_y = 0; & \quad R_A - 40 - 40 + 30 = 0 \\ & \quad R_A = 50 \text{ kN} \\ \text{ZERO BARS: } & \quad CI, EH, EF, FG, AI \end{aligned}$$

BEGIN ANALYSIS FROM SUPPORT A.
INVESTIGATE JOINT I.



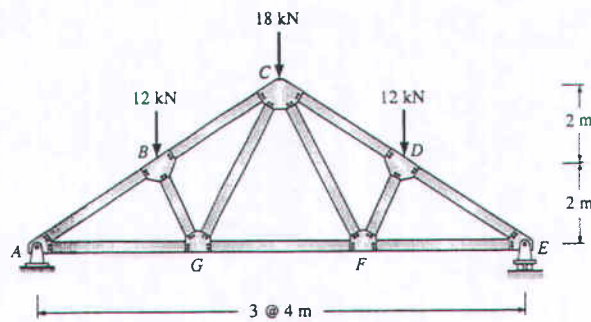
$$\begin{aligned} \sum F_x = 0; & \quad F_{DIx} - 50 = 0 \quad F_{DIx} = 50 \text{ kN TENSION} \\ \uparrow \sum F_y = 0; & \quad 50 - 40 + F_{DIy} = 0 \\ & \quad F_{DIy} = -10 \text{ kN COMP.} \end{aligned}$$

RESULTS ARE INCONSISTENT!

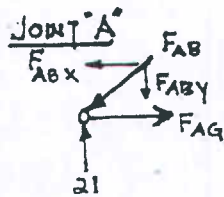
- (1) MAGNITUDES MUST BE THE SAME. SINCE BARS HAVE 45° SLOPE.
2. FORCES MUST BOTH BE TENSION OR COMPRESSION.

SINCE EQUILIBRIUM NOT POSSIBLE, STRUCTURE IS UNSTABLE.

P4.26 Determine the forces in all bars of the truss. Indicate tension or compression.



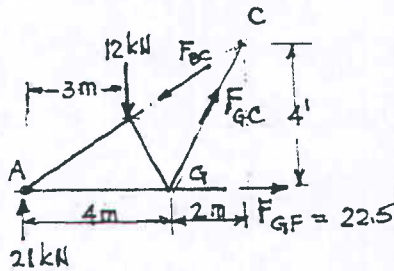
P4.26



$$\begin{aligned} \sum F_y &= 0 \\ 21 - F_{ABY} &= 0 \\ F_{ABY} &= 21 \text{ kN} \end{aligned}$$

$$\begin{aligned} -\frac{F_{ABX}}{3} &= \frac{F_{ABY}}{2} \\ F_{ABX} &= \frac{3}{2}(21) = 31.5 \text{ kN} \end{aligned}$$

CUT A SECTION 1-1

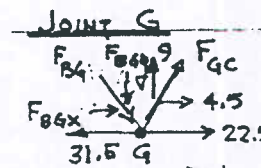
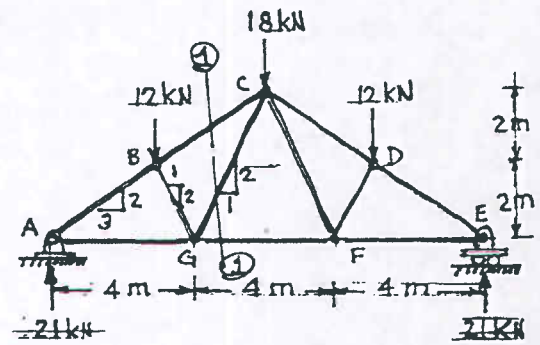


$$\begin{aligned} \sum M_C &= 0 = 21 \times 6 - 12 \times 3 - 4 F_{GF} \\ F_{GF} &= 22.5 \text{ kN} \end{aligned}$$

EXTEND F_{GC} TO G AND BREAK INTO COMPONENTS. $\sum M_A = 0$

$$\begin{aligned} \sum M_A &= 0 = 12 \times 3 - F_{GCY} \times 4 \\ F_{GCY} &= 9 \text{ kN TENSION} \end{aligned}$$

$$9 = \frac{F_{GCY}}{4} = \frac{F_{GCX}}{2} \therefore F_{GCX} = 4.5 \text{ kN}$$



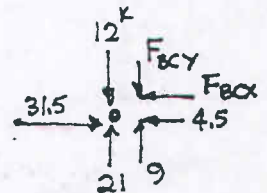
$$\begin{aligned} \sum F_x &= 0 = 22.5 - 31.5 + 4.5 \\ &\quad + F_{BGX} \end{aligned}$$

$$F_{BGX} = 4.5 \text{ kN} \rightarrow \text{COMPR}$$

$$\sum F_y = 0 = -F_{BGY} + 9 = 0$$

$$F_{BGY} = 9 \text{ kN} \downarrow \text{COMPR}$$

JOINT B

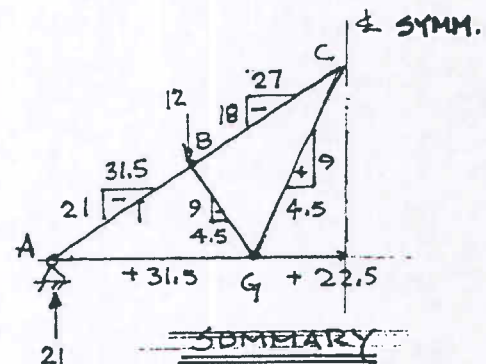


$$\sum F_x = 0 = 31.5 - 4.5 - F_{BCX}$$

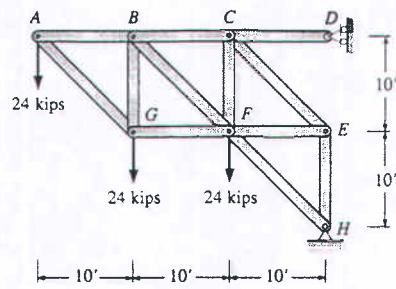
$$F_{BCX} = 27 \text{ kN COMPR}$$

$$\sum F_y = 0 = 21 + 9 - F_{BCY} - 12$$

$$F_{BCY} = 18 \text{ kN COMPR}$$

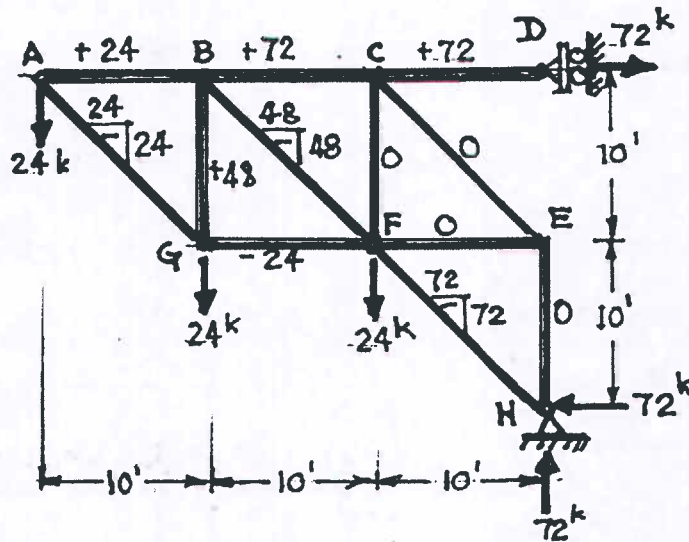


P4.27 Determine the forces in all bars of the truss.
Indicate tension or compression.

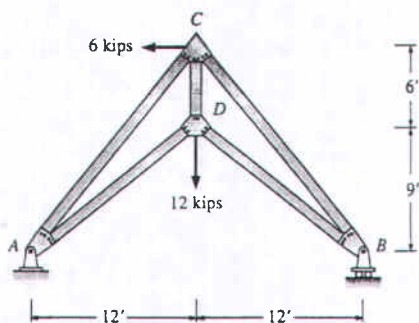


P4.27

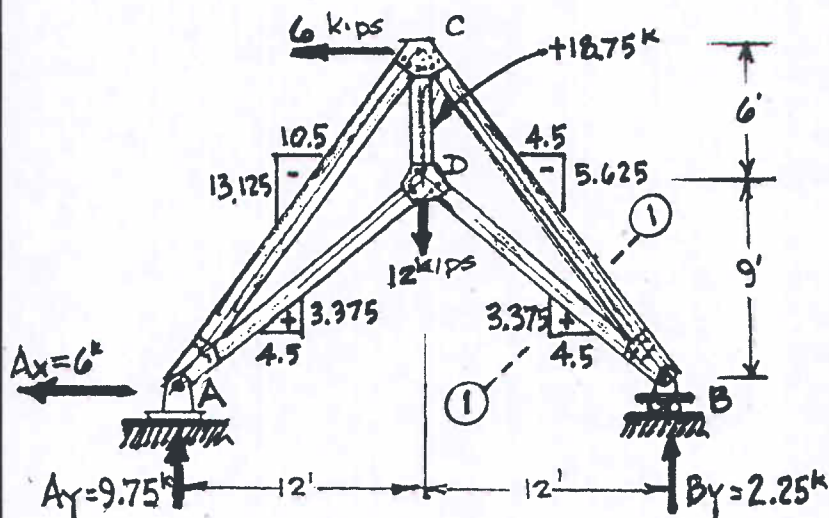
Solve by METHOD OF JOINTS



P4.28 Determine the forces in all bars of the truss. Indicate tension or compression.



P4.28



P 4.28

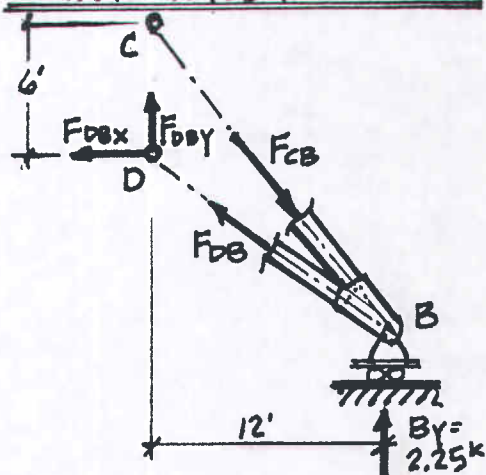
$$\sum \bar{M}_A = 0; \quad -6k(15') + 12k(12') - B_Y(24') = 0$$

$$\underline{B_Y = 2.25k \uparrow}$$

$$+\uparrow \sum F_Y = 0; \quad A_Y + 2.25k - 12k = 0$$

$$\underline{A_Y = 9.75k \uparrow}$$

FREEBODY RIGHT OF SECTION ①
COMPUTE FORCE IN BAR DB.



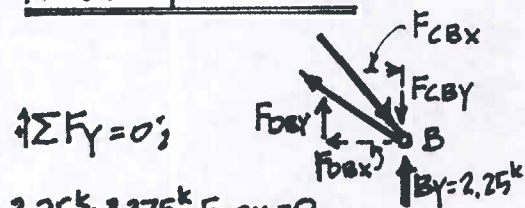
$$\sum \bar{M}_C = 0; \quad F_{DBX}(6') - 2.25(12') = 0$$

$$\underline{F_{DBX} = 4.5k}$$

$$\frac{F_{DBX}}{12'} = \frac{F_{DBY}}{9'}$$

$$\underline{F_{DBY} = 3.375k \text{ (TENSION)}}$$

FREEBODY JOINT B



$$+\uparrow \sum F_Y = 0;$$

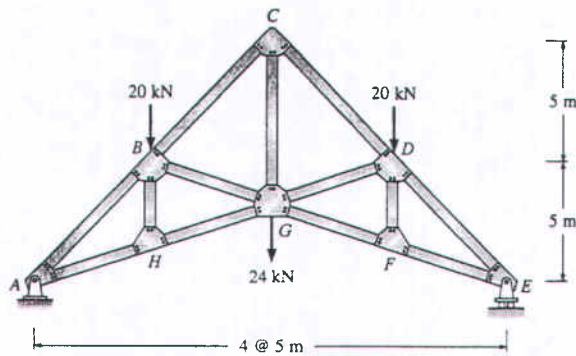
$$2.25k + 3.375k - F_{CBY} = 0$$

$$\underline{F_{CBY} = 5.625k}$$

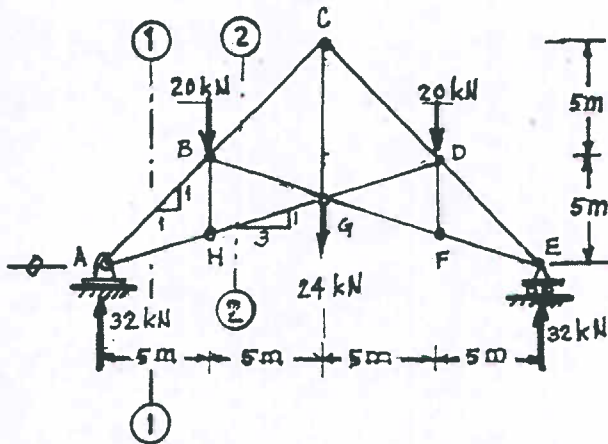
$$\frac{F_{CBY}}{15'} = \frac{F_{CBX}}{12'}$$

$$\underline{F_{CBX} = 4.5k \text{ (COMPRESSION)}}$$

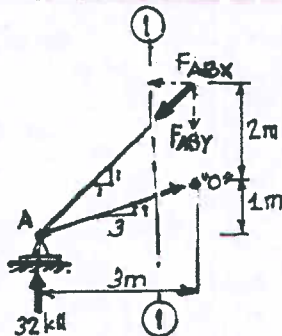
P4.29 Determine the forces in all bars of the truss.
Indicate tension or compression.



