

FUNDAMENTALS OF STRUCTURAL ANALYSIS

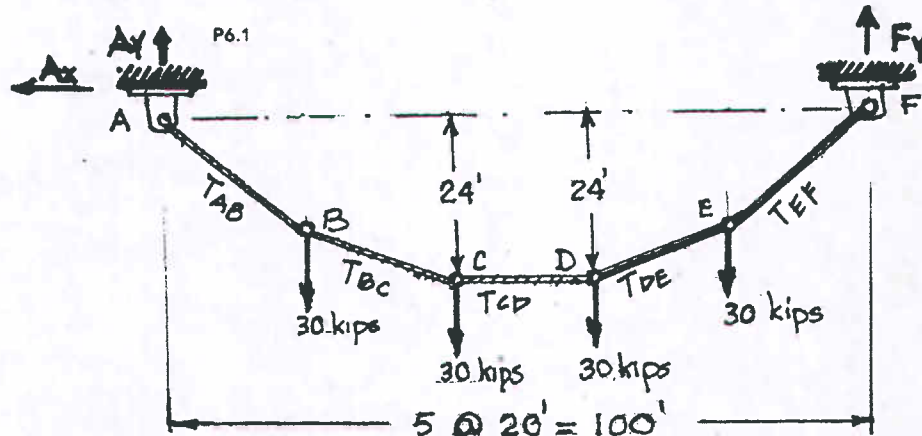
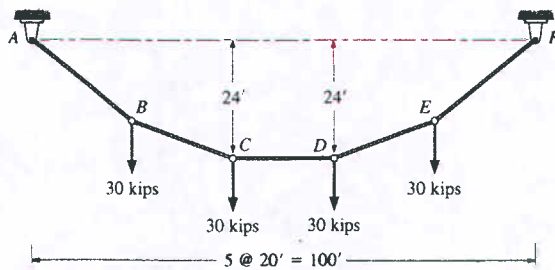
3rd Edition

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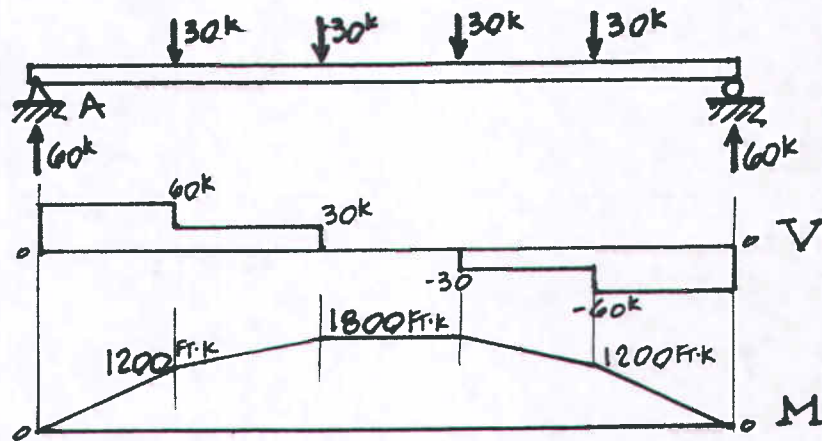
SOLUTIONS MANUAL

CHAPTER 6: CABLES

P6.1. Determine the reactions at the supports, the magnitude of the cable sag at joints B and E, the magnitude of the tension force in each segment of the cable, and the total length of the cable in Figure P6.1.



USE GENERAL CABLE THEOREM:



$Hh = M$ @ C $H(24') = 1800 \text{ Ft.k}$ THUS $H = 75k$ AND $A_x = H = 75k$

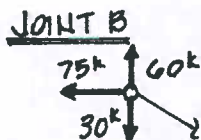
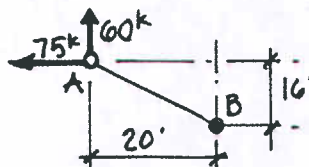
@ B $75k(h) = 1200 \text{ Ft.k}$ THUS $h = 16 \text{ FT @ B}$

CHECK $M @ B$: $\sum M_B = 0$; $75k(16') - 60k(20') = 0 \quad \checkmark \text{ OK}$

