

FUNDAMENTALS OF STRUCTURAL ANALYSIS

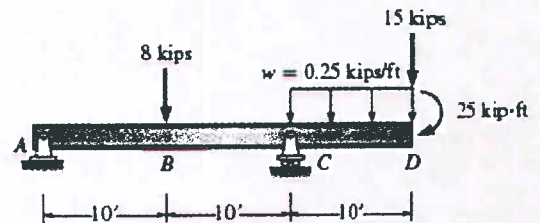
3rd Edition

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SOLUTIONS MANUAL

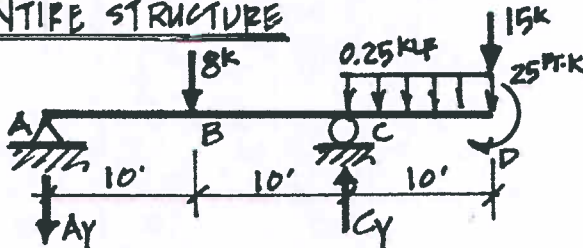
CHAPTER 5: BEAMS AND FRAMES

P5.1. Write the equations for shear and moment between points B and C as a function of distance x along the longitudinal axis of the beam in Figure P5.1 for (a) origin of x at point A, and (b) origin of x at D.



P5.1

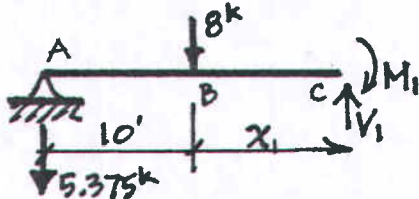
ENTIRE STRUCTURE



$$\begin{aligned}\sum M_A &= 0; \\ 8(10') + 0.25(10')(25') + 15(30') + 25 \text{ Ft}\cdot\text{k} - C_y(20') &= 0 \\ C_y &= 30.875 \text{ k}\uparrow\end{aligned}$$

$$\begin{aligned}\sum F_y &= 0; \\ A_y - 8 \text{ k} - 0.25(10') - 15 + 30.875 \text{ k} &= 0 \\ A_y &= 5.375 \text{ k}\downarrow\end{aligned}$$

FREEBODY SEGMENT AC



SEGMENT BC: ORIGIN @ A:

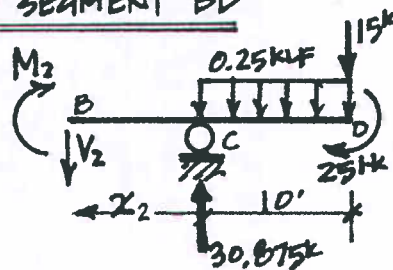
$$\begin{aligned}\sum M_1 &= 0; \\ M_1 - 8(x_1) - 5.375(10' + x_1) &= 0\end{aligned}$$

$$M_1 = 53.75 + 13.375(x_1)$$

$$M @ C = 53.75 + 13.375(10') = -187.5 \text{ Ft}\cdot\text{k}$$

$$\begin{aligned}\sum F_y &= 0; \\ -5.375 \text{ k} - 8 \text{ k} + V_1 &= 0 \\ V_1 &= 13.375 \text{ k}\uparrow\end{aligned}$$

FREEBODY SEGMENT BD



SEGMENT BC: ORIGIN @ D:

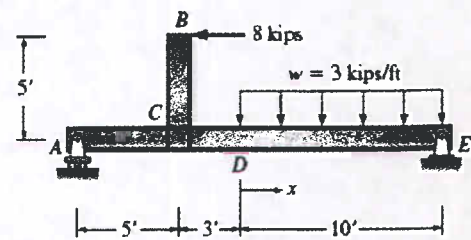
$$\begin{aligned}\sum M_2 &= 0; \\ M_2 - 30.875(x_2) + 0.25(10')(5' + x_2) + 15(10' + x_2) + 25 \text{ Ft}\cdot\text{k} &= 0\end{aligned}$$

$$M_2 = 187.5 - 13.375(x_2)$$

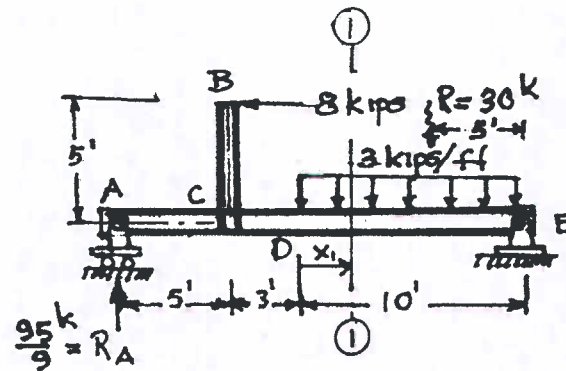
$$M @ B = -53.75 \text{ Ft}\cdot\text{k}$$

$$\begin{aligned}\sum F_y &= 0; \\ -V_2 + 30.875 \text{ k} - 0.25(10') - 15 \text{ k} &= 0 \\ V_2 &= -13.375 \text{ k}\downarrow\end{aligned}$$

P5.2. Write the equations for shear and moment between points *D* and *E*. Select the origin at *D*.



P5.2

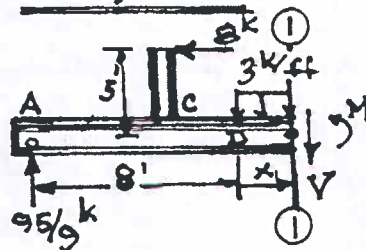


compute R_A : $\sum M_E = 0$

$$R_A 18' - 8'(5') - 30'(5') = 0$$

$$R_A = 190/18 = \underline{\underline{95/9 \text{ kips}}}$$

$$V = \underline{\underline{95/9 - 3x_1}} \quad \underline{\underline{\text{Ans.}}}$$



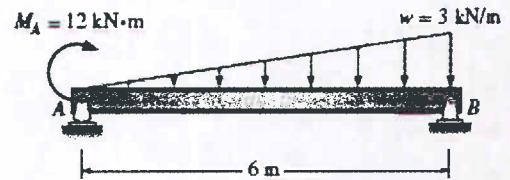
SUM MOMENTS ABOUT CUT ①: $\sum M_D = 0$

$$95/9 (8+x_1) - 8'(5') - 3x_1 \left(\frac{x_1}{2} \right) - M = 0$$

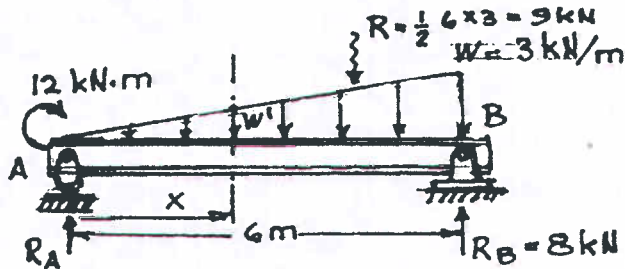
$$\underline{\underline{740/9 + 95/9 x_1 - 40 - 3/2 x_1^2 = M}}$$

$$\underline{\underline{M = \underline{\underline{400/9 + 95/9 x_1 - 1.5 x_1^2}}}} \quad \underline{\underline{\text{Ans.}}}$$

P5.3. Write the equations for shear and moment between points A and B. Select the origin at A. Plot the graph of each force under a sketch of the beam. The rocker at A is equivalent to a roller.

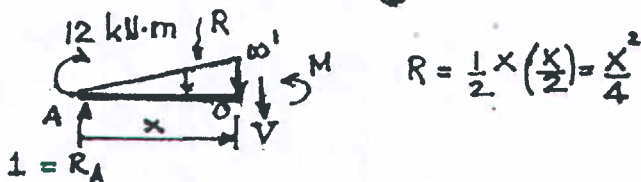


P5.3



$$\sum M_B = 0; 12 \text{ kN}\cdot\text{m} + R_A(6\text{ m}) - (9\text{ kN})(2\text{ m}) = 0$$

$$R_A = \frac{6}{6} = 1 \text{ kN} \uparrow$$



$$\frac{w'}{x} = \frac{3}{6} \therefore w' = \frac{x}{2}$$

$$\sum F_y = 0; 1\text{ kN} - \frac{1}{2}xw' - V = 0$$

$$V = 1 - \frac{1}{2}x\left(\frac{x}{2}\right)$$

$$V = 1 - \frac{x^2}{4} \quad \text{Ans.}$$

$$\sum M_O = 0; 1(x) + 12 - \frac{x^2}{4}\left(\frac{x}{3}\right) - M = 0$$

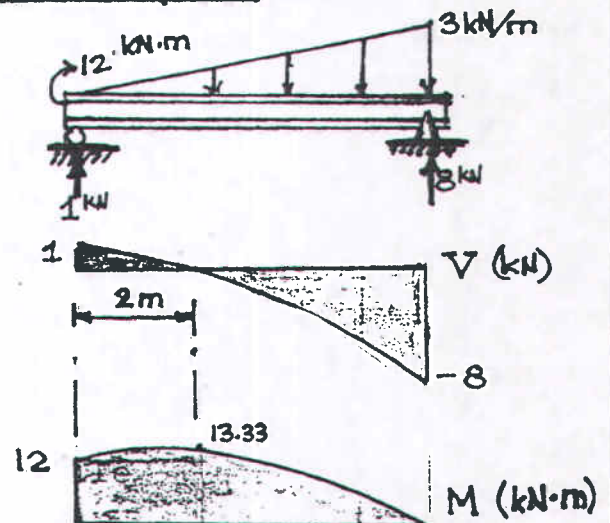
$$M = 12 + x - \frac{x^3}{12}$$

$$M = 12 + x - \frac{x^3}{12} \quad \text{Ans}$$

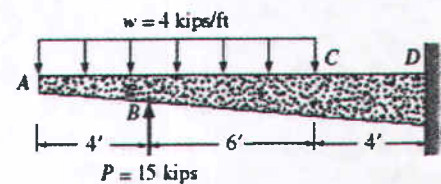
$$\text{CHECK @ } x=6: M = 12 + 6 - \frac{36(6)}{12}$$

$$M = 18 - 18 = 0 \quad \text{O.K.}$$

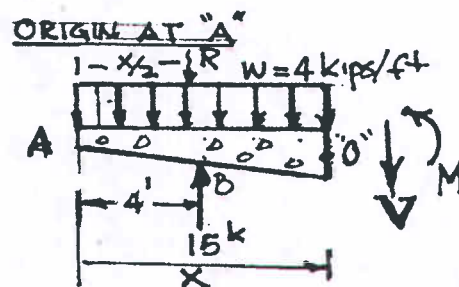
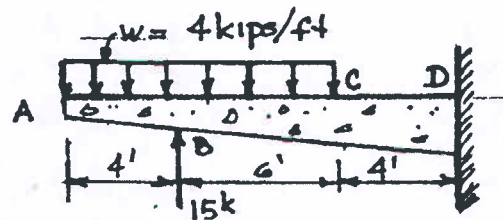
V and M diagrams



P5.4. Write the equations for shear V and moment M between points B and C . Take the origin at point A . Evaluate V and M at point C , using the equations.



P5.4



EQ FOR SHEAR $\sum F_y \uparrow = 0$

$$0 = 15 - 4x - V$$

$$V = 15 - 4x$$

V at C $x = 10'$

$$V_C = 15 - 4(10) = -25 \text{ kips Ans}$$

EQ FOR MOMENT $\sum M_O \rightarrow = 0$

$$- R \cdot \frac{x}{2} + 15(x - 4) - M = 0$$

$$M = -4x\left(\frac{x}{2}\right) + 15x - 60$$

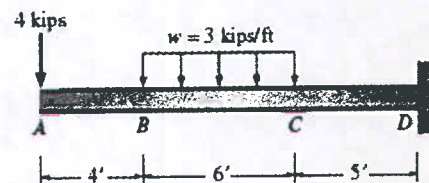
$$M = -2x^2 + 15x - 60 \text{ Ans.}$$

$$\text{AT } x = 10; M_C = -2(10)^2 + 15(10) - 60$$

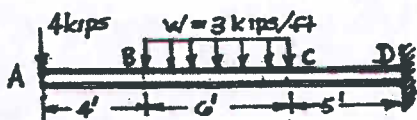
$$= -200 + 150$$

$$M_C = -110 \text{ kip}\cdot\text{ft Ans.}$$

P5.5. Write the equations for moment between points B and C as a function of distance x along the longitudinal axis of the beam in Figure P5.5 for (a) origin of x at A and (b) origin of x at B.

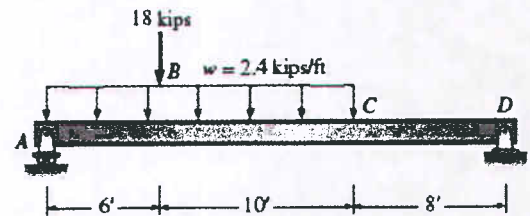


P5.5



FREE BODY DIAGM	SEGMENT	ORIGIN	RANGE	V EQUATION	M EQUATION
	BC	A	$4' \leq x_1 \leq 10'$	$+\uparrow \Sigma F_y = 0;$ $-4^k - V_1 - 3^k(x_1 - 4') = 0$ $\underline{V_1 = 8 - 3x_1}$	$\sum \bar{M}_0 = 0;$ $-4x_1 - M_1 - 3(x_1 - 4')\left(\frac{x_1 - 4'}{2}\right) = 0$ $\underline{M_1 = -\frac{3}{2}(x_1 - 4')^2 - 4x_1}$
	BC	B	$0 \leq x_2 \leq 6'$	$+\uparrow \Sigma F_y = 0;$ $-4^k - V_2 - 3x_2 = 0$ $\underline{V_2 = -4 - 3x_2}$	$\sum \bar{M}_0 = 0;$ $-4^k(x_2 + 4') - M_2 - 3x_2\left(\frac{x_2}{2}\right) = 0$ $\underline{M_2 = -16 - 4x_2 - \frac{3}{2}x_2^2}$

P5.6. Write the equations required to express the moment along the entire length of beam in Figure P5.6. Use an origin at point A, and then repeat computations using an origin at point D. Verify that both procedures give the same value of moment at point C.



P5.6

Diagram of beam AD with origin at A. The beam has a pin support at A and a roller support at D. A downward point load of 18 k is applied at point B, which is 6 feet from A. A uniformly distributed load of $w = 2.4 \text{ kips/ft}$ is applied from point B to point C, which is 10 feet from B. The distance from point C to point D is 8 feet. The total length of the beam is 24 feet.

ORIGIN AT A

Reactions: $R_A = 39.1 \text{ k}$, $R_D = 17.3 \text{ k}$

Section A-B: $M_1 = 39.1x_1 - 2.4x_1\left[\frac{x_1}{2}\right] = 39.1x_1 - 1.2x_1^2$

Section B-C: $M_2 = 39.1x_2 - 2.4x_2\left[\frac{x_2}{2}\right] - 18(x_2 - 6) = 39.1x_2 - 1.2x_2^2 - 108$

Section C-D: $M_3 = 39.1x_3 - 18(x_3 - 6) - 2.4 \times 16(8 + x_3 - 16) = 415.2 - 17.3x_3$

ORIGIN AT D

Reactions: $R_A = 39.1 \text{ k}$, $R_D = 17.3 \text{ k}$

Section CD: $M_4 = 17.3x_4$

Section BC: $M_5 = 17.3x_5 - (x_5 - 8)2.4\left(\frac{x_5 - 8}{2}\right) = 17.3x_5 - 1.2(x_5 - 8)^2$

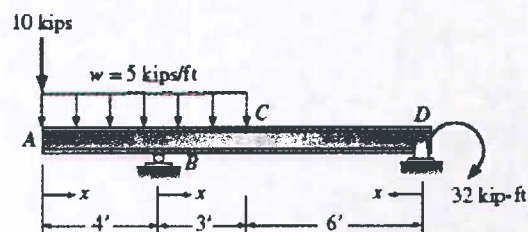
Section BA: $M_6 = 17.3x_6 - 18(x_6 - 8) - 2.4(x_6 - 8)\left(\frac{x_6 - 8}{2}\right) = -1.2(x_6 - 8)^2 - 0.7x_6 + 324$

CHECK
MOMENT AT C

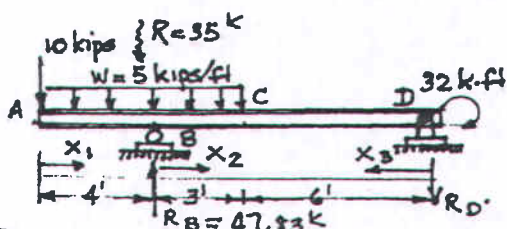
$M_4 = 17.3 \times 8 = 138.4 \text{ kip-ft}$

$M_5 = 21.1 \times 16 - 1.2(16)^2 + 108 = 138.4 \text{ kip-ft}$

P5.7. Write the equations for shear and moment using the origins shown in the figure. Evaluate the shear and moment at C, using the equations based on the origin at point D.

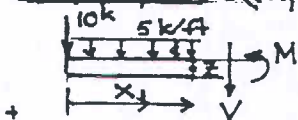


P5.7



$$\begin{aligned} \sum M_D = 0 &= -10 \times 13 - 35 \times 9.5 + 32 + 9R_B \\ R_B &= 47.83 \text{ kips} \\ \uparrow \sum F_y = 0 &= 47.83 - 10 - 35 - R_D, R_D = 2.83 \end{aligned}$$

SEGMENT AB: $0 \leq x_1 \leq 4$

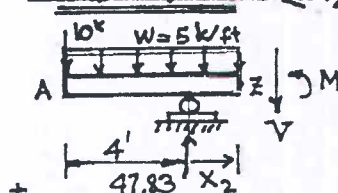


$$\uparrow \sum F_y = 0 = -10 - 5x_1 - V \Rightarrow V = -10 - 5x_1$$

$$\sum M_{x_1} = 0 = 10x_1 + 5x_1 \cdot \frac{x_1}{2} + M$$

$$M = -10x_1 - \frac{5}{2}x_1^2$$

SEGMENT BC: $0 \leq x_2 \leq 3$



$$\uparrow \sum F_y = 0 = -10 - (4 + x_2)5 + 47.83 - V$$

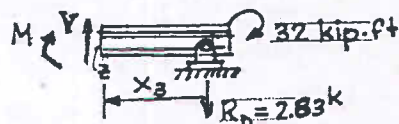
$$V = -30 - 5x_2 + 47.83$$

$$V = 17.83 - 5x_2$$

$$\sum M_{x_2} = 0 = -M - 10(4 + x_2) - 5\left(\frac{4 + x_2}{2}\right)^2 + 47.83x_2$$

$$M = -40 + 37.83x_2 - \frac{5}{2}(4 + x_2)^2$$

SEGMENT DC: $0 \leq x_3 \leq 6$



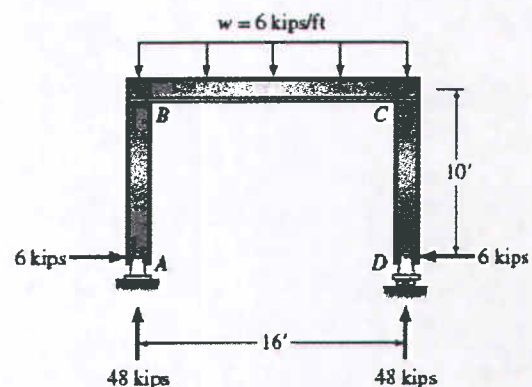
$$\uparrow \sum F_y = 0; V - 2.83 = 0; V = 2.83 \text{ k}$$

$$\sum M_{x_3} = 0; M + 32 + 2.83x_3 = 0$$

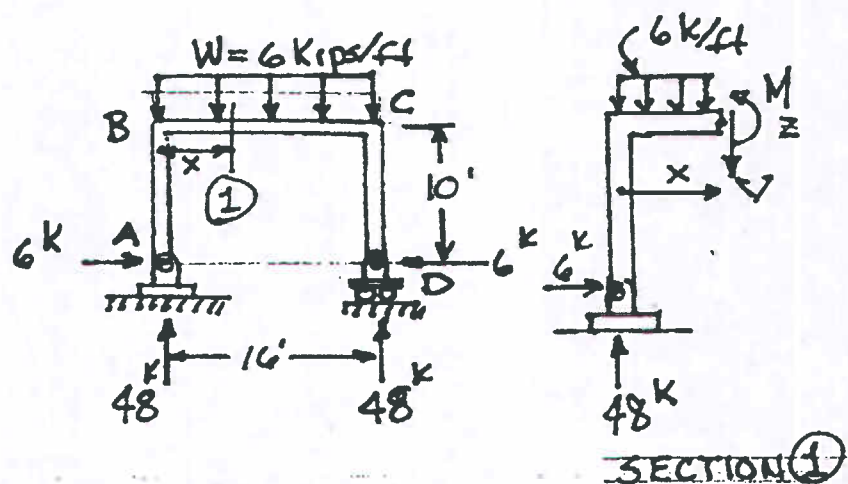
$$M = -32 - 2.83x_3$$

$$M_C (x_3 = 6') = -49 \text{ ft} \cdot \text{k}$$

P5.8. Write the equation for moment between points B and C for the rigid jointed frame in Figure P5.8.