P4.29



FREEBODY LEFT OF SECTION ①



COMPUTE F_{AB}

$$\sum M_A = 0; \quad 32 \text{ kN}(3 \text{ m}) - F_{ABx}(2 \text{ m}) = 0$$

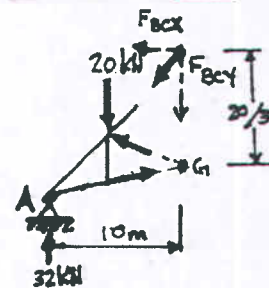
$$F_{ABx} = 48 \text{ kN COMPRESS.}$$

$$F_{ABx}/1 = F_{ABY}/1$$

$$F_{ABY} = 48 \text{ kN}$$

ZERO BARS: BH & DF

FREEBODY LEFT OF SECTION ②



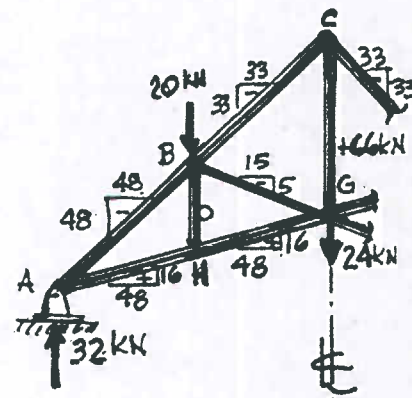
$$\sum M_A = 0;$$

$$32 \text{ kN}(10 \text{ m}) - 20(5 \text{ m}) - F_{BCx}(20/3) = 0$$

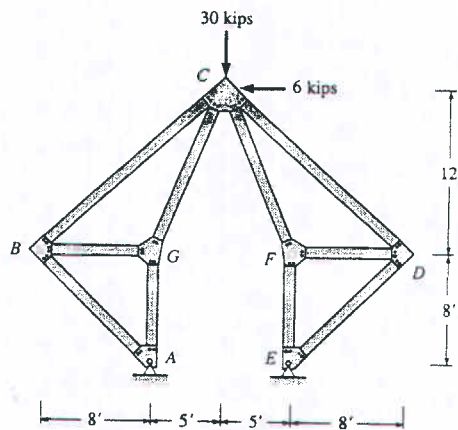
$$F_{BCx} = 33 \text{ kN COMPRESSION} \quad \therefore$$

$$F_{BCY} = 33 \text{ kN}$$

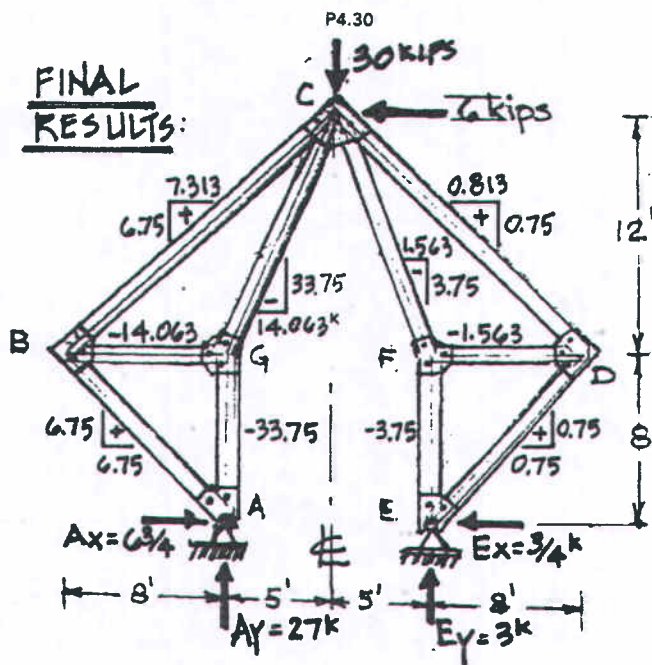
FINAL RESULTS, SYMMETRICAL ABOUT CENTERLINE &:



P4.30 Determine the forces in all bars of the truss. Indicate tension or compression.



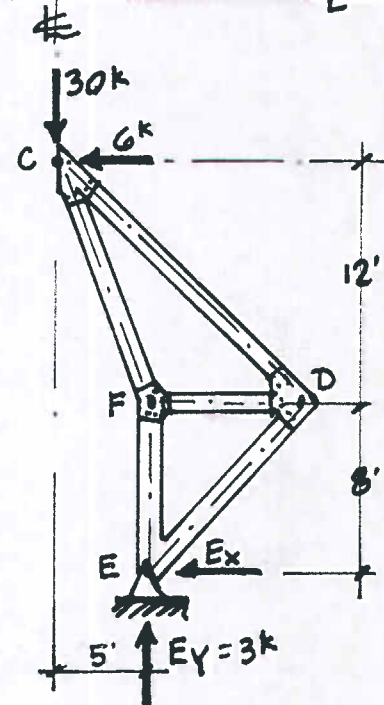
FINAL RESULTS:



$$\begin{aligned} \sum M_A &= 0; \\ 30k(5') - 6k(20') - E_y(10') &= 0 \\ E_y &= 3k \uparrow \end{aligned}$$

$$\begin{aligned} \sum F_y &= 0; \\ A_y - 30k + 3k &= 0 \\ A_y &= 27k \uparrow \end{aligned}$$

FREEBODY RIGHT SIDE OF CENTERLINE Φ :

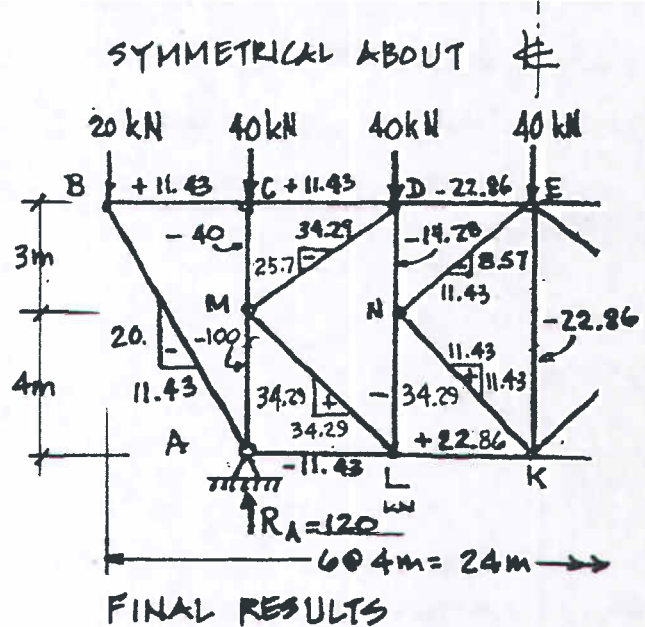
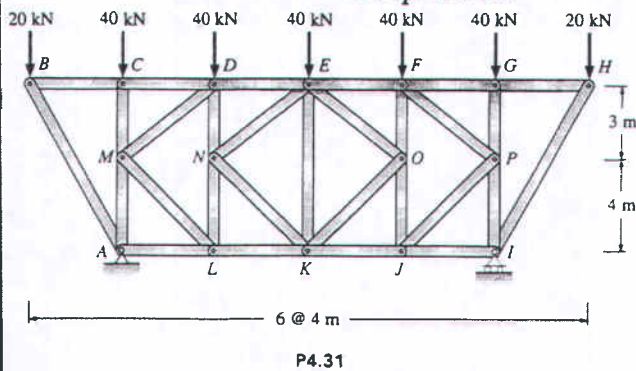


$$\begin{aligned} \sum M_C &= 0; \\ E_x(20') - 3k(5') &= 0 \\ E_x &= 3/4k \leftarrow \end{aligned}$$

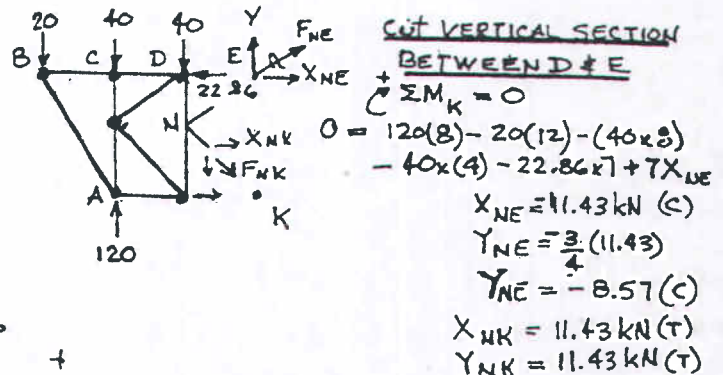
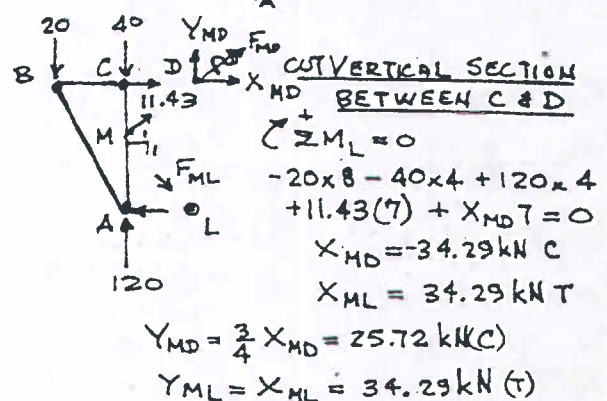
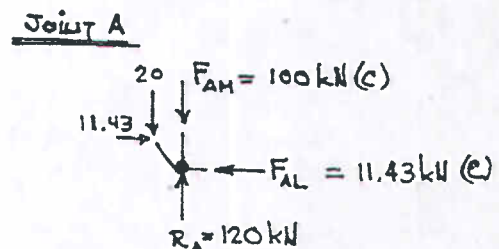
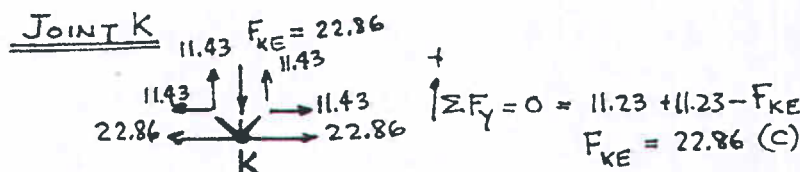
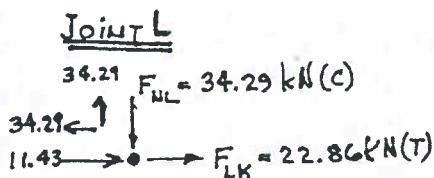
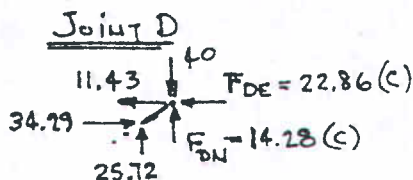
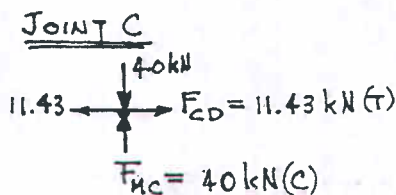
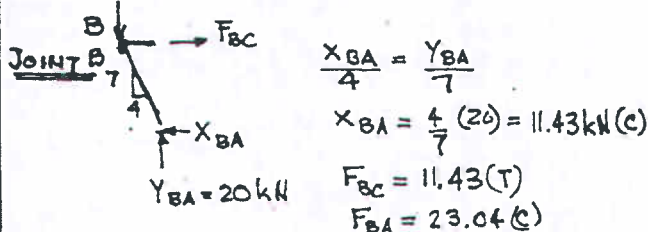
ENTIRE STRUCTURE:

$$\begin{aligned} \sum F_x &= 0; \\ -6k - 3/4k + A_x &= 0 \\ A_x &= 6 3/4k \rightarrow \end{aligned}$$

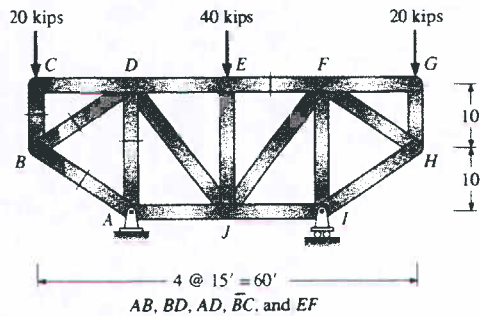
P4.31 Determine the forces in all bars of the truss. Indicate tension or compression.