$T_{AB} = T_{EF} = \sqrt{(60k)^2 + (75k)^2} = 96 \text{ kips}$

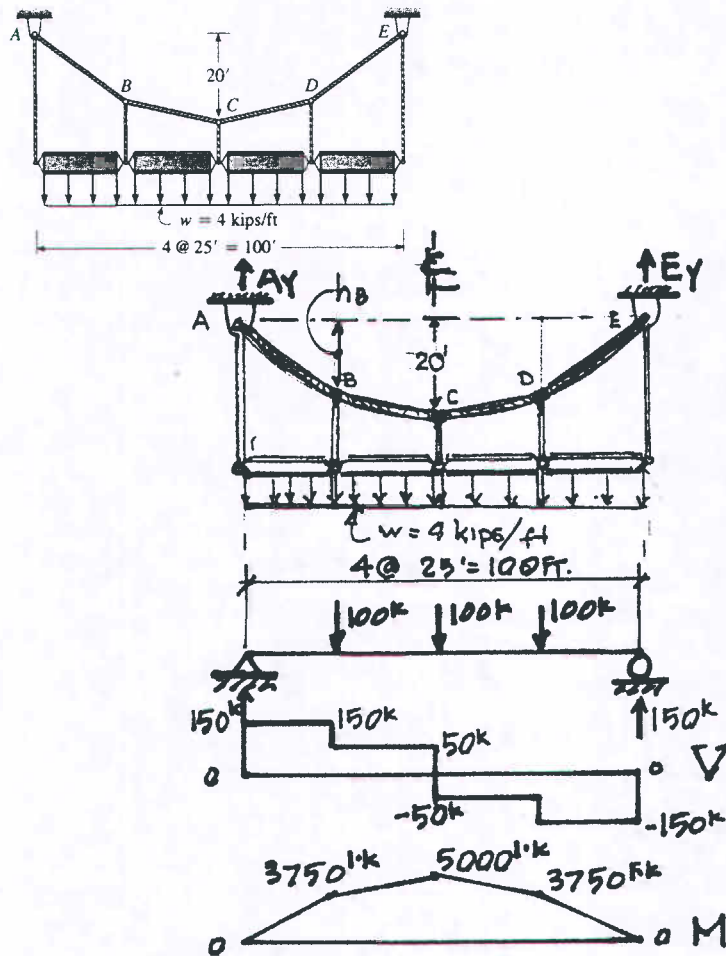
$T_{DE} = T_{BC} = \sqrt{(30k)^2 + (75k)^2} = 80.78 \text{ kips}$

$T_{CD} = H = 75k$



LENGTH OF CABLE $= \sqrt{(16')^2 + (20')^2}(2) + \sqrt{(8')^2 + (20')^2}(2) + 20 = 114.3 \text{ FT.}$

P6.2. The cable in Figure P6.2 supports four simply supported girders uniformly loaded with 4 kips/ft. (a) Determine the minimum required area of the main cable $ABCDE$ if the allowable stress is 60 kips/in². (b) Determine the cable sag at point B .



USE GENERAL CABLE THEOREM.

LOAD TO MAIN CABLE @
POINTS B, C AND D =
 $25' (4 \text{ k/ft}) = \underline{100 \text{ kips}}$

(a.) @ C AT CENTERLINE ϕ : $H \cdot 20' = 5000 \text{ ft} \cdot \text{k}$
 $H = \underline{250 \text{ kips}}$

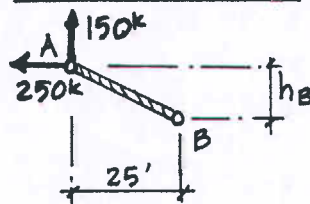
MAX. THRUST AT SUPPORTS:

$$T = \sqrt{(150 \text{ k})^2 + (250 \text{ k})^2} = \underline{291.5 \text{ kips}}$$

AREA OF CABLE REQUIRED = $T / 60 \text{ ksi}$

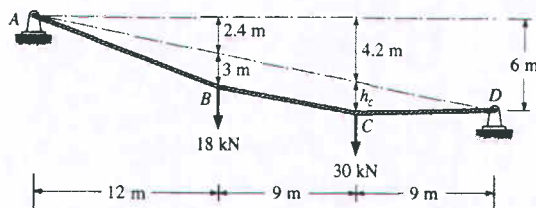
$$\text{Area} = 291.5 \text{ k} / 60 \text{ ksi} = \underline{4.86 \text{ in}^2}$$

(b.) CABLE SAG h_B :

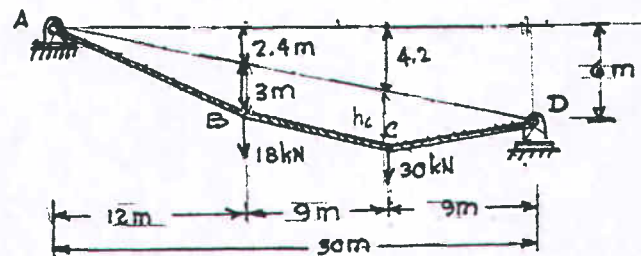


$$\begin{aligned} \sum M_B &= 0; \\ 150 \text{ k}(25') - 250 \text{ k} h_B &= 0 \\ h_B &= \underline{15 \text{ ft.}} \end{aligned}$$

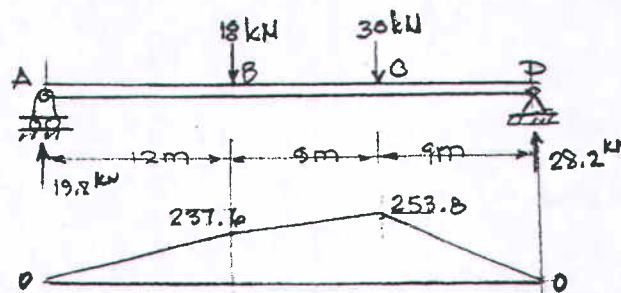
P6.3. Determine the reactions at supports A and D, the maximum tension in the cable, and the magnitude of the cable sag at point C in Figure P6.3.