P5.8

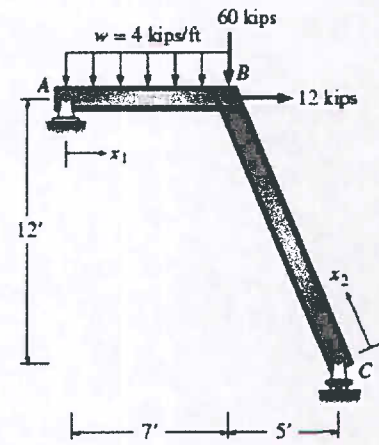


MEMBER BC $0 \leq x \leq 16'$

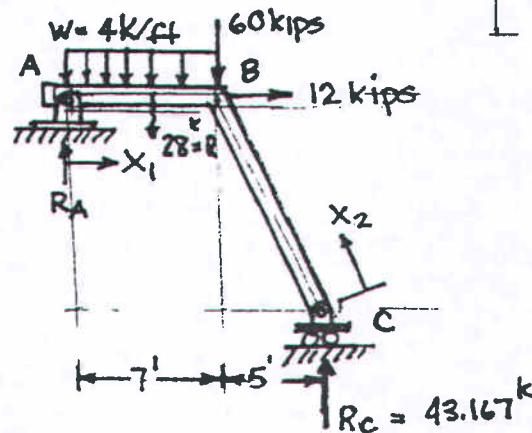
$$\sum M_z = 0 = 48x - 6 \times 10 - 6x\left(\frac{x}{2}\right) - M$$

$$\underline{\underline{M = -60 + 48x - 3x^2}}$$

P5.9. Write the equations for moment as a function of distance along the longitudinal axes for members AB and BC of the frame in Figure P5.9. Origins for each member are shown.



P5.9



$$\sum M_A = 0 = 28 \times 3.5 + 60 \times 7 - 12 R_C$$

$$R_C = 43.167 \text{ k} \uparrow$$

$$\sum F_y = 0 = R_A - 28 - 60 + 43.167$$

$$R_A = 44.833 \text{ k} \uparrow$$

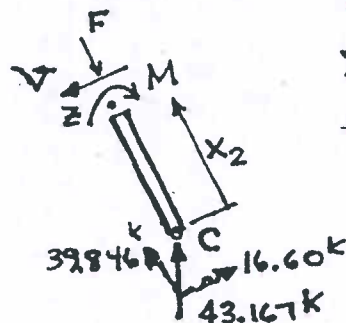
EQUATIONS FOR MOMENT

MEMBER AB ORIGIN "A" $0 \leq x_1 \leq 7$

$$M = 44.833 x_1 - 4 x_1 \left(\frac{x_1}{2} \right)$$

$$V = 44.833 - 4 x_1$$

MEMBER CB BREAK R_C INTO COMPONENTS

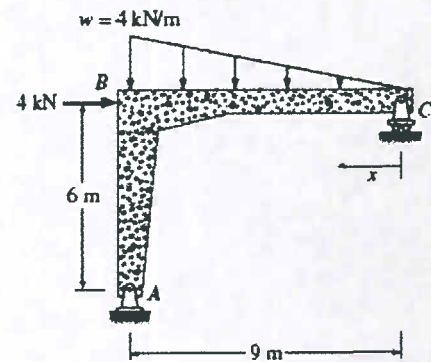


$$\sum M_2 = 0$$

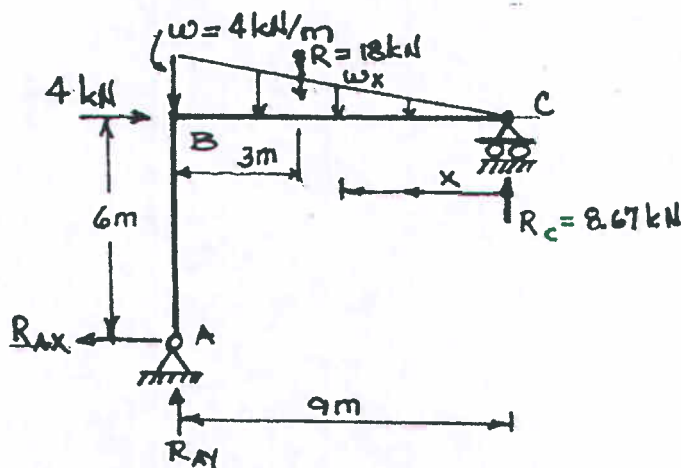
$$0 = M - 16.60 x_2$$

$$M = 16.60 x_2$$

P5.10. Write the equations for shear and moment between points B and C for the rigid frame in Figure P5.10. Select the origin at point C.



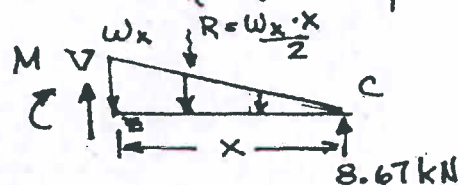
P5.10



REACTIONS $\sum F_x = 0$; $R_{Ax} = 4 \text{ kN} \leftarrow$

$\sum M_A = 0$; $4 \times 6 + 18 \times 3 - R_c \times 9 = 0$
 $R_c = 8.67 \text{ kN} \uparrow$

$\sum F_y = 0$; $R_{Ay} - 18 + 8.67 = 0$
 $R_{Ay} = 9.33 \text{ kN} \uparrow$



$\frac{w_x}{x} = \frac{4}{9}$; $w_x = \frac{4}{9}x$

SHEAR $\sum F_y = 0$;
 $V - \frac{w_x \cdot x}{2} + 8.67 = 0$

$V = \frac{4}{9}x \cdot \frac{x}{2} - 8.67$

$V = \frac{2}{9}x^2 - 8.67$

MOMENT $\sum M_z = 0$;

$M + w_x \cdot \frac{x}{2} \left(\frac{x}{3} \right) - 8.67x = 0$

$M = 8.67x - \left(\frac{4}{9}x \right) \frac{x^2}{6}$

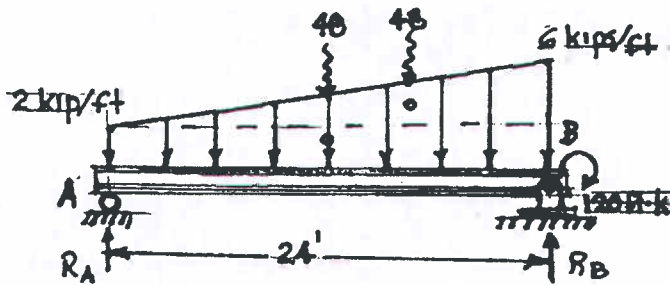
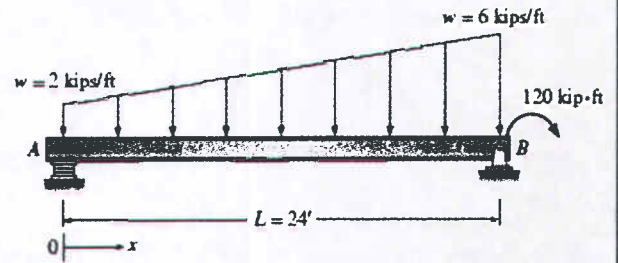
$M = 8.67x - \frac{2x^3}{27}$

P5.11. Consider the beam shown in Figure P5.11. The elastomeric pad at support A is equivalent to a roller.

(a) Write the equations for shear and moment in terms of x . Select an origin at A.

(b) Locate the section of maximum moment.

(c) Calculate M_{max} .



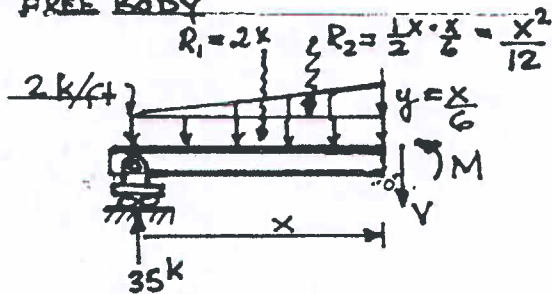
$$(a) \quad \Sigma M_A = 0; \quad 48 \times 12 + 48 \times 16 + 120 - 24 R_B = 0$$

$$R_B = 61 \text{ kips} \uparrow$$

$$\Sigma F_y = 0; \quad 61 - 48 - 48 + R_A = 0$$

$$R_A = 35 \text{ kips} \uparrow$$

FREE BODY



$$\text{EQ FOR } V: \quad \Sigma F_y = 0; \quad 35 - 2x - \frac{1}{2}x \cdot \frac{x}{6} - V = 0$$

$$V = 35 - 2x - \frac{x^2}{12}$$

EQ M:

$$(\Sigma M_o = 0); \quad 35x - 2x \cdot \frac{x}{2} - \frac{x^2}{12} \cdot \frac{x}{3} - M = 0$$

$$M = 35x - x^2 - \frac{x^3}{36}$$

(b) LOCATE SECTION OF MAX M

$$V = 0 = 35 - 2x - \frac{x^2}{12} \quad (1)$$

$$0 = x^2 + 24x - 420$$

complete square $(x + 12)^2 = 420 + 12^2 = 564$

$$x + 12 = 23.75$$

$$x = 11.75 \text{ ft}$$

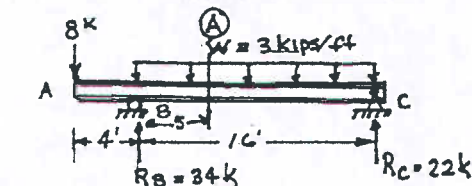
$$(c) \quad M_{max} = 35(11.75) - (11.75)^2 - \frac{(11.75)^3}{36}$$

$$= 411.25 - 138.06 - 45.06$$

$$= 228.13 \text{ Kip.ft}$$

P5.12. Consider the beam shown in Figure P5.12.

- Write the equations for shear and moment for the beam using an origin at end A.
- Using the equations, evaluate the moment at section A.
- Locate the point of zero shear between B and C.
- Evaluate the maximum moment between points B and C.
- Write the equations for shear and moment using an origin at C.
- Evaluate the moment at section A.
- Locate the section of maximum moment and evaluate M_{max} .
- Write the equations for shear and moment between B and C using an origin at B.
- Evaluate the moment at section A.



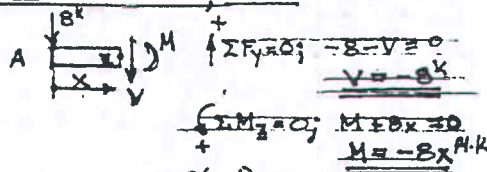
$$+\circlearrowleft \sum M_B = 0; -8 \times 4 - 48 \times 8 + 16 R_C = 0$$

$$R_C = 22 \text{ kips}$$

$$+\uparrow \sum F_y = 0; -8 - 48 + R_B + 22 = 0$$

$$R_B = 34 \text{ kips}$$

(a) A-B ORIGIN AT A, $0 \leq x \leq 4$



$$+\uparrow \sum F_y = 0; -8 - V = 0$$

$$V = -8 \text{ k}$$

$$+\circlearrowleft \sum M = 0; M - 8x = 0$$

$$M = -8x \text{ kip}\cdot\text{ft}$$

$$+\uparrow \sum F_y = 0; -8 + 34 - 3(x-4) - V = 0$$

$$V = -3x + 38 \quad (\text{EQ 1})$$

$$+\circlearrowleft \sum M = 0; 8x - 34(x-4) + 3(x-4)(x-4) + M = 0$$

$$M = -15x^2 + 38x - 160 \quad (\text{EQ 2})$$

(b) Moment at A set $x = 0$

$$M_A = -15(0)^2 + 38(0) - 160$$

$$= -160$$

$$= -121.5 + 34(2) - 160$$

$$= 60.5 \text{ kip}\cdot\text{ft}$$

(c) Locate point $V = 0$ B-C

USE EQ 1: $0 = -3x + 38$

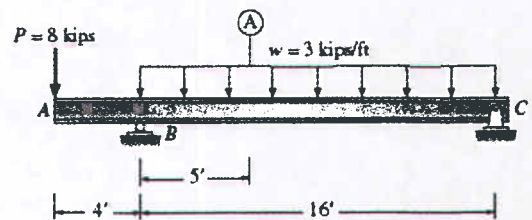
$$x = 12.67'$$

(d) M_{max} : set $x = 12.67$ in (EQ 2)

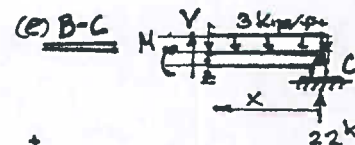
$$M_{max} = -15(12.67)^2 + 38(12.67) - 160$$

$$= -240.79 + 481.46 - 160$$

$$M_{max} = 80.67 \text{ kip}\cdot\text{ft}$$



P5.12

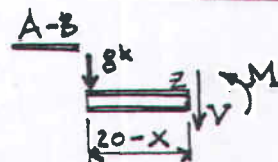


$$+\uparrow \sum F_y = 0; V - 3x + 22 = 0$$

$$V = -22 + 3x$$

$$+\circlearrowleft \sum M = 0; M + 3x \cdot \frac{x}{2} - 22x = 0$$

$$M = 22x - \frac{3}{2}x^2$$



$$+\uparrow \sum F_y = 0; -8 - V = 0$$

$$V = -8 \text{ k}$$

$$+\circlearrowleft \sum M = 0; -8(20-x) - M = 0$$

$$M = 8x - 160$$

(f) M at section A, let $x = 11'$

$$M = 22(11) - \frac{3}{2}(11)^2 = 60.5 \text{ kip}\cdot\text{ft}$$

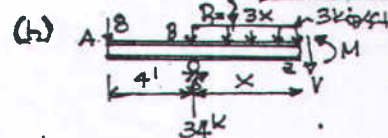
(g) M_{max} , set $V = 0; -22 + 3x = 0$

$$x = \frac{22}{3} = 7.33'$$

$$+ M_{max} = 22 \times 7.33 - \frac{3}{2}(7.33)^2$$

$$= 141.26 - 80.59$$

$$M_{max} = 80.67 \text{ kip}\cdot\text{ft}$$



$$+\uparrow \sum F_y = 0; -8 + 34 - 3x - V = 0$$

$$V = 26 - 3x$$

$$+\circlearrowleft \sum M = 0; -8(4+x) + 34x - 3x \cdot \frac{x}{2} - M = 0$$

$$M = -32 + 26x - \frac{3}{2}x^2$$

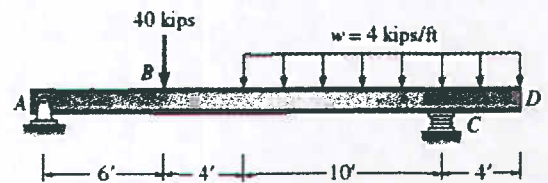
(h) Moment at section A

Let $x = 5'$

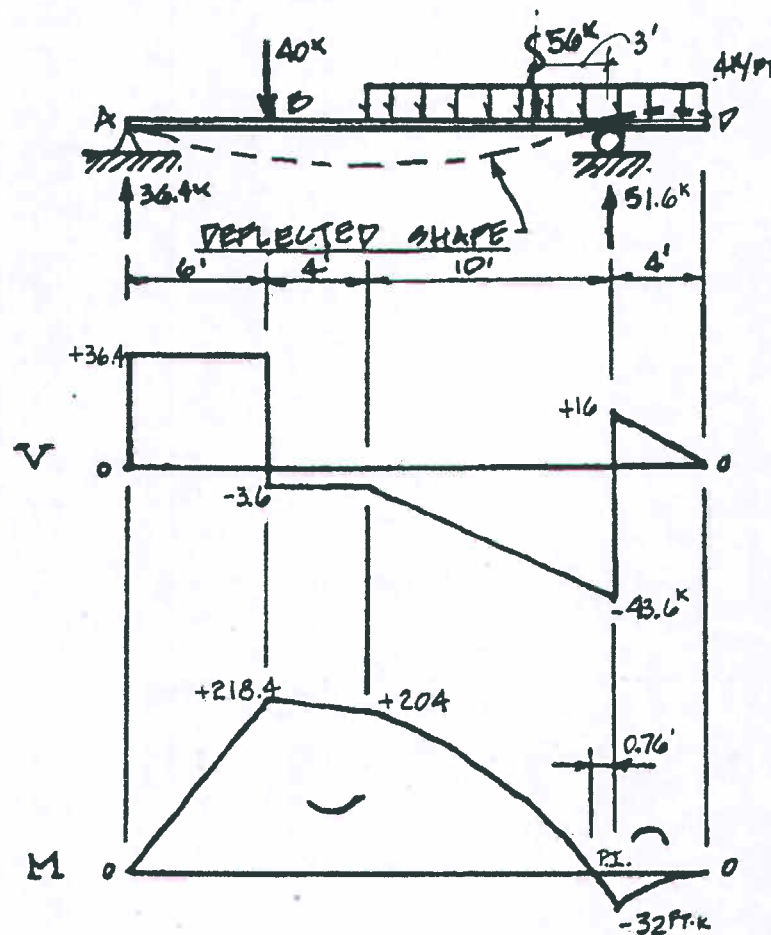
$$M = -32 + 26(5) - \frac{3}{2}(5)^2$$

$$M = 60.5 \text{ kip}\cdot\text{ft}$$

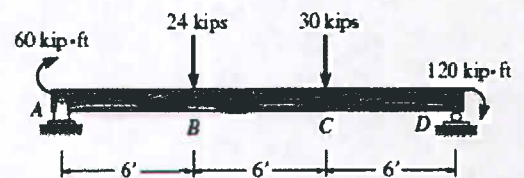
P5.13. For each beam, draw the shear and moment diagrams, label the maximum values of shear and moment, **locate** points of inflection, and make an accurate sketch of the deflected shape.



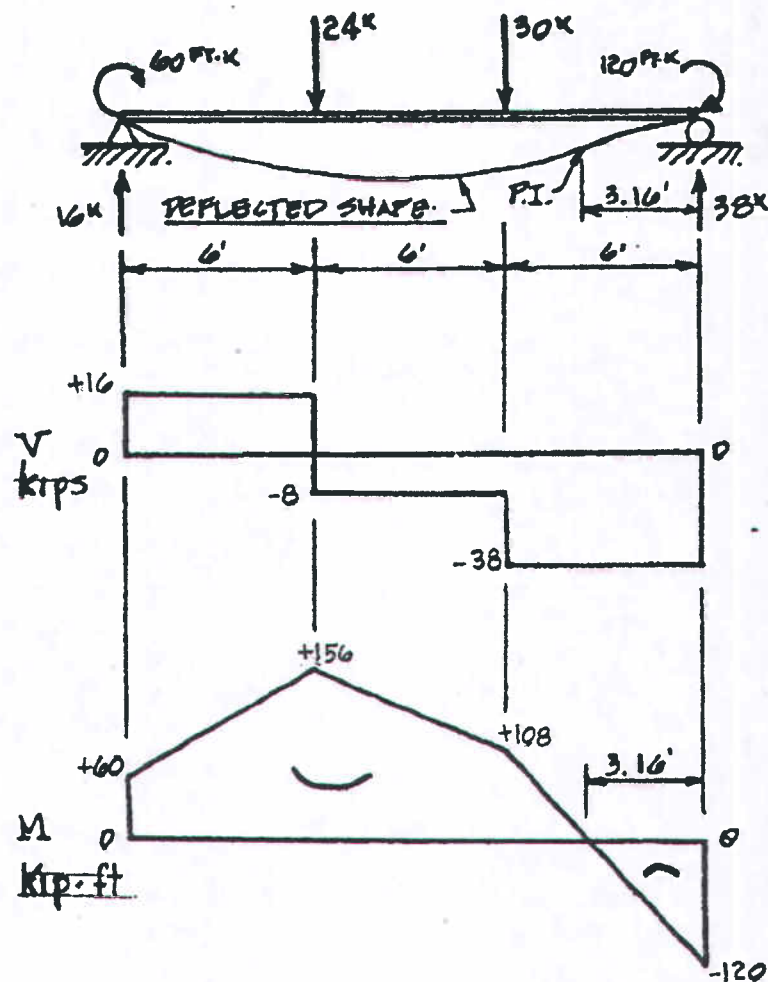
P5.13



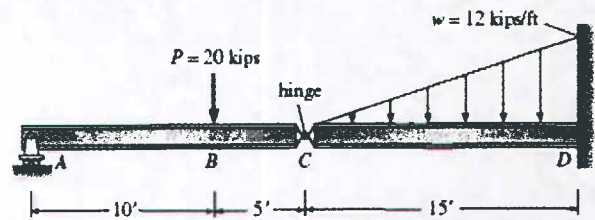
P5.14. For each beam, draw the shear and moment diagrams, label the maximum values of shear and moment, locate points of inflection, and make an accurate sketch of the deflected shape.