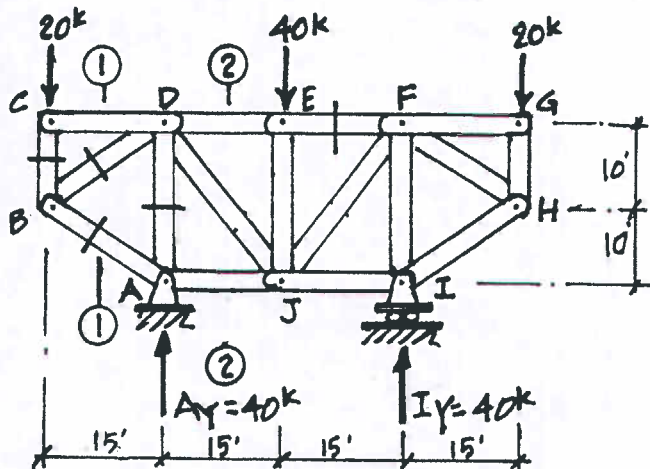
REACTIONS: $R_A = R_I = \frac{240}{2} = 120 \text{ kN}$



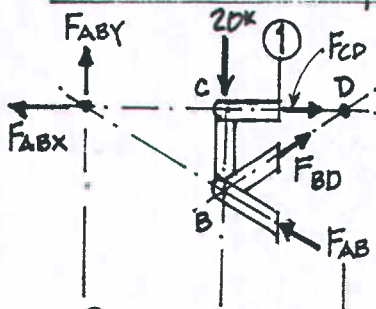
P4.32 Using the method of sections, determine the forces in the bars listed below each figure.



P4.32



FREE BODY LEFT OF SECTION ①

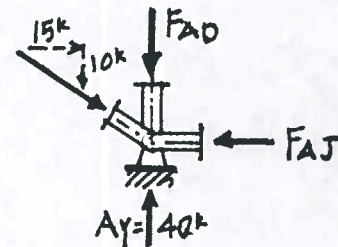


$$\begin{aligned} \sum M_D = 0; & \quad -20k(15') + F_{ABY}(30') = 0 \\ & \quad F_{ABY} = 10k \text{ COMPRESSION} \\ & \quad F_{ABX} = \left(\frac{15}{10}\right) F_{ABY} = 15k \\ & \quad F_{AB} = \sqrt{(10)^2 + (15)^2} = \underline{18k \text{ COMPR.}} \end{aligned}$$

$$\begin{aligned} \sum F_Y = 0; & \quad -20k + F_{ABY} + F_{BDY} = 0 \\ & \quad F_{BDY} = 10k \text{ TENSION} \\ & \quad F_{BDX} = \left(\frac{15}{10}\right) F_{BDY} = 15k \\ & \quad F_{BD} = \underline{18k \text{ TENSION}} \end{aligned}$$

BY OBSERVATION $F_{BC} = 20k \text{ COMPR.}$

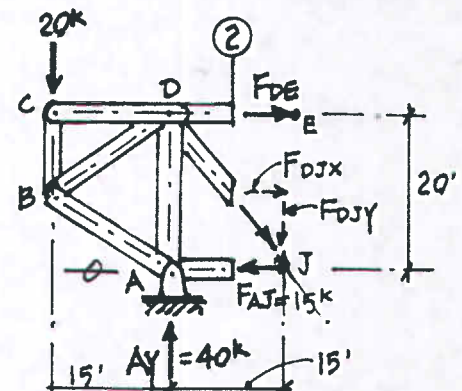
JOINT A, COMPUTE F_{AD} :



$$\begin{aligned} \sum F_Y = 0; & \quad -F_{AD} + 40k - 10k = 0 \\ & \quad F_{AD} = \underline{30k \text{ COMPR}} \end{aligned}$$

$$\begin{aligned} \sum F_X = 0; & \quad 15k - F_{AJ} = 0 \\ & \quad F_{AJ} = \underline{15k \text{ COMPR}} \end{aligned}$$

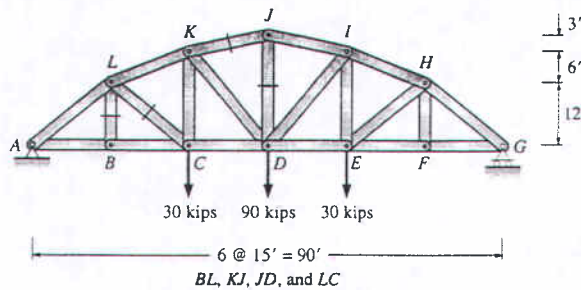
FREEBODY LEFT OF SECTION ②:



$$\begin{aligned} \sum F_Y = 0; & \quad -20k + 40k - F_{DJY} = 0 \\ & \quad F_{DJY} = 20k \text{ TENSION} \\ & \quad F_{DJX} = \left(\frac{15}{20}\right) F_{DJY} = \underline{15k} \end{aligned}$$

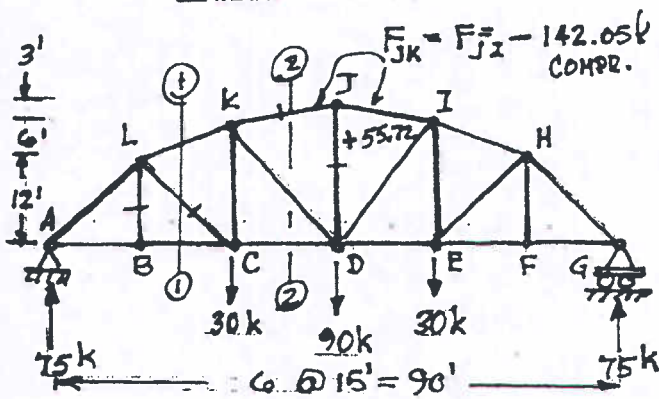
$$\begin{aligned} \sum F_X = 0; & \quad -F_{AJ} + F_{DJX} + F_{DE} = 0 \\ & \quad -15k + 15 + F_{DE} = 0 \\ & \quad F_{DE} = \underline{0} \end{aligned}$$

P4.33 Using the method of sections, determine the forces in the bars listed below each figure.

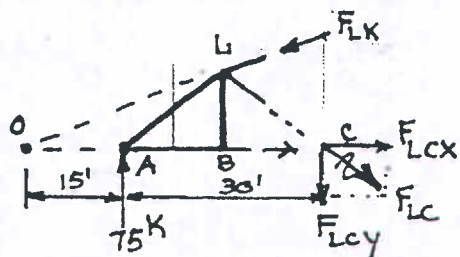


P4.33

COMPUTE FORCES IN BL, KJ, JD and LC.



FORCE IN BAR LC CUT SECTION 1-1



$$\sum M_O = 0 = -75 \times 15 + F_{LCY} \times 45$$

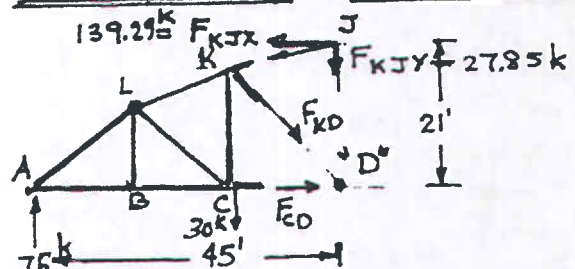
$$F_{LCY} = 25 \text{ kips}$$

$$\frac{F_{LCY}}{12} = \frac{F_{LCX}}{15} \therefore F_{LCX} = \frac{15}{12} (25) = 31.25 \text{ kips}$$

$$F_{LC} = \sqrt{(25)^2 + (31.25)^2}$$

$$F_{LC} = 40.02 \text{ kips TENSION}$$

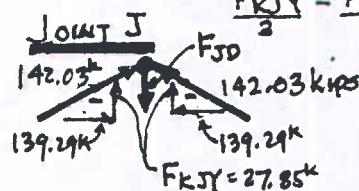
FORCE IN BAR KJ CUT SECTION 2-2



$$\sum M_D = 0 = 75 \times 45 - F_{KJX} \times 21 - 30 \times 15$$

$$F_{KJX} = 139.29 \text{ kips}$$

$$F_{KJY} = \frac{F_{KJX}}{15} \therefore F_{KJY} = \frac{139.29}{5} = 27.85 \text{ kips}$$



$$F_{KJ} = F_{JI} = \sqrt{(139.29)^2 + (27.85)^2} = 142.03 \text{ k COMPRESSION}$$

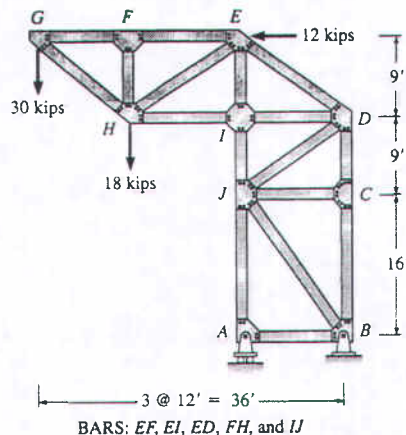
$$\sum F_y = 0;$$

$$-(F_{KJY} + F_{JIY}) + F_{JD} = 0$$

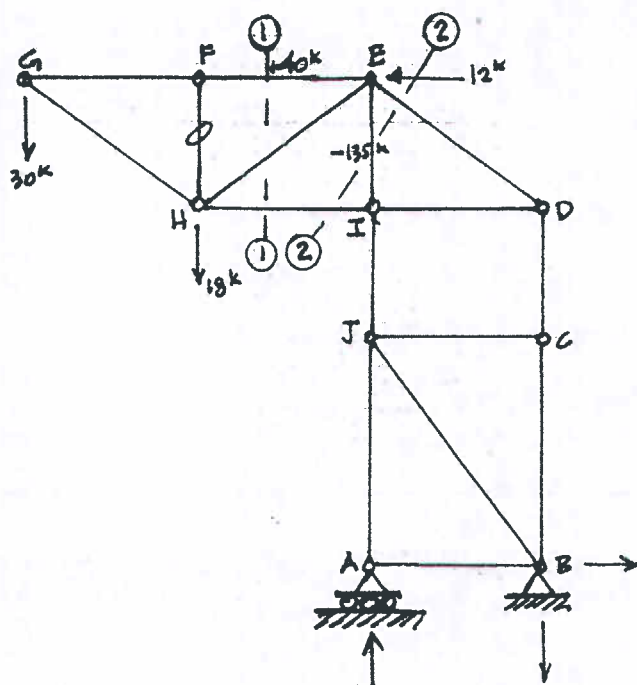
$$F_{JD} = 55.72 \text{ k TENSION}$$

$$\text{BY INSPECTION } F_{BL} = 0$$

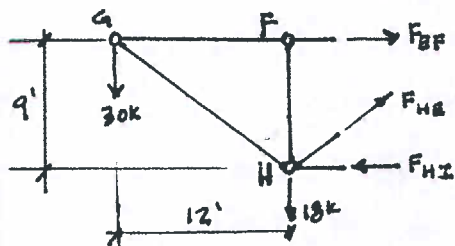
P4.34 Using the method of sections, determine the forces in the bars listed below each figure.