P6.3



USE GENERAL CABLE THEOREM:



COMPUTE H IN CABLE $Hh_2 = M$

@ B: $H(3\text{m}) = 237 \text{ kN}\cdot\text{m}$ THUS $H = 79.2 \text{ kN}$

COMPUTE h_c CABLE SAG

$H \cdot h_c = 253.8 \text{ kN}\cdot\text{m}$

$h_c = 253.8 / 79.2 \text{ kN}$

$h_c = 3.205 \text{ m}$

$\sum M_A = 0;$

$18\text{kN}(12\text{m}) + 30\text{kN}(21\text{m}) - 79.2(6\text{m}) - D_y(30\text{m}) = 0$

$D_y = 12.4 \text{ kN} \uparrow$

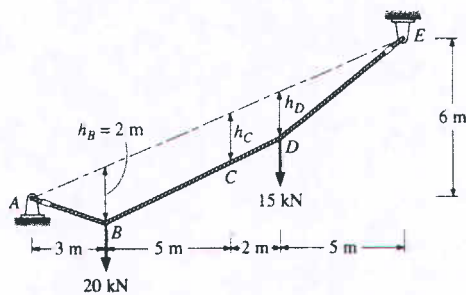
$\sum F_y = 0; -48 + 12.4 + A_y = 0$

$A_y = 35.6 \text{ kN} \uparrow$

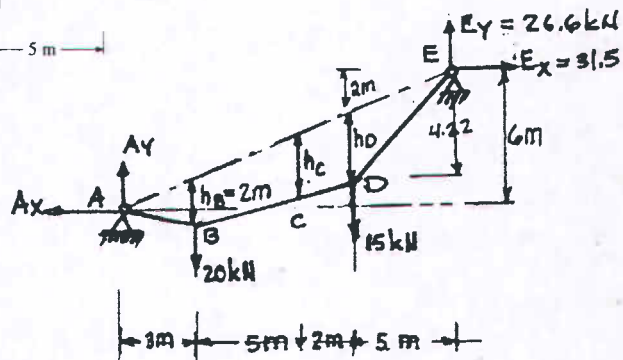
MAX. TENSION IN CABLE

$T_{AB} = \sqrt{(35.6)^2 + (79.2)^2} = 86.8 \text{ kips}$

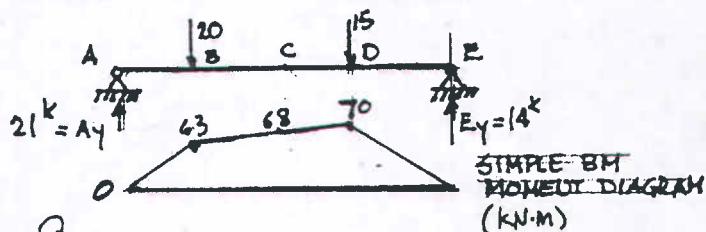
- P6.4.** (a) Determine the reactions at supports A and E and the maximum tension in the cable in Figure P6.4.
 (b) Establish the cable sag at points C and D.



P6.4



~~USE GENERAL CABLE THEOREM~~



$$\sum M_A = 0; 20(3) + 15(10) - E_y(15) = 0$$

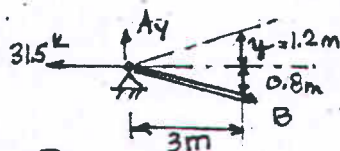
$$E_y = 14 \text{ kN}$$

$$\sum F_y = 0; A_y - 20 - 15 + E_y = 0$$

$$A_y = 21 \text{ kN} \uparrow$$

$$\text{At B: } H \cdot h_B = M_B = 63 \text{ kN}\cdot\text{m}$$

$$H = \frac{63}{2} = 31.5 \text{ kN}$$



$$\frac{y}{3} = \frac{6}{15}$$

$$y = 1.2 \text{ m}$$

$$(a) \sum M_B = 0; 31.5(0.8) - A_y(3) = 0$$

$$A_y = 8.4 \text{ kN} \text{ CABLE SUPPORT REACTION}$$

$$\sum F_y = 0; 8.4 - 20 - 15 + E_y = 0$$

$$E_y = 26.6 \text{ kN} \text{ CABLE SUPPORT REACTION}$$

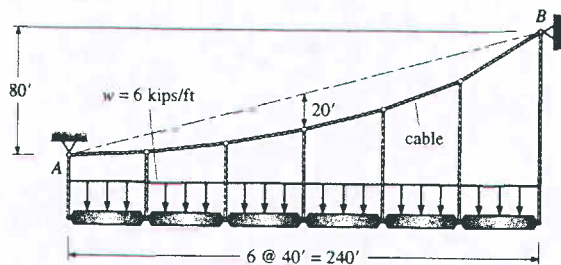
MAX TENSION IN SEGMENT DE

$$T_{\max} = \sqrt{(26.6)^2 + (31.5)^2} = 41.23 \text{ kN}$$

$$(b) h_C: H \cdot h_C = M_C \therefore h_C = \frac{M_C}{H} = \frac{68}{31.5} = 2.16 \text{ m}$$

$$h_D: H \cdot h_D = M_D \therefore h_D = \frac{M_D}{H} = \frac{70}{31.5} = 2.22 \text{ m}$$

P6.5. Compute the support reactions and the maximum tension in the main cable in Figure P6.5. The hangers can be assumed to provide a simple support for the suspended beams.