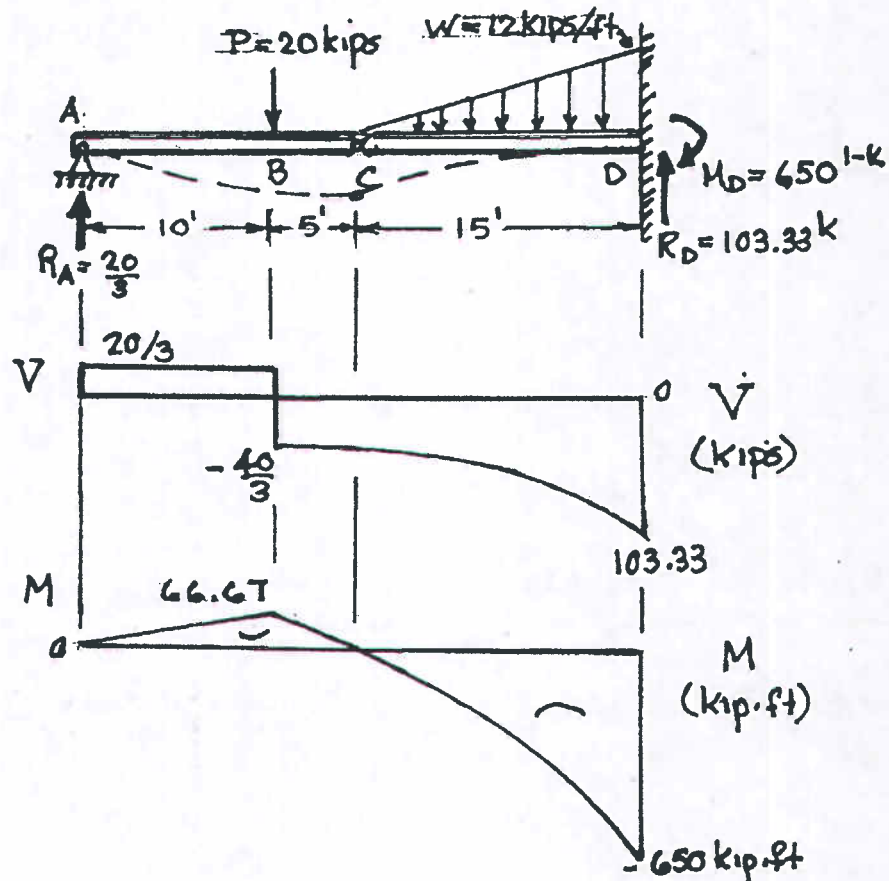
P5.14



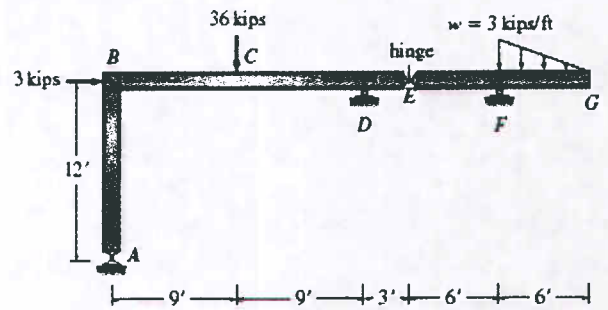
P5.15. For each beam, draw the shear and moment diagrams, label the maximum values of shear and moment, locate points of inflection, and make an accurate sketch of the deflected shape.



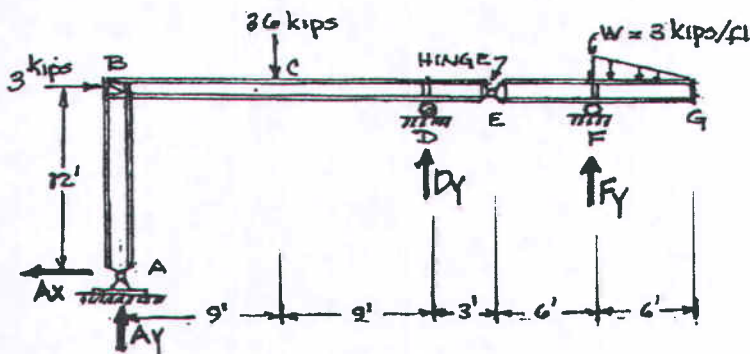
P5.15



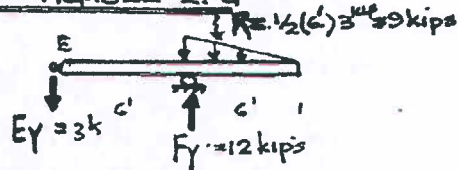
P5.16. Draw the shear and moment diagrams for all members of the frame in Figure P5.16. Sketch the deflected shape.



P5.16

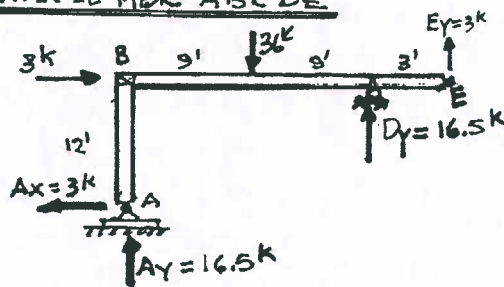


ANALYZE MEMBER EFG

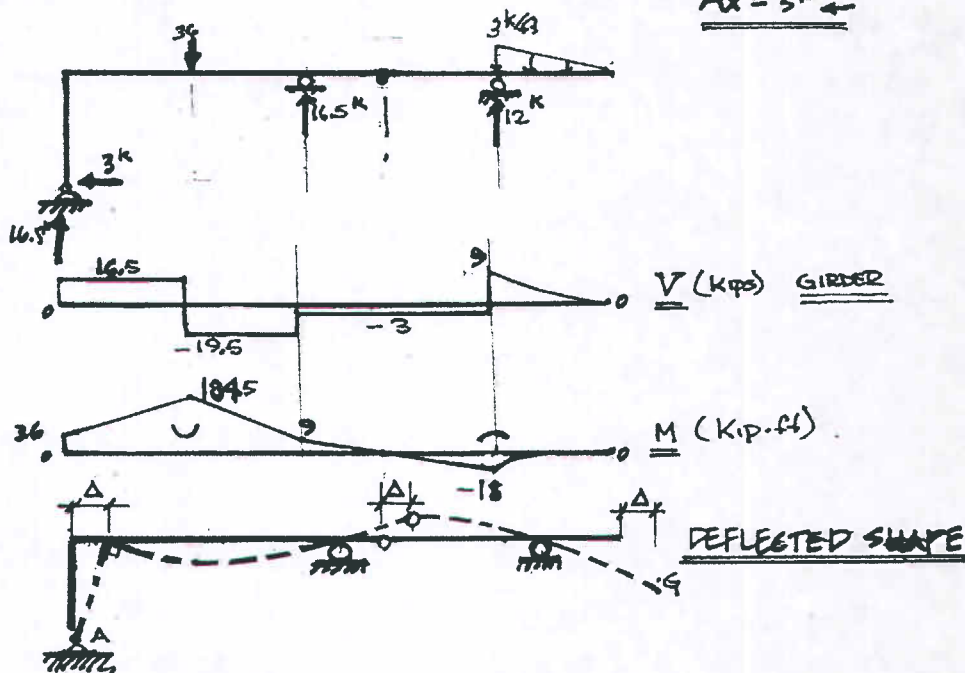
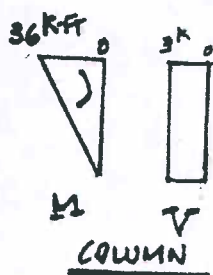


$$\begin{aligned} \sum M_E &= 0; \quad 9'(8) - F_y(6) = 0 \\ F_y &= 12 \text{ kips } \uparrow \\ \uparrow \sum F_y &= 0; \quad -9k + 12k - E_y = 0 \\ E_y &= 3k \downarrow \end{aligned}$$

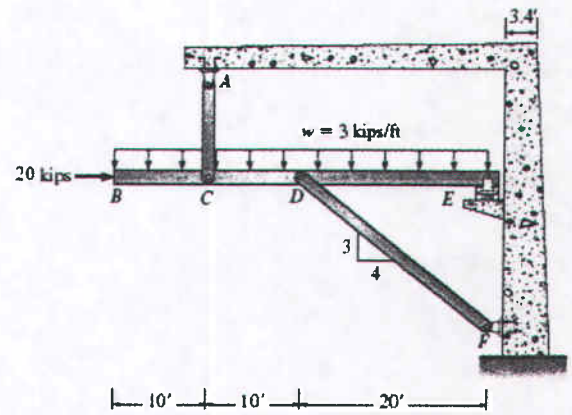
ANALYZE MBR ABCDE



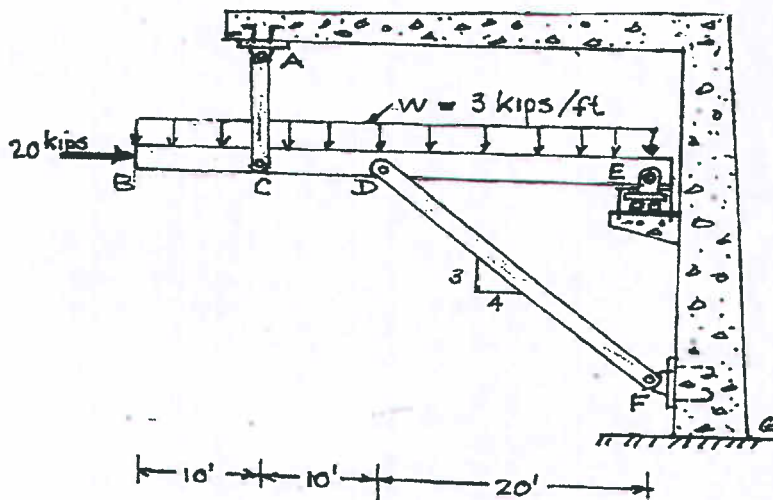
$$\begin{aligned} \sum M_A &= 0; \quad 3'(12') + 36'(9) - D_y(18) - 3'(21) = 0 \\ D_y &= 16.5 \text{ k } \uparrow \\ \uparrow \sum F_y &= 0; \quad A_y - 36k + 16.5k + 3k = 0 \\ A_y &= 16.5 \text{ kips } \uparrow \\ \rightarrow \sum F_x &= 0; \quad 3k - A_x = 0 \\ A_x &= 3k \leftarrow \end{aligned}$$



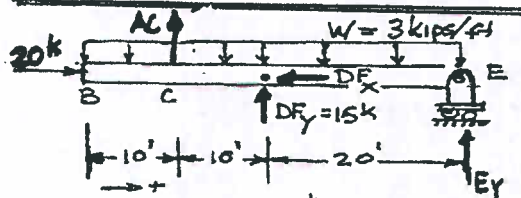
P5.17. Draw the shear and moment diagrams for girder BCDE and sketch its deflected shape. The support at E may be treated as a roller, and connections at joints A, C, D, and F may be treated as frictionless pins.



P5.17



ANALYZE MEMBER BCDE



$$\sum F_x = 0; 20^k - D_F^x = 0$$

$$D_F^x = 20 \text{ kips} \leftarrow$$

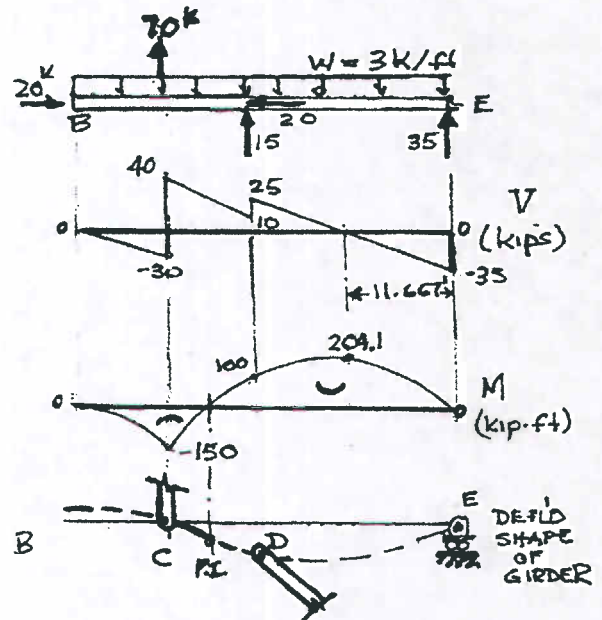
$$D_F^y = 15 \text{ kips} \uparrow$$

$$\sum M_E = 0; 30'AC - 120^k(20') + D_F^y(20') = 0$$

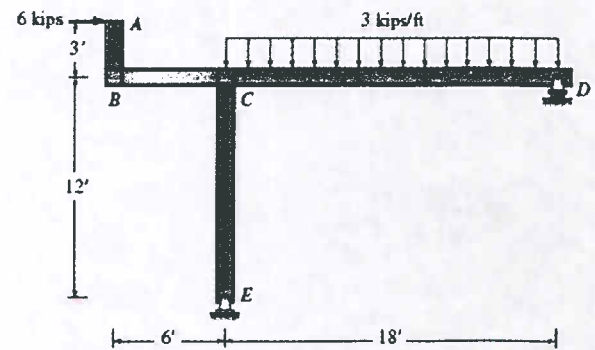
$$AC = 70 \text{ kips} \uparrow$$

$$\sum F_y = 0; 70^k + 15^k - 3 \left(\frac{40'}{4} \right) + E_y = 0$$

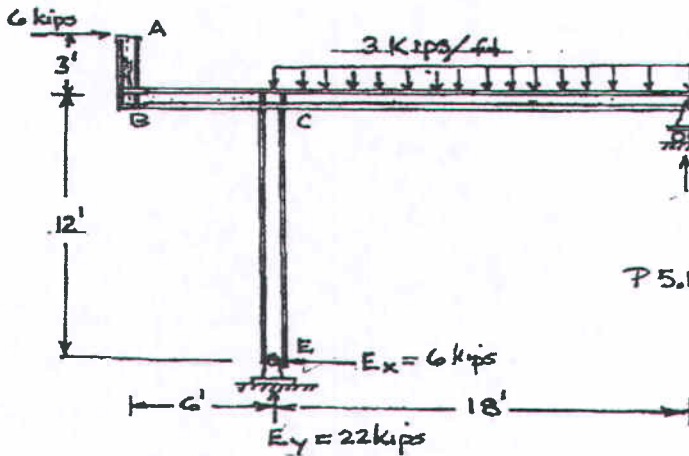
$$E_y = 35 \text{ kips} \uparrow$$



P5.18. Draw the shear and moment curves for each member of the frame in Figure P5.18. Sketch the deflected shape.



P5.18



P5.18

COMPUTE REACTIONS:

$$\sum M_E = 0;$$

$$6k(15') + 3k/ft(18')(9') - D_y(18') = 0$$

$$D_y = 32k \uparrow$$

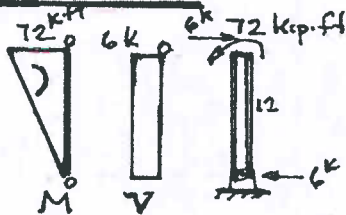
$$+\uparrow \sum F_y = 0;$$

$$-3k/ft(18') + 32k + E_y = 0$$

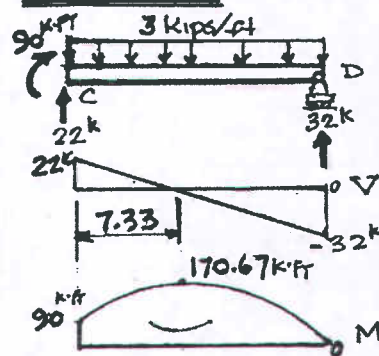
$$E_y = 22k \uparrow$$

$$\sum F_x = 0; \quad E_x = 6k \rightarrow$$

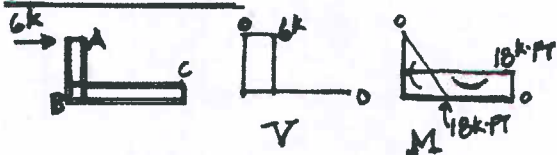
MEMBER "CE"



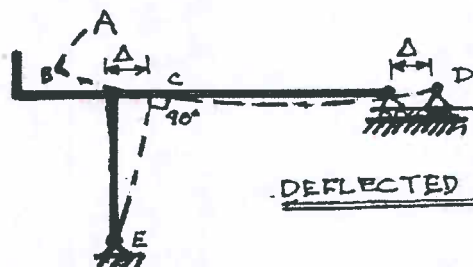
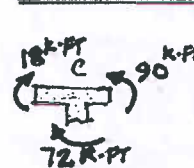
MEMBER "CD"



MEMBER "ABC"

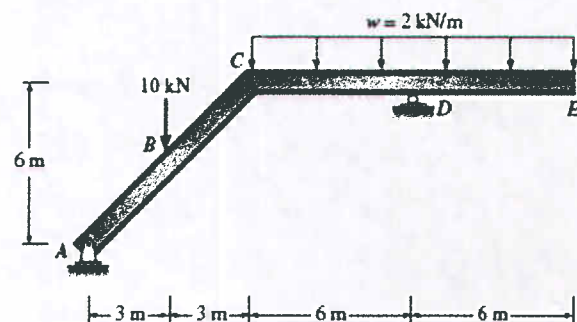


JOINT "C" MOMENTS

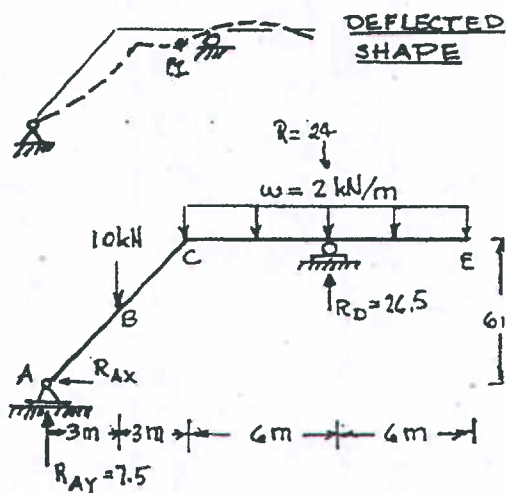


DEFLECTED SHAPE

P5.19. Draw the shear and moment curves for each member of the frame in Figure P5.19. Sketch the deflected shape.



P5.19



REACTIONS

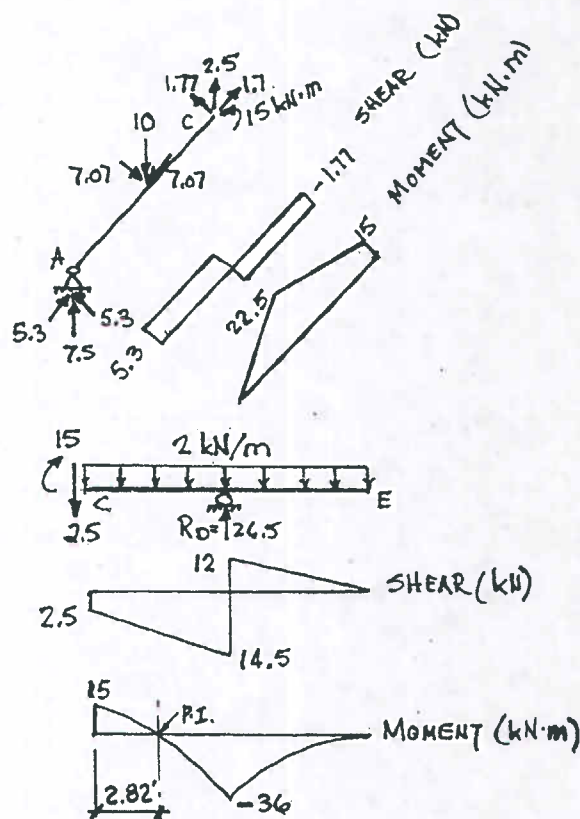
$$\sum F_x = 0: R_{Ax} = 0$$

$$\sum M_A = 0: 10 \times 3 + 24(12) - R_D(12) = 0$$

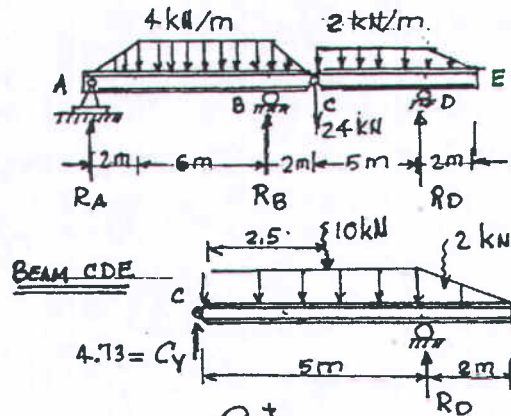
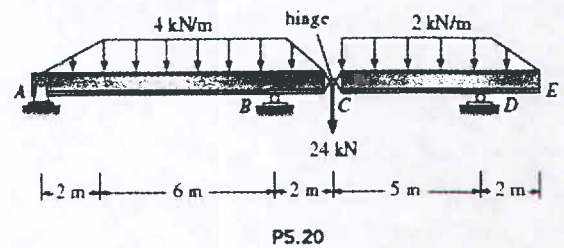
$$R_D = 26.5 \text{ kN} \uparrow$$

$$\sum F_y = 0: R_{Ay} - 34 + 26.5 = 0$$

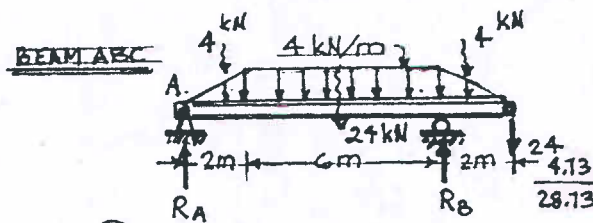
$$R_{Ay} = 7.5 \text{ kN} \uparrow$$



P5.20. Draw the shear and moment curves for the beam in Figure P5.20. Sketch the deflected shape.