P4.34



FREEBODY LEFT OF SECTION (1) TO COMPUTE FORCE IN BAR "EF":



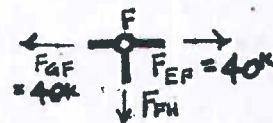
$$\sum M_H = 0;$$

$$F_{EF}(9') - 30k(12') = 0$$

$$F_{EF} = 40k$$

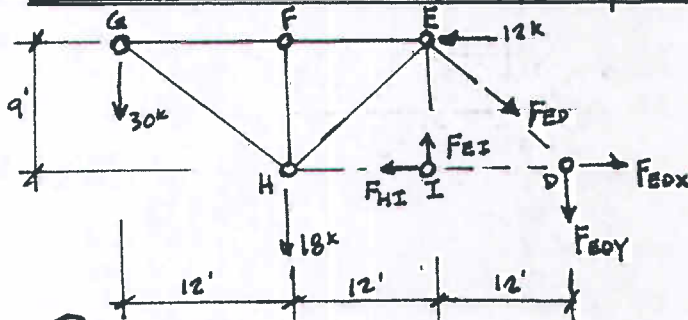
TENSION

FREEBODY JOINT "F":



$$+\sum F_y = 0; \quad F_{FI} = 0$$

FREEBODY LEFT OF SECTION (2) TO COMPUTE BAR FORCES IN BARS EI & ED:



$$\sum M_D = 0;$$

$$-30k(36') - 18k(24') - 12k(9') + F_{EI}(12') = 0$$

$$F_{EI} = 135k \text{ COMPRESSION}$$

$$\sum M_I = 0;$$

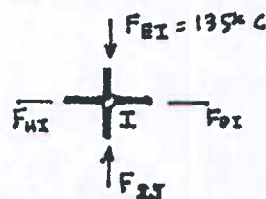
$$-30k(24') - 18k(12') - 12k(9') + F_{EDY}(12') = 0$$

$$F_{EDY} = 87k \downarrow$$

$$F_{EDX}/4' = 87k/3' \quad \text{THUS} \quad F_{EDX} = 116k$$

$$F_{ED} = 145k \text{ TENSION}$$

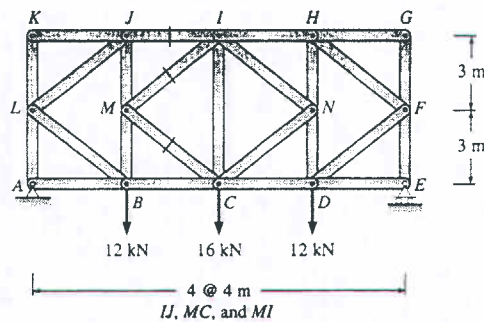
FREEBODY JOINT I:



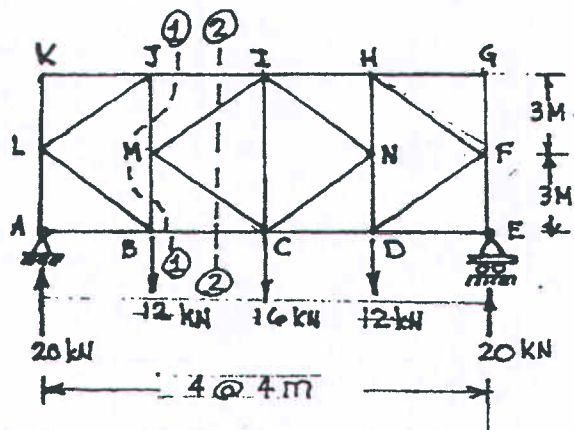
$$+\sum F_y = 0; \quad -F_{FI} + F_{IJ} = 0$$

$$F_{IJ} = 135k \text{ COMPRESSION}$$

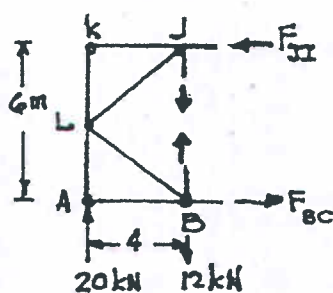
P4.35 Using the method of sections, determine the forces in the bars listed below each figure.



P4.35

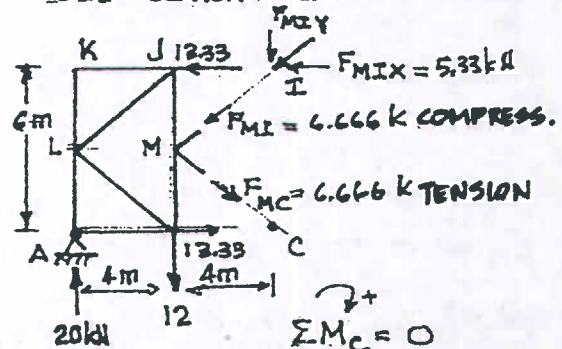


FORCES IN TOP AND BOTTOM CHORDS
SEE SECTION 1-1 ABOVE.



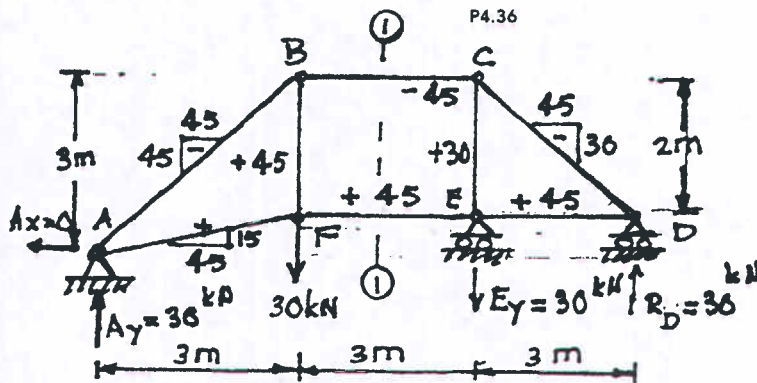
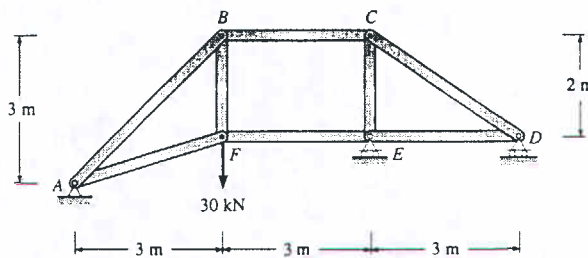
$$\begin{aligned}\sum M_A &= 0 \\ 0 &= 20 \times 4 - F_{JI} \times 6 \\ F_{JI} &= 13.33 \text{ kN (Compe)} \\ \sum F_x &= 0 \Rightarrow F_{BI} = 13.33 \\ F_{BI} &= 13.33 \text{ kN (T)}\end{aligned}$$

FORCES IN DIAGONAL BARS MI and MC
SEE SECTION 2-2



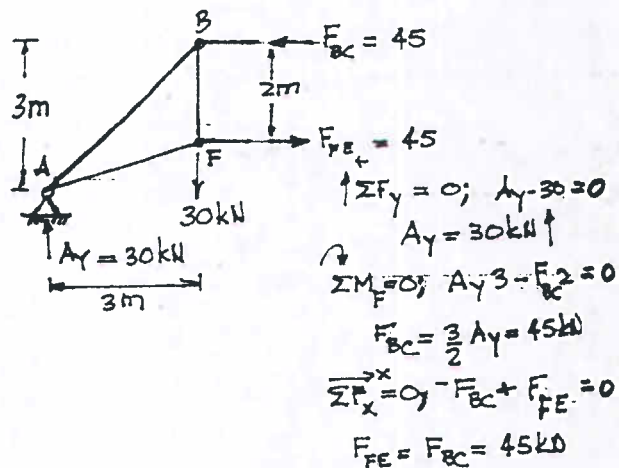
$$\begin{aligned}0 &= 20 \times 8 - 12 \times 4 - 13.33 \times 6 \\ &\quad - F_{MI} \times 6 \\ 6 F_{MI} &= 160 - 48 - 80 \\ F_{MI} &= 5.33 \text{ kN (Compe)} \\ \sum F_y &= 0 = 20 - 12 - F_{MIY} - F_{MCIY} \\ \frac{F_{MIY}}{3} &= \frac{F_{MIx}}{4} \therefore F_{MIY} = 4 \text{ kN (Compe)} \\ F_{MCx} &= 5.33 \text{ kN (T)} \quad F_{MCIY} = 4 \text{ kN} \\ F_{MC} &= \sqrt{F_{MCx}^2 + F_{MCIY}^2} = 6.66 \text{ kN}\end{aligned}$$

P4.36 Determine the forces in all bars of the truss. Indicate tension or compression. *Hint:* Start with the method of sections.



$$\sum F_x = 0 \therefore \underline{A_x = 0}$$

CUT SECTION ① - ①

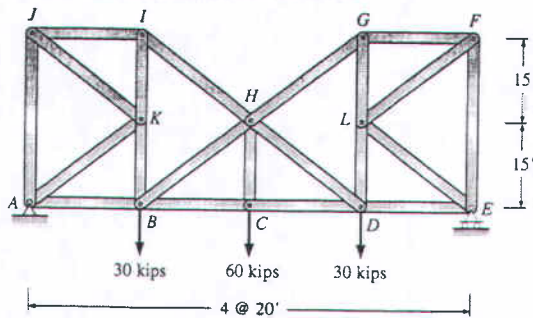


ENTIRE STRUCTURE

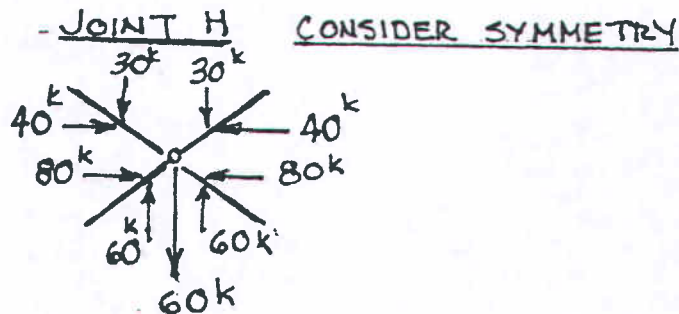
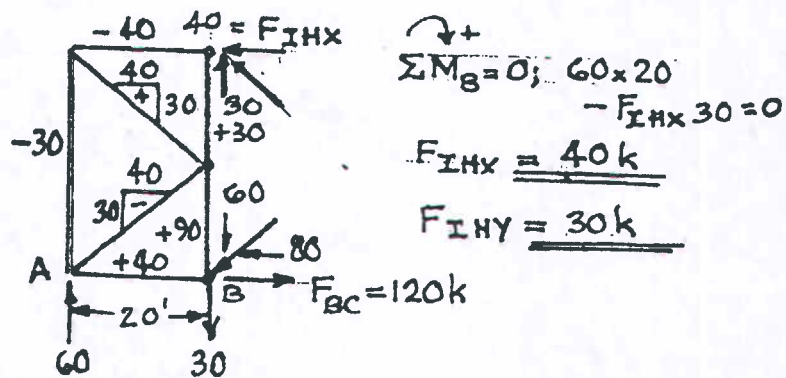
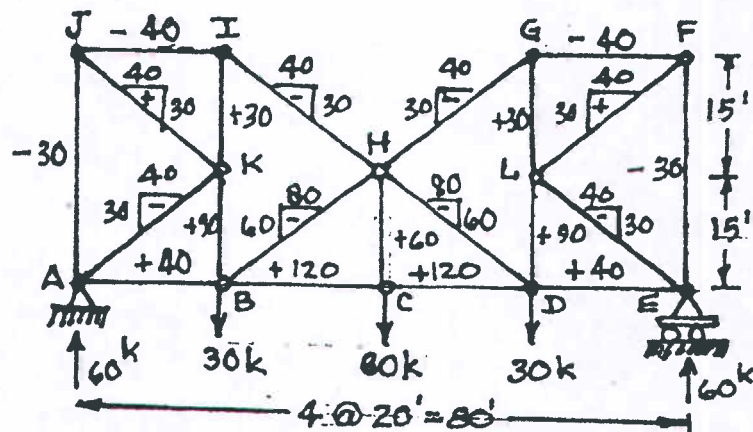
$$\begin{aligned} \sum M_D = 0; & \quad 30 \times 9 - 30 \times 6 - E_y \times 3 = 0 \\ & \quad \underline{E_y = 30 \text{ kN} \downarrow} \\ \sum F_y = 0; & \quad 30 - 30 - 30 + R_D = 0 \\ & \quad \underline{R_D = 30 \text{ kN} \uparrow} \end{aligned}$$

FINAL RESULTS SHOWN ABOVE

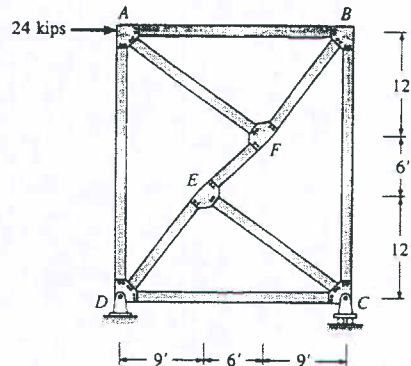
P4.37 Determine the forces in all bars of the truss. Indicate tension or compression. *Hint:* Start with the method of sections.