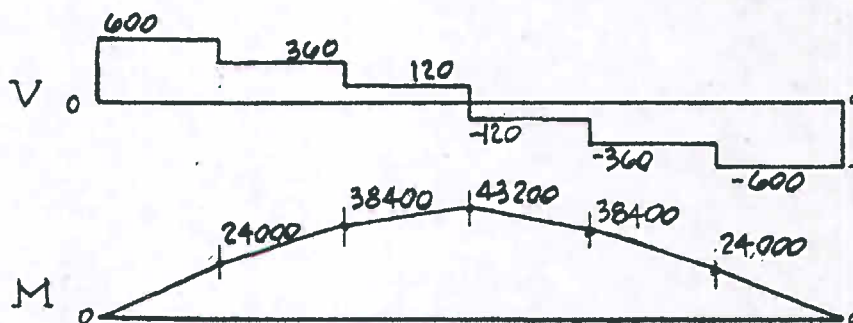
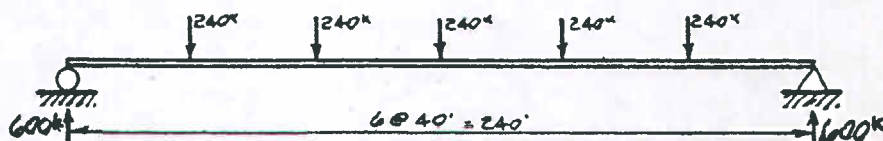
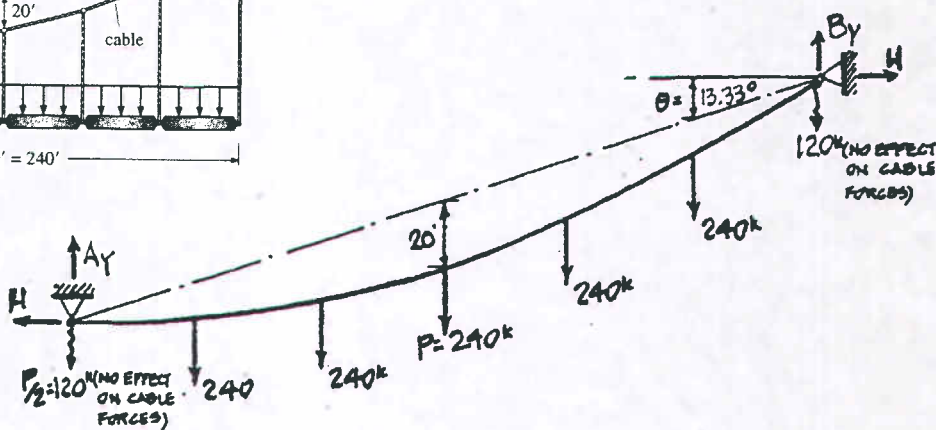


P6.5

USE GENERAL CABLE THEOREM

COMPUTE LOADS TO MAIN CABLE

$$P = 6 \text{ k/ft} (40') = \underline{240 \text{ k}}$$



$$H h_2 = M_2$$

$$H (20) = 43200$$

$$H = \underline{2160 \text{ k}}$$

COMPUTE B_y (NEGLECT END CABLE FORCES)

$$\sum M_A = 0 \quad 240(40 + 80 + 120 + 160 + 200) - B_y(240) + 2160(80) = 0$$

$$B_y = \underline{1320 \text{ k}}$$

$$\text{MAX } T = \sqrt{(2160)^2 + (1320)^2}$$

$$T_{\text{MAX}} = \underline{2531.4 \text{ k}}$$

REACTIONS @ "B"

$$B_y \text{ TOTAL} = 1320 + 120$$

$$B_y = \underline{1440 \text{ k}}$$

$$B_x = H \rightarrow$$

$$B_x = \underline{2160 \text{ k}}$$

REACTIONS @ "A"

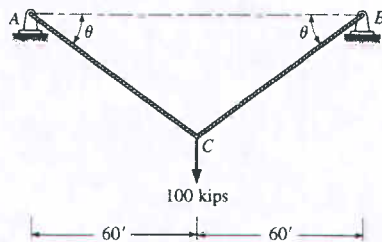
$$\sum F_y = 0 \quad 240(6) - 1440 - A_y = 0$$

$$A_y = \underline{0}$$

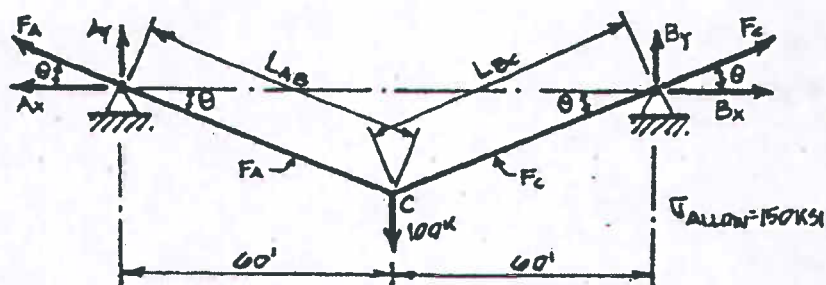
$$A_x = H \rightarrow$$

$$A_x = \underline{2160 \text{ k}}$$

P6.6. What value of θ is associated with the minimum volume of cable material required to support the 100-kip load in Figure P6.6? The allowable stress in the cable is 150 kips/in².