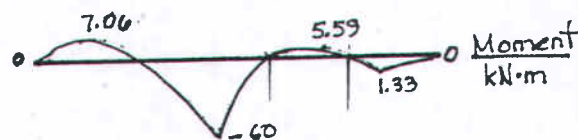
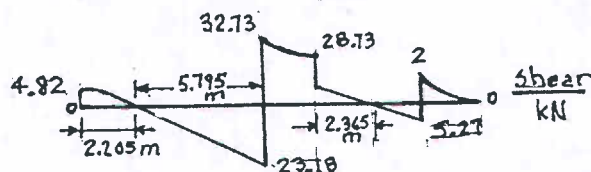
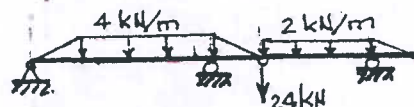


$$\begin{aligned}\sum M_C = 0 &= 10 \times 2.5 + 2 \times 5.67 - 5R_D \\ R_D &= 7.268 \text{ kN} \uparrow \\ \sum F_y = 0 &= C_y + R_D - 12 \\ C_y &= 12 - R_D = 4.73 \text{ kN} \uparrow\end{aligned}$$

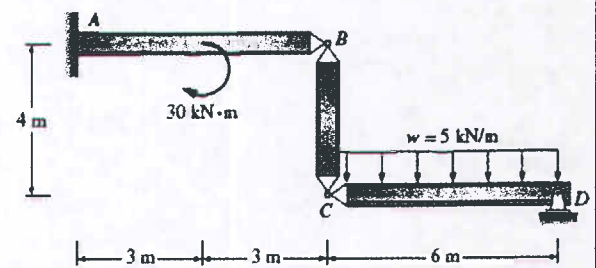


$$\begin{aligned}\sum M_A = 0 &= 4 \times \frac{1}{2} + 24 \times 5 + 8.67(4) + 28.73(10) - R_B \times 8 \\ R_B &= 55.91 \text{ kN}\end{aligned}$$

$$\sum F_y = 0 = R_A - 4 - 24 - 4 - 28.73 ; R_A = 4.82 \text{ kN}$$

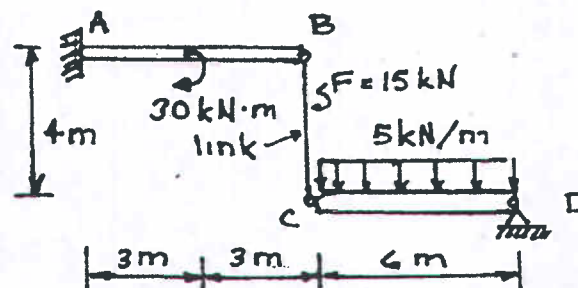


P5.21. Draw the shear and moment curves for each member of the frame in Figure P5.21. Sketch the deflected shape hinges at B and C .

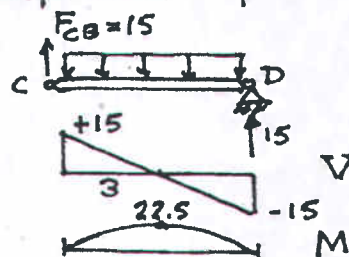


P5.21

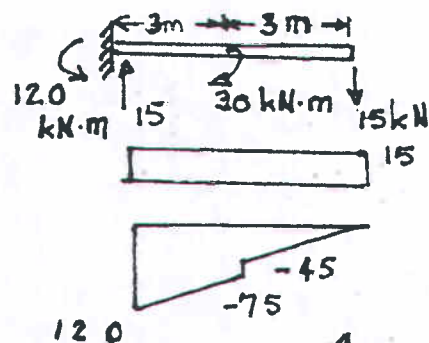
Link BC CARRIES AXIAL LOAD ONLY.



MEMBER CD



MEMBER AB



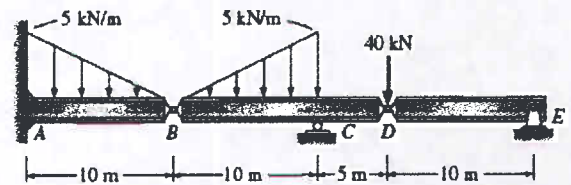
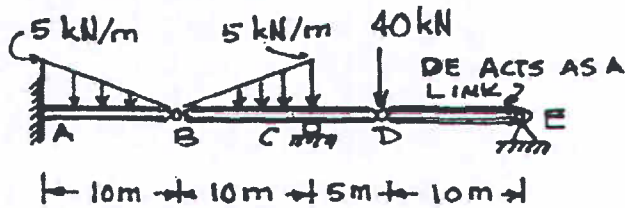
V (kN)

M (kN·m)

DEFLECTED
SHAPE



P5.22. Draw the shear and moment curves for each member of the frame in Figure P5.22. Sketch the deflected shape. Fixed end at A.

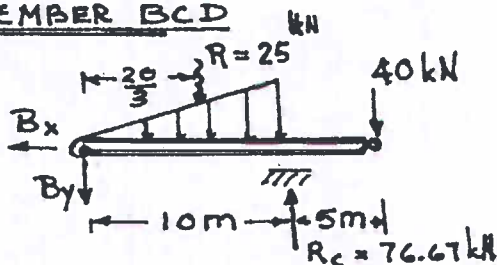


P5.22

MEMBER DE

ALL FORCES = ZERO
SINCE NO LOADS APPLIED

MEMBER BCD



REACTIONS

$$\sum M_B = 0: 25 \times \frac{20}{3} + 40(15) - 10R_C = 0$$

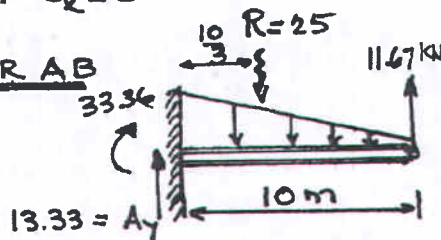
$$R_C = 76.67 \text{ kN}$$

$$\sum F_y = 0: -B_y - 25 - 40 + 76.67 = 0$$

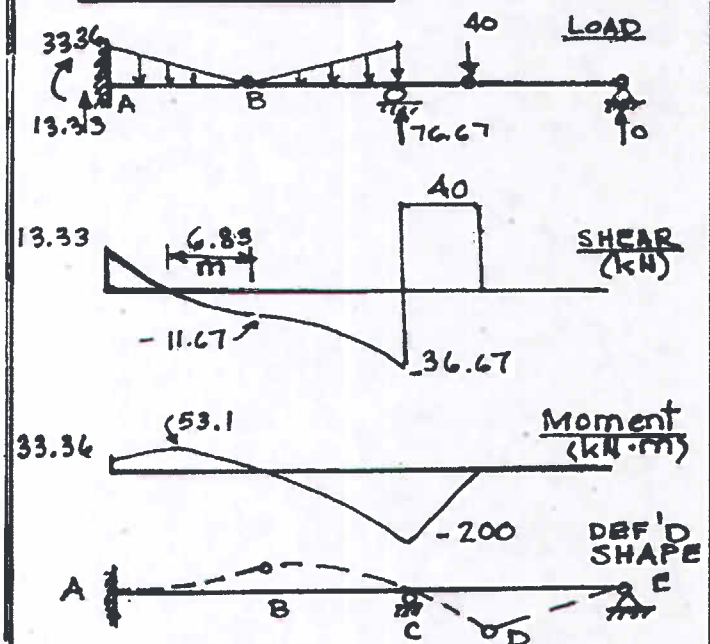
$$B_y = 11.67 \text{ kN} \downarrow$$

$$\sum F_x = 0: B_x = 0$$

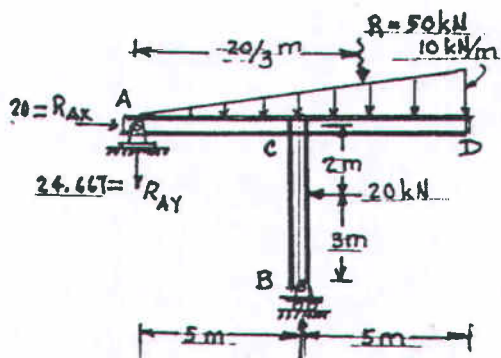
MEMBER AB



ENTIRE STRUCTURE



P5.23. Draw the shear and moment curves for each member of the frame in Figure P5.23. Sketch the deflected shape.

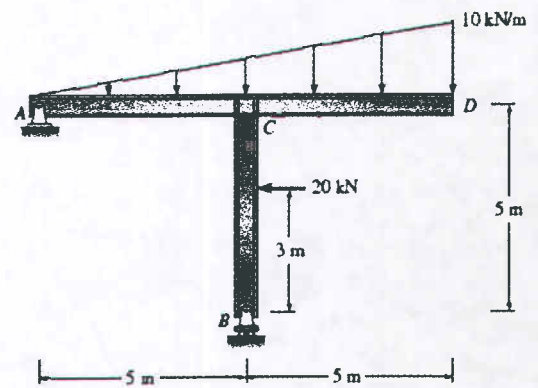
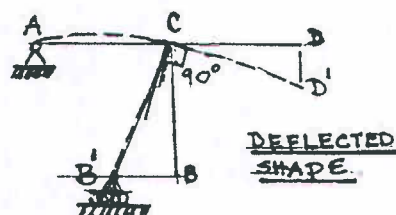
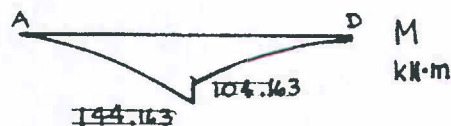
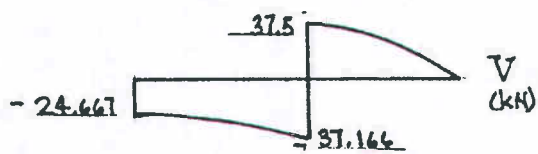
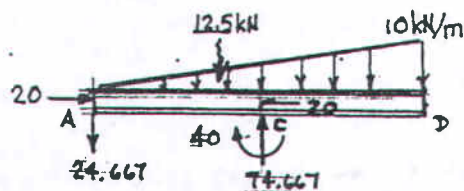
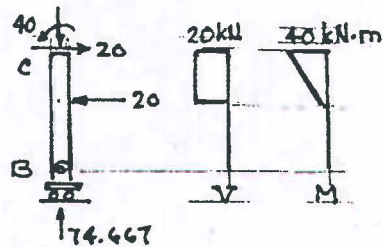


$$\sum M_A = 0; 50 \times \frac{20}{3} + 20 \times 2 - 5R_B = 0$$

$$R_B = 74.667 \text{ kN}$$

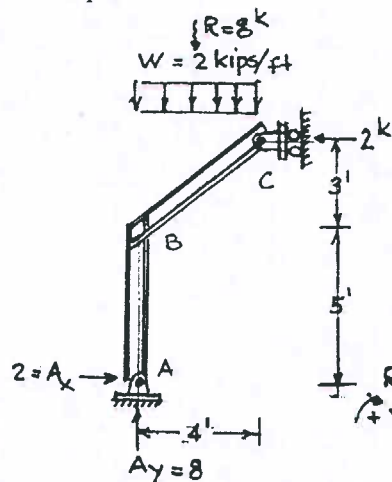
$$\sum F_y = 0; -50 + 74.667 - R_{Ay} = 0$$

$$R_{Ay} = 24.667$$



P5.23

P5.24. Draw the shear and moment curves for each member of the frame in Figure P5.24. Sketch the deflected shape.



REACTIONS:

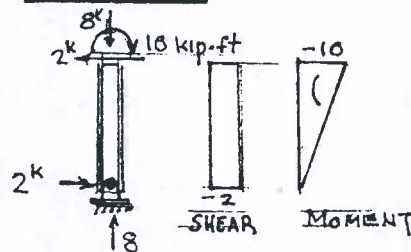
$$\sum M_A = 0 = -C_x \cdot 8 + 8 \times 2$$

$$C_x = 2 \text{ kips}$$

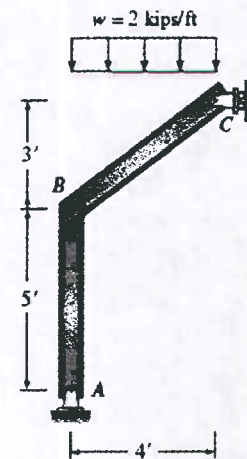
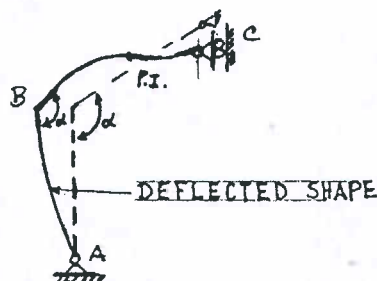
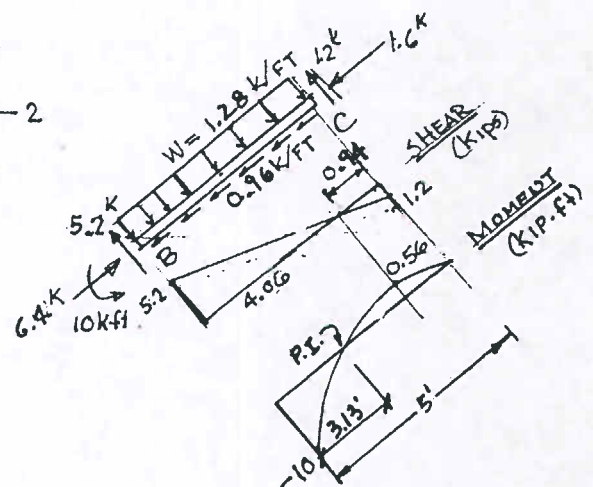
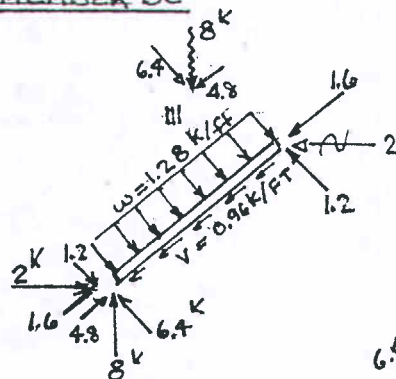
$$\sum F_x = 0 = A_x - 2 \therefore A_x = 2 \text{ kips}$$

$$\sum F_y = 0 \quad A_y = 8 \text{ kips}$$

MEMBER AB

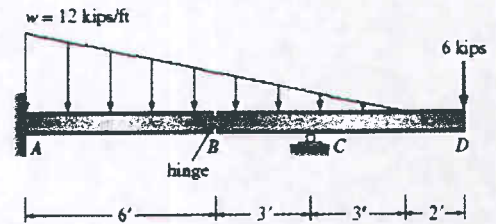


MEMBER BC

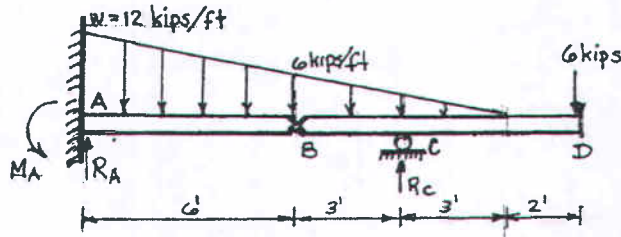


P5.24

P5.25. Draw the shear and moment curves for each member of the beam in Figure P5.25. Sketch the deflected shape. The shear connection at B acts as a hinge.



P5.25



BCD

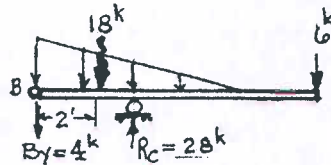
$$\sum M_B = 0;$$

$$0 = 18 \times 2 + 4 \times 8 - R_C \times 3$$

$$R_C = 28 \text{ kips}$$

$$\sum F_y = 0 = -18 - B_y + 28 - 4$$

$$B_y = 4 \text{ k}$$



AB

$$\sum F_y = 0;$$

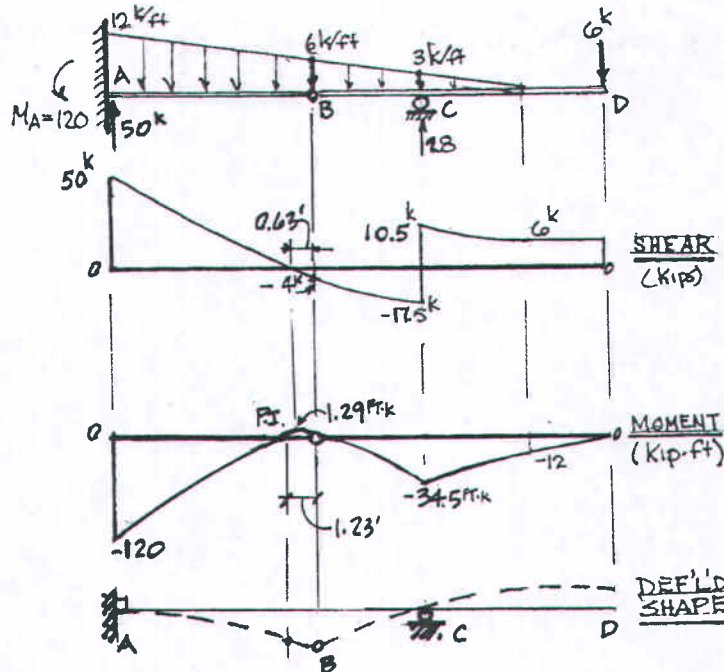
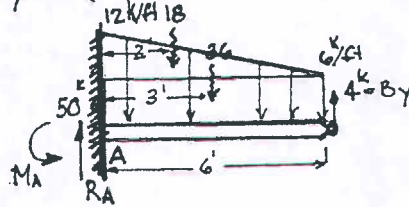
$$0 = -18 - 36 + 4 + R_A$$

$$R_A = 50 \text{ kips}$$

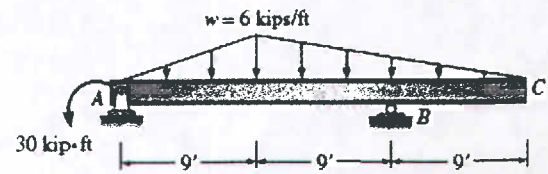
$$\sum M_A = 0;$$

$$0 = 18 \times 2 + 36 \times 3 - 4 \times 6 - M_A$$

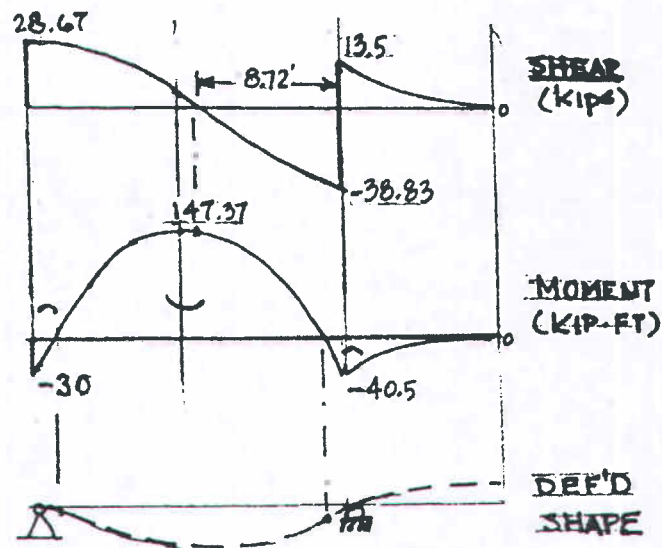
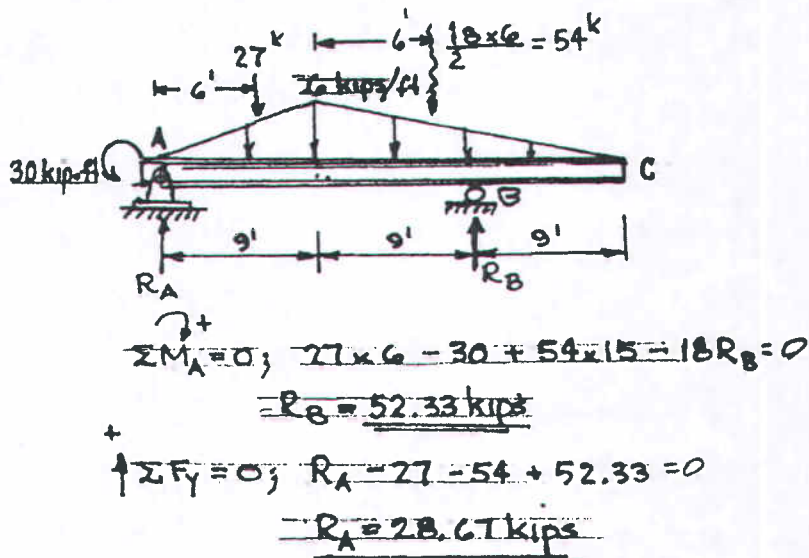
$$M_A = 120 \text{ kip}\cdot\text{ft}$$



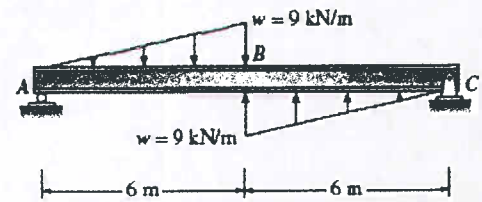
P5.26. Draw the shear and moment curves for the beam in Figure P5.26. Sketch the deflected shape.