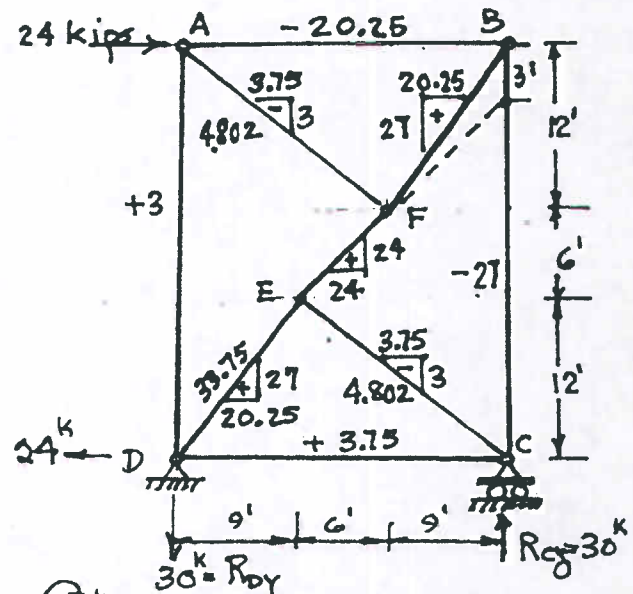
P4.37



P4.38 Determine the forces in all bars of the truss. Indicate tension or compression. *Hint:* Start with the method of sections.

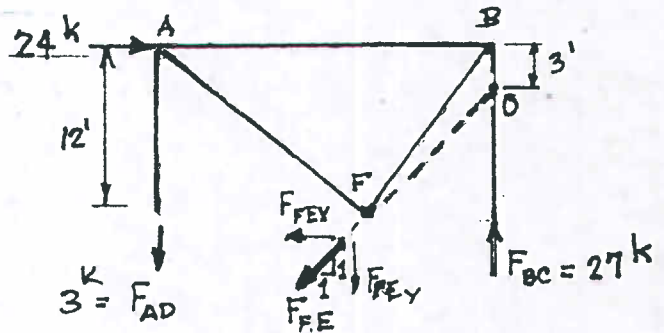


P4.38



$$\sum M_D = 0 = 24 \times 30 - R_C \times 24$$

$$R_C = \underline{30 \text{ kips}}$$



$$\sum M_D = 0 = 24 \times 3' - F_{AD} \times 24$$

$$F_{AD} = \underline{3 \text{ kips}} \text{ TENSION}$$

$$\sum M_F = 0 = 24 \times 12' - 3 \times 15' - F_{BC} \times 9'$$

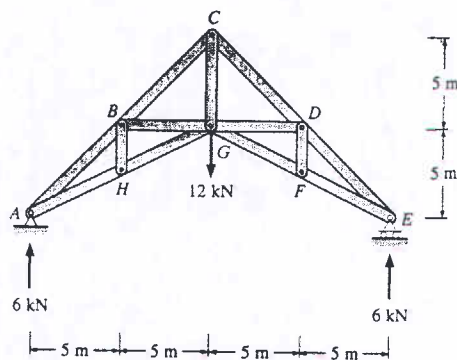
$$F_{BC} = \underline{27 \text{ kips}} \text{ compr.}$$

$$\sum F_x = 0 = 24 \text{ k} - F_{FEX} \quad \therefore F_{FEX} = 24 \text{ k}$$

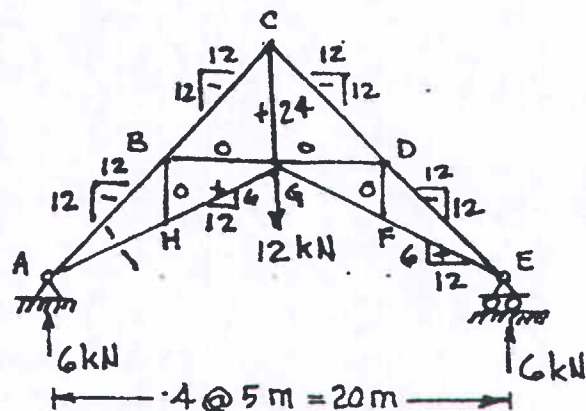
COMPLETE BY METHOD OF JOINTS

SEE TOP FIGURE

P4.39 Determine the forces in all bars of the truss.
Indicate tension or compression.



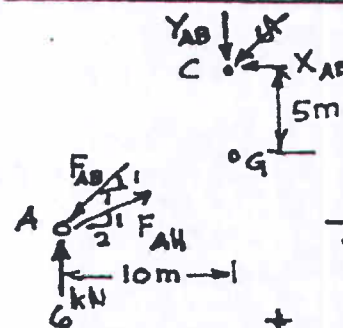
P4.39



ZERO BARS: BH, BG, DG, DF

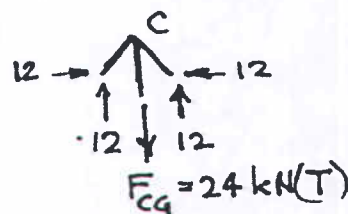
METHOD OF SECTIONS

CUT FBD

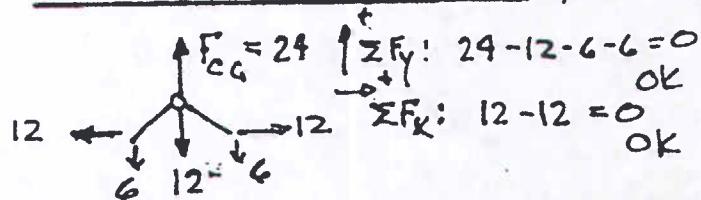


$$\begin{aligned} \sum M_G = 0; \\ 6 \times 10 - X_{AB} 5 = 0 \\ X_{AB} = 12 \text{ kN (C)} \\ Y_{AB} = X_{AB} = 12 \text{ kN} \\ \sum F_x = 0; X_{AH} - X_{AB} = 0 \\ X_{AH} = 12 \text{ kN T} \\ \sum F_y = 0; 6 - Y_{AB} + Y_{AH} = 0 \\ Y_{AH} = 12 - 6 = 6 \text{ kN (T)} \end{aligned}$$

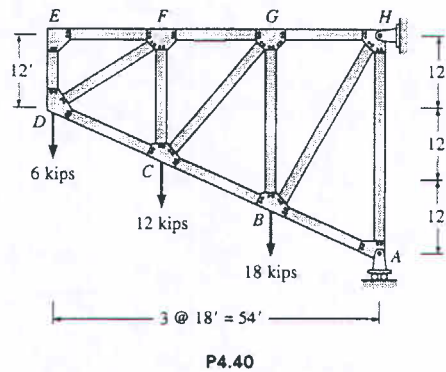
JOINT C



AS A CHECK CONSIDER JOINT G

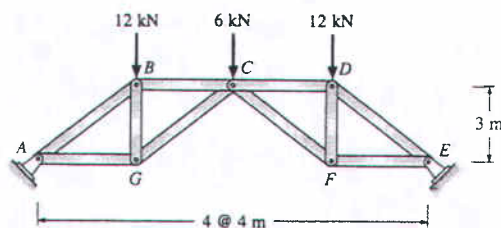


P4.40 Determine the forces in all bars of the truss.
Indicate tension or compression.

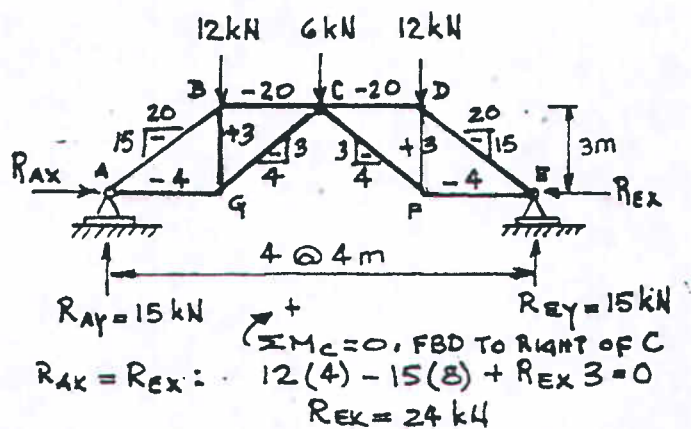


STRUCTURE UNSTABLE.
REACTIONS FORM A CONCURRENT
FORCE SYSTEM.

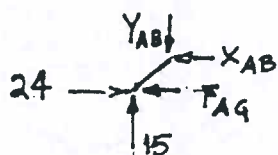
P4.41 Determine the forces in all bars of the truss. Indicate **tension** or **compression**.



P4.41

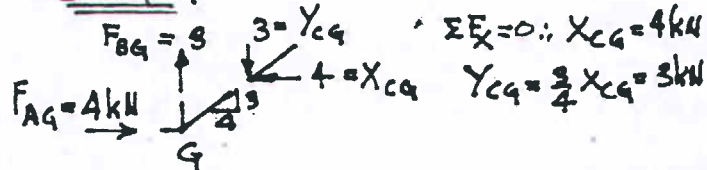


JOINT A



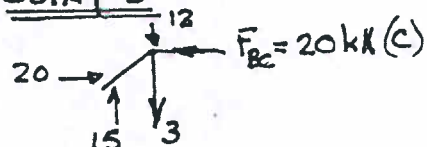
$$\begin{aligned} \sum F_y = 0: & \quad Y_{AB} = 15 \text{ kN} \quad (+) \\ & \quad X_{AB} = \frac{4}{3} 15 = 20 \text{ kN} \\ \sum F_x = 0: & \quad 24 - 20 - F_{AG} = 0 \\ & \quad F_{AG} = 4 \text{ (C)} \end{aligned}$$

JOINT G

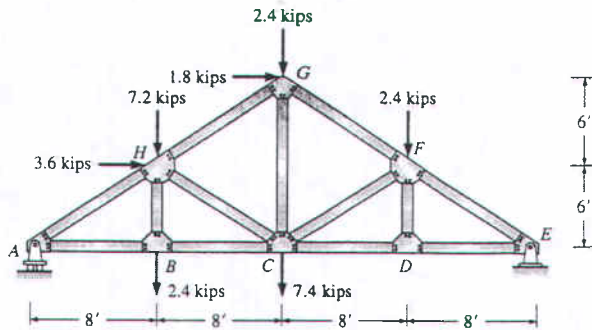


$$\begin{aligned} \sum F_x = 0: & \quad X_{CG} = 4 \text{ kN} \\ & \quad Y_{CG} = \frac{3}{4} X_{CG} = 3 \text{ kN} \end{aligned}$$

JOINT B

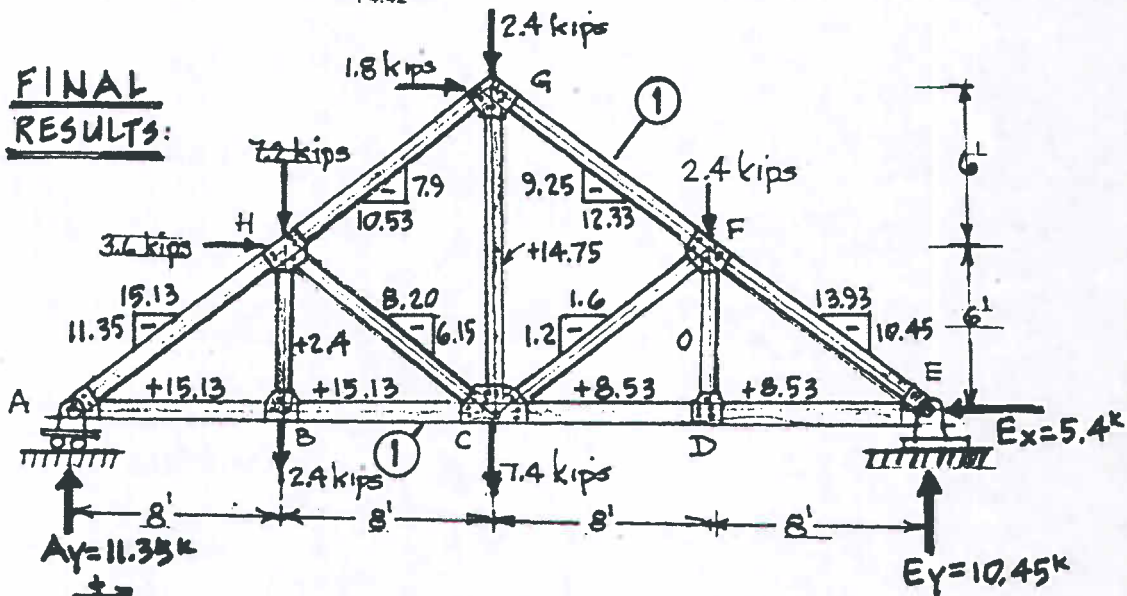


P4.42 Determine the forces in all bars of the truss.
Indicate tension or compression.