P6.6



COMPUTE VALUE OF θ THAT MINIMIZES THE VOLUME OF THE CABLE

BY SYMMETRY $F_A = F_C = F$ AND $A_Y = B_Y$

$$A_Y / F_A = \sin \theta$$

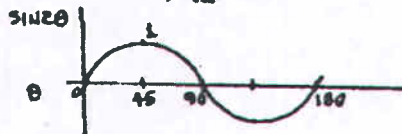
$$\text{AREA REQUIRED} = F / \sigma_{\text{ALLOW}} = 50 / \sin \theta \text{ (150)}$$

$$\text{LENGTH } L = L_{AB} = L_{BC} = 60'(12") / \cos \theta$$

$$\text{VOLUME } V = \text{AREA}(2L) = \left(\frac{1}{33 \sin \theta} \right) (2(720) / \cos \theta)$$

IDENTITIES: $\sin 2\theta = 2 \sin \theta \cos \theta$
 $\sin \theta \cos \theta = \frac{1}{2} \sin 2\theta$

$$\text{VOLUME } V = 480 / \left(\frac{1}{2} \sin 2\theta \right)$$



TO MINIMIZE VOLUME $\sin 2\theta = 1$

$$A_Y = C_Y = 50 \text{ K}$$

$$F_A = 50 / \sin \theta$$

$$\text{AREA} = \frac{1}{33 \sin \theta}$$

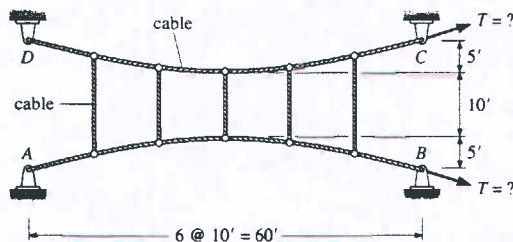
$$L = 720 / \cos \theta$$

$$V = 480 / \cos \theta \sin \theta \text{ in}^3$$

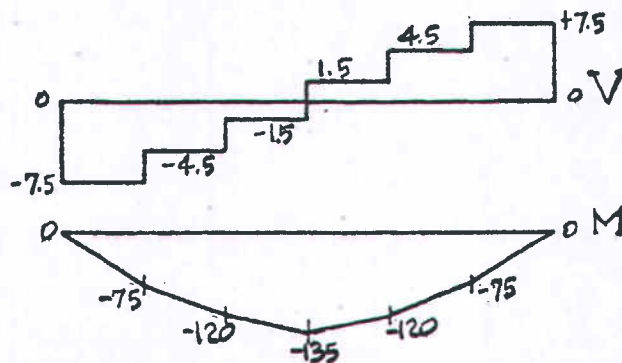
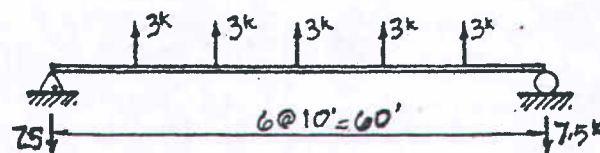
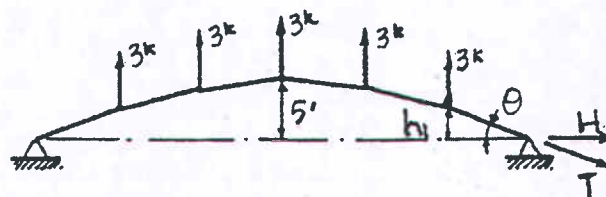
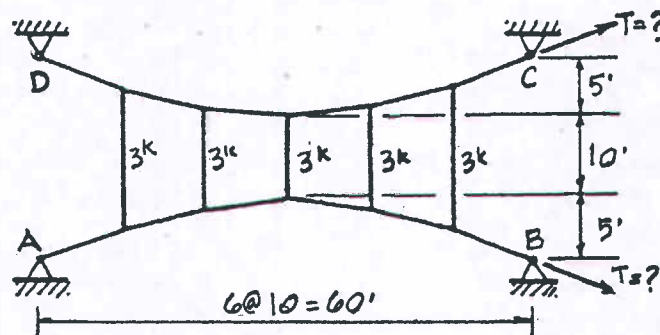
$$V = 960 / \sin 2\theta \text{ in}^3$$

$$\theta = 45^\circ$$

P6.7. The cables in Figure P6.7 have been dimensioned so that a 3-kip tension force develops in each vertical strand when the main cables are tensioned. What value of jacking force T must be applied at supports B and C to tension the system?



P6.7



COMPUTE JACKING FORCE T :