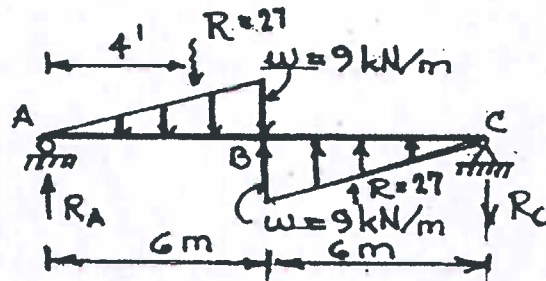
P5.26



P5.27. Draw the shear and moment curves for the beam in Figure P5.27. Sketch the deflected shape.



P5.27



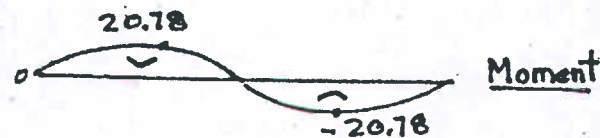
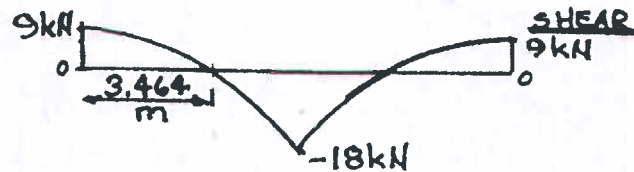
REACTIONS

RESULTANT OF distrib loads each side of $\oint$

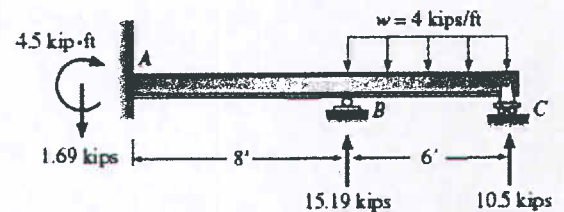
$$\curvearrowleft + R = \frac{1}{2} 6 \times 9 = 27 \text{ kN}$$

$$\sum M_A = 0: 27 \times 4 = 27(8) + 12 R_C = 0$$

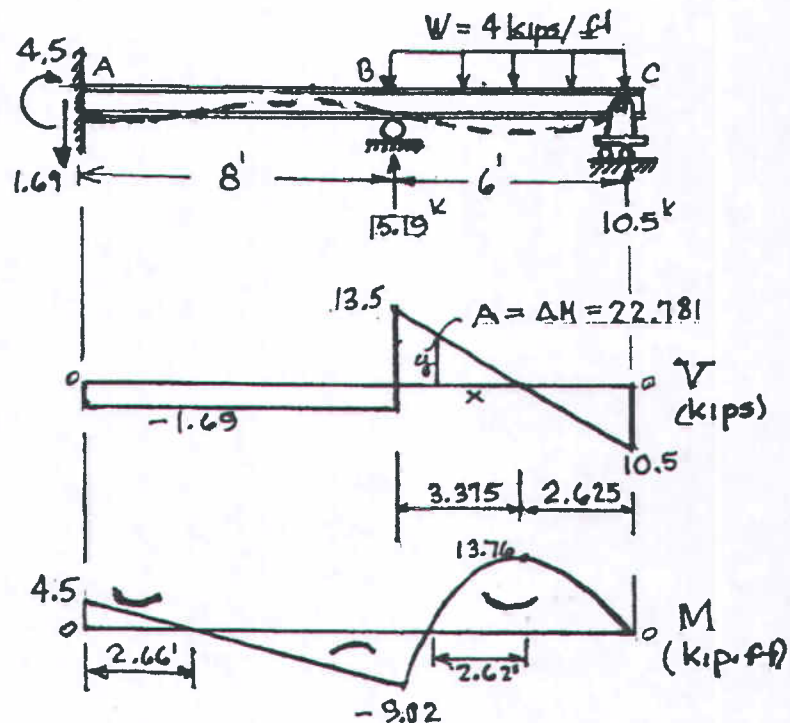
$$R_C = 9 \text{ kN} \uparrow, R_A = 9 \text{ kN} \uparrow$$



P5.28. Draw the shear and moment curves for the indeterminate beam in Figure P5.28. Reactions are given. Sketch the deflected shape.



P5.28



$$\Delta M_{AB} = -1.69 \times 8 = -13.52 \text{ kip}\cdot\text{ft}$$

$$4.5 - 13.52 = -9.02 \text{ kip}\cdot\text{ft}$$

LOCATE POINT OF INFLECTION

$$\frac{13.5}{y} = \frac{3.375}{x}$$

$$4x = y$$

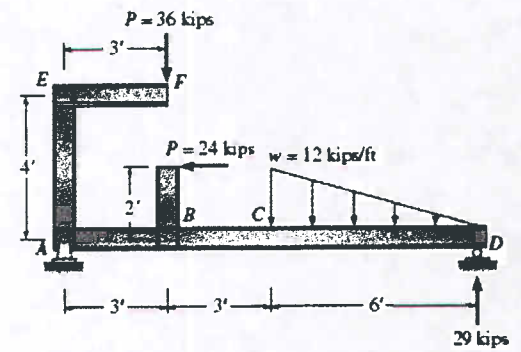
$\Delta M = \text{AREA } V\text{-DIAGRAM}$

$$13.76 = \frac{1}{2} y x = \frac{1}{2} (4x) x$$

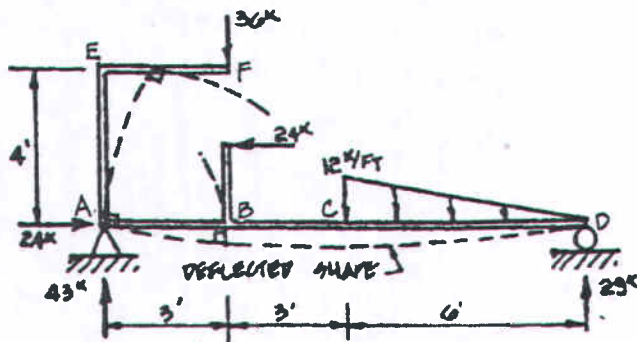
$$x^2 = 13.76/2 = 6.88$$

$$\underline{\underline{x = 2.62'}}$$

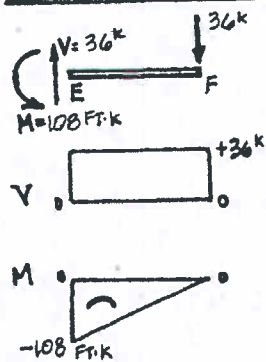
P5.29. Draw the shear and moment curves for the beam in Figure P5.29. Sketch the deflected shape.



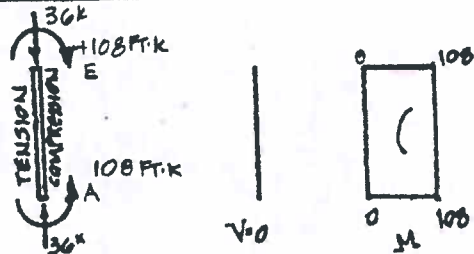
P5.29



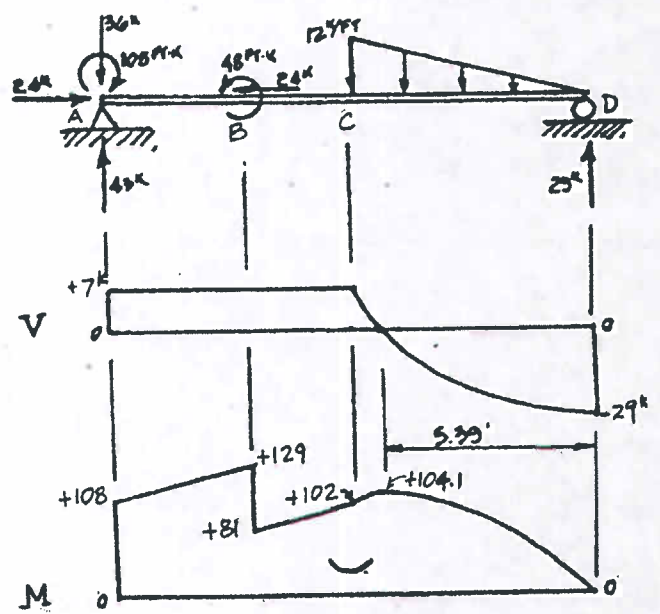
ISOLATE CANTILEVER EF



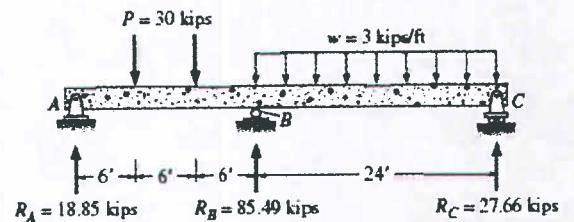
ISOLATE AE



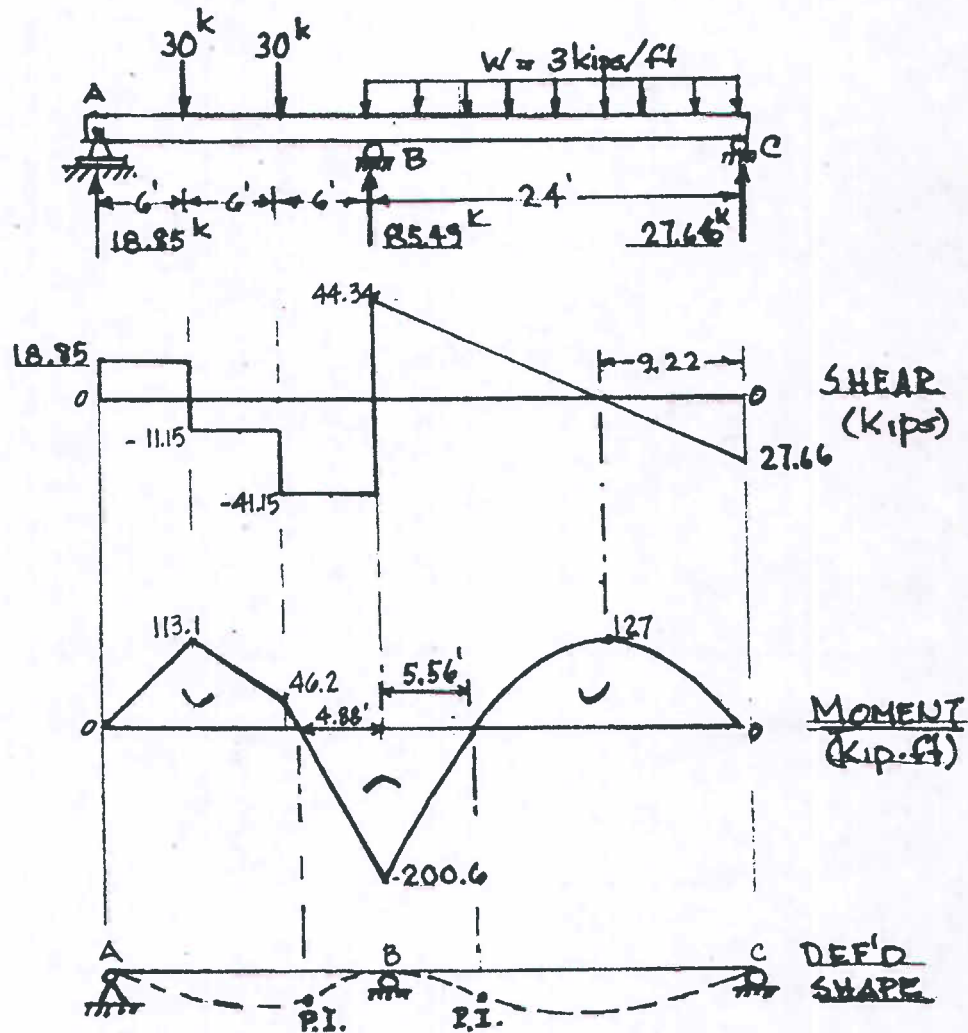
ENTIRE STRUCTURE



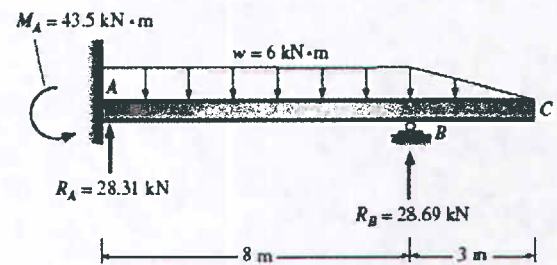
P5.30. Draw the shear and moment curves for the beam in Figure P5.30 (reactions given). Locate all points of zero shear and moment. Sketch the deflected shape.



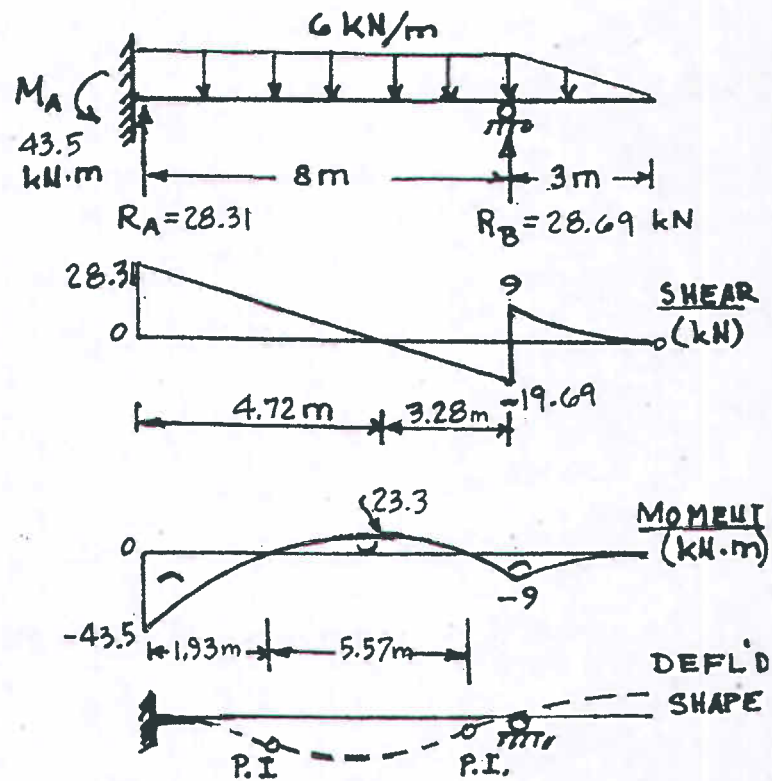
P5.30



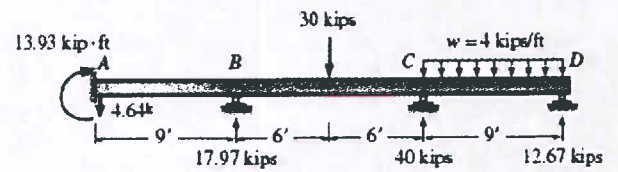
P5.31. Draw the shear and moment diagrams for each indeterminate beam. Reactions are given. Label maximum values of shear and moment. Locate all inflection points, and sketch the deflected shape.



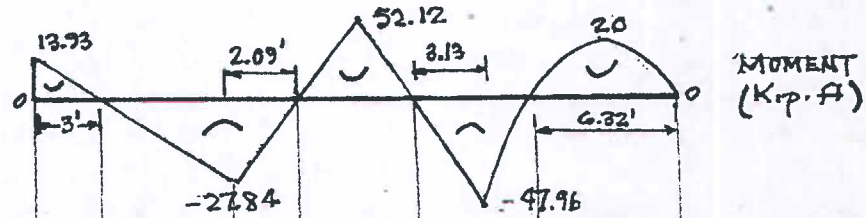
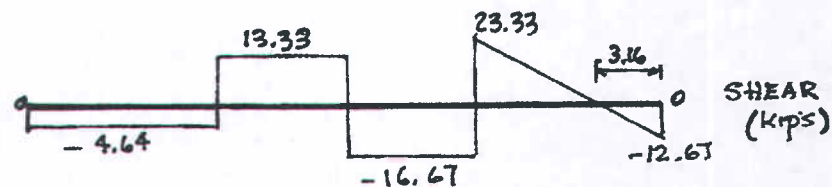
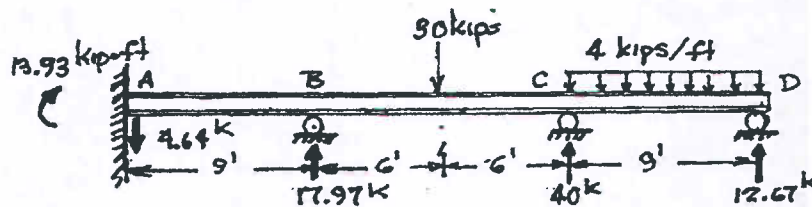
P5.31



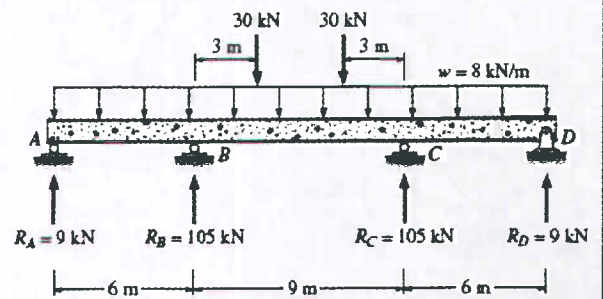
P5.32. Draw the shear and moment diagrams for each indeterminate beam. Reactions are given. Label maximum values of shear and moment. Locate all inflection points, and sketch the deflected shape.



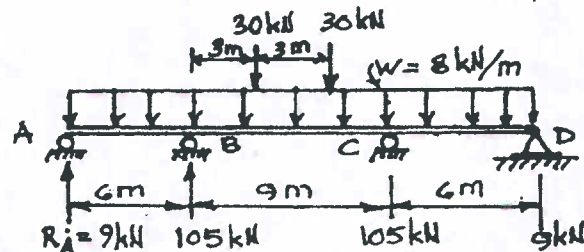
P5.32



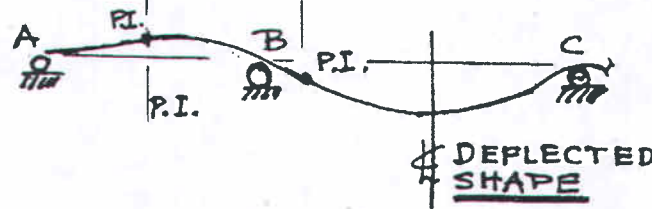
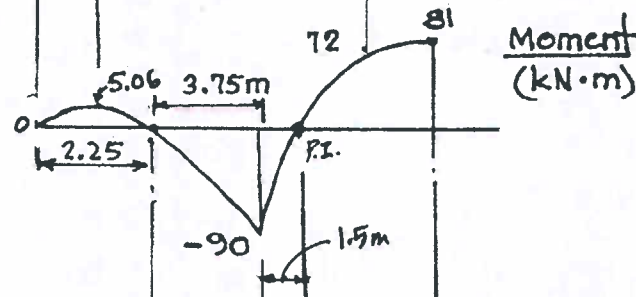
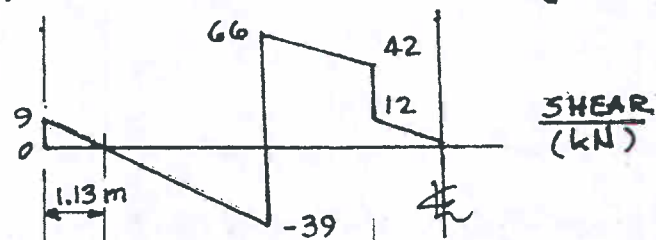
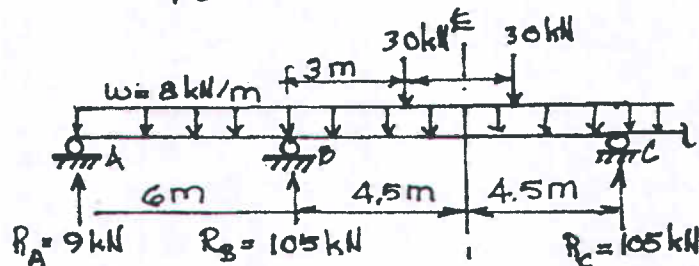
P5.33. Draw the shear and moment diagrams for Figure P5.33. Sketch the deflected shape.