P4.42

FINAL RESULTS:

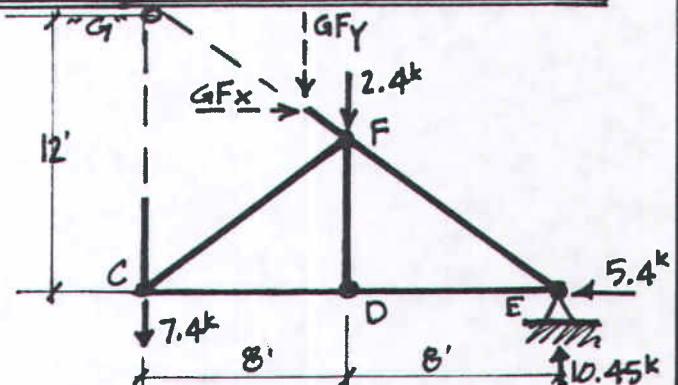


$$\begin{aligned} \sum F_x = 0; \quad 3.6k + 1.8k - E_x = 0 \quad \underline{E_x = 5.4k \leftarrow} \\ \sum M_A = 0; \quad 3.6k(6') + 7.2k(8') + 1.8k(12') + 2.4k(8' + 16' + 24') + 7.4k(16') - E_y(32') = 0 \\ \underline{E_y = 10.45k \uparrow} \\ \uparrow \sum F_y = 0; \quad -7.2k - 3(2.4k) - 7.4k + 10.45k + A_y = 0 \\ \underline{A_y = 11.35k \uparrow} \end{aligned}$$

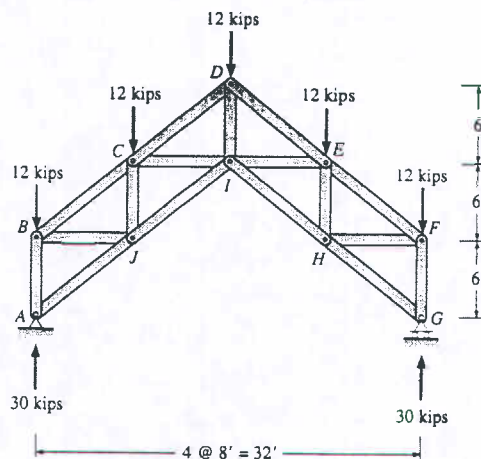
FREEBODY RIGHT SIDE OF SECTION ① COMPUTE FORCE IN BAR GF:

$$\begin{aligned} \sum M_C = 0; \\ GF_x(12') + 2.4k(8') - 10.45k(16') = 0 \\ \underline{GF_x = 12.33k \text{ COMPR}} \\ GF_y/6' = GF_x/8' \quad \underline{GF_y = 9.25k} \end{aligned}$$

REMAINING BAR FORCES SOLVED BY METHOD OF JOINTS.

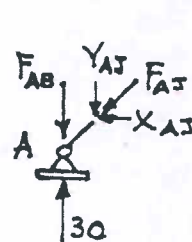


P4.43 Determine the forces in all bars of the truss. Indicate tension or compression.



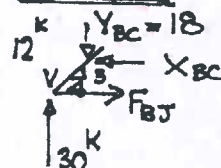
P4.43

JOINT A



$$\begin{aligned} \rightarrow + \quad \Sigma F_x = 0: \quad X_{AJ} &= 0 \\ \therefore Y_{AJ} &= F_{AJ} = 0 \\ \uparrow \Sigma F_y = 0: \quad 30 - F_{AB} &= 0 \\ \therefore F_{AB} &= 30 \text{ (C)} \end{aligned}$$

JOINT B



$$\begin{aligned} X_{BC} &= \frac{4}{3} Y_{BC} = 24 \text{ k (C)} \\ F_{BC} &= \frac{5}{3} Y_{BC} = 30 \text{ k (C)} \\ F_{BJ} &= X_{BC} = 24 \text{ k (T)} \end{aligned}$$

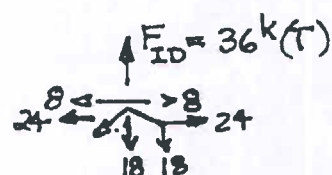
JOINT J

$$\begin{aligned} (C) 18 \text{ k} &= F_{CJ} \quad Y_{JI} = 18 \text{ k} \quad F_{JI} = \frac{5}{3} Y_{JI} = 30 \text{ k (T)} \\ 24 \text{ k} &\rightarrow X_{JI} = 24 \text{ k (T)} \end{aligned}$$

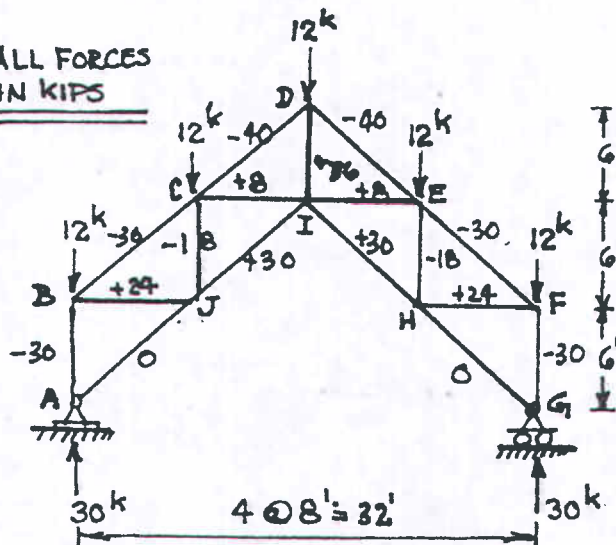
JOINT C

$$\begin{aligned} \uparrow \Sigma F_y = 0: \quad 18 + 18 - 12 - Y_{CD} &= 0 \\ Y_{CD} &= 24 \text{ k (C)} \\ F_{CD} &= \frac{5}{3} (Y_{CD}) = 40 \text{ k} \\ X_{CD} &= \frac{4}{3} Y_{CD} = 32 \text{ k} \\ \rightarrow + \quad \Sigma F_x = 0: \quad 24 - 32 + F_{CI} &= 0 \\ F_{CI} &= 8 \text{ k (T)} \end{aligned}$$

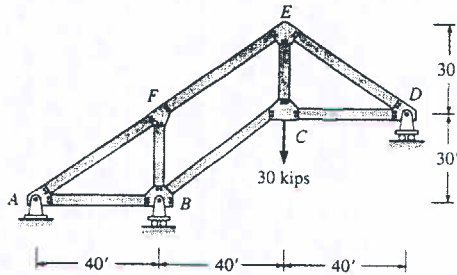
JOINT I



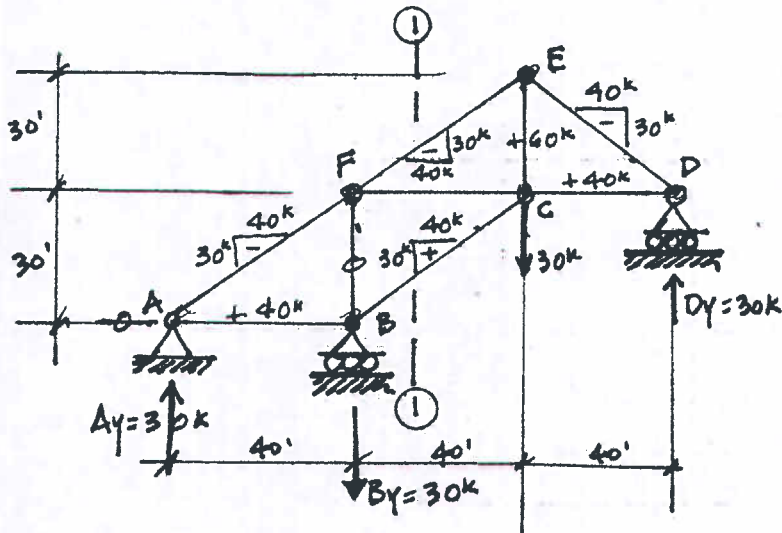
ALL FORCES
IN KIPS



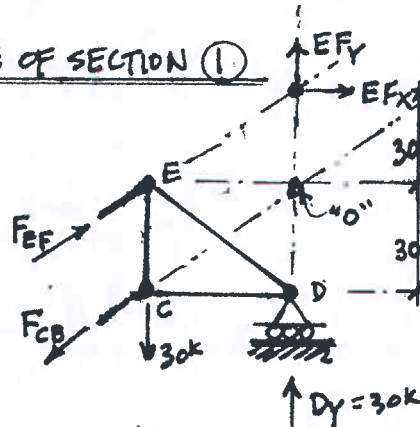
P4.44 Determine the forces in all bars of the truss. Indicate tension or compression.



P4.44



FBD : RIGHT SIDE OF SECTION ①



$$\sum M_{10''} = 0;$$

$$\sum F_x = 0;$$

$$\sum F_y = 0;$$

$$EF_x (30') - 30k (40') = 0$$

$$EF_x = 40k \rightarrow$$

$$EF_y = 40k \left(\frac{3}{4} \right) = 30k \uparrow$$

$$EF_x - CB_x = 0$$

$$CB_x = 40k \leftarrow$$

$$CB_y = 40k \left(\frac{3}{4} \right) = 30k \downarrow$$

$$\begin{aligned} -30k + 30k - 30k + D_y &= 0 \\ D_y &= 30k \uparrow \end{aligned}$$

ENTIRE STRUCTURE :

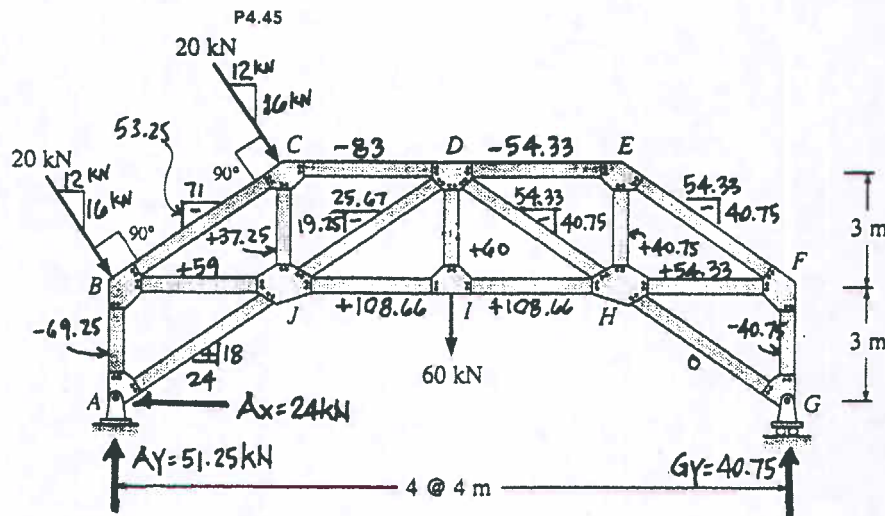
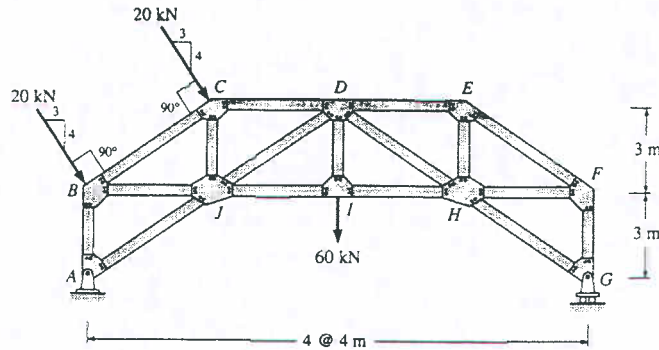
$$\begin{aligned} \sum M_A &= 0; \\ -B_y (40') + 30k (80') - 30k (120') &= 0 \\ B_y &= 30k \downarrow \end{aligned}$$

$$\begin{aligned} \sum F_y &= 0; \\ -30k - 30k + 30k + A_y &= 0 \\ A_y &= 30k \uparrow \end{aligned}$$

$$\begin{aligned} \sum F_x &= 0; \\ A_x &= 0 \end{aligned}$$

REMAINING BAR FORCES
SOLVED BY METHOD OF
JOINTS.