$$Hh = M; \quad H = 135 \text{ ft-k} / 5 \text{ ft}$$

$$H = 27 \text{ kips}$$

$$Hh_i = 75 \text{ ft-k}$$

$$h_i = 75 \text{ ft-k} / 27 \text{ k}$$

$$h_i = 2.78 \text{ ft}$$

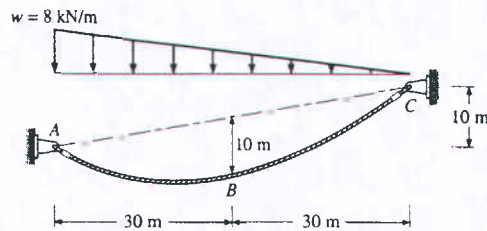
$$\tan \theta = h_i / 10 \text{ ft}$$

$$\theta = 15.52^\circ$$

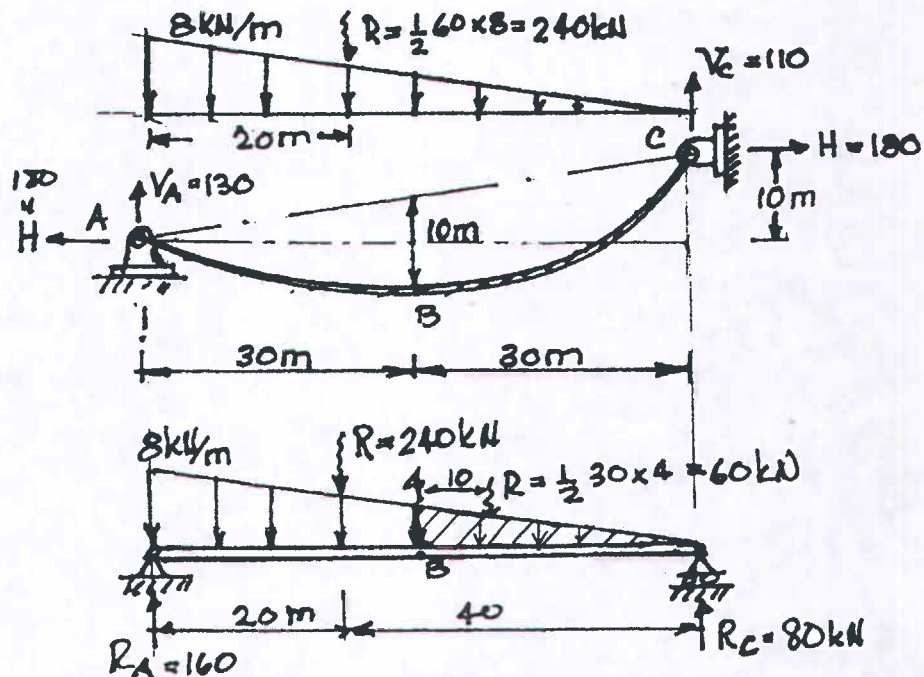
$$T = H / \cos \theta = 27 \text{ k} / 0.964$$

$$T = 28.02 \text{ kips}$$

P6.8. Compute the support reactions and the maximum tension in the cable in Figure P6.8.



P6.8



at midspan

$$M_B = 80 \times 30 - 60 \times 10 = 1800 \text{ kN}\cdot\text{m}$$

$$H \cdot h = 1800$$

$$H = 180 \text{ kN}$$

$$\sum M_A = 0 \quad \text{SEE CABLE}$$

$$0 = 240 \times 20 + 180 \times 10 - V_C \cdot 60$$

$$V_C = 110 \text{ kN}$$

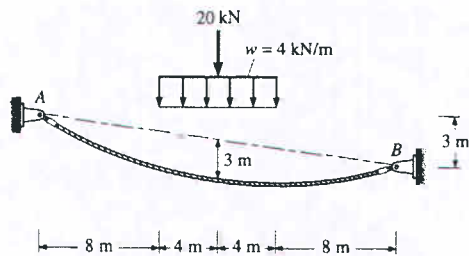
$$\sum F_y = 0 = V_A - 240 + 110$$

$$V_A = 130 \text{ kN}$$

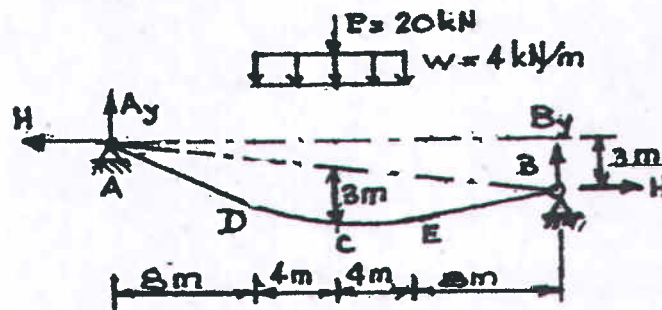
MAX CABLE FORCE AT SUPPORT "A"

$$T_{\max} = \sqrt{180^2 + 130^2} = 222.04 \text{ kN}$$

P6.9. Compute the support reactions and the maximum tension in the cable in Figure P6.9.



P6.9



COMPUTE REACTIONS

$$\sum M_A = 0: 20 \times 12 + (4 \times 8)12 - B_y 24 - H 3 = 0 \quad (1)$$

$$\sum M_C = 0 \text{ (FREE BODY TO RIGHT OF 'C')} \\ (4 \times 4)2 - B_y 12 + H 1.5 = 0 \quad (2)$$

SOLVING (1) and (2) GIVES

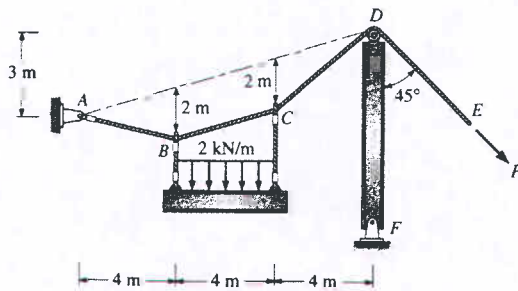
$$B_y = \underline{14.33 \text{ kN}}; \quad H = \underline{93.33 \text{ kN}}$$

$$\sum F_y = 0: A_y + 14.33 - 20 - 4 \times 8 \\ A_y = \underline{37.67 \text{ kN}}$$

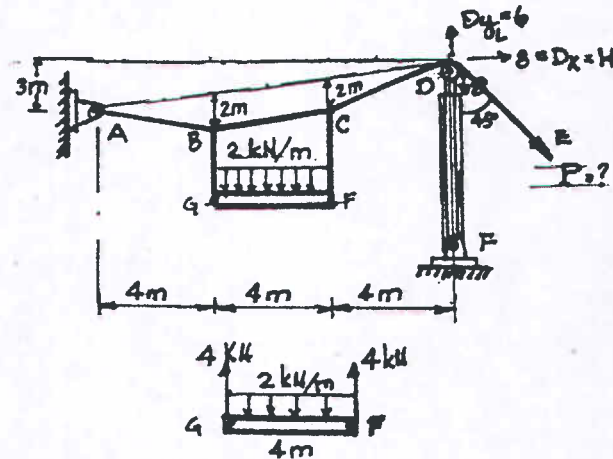
MAX. TENSION OCCURS AT 'A', POINT OF MAXIMUM SLOPE:

$$T_{\text{MAX}} = \sqrt{93.33^2 + 37.67^2} \\ = \underline{100.65 \text{ kN}}$$

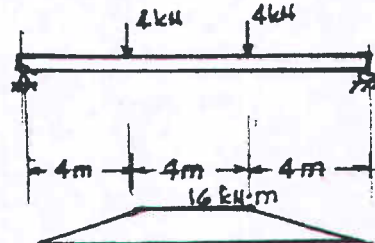
P6.10. A cable *ABCD* is pulled at end *E* by a force *P* (Figure P6.10). The cable is supported at point *D* by a rigid member *DF*. Compute the force *P* that produces a sag of 2 m at points *B* and *C*. The horizontal reaction at support *F* is zero. **Compute** the vertical reaction at *F*.



P6.10



USE GENERAL CABLE THEORY.



$$H \cdot h = M = 16 \text{ kN} \cdot \text{m}$$

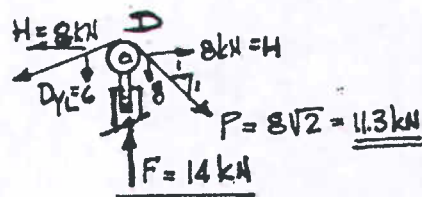
$$\text{AT "C"} \quad H = \frac{16}{2} = 8 \text{ k}$$

TOP FIGURE

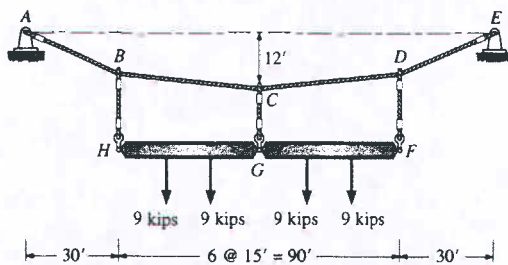
$$\sum M_A = 0; \quad 4 \times 4 + 4 \times 8 + 8 \times 3 - D_{yL} \cdot 2 = 0$$

$$D_{yL} = 6 \text{ kN}$$

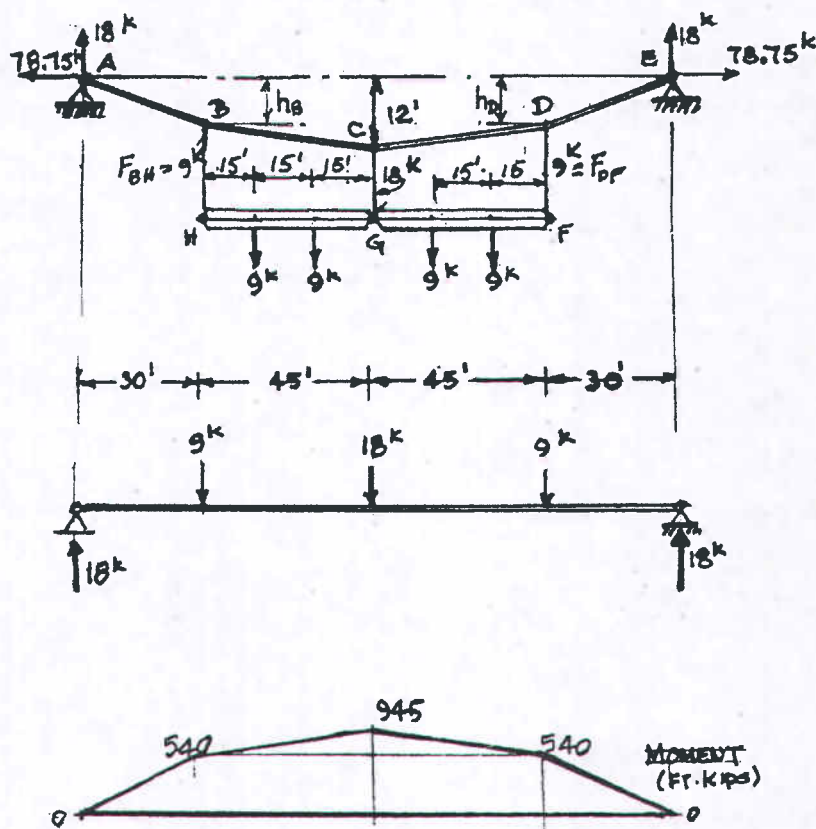
SUPPORT AT D



P6.11. Compute the support reactions and the maximum tension in the cable in Figure P6.11. The sag at midspan is 12 ft. Each hanger can be assumed to provide a simple support for the suspended beam. Determine the sag at points B and D.