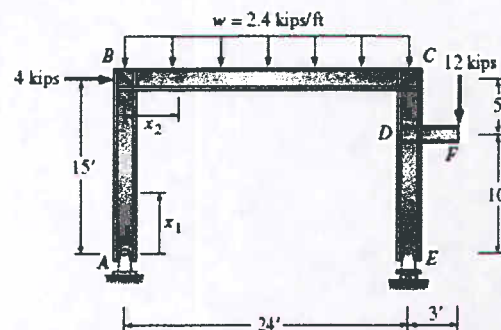
P5.33



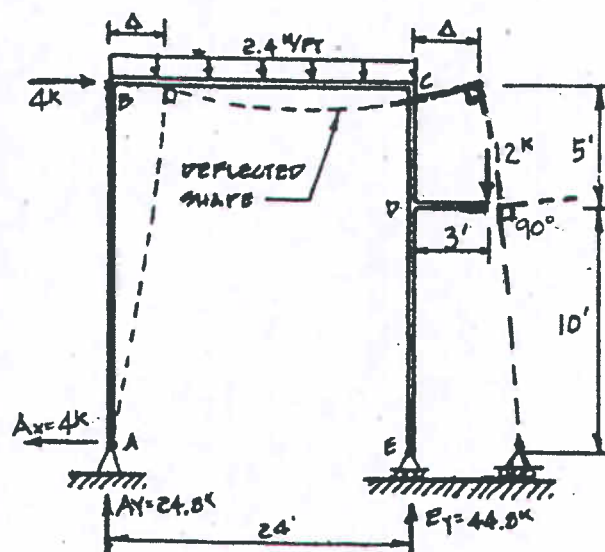
Since structure symmetrical
Analyse Left half.



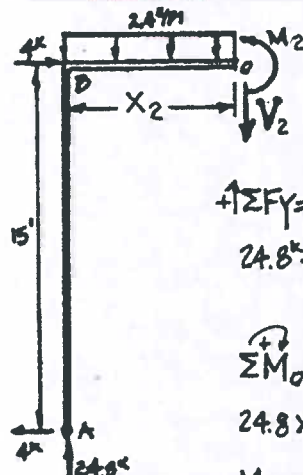
P5.34. (a) Draw the shear and moment curves for the frame in Figure P5.34. Sketch the deflected shape. (b) Write the equations for shear and moment in column AB. Take the origin at A. (c) Write the shear and moment equations for girder BC. Take the origin at joint B.



P5.34



SEGMENT ABC; ORIGIN @ B; RANGE $0 \leq x_2 \leq 24$



$$+\uparrow \Sigma F_y = 0;$$

$$24.8 - 2.4x_2 - V_2 = 0$$

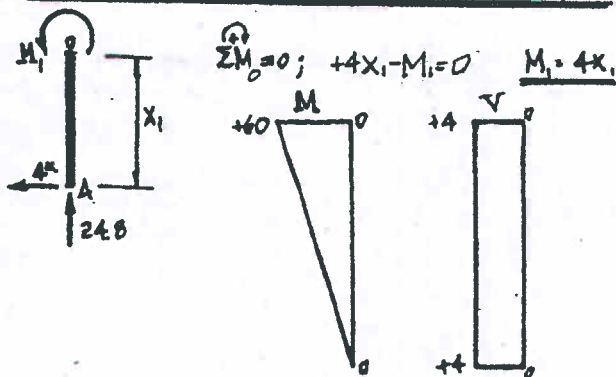
$$V_2 = 24.8 - 2.4x_2$$

$$+\circlearrowleft \Sigma M_B = 0;$$

$$24.8x_2 + 4(15) - 2.4x_2\left(\frac{x_2}{2}\right) - M_2 = 0$$

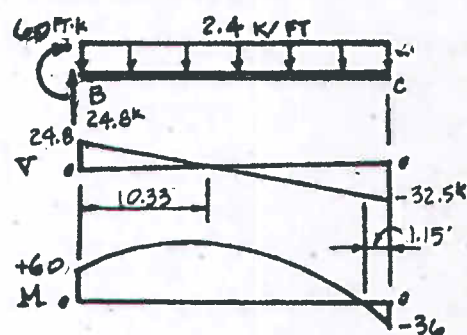
$$M_2 = 60 + 24.8x_2 - 1.2x_2^2$$

SEGMENT AB; ORIGIN @ A; RANGE $0 \leq x_1 \leq 15$

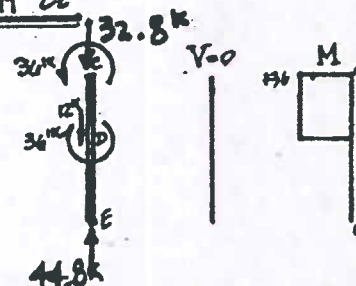


$$+\circlearrowleft \Sigma M_A = 0; +4x_1 - M_1 = 0 \quad M_1 = 4x_1$$

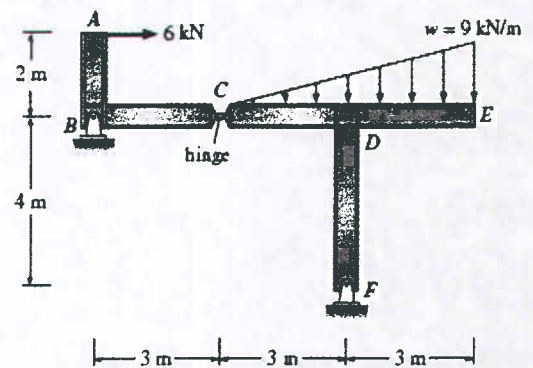
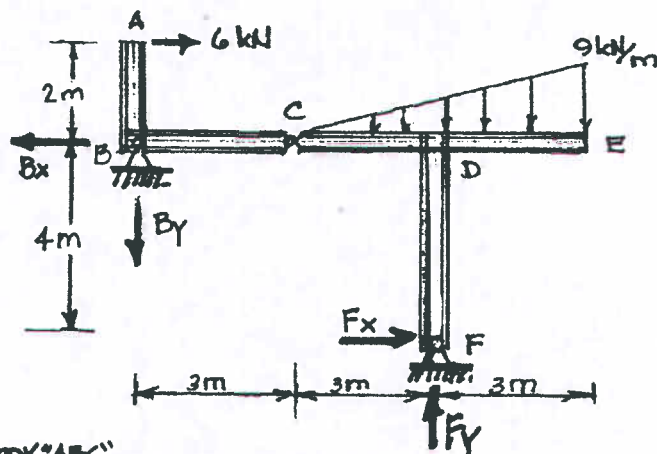
SEGMENT BC



SEGMENT CE

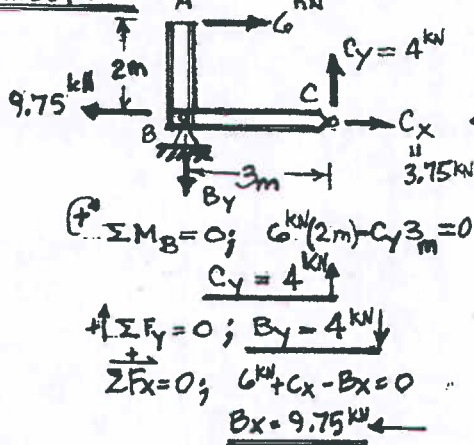


P5.35. Draw the shear and moment curves for each member of the frame in Figure P5.35. Sketch the deflected shape. Joints B and D are rigid.

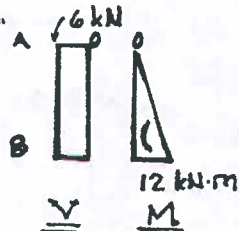


P5.35

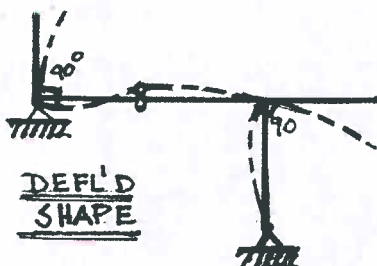
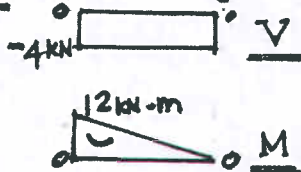
FREEBODY "ABC"



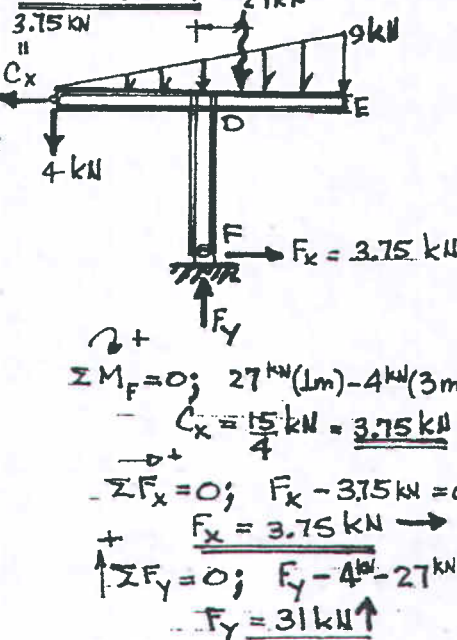
MBR "AB"



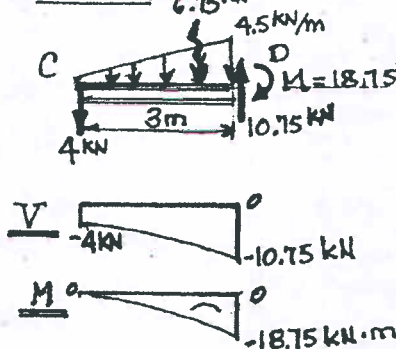
MBR "BC"



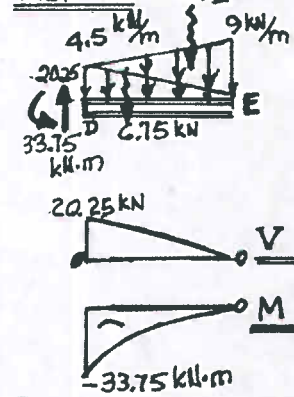
FREEBODY "CDEF"



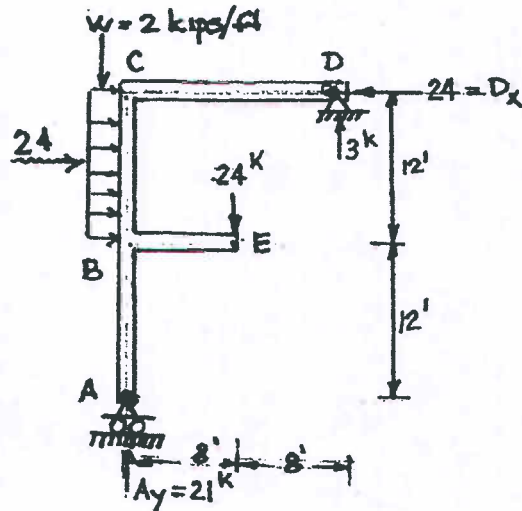
MBR "CD"



MBR "DE"



P5.36. Draw the **moment** diagrams for each member of the frame in Figure P5.36. Sketch the deflected shape of the frame. Joints B and C are rigid.

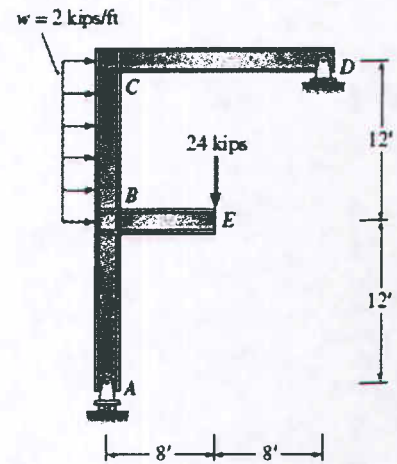
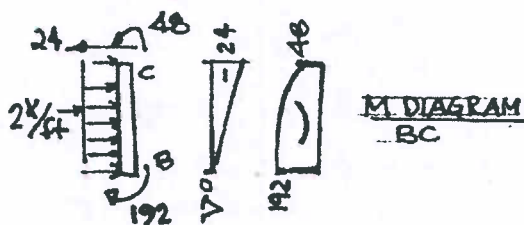
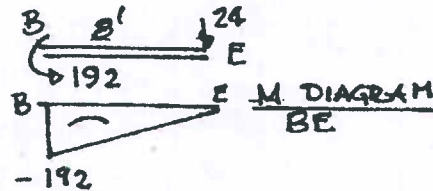
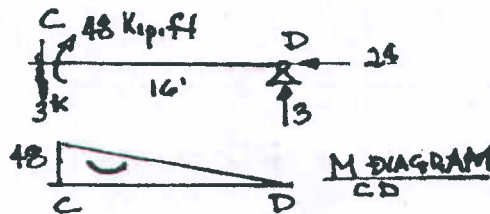


$$\sum M_D = 0; \quad +A_y 16 - 24 \times 8 - 24 \times 6 = 0$$

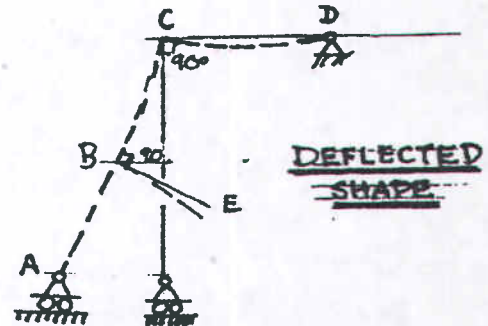
$$\rightarrow A_y = \underline{21 \text{ kips}}$$

$$+\sum F_x = 0 = -24 - D_x \therefore D_x = \underline{24 \text{ kips}}$$

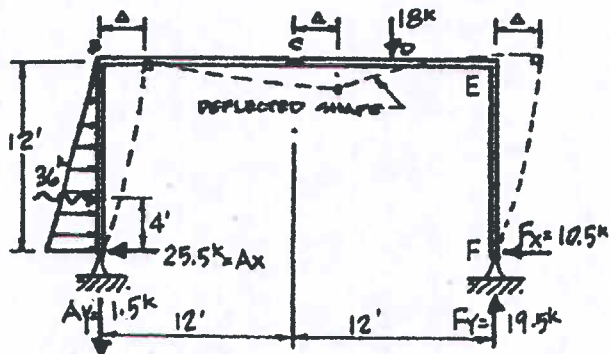
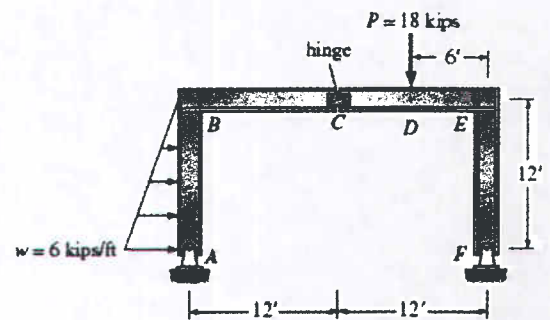
$$+\sum F_y = 0 = -24 + 21 + D_y \therefore D_y = \underline{3 \text{ k}}$$



P5.36



P5.37. Draw the shear and moment curves for each member of the frame in Figure P5.37. Sketch the deflected shape. Treat the shear plate connection at C as a hinge.



ENTIRE STRUCTURE

$$\begin{aligned} \sum M_A = 0; & 36(4') + 18(18') - F_y(24') = 0 & F_y = 19.5k \uparrow \\ + \uparrow \sum F_y = 0; & -A_y - 18k + 19.5k = 0 & A_y = 1.5k \downarrow \end{aligned}$$

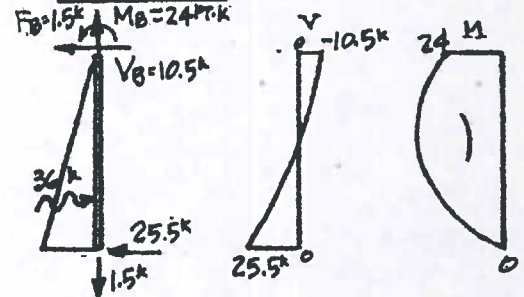
ISOLATE SEGMENT CDE

$$\begin{aligned} \sum M_C = 0; & 18k(6') - 19.5k(12') + F_x(12') = 0 \\ & F_x = 10.5k \leftarrow \end{aligned}$$

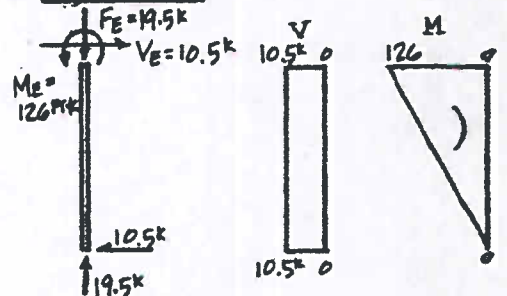
ENTIRE STRUCTURE

$$\begin{aligned} \sum F_x = 0; & 36 - 10.5k - A_x = 0 \\ & A_x = 25.5k \leftarrow \end{aligned}$$

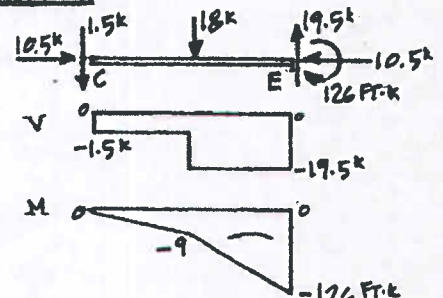
SEGMENT AB



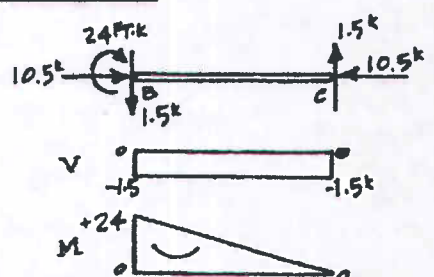
SEGMENT EF



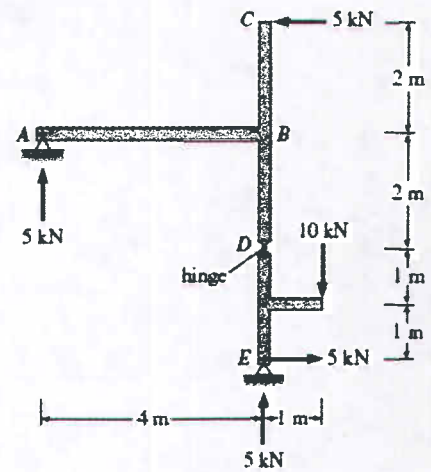
SEGMENT CE



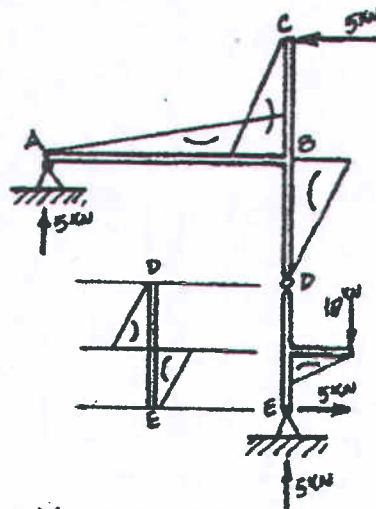
SEGMENT BC



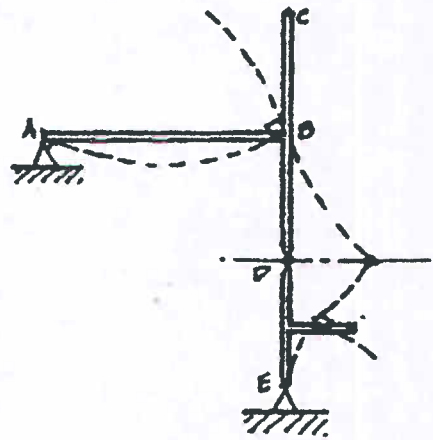
P5.38. (a) Make an accurate sketch of the deflected shape of the frame in Figure P5.38. Pay careful attention to curvature and displacement. Joint B is rigid. (b) Draw a free body of joint B and show all forces.