P4.45 Determine the forces in all bars of the truss. Indicate tension or compression.



REACTIONS

$$\sum M_A = 0: 12 \times 3 + 12 \times 6 + 16 \times 4 + 60 \times 8 - 16 R_G = 0$$

$$R_G = 40.75 \text{ kN}$$

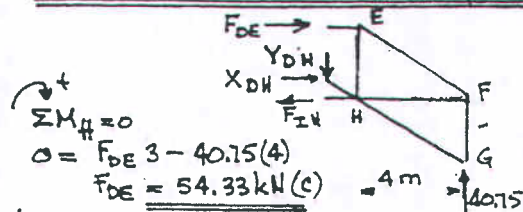
$$\sum F_y = 0: R_{Ay} + 40.75 - 32 - 60 = 0$$

$$R_{Ay} = 51.25 \text{ kN}$$

$$\sum F_x = 0: 12 + 12 - R_{Ax} = 0$$

$$R_{Ax} = 24 \text{ kN}$$

CUT VERTICAL SECTION TO RIGHT OF D



$$\sum M_H = 0$$

$$0 = F_{DE} \cdot 3 - 40.75(4)$$

$$F_{DE} = 54.33 \text{ kN (c)}$$

$$\sum F_y = 0: 40.75 - Y_{DH} = 0 \Rightarrow Y_{DH} = 40.75 \text{ kN}$$

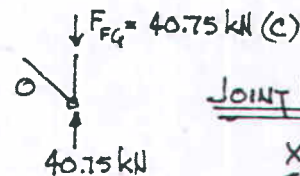
$$\frac{Y_{DH}}{3} = \frac{X_{DH}}{4} \therefore X_{DH} = \frac{4}{3}(40.75)$$

$$X_{DH} = 54.33 \text{ kN (c)}$$

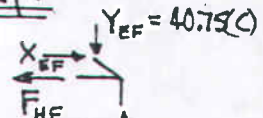
$$\sum F_x = 0 = 54.33 + 54.33 - F_{IH}$$

$$F_{IH} = 108.66 \text{ kN (T)}$$

JOINT G



JOINT F



$$X_{EF} = \frac{4}{3} Y_{EF}$$

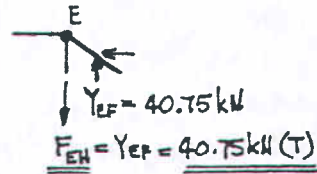
$$= 54.33 \text{ kN (c)}$$

$$F_{HF} = X_{EF} = 54.33 \text{ kN (T)}$$

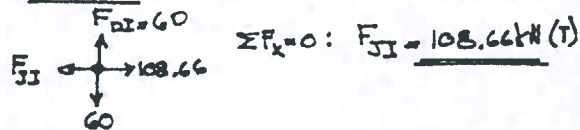
P4.45 CONTINUED

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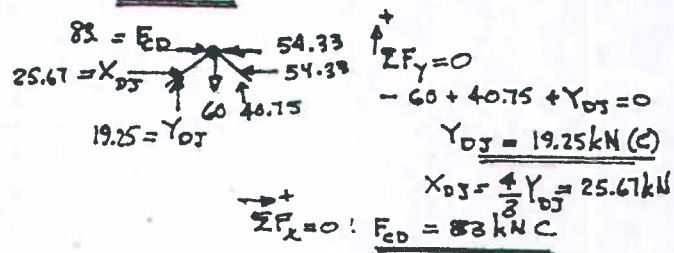
JOINT E



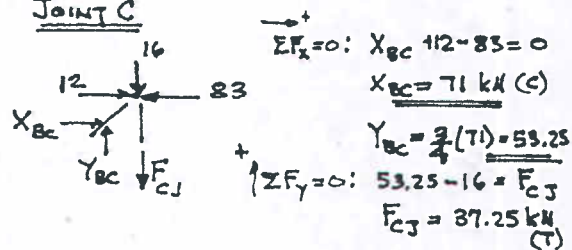
JOINT I



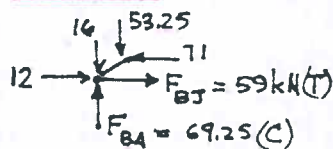
JOINT D



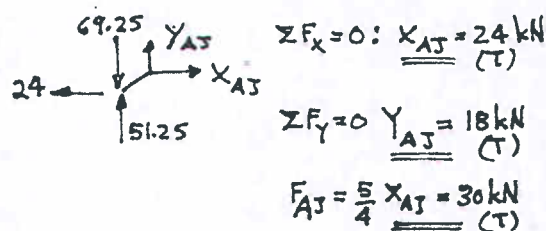
JOINT C



JOINT B

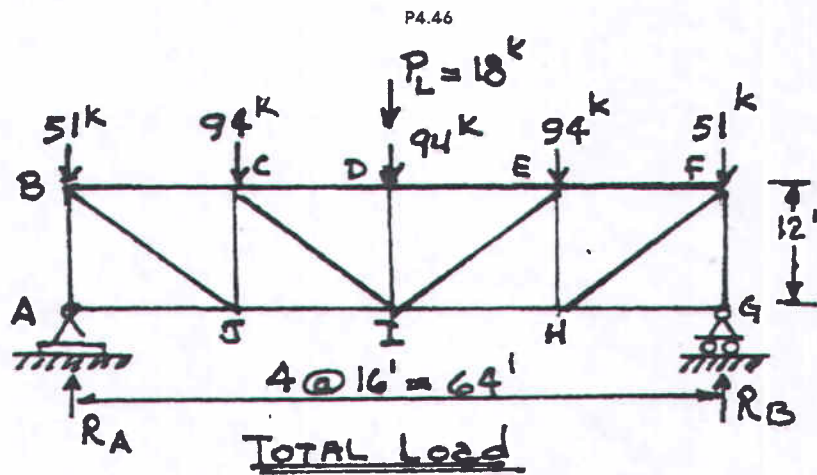
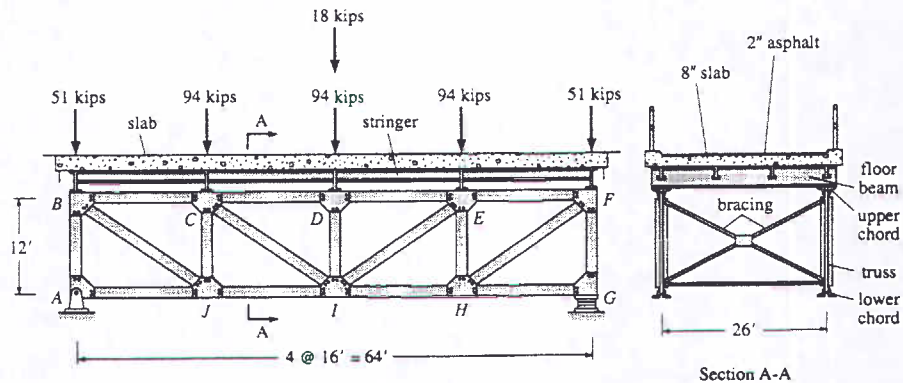


JOINT A

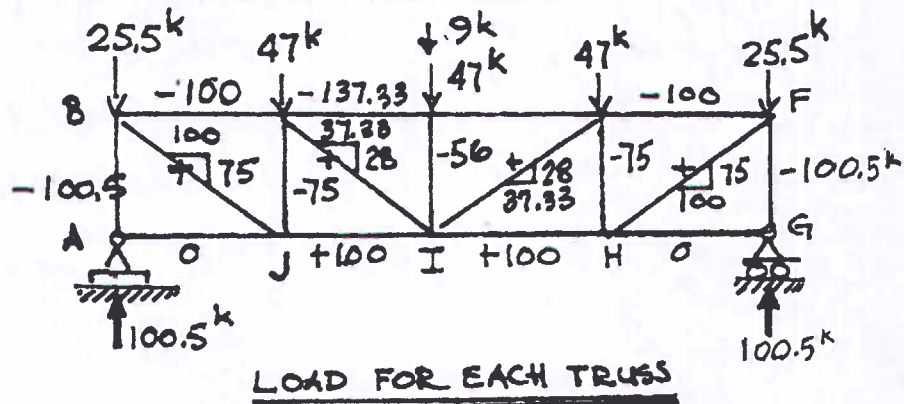


P4.46. A two-lane highway bridge, supported on two deck trusses that span 64 ft, consists of an 8-in reinforced concrete slab supported on four steel stringers. The slab is protected by a 2-in wearing surface of asphalt. The 16-ft-long stringers frame into the floor beams, which in turn transfer the live and dead loads to the panel points of each truss. The truss, bolted to the left abutment at point A, may be treated as pin supported. The right end of

the truss rests on an elastomeric pad at G. The elastomeric pad, which permits only horizontal displacement of the joint, can be treated as a roller. The loads shown represent the total dead and live loads. The 18-kip load is an additional live load that represents a heavy wheel load. Determine the force in the lower chord between panel points I and J, the force in member JB, and the reaction applied to the abutment at support A.



TWO TRUSSES SUPPORT LOAD.

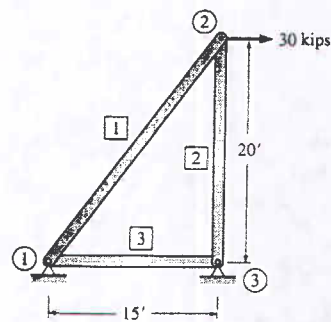


P4.47 Computer analysis of a truss. The purpose of this study is to show that the *magnitude of the joint displacements* as well as the magnitude of the forces in members may control the proportions of structural members. For example, building codes typically specify maximum permitted displacements to ensure that excessive cracking of attached construction, such as exterior walls and windows, does not occur (see Photo 1.1 in Sec. 1.3).

A preliminary design of the truss in Figure P4.47 produces the following bar areas: member 1, 2.5 in^2 ; member 2, 3 in^2 ; and member 3, 2 in^2 . Also $E = 29,000 \text{ kips/in}^2$.

Case 1: Determine all bar forces, joint reactions, and joint displacements, assuming pin joints. Use the computer program to plot the deflected shape.

Case 2: If the maximum horizontal displacement of joint 2 is not to exceed 0.25 in , determine the minimum required area of the truss bars. For this case assume that all truss members have the *same* cross-sectional area. Round the area to the nearest whole number.



P4.47

P4.47

Case 1

Member Section Forces					
	Member Label	S...	Axial[k]	Shear[k]	Moment[k-ft]
1	M1	1	-50	0	0
2		2	-50	0	0
3	M2	1	40	0	0
4		2	40	0	0
5	M3	1	0	0	0
6		2	0	0	0

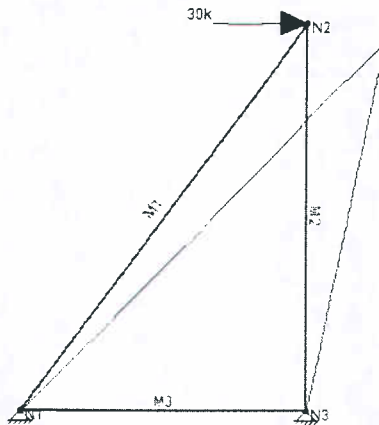
Joint Reactions				
	Joint Label	X [k]	Y [k]	MZ [k-ft]
1	N1	-30	-40	0
2	N3	0	40	0
3	Totals:	-30	0	0

Joint Deflections				
	Joint Label	X [in]	Y [in]	Rotation [rad]
1	N1	0	0	0
2	N2	.492	-.11	0
3	N3	0	0	0

P4.47 CONTINUED

P4.47 CONTINUED

Deflected Shape (horizontal deflection at joint 2 = 0.492 in.)