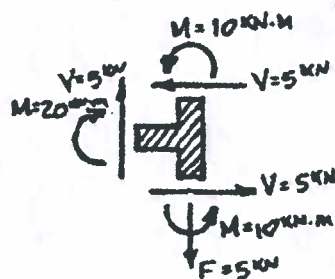
P5.38



M DIAGRAMS

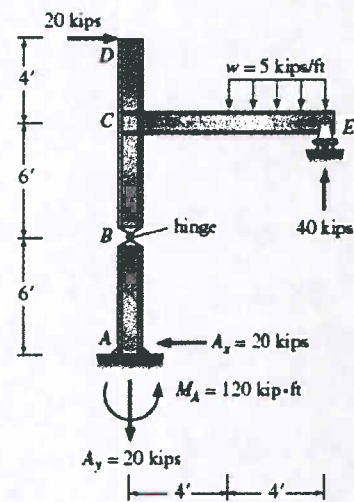


DEFLECTED SHAPE

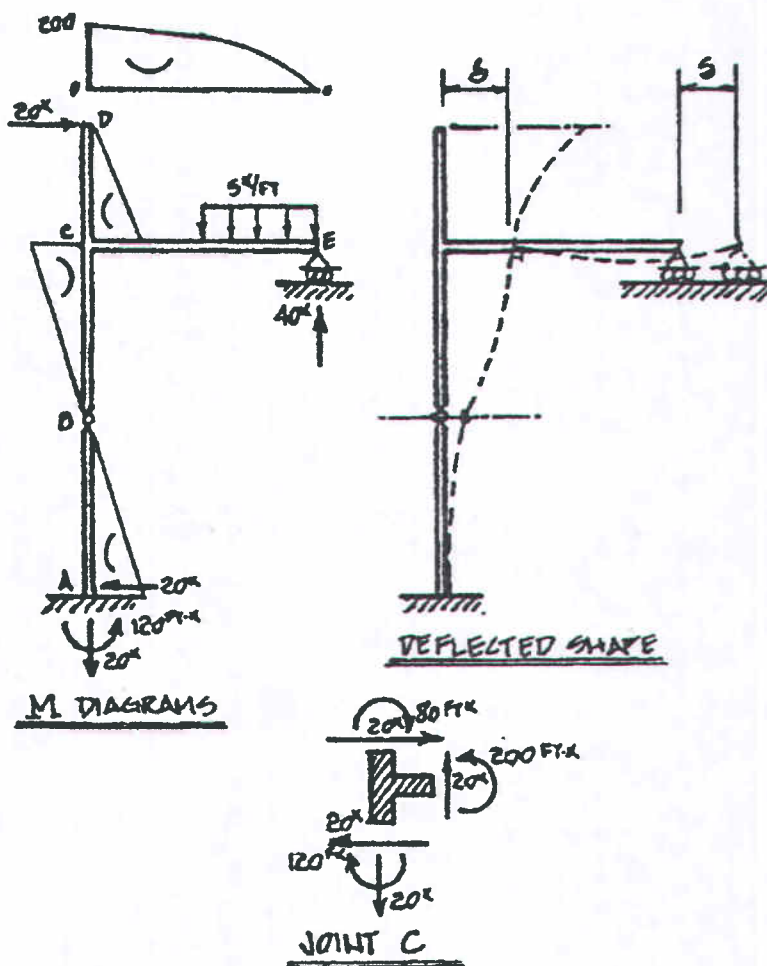


JOINT B

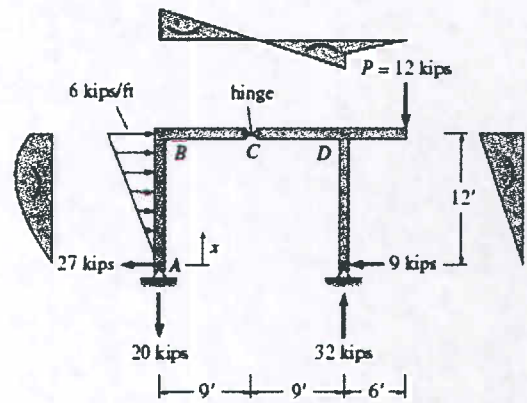
P5.39. For the frame in Figure P5.39, draw the shear and moment curves for all members. Next draw an accurate sketch of the deflected shape of the frame. Show all forces acting on a free-body diagram of joint C. (Joint C is a rigid joint.) Fixed support at A.



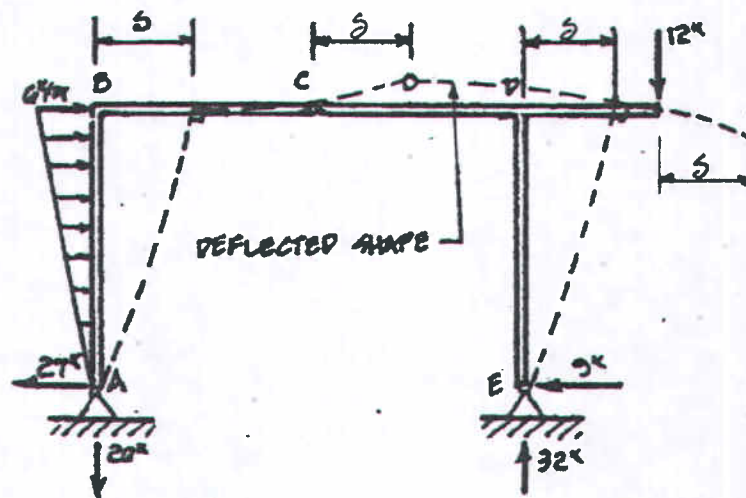
P5.39



P5.40. (a) Sketch the deflected shape of the frame in Figure P5.40 accurately. Reactions and **moment curves** are given. Curvature is also indicated. Joints *B* and *D* are rigid. The hinge is located at point *C*. (b) Using an origin at *A*, write the equations for shear and moment in member *AB* in terms of the applied load and the distance *x*.



P5.40



SEGMENT AB



$$y/x = 6/12 ; y = x/2$$

$$R = y(x/2) = x^2/4$$

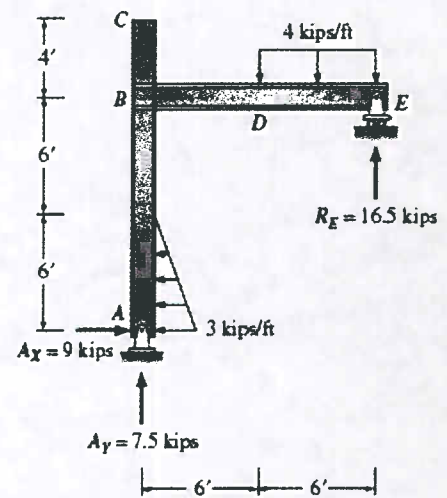
$$\sum F_x = 0, V + x^2/4 - 27$$

$$\underline{V = 27 - x^2/4}$$

$$\sum M_o = 0, 27x - (x^2/2)(x/3) - M = 0$$

$$\underline{M = 27x - x^3/6}$$

P5.41. Draw the shear and moment curves for all members of the frame in Figure P5.41. Sketch the deflected shape (reactions given).

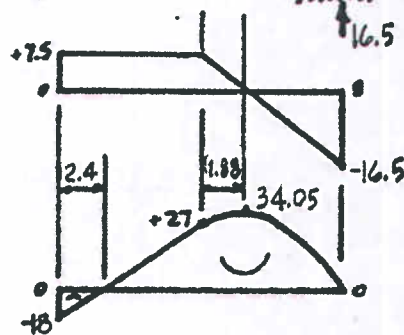


SEGMENT DE



SHEAR
CURVE

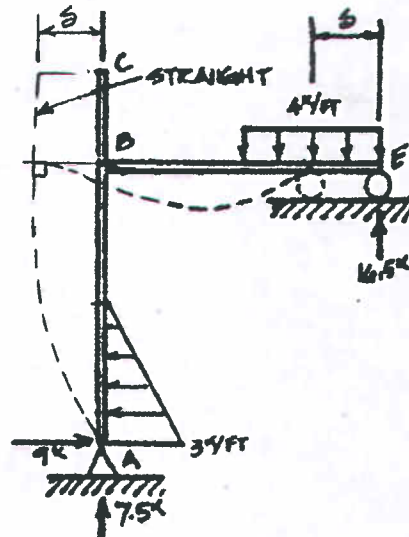
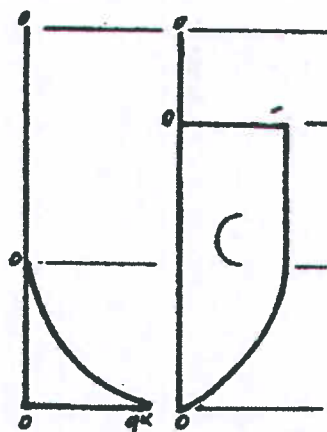
MOMENT
CURVE



SEGMENT ABC

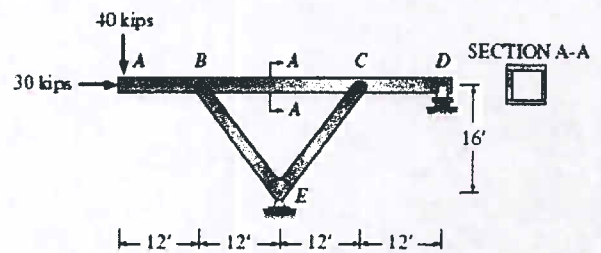
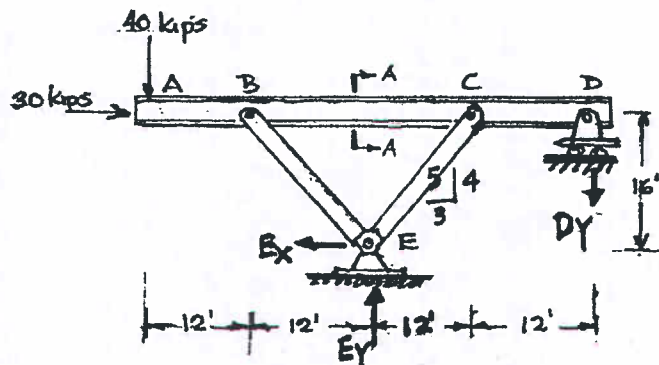
SHEAR
CURVE

MOMENT
CURVE



DEFLECTED SHAPE

P5.42. The hollow box beam $ABCD$ in Figure P5.42 is supported by a roller at point D and two links BE and CE . Compute all reaction, draw the shear and moment curves for the beam, and sketch the deflected shape of the structure.

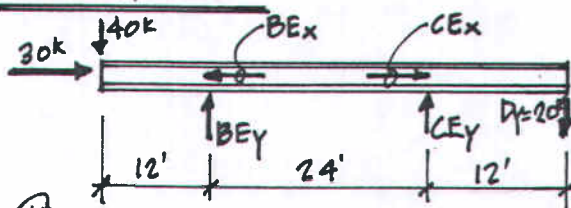


P5.42

ENTIRE STRUCTURE: COMPUTE REACTIONS

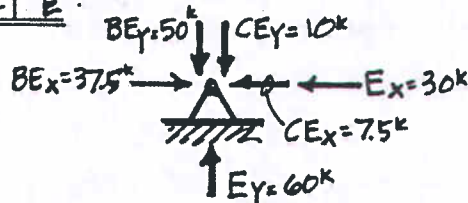
$$\begin{aligned} \sum F_x = 0; \quad 30k - E_x &= 0 & E_x &= 30k \leftarrow \\ \sum M_E = 0; \quad 30k(16') - 40k(24') + D_y(24') &= 0 & D_y &= 20k \downarrow \\ \sum F_y = 0; \quad -40k + E_y - 20k &= 0 & E_y &= 60k \uparrow \end{aligned}$$

MEMBER "ABCD"

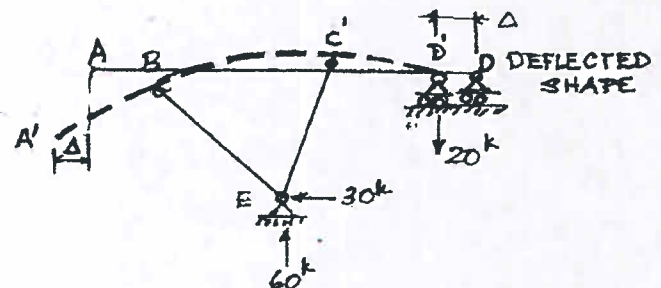
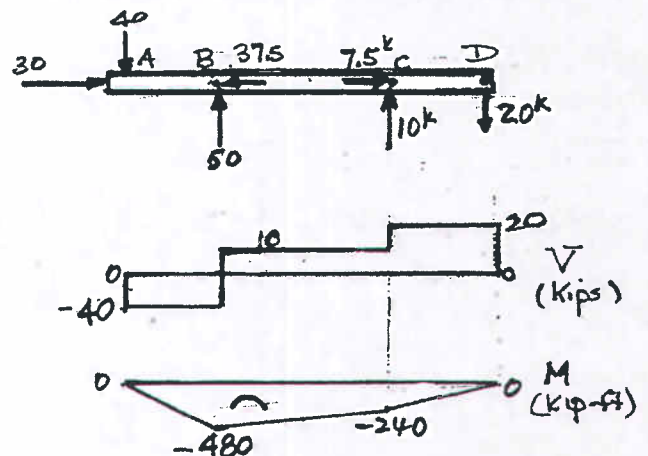


$$\begin{aligned} \sum M_B = 0; \quad -40k(12') - CE_y(24') + 20k(36') &= 0 & CE_y &= 10k \uparrow \\ CE_x/3 = CE_y/4 & & CE_x &= 7.5k \rightarrow \\ \sum F_x = 0; \quad 30k - BE_x + 7.5k &= 0 & BE_x &= 37.5k \leftarrow \\ \sum F_y = 0; \quad -40k + BE_y + 10k - 20k &= 0 & BE_y &= 50k \uparrow \end{aligned}$$

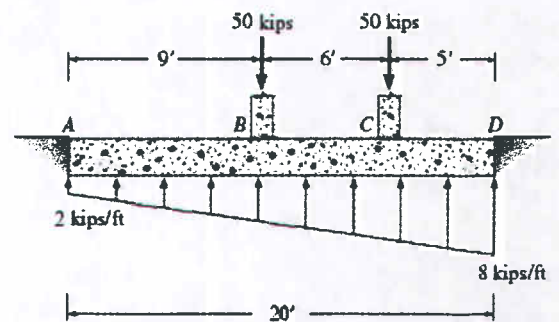
SUPPORT "E"



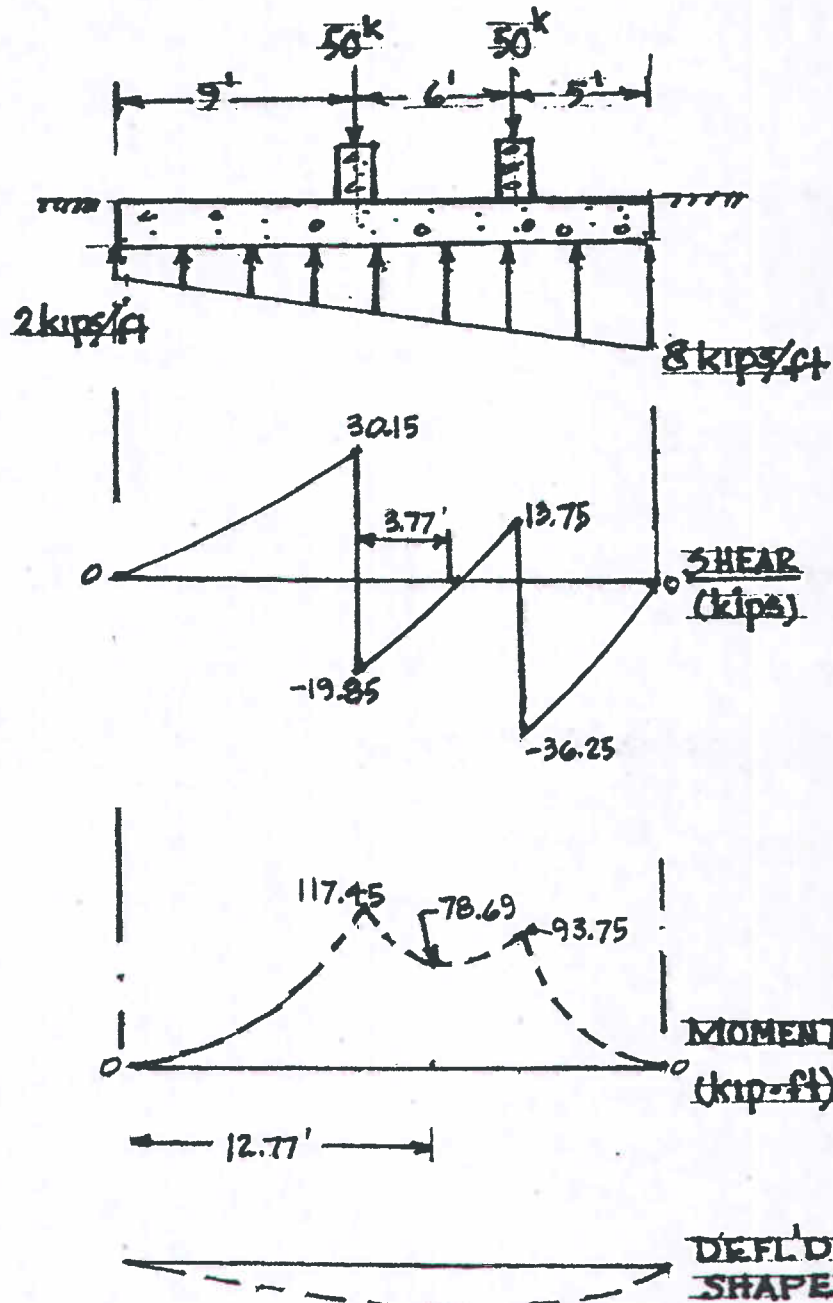
SHEAR & MOMENT CURVES MBR "ABCD":



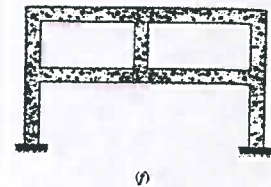
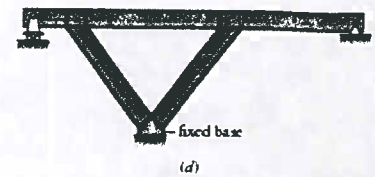
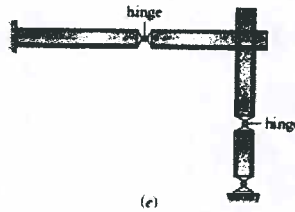
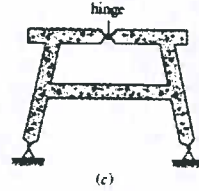
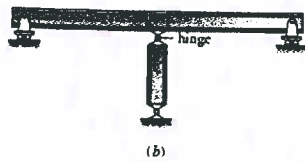
P5.43. The two concentrated loads, supported on the combined footing in Figure P5.43, produce a trapezoidal distribution of soil pressure. Construct the shear and moment diagrams. Label all ordinates of the diagrams. Sketch the deflected shape.



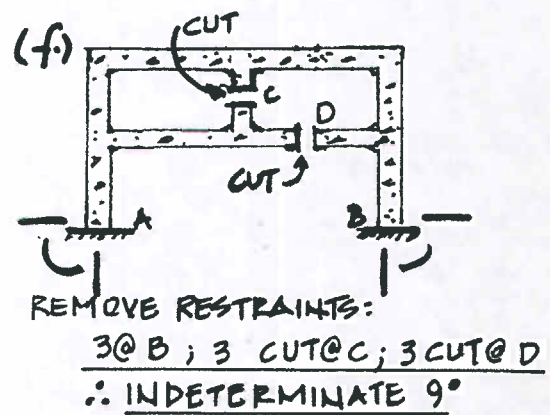
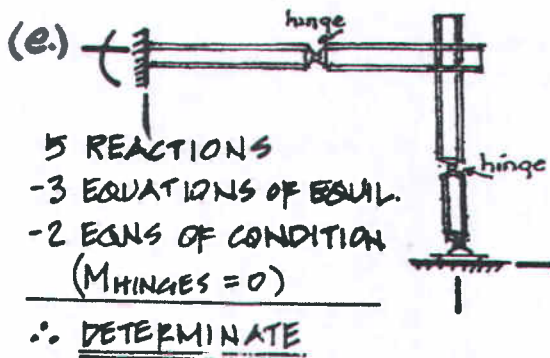
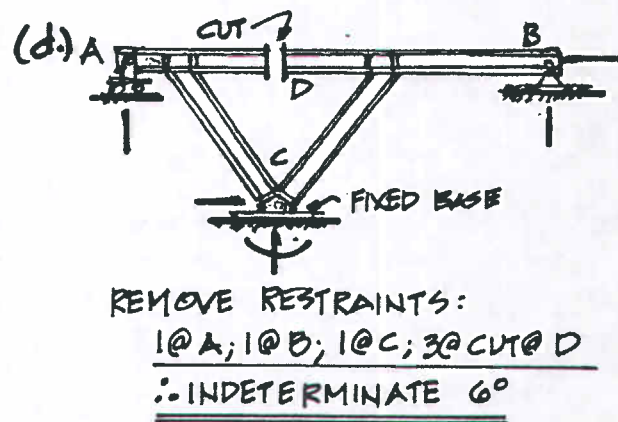
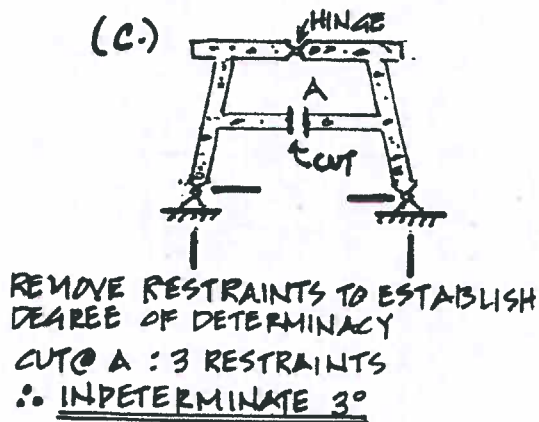
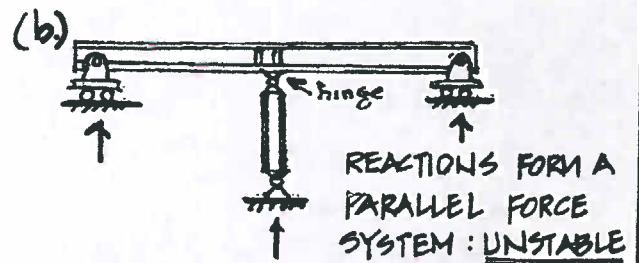
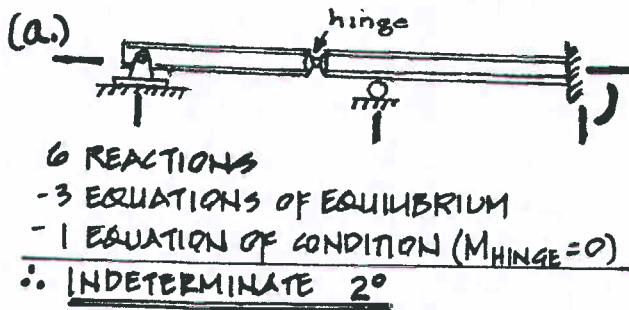
P5.43



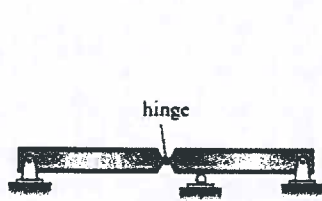
P5.44. Classify the structures in Figure P5.44. Indicate whether stable or unstable. If stable, indicate whether determinate or indeterminate. If indeterminate, give the degree.



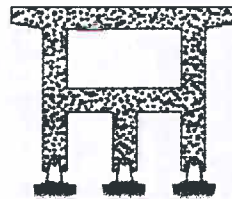
P5.44



P5.45. Classify the structures in Figure P5.45. Indicate whether stable or unstable. If stable, indicate whether determinate or indeterminate. If indeterminate, give the degree.



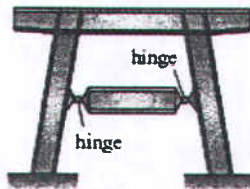
(a)



(b)



(c)

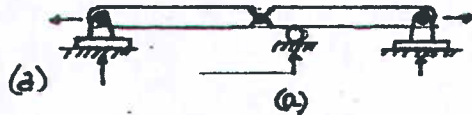


(d)



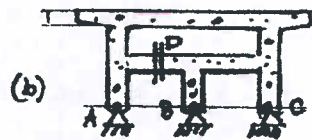
(e)

P5.45



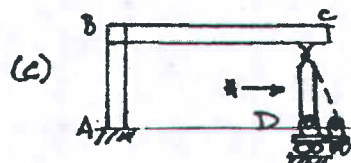
(a)

5 REACTIONS
3 E.O. OF EQUIL.
1 CONDITION EQ ($M=0$ AT HINGE)
INDETERMATE 1°



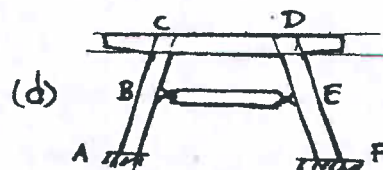
(b)

REMOVE RESTRAINTS
2 at B, 1 at C
3, COI AT D
∴ INDET 6°



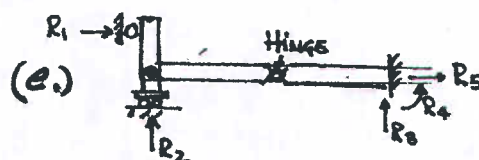
(c)

UNSTABLE:
MEMBER CD
CAN NOT CARRY A
LATERAL LOAD, BUT
FRAME ABC IS STABLE



(d)

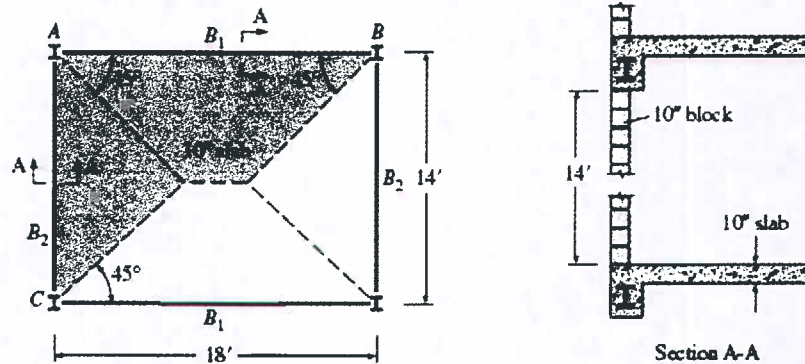
REMOVE RESTRAINTS
MEMBER BE IS A TWO FORCE
ELEMENT 1 HORIZ RESTRAINT
REMOVE SUPPORT F
∴ INDET 4°



(e)

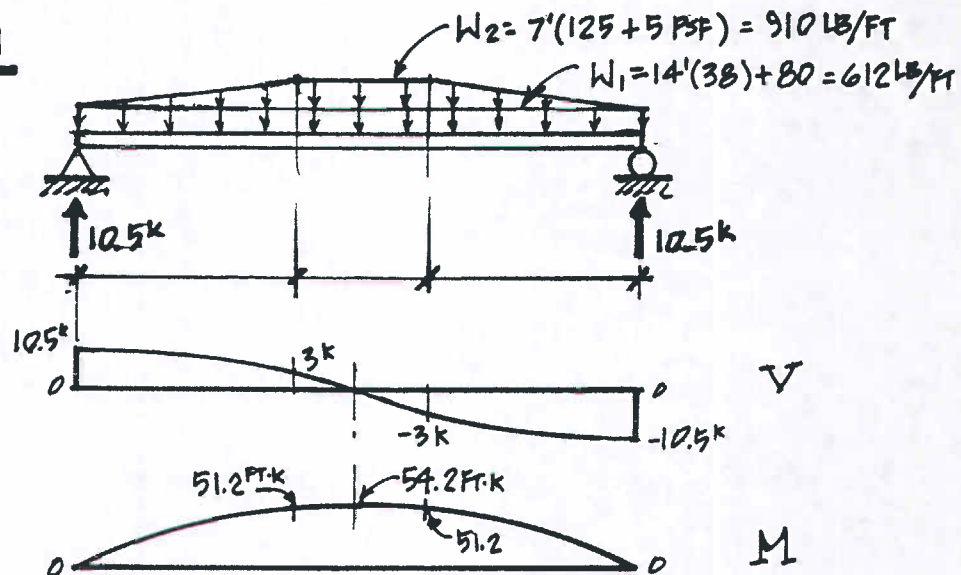
5 REACTIONS
3 E.O. OF EQUIL.
1 CONDITION EQ AT
HINGE
∴ INDET 1°

P5.46. The corner panel of a typical floor of a warehouse is shown in Figure P5.46. It consists of a 10-in-thick reinforced concrete slab supported on steel beams. The slab weighs 125 lb/ft^2 . The weight of light fixtures and utilities suspended from the bottom of the slab is estimated to be 5 lb/ft^2 . The exterior beams B_1 and B_2 support a 14-ft-high masonry wall constructed of lightweight, hollow concrete block that weighs 38 lb/ft^2 . We assume that the tributary area for each beam is shown by the dashed lines in Figure P5.46, and the weight of the beams and their fireproofing is estimated to be 80 lb/ft . Draw the shear and moment diagrams produced by the total dead load for beams B_1 and B_2 .

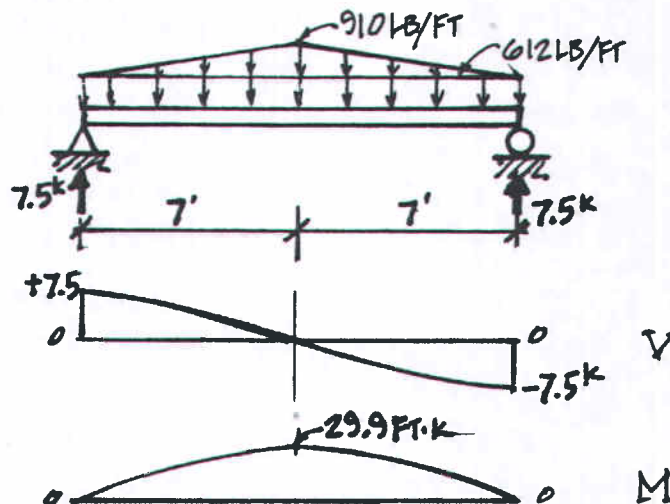


P5.46

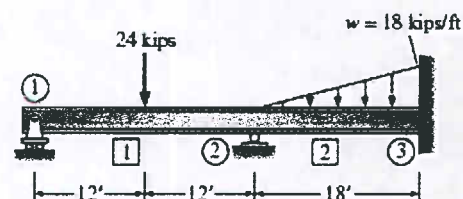
BEAM 1



BEAM 2



P5.47. Computer analysis of a continuous beam. The continuous beam in Figure P5.47 is constructed from a W18 106 wide flange steel beam with $A = 31.1 \text{ in}^2$ and $I = 1910 \text{ in}^4$. Determine the reactions, plot the shear and moment diagrams and the deflected shape. Evaluate the deflections. Neglect weight of beam. $E = 29,000 \text{ ksi}$.

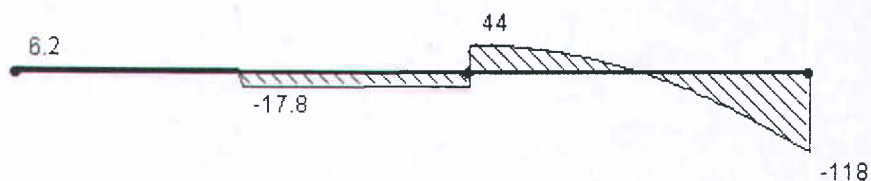


P5.47

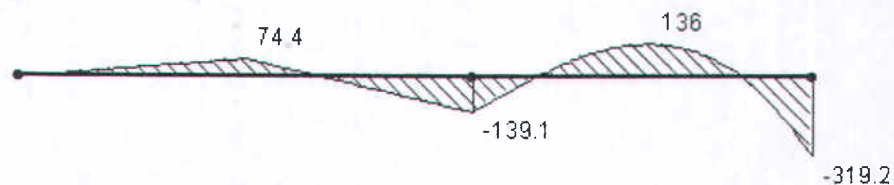
Reactions

Joint Reactions				
	Joint Label	X [k]	Y [k]	MZ [k-ft]
1	N3	0	118.008	-319.248
2	N1	0	6.204	0
3	N2	0	61.788	0
4	Totals:	0	186	-319.248

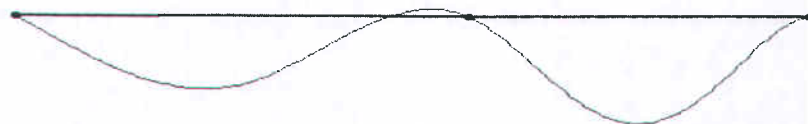
Shear Diagram (kips)



Moment Diagram (kip-ft)



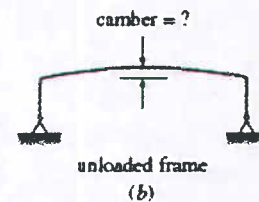
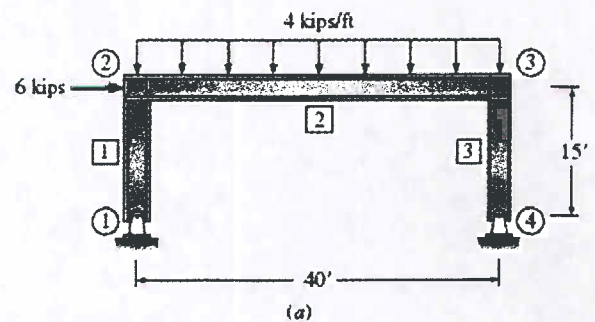
Deflected Shape (Deflection at 24-kip loading point = 0.059 in.)



P5.48 Computer analysis. The legs and girder of the rigid frame in Figure P5.48a are fabricated from a W18×130 wide flange steel beam: $A = 38.2 \text{ in}^2$ and $I = 2460 \text{ in}^4$. The frame is to be designed for a uniform load of 4 kips/ft and a lateral wind load of 6 kips; $E = 29,000 \text{ kips/in}^2$. The weight of the girder is included in the 4 kips/ft.

(a) Compute the reactions; plot the deflected shape and the shear and moment curves for the legs and girder, using the computer program. (Set the number of sections equal to 7 for all members.)

(b) To avoid ponding of rainwater on the roof, the girder is to be fabricated with a camber equal to the deflection at midspan of the roof girder produced by the loads shown. Determine the camber (see Figure P5.48b).

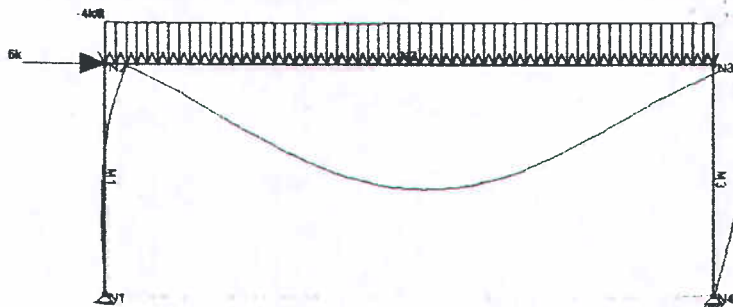


P5.48

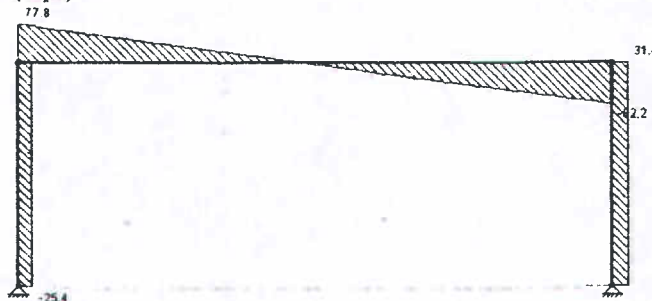
(a)

Joint Reactions			
Joint Label	X [k]	Y [k]	MZ [k-ft]
N1	25.395	77.75	0
N4	-31.395	82.25	0
Totals:	-6	160	0

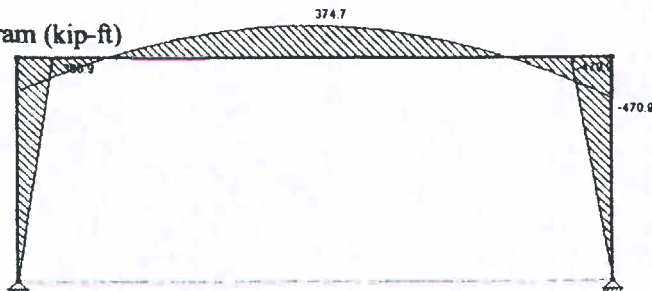
Deflected Shape (vertical deflection at girder midspan = 1.179 in.):



Shear Diagram (kips):



Moment Diagram (kip-ft)



(b)

Vertical deflection at midspan of girder = 1.179 in. Round the camber to $1\frac{1}{4}$ in. for practical applications.

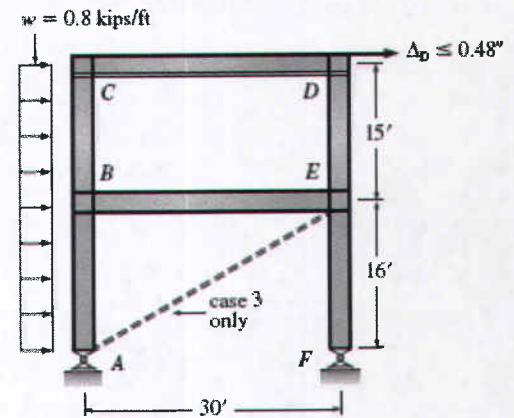
P5.49 Computer investigation of wind load on a building frame.

Case 1: The columns and girders of the building frame in Figure P5.49 have been designed initially for vertical load as specified by the building code. Floor beams are connected to columns by rigid joints. As part of the design, the building frame must be checked for lateral deflection under the 0.8 kips/ft wind load to ensure that lateral displacement will not damage the exterior walls attached to the structural frame. If the code requires that the maximum lateral deflection at the top of the roof not exceed 0.48 in. to prevent damage to the exterior walls, is the building frame sufficiently stiff to satisfy this requirement?

Case 2: If the bases of the columns at point A and F are attached to the foundations by fixed supports instead of pin connections, how much is the lateral deflection at joint D reduced?

Case 3: If a pin connected diagonal bar with a 2 inch 2 inch square cross section running from support A to joint E is added, determine the lateral deflection at joint D . Assume pin supports at joints A and F .

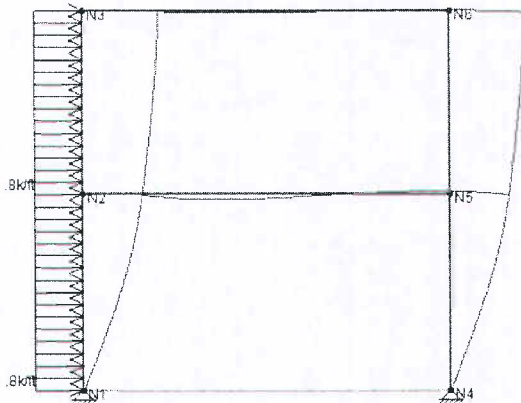
For the columns $I = 640 \text{ in}^4$ and $A = 17.9 \text{ in}^2$, for the girders $I = 800 \text{ in}^4$ and $A = 11.8 \text{ in}^2$, and for the diagonal brace $A = 4 \text{ in}^2$. Use $E = 29,000 \text{ kips/in}^2$.



P5.49

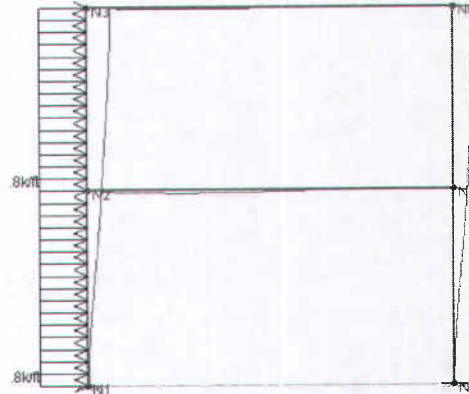
Case 1:

$\Delta_D = 2.784 \text{ in.}$ (frame is not stiff enough)



Case 2:

$\Delta_D = 0.834 \text{ in.}$ (frame is still not stiff enough)



Case 3:

$\Delta_D = 0.272 \text{ in.}$ (braced frame is very effective in reducing lateral stiffness)