Case 2

From the computer analysis, the horizontal deflection at joint 2 is 1.303 in if the area of all truss members is set equal to 1 in². To limit the deflection to 0.25 in, the minimum required area is $(1.0 \text{ in}^2) \times (1.303/0.25) = 5.212 \text{ in}^2$. Round this area to 6 in².

P4.48. Computer study. The objective is to compare the behavior of a determinate and an indeterminate structure.

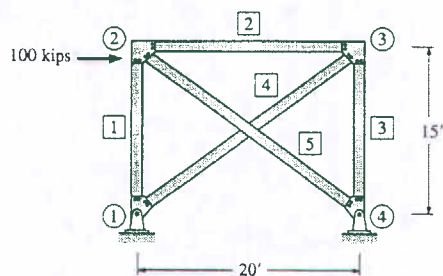
The forces in members of *determinate* trusses are not affected by member stiffness. Therefore, there was no need to specify the cross-sectional properties of the bars of the determinate trusses we analyzed by hand computations earlier in this chapter. In a *determinate* structure, for a given set of loads, only one load path is available to transmit the loads into the supports, whereas in an *indeterminate structure*, multiple load paths exist (see Sec. 3.10). In the case of trusses, the axial stiffness of members (a function of a member's cross-sectional area) that make up each load path will influence the magnitude of the force in **each** member of the load path. We examine this aspect of behavior by varying the properties of certain members of the **indeterminate** truss shown in Figure P4.48. Use $E = 29,000 \text{ kips/in}^2$.

Case 1: Determine the reactions and the forces in members 4 and 5 if the area of all bars is 10 in^2 .

Case 2: Repeat the analysis in **Case 1**, this time increasing the area of member 4 to 20 in^2 . The area of all other bars remains 10 in^2 .

Case 3: Repeat the analysis in **Case 1**, increasing the area of member 5 to 20 in^2 . The area of all other bars remains 10 in^2 .

What conclusions do you reach from the above study?



P4.48

Case 1

Joint Reactions				
	Joint Label	X [k]	Y [k]	M [k]
1	N1	-41.304	-75	
2	N4	-58.696	75	
3	Totals:	-100	0	

Member Section Forces					
	Member Label	S...	Axial[k]	Shear[k]	Moment
1	M1	1	-44.022	0	0
2		2	-44.022	0	0
3	M2	1	41.304	0	0
4		2	41.304	0	0
5	M3	1	30.978	0	0
6		2	30.978	0	0
7	M4	1	-51.63	0	0
8		2	-51.63	0	0
9	M5	1	73.37	0	0
10		2	73.37	0	0

Case 2

Joint Reactions				
	Joint Label	X [k]	Y [k]	M [k]
1	N1	-49.755	-75	
2	N4	-50.245	75	
3	Totals:	-100	0	

Member Section Forces					
	Member Label	S...	Axial[k]	Shear[k]	Moment
1	M1	1	-37.684	0	0
2		2	-37.684	0	0
3	M2	1	49.755	0	0
4		2	49.755	0	0
5	M3	1	37.316	0	0
6		2	37.316	0	0
7	M4	1	-62.193	0	0
8		2	-62.193	0	0
9	M5	1	62.807	0	0
10		2	62.807	0	0

Case 3

Joint Reactions				
	Joint Label	X [k]	Y [k]	M [k]
1	N1	-29.296	-75	
2	N4	-70.704	75	
3	Totals:	-100	0	

Member Section Forces					
	Member Label	S...	Axial[k]	Shear[k]	Moment
1	M1	1	-53.028	0	0
2		2	-53.028	0	0
3	M2	1	29.296	0	0
4		2	29.296	0	0
5	M3	1	21.972	0	0
6		2	21.972	0	0
7	M4	1	-36.62	0	0
8		2	-36.62	0	0
9	M5	1	88.38	0	0
10		2	88.38	0	0

P4.48 CONTINUED

P4.48 CONTINUED

Observations

Case 1: Cutting a horizontal section at the mid-height of the building, it can be shown that the applied horizontal load (100 kips), which can be either a wind or earthquake load in practical applications, has to be resisted (or be balanced) by the horizontal components of two diagonal member forces. (Recall that two columns don't resist horizontal load because they are oriented in the vertical direction.) If the beam (Member 2) were stiff, i.e., with a large cross sectional area such that it can not be shortened easily, joints 2 and 3 would deflect to the right by approximately the same amount, which would have resulted in about the same axial forces in Members 4 and 5, except that Member 4 is in compression and Member 5 is in tension. But the cross sectional area of the beam is of the same order as the other members, the shortening of beam causes Joint 3 to deflect less than Joint 2. As a consequence, the diagonal Member 4 is stretched less with a lower axial force than Member 5.

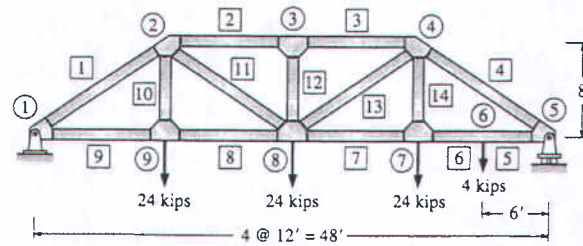
Case 2 and 3: As a general rule, the stiffer part of the structure attracts (i.e., resists) more forces. Since the applied load is to be resisted by the horizontal component of the two diagonal member forces, increasing the cross sectional area (i.e., member stiffness) of Member 4 (Case 2) or Member 5 (Case 3) would result in a higher axial force in that member.

Note that the vertical reactions at supports 1 and 4 are not affected by the change of load path. To prove that the vertical reaction at support 4 is always equal to 75 kips, simply consider the whole structure as a freebody and take moment about support 1.

Practical Example

P4.49. Computer analysis of a truss with rigid joints. The truss in Figure P4.49 is constructed of square steel tubes welded to form a structure with rigid joints. The top chord members 1, 2, 3, and 4 are

$4 \times 4 \times \frac{1}{4}$ inch square tubes with $A = 3.59 \text{ in}^2$ and $I = 8.22 \text{ in}^4$. All other members are $3 \times 3 \times \frac{1}{4}$ inch square tubes with $A = 2.59 \text{ in}^2$ and $I = 3.16 \text{ in}^4$. Use $E = 29,000 \text{ kips/in}^2$.



P4.49

(a) Considering all joints as rigid, compute the axial forces and moments in all bars and the deflection at midspan when the three 24-kip design loads act at joints 7, 8, and 9. (Ignore the 4-kip load.)

(b) If a hoist is also attached to the lower chord at the midpoint of the end panel on the right (labeled joint 6*) to raise a concentrated load of 4 kips, determine the forces and moments in the lower chord (members 5 and 6). If the maximum stress is not to

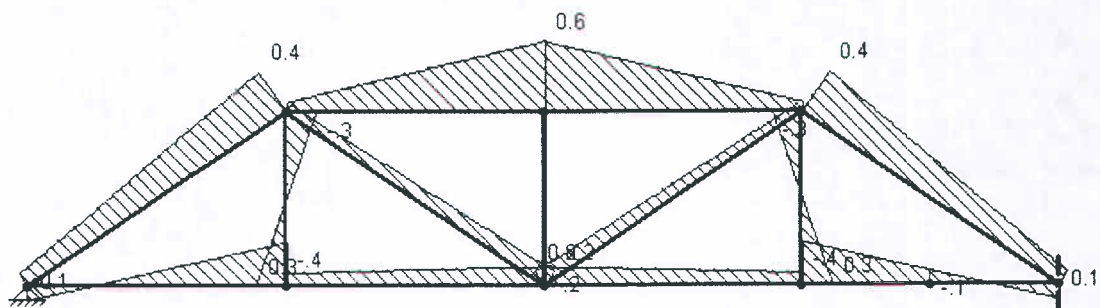
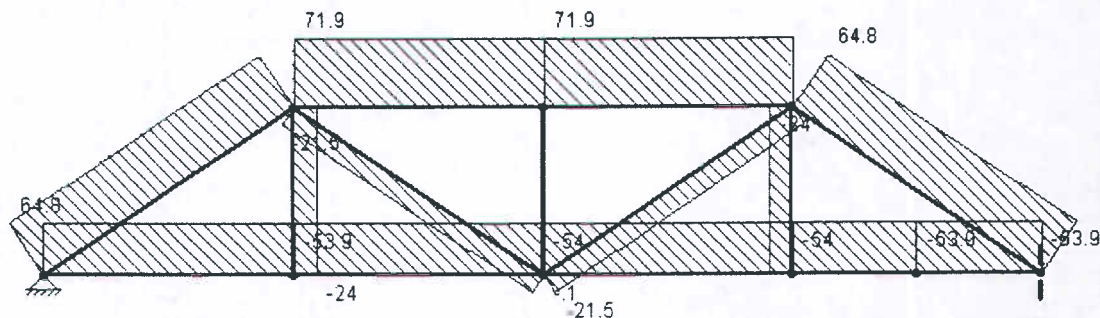
exceed 25 kips/in^2 , can the lower chord support the 4-kip load safely in addition to the three 24-kip loads? Compute the maximum stress, using the equation

$$\sigma = \frac{F}{A} + \frac{Mc}{I}$$

where $c = 1.5 \text{ in}$ (one-half the depth of the lower chord).

*Note: If you wish to compute the forces or deflection at a particular point of a member, designate the point as a joint.

(a)

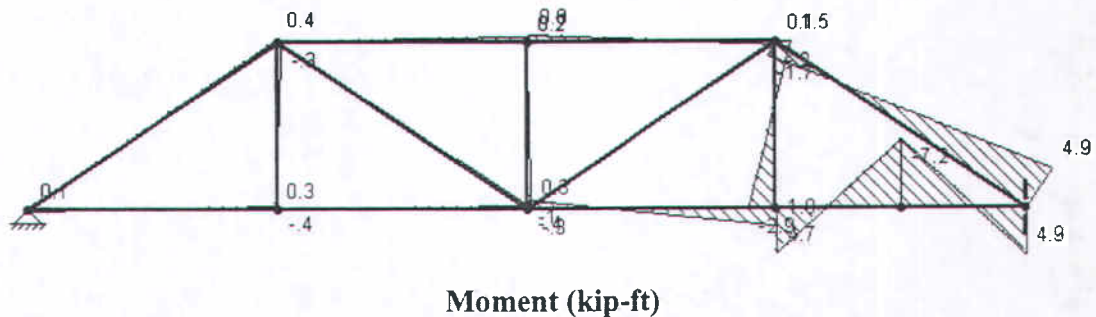
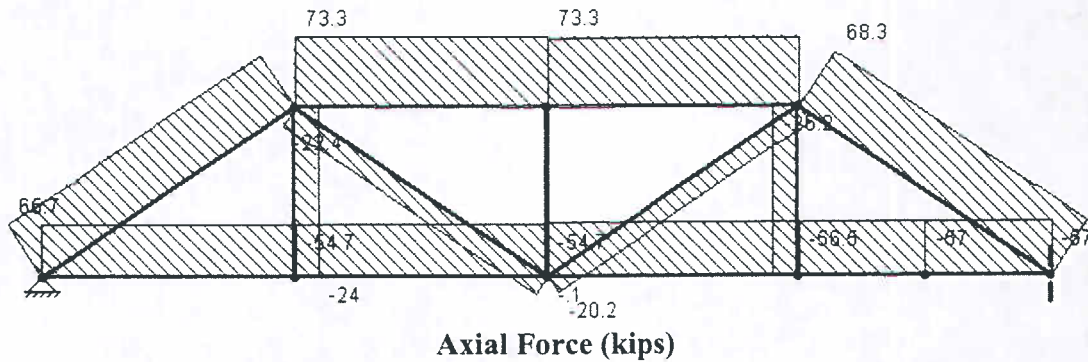


Vertical deflection at midspan (Joint 8) = 0.892 in (downward)

P4.49 CONTINUED

P4.49 CONTINUED

(b)



Axial force in Members 5 and 6: $F = 57.04$ kips

Maximum moment (at Joint 6): $M = 7.22$ kip-ft = 86.64 kip-in

Maximum stress: $\sigma = \frac{F}{A} + \frac{Mc}{I} = \frac{57.04}{2.59} + \frac{86.64(1.5)}{3.16} = 22.02 + 41.13 = 63.15 \text{ ksi} > 25 \text{ ksi (NG)}$

Observation: A truss is in general an efficient system if loads are applied at joints only. Nevertheless, a truss in an industrial building may be damaged or fail if workers connect hoists at or near the center of a panel.