P6.11



$$H \cdot h = M$$

$$H \cdot 12' = 945 \text{ Ft} \cdot \text{k}$$

$$H = 78.75 \text{ k}$$

Sag at D (BY SYMMETRY $h_B = h_D$)

$$H \cdot h_D = M$$

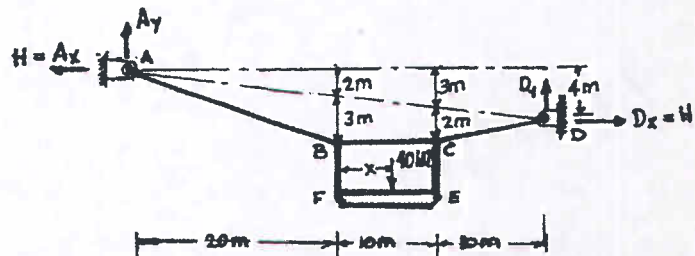
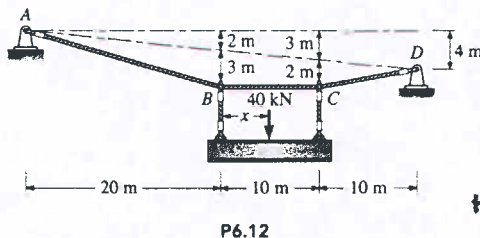
$$h_D = \frac{540 \text{ Ft} \cdot \text{k}}{78.75 \text{ k}} = 6.86 \text{ Ft}$$

MAX. TENSION
AT A OR E

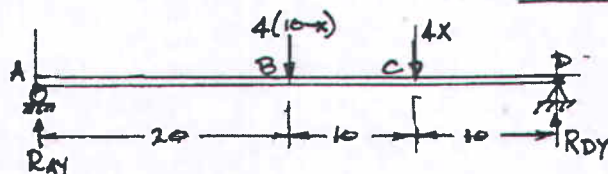
$$T_{\text{max}} = \sqrt{78.75^2 + 18^2}$$

$$= 80.78 \text{ kips}$$

P6.12. Determine the location of the 40-kN load such that sags at points B and C are 3 m and 2 m, respectively. Determine the maximum tension in the cable and the reactions at supports A and D.



$$\begin{aligned} \sum M_F = 0 & \quad 40x = F_{CE}10 \\ F_{CE} &= 4x \\ \sum F_y = 0 & \quad F_{BF} - 40 + 4x = 0 \\ F_{BF} &= 4(10-x) \end{aligned}$$



$$\sum M_A = 0; (40-4x)20 + 4x30 - R_D40 = 0$$

$$4R_D = 80 - 8x + 12x$$

$$D_y = R_D = 20 + x$$

$$M_C = R_D10 = 200 + 10x$$

$$\sum F_y = 0; R_A - 40 + 4x - 4x + 20 + x = 0$$

$$A_y = R_A = 20 - x$$

$$M_B = R_A(20) = 400 - 20x$$

USE CABLE THEOREM.

$$H \cdot h_B^3 = M_B = 400 - 20x$$

$$3H = 400 - 20x \quad \text{EQ 1}$$

$$H \cdot h_C^2 = 200 + 10x$$

$$2H = 200 + 10x \quad \text{EQ 2}$$

$$H = 100 + 5x \quad \text{EQ 2A}$$

substi EQ 2A INTO EQ 1

$$3(100 + 5x) = 400 - 20x$$

$$35x = 100$$

$$x = 2.86 \text{ m}$$

$$\text{substi } x \text{ into EQ 2A} \quad \therefore H = 100 + 5(2.86) = 114.3 \text{ kips}$$

$$\text{REACTIONS: } \sum M_A = 0; 40(20 + 2.86) - H4 - D_y40 = 0$$

$$\frac{40}{40} D_y = \frac{40(20 + 2.86) - 4(114.3)}{40}$$

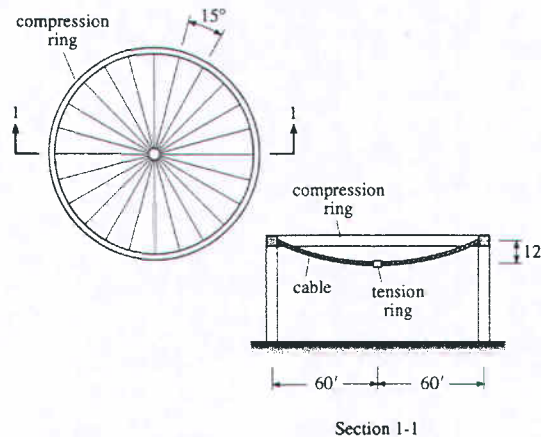
$$D_y = 20 + 2.86 - 11.43 = 11.43 \text{ kN} \uparrow$$

$$A_y = 40 - D_y = 40 - 11.43 = 28.57 \text{ kN} \uparrow$$

$$\text{MAX TENSION: } T = \sqrt{A_y^2 + H^2} = \sqrt{(28.57)^2 + (114.3)^2} = 117.82 \text{ kN}$$

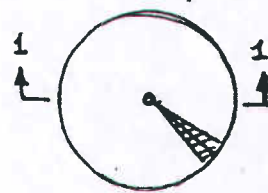
Practical Application

P6.13. The cable-supported roof for a summer theater, shown in Figure P6.13, is composed of 24 equally spaced cables that span from a tension ring at the center to a compression ring on the perimeter. The tension ring lies 12 ft below the compression ring. The roof weighs 25 lb/ft² based on the horizontal projection of the roof area. If the sag at midspan of each cable is 4 ft, determine the tensile force each cable applies to the compression ring. What is the required area of each cable if the allowable stress is 110 kips/in²? Determine the weight of the tension ring required to balance the vertical components of the cable forces.

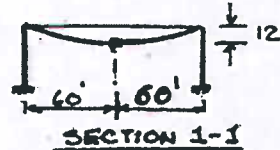


P6.13

NOTE: WE ASSUME WEIGHT OF TENSION RING SUPPLIES THE FORCE TO BALANCE ΣR_A .



GIVEN: CABLE SAG at midspan = 4'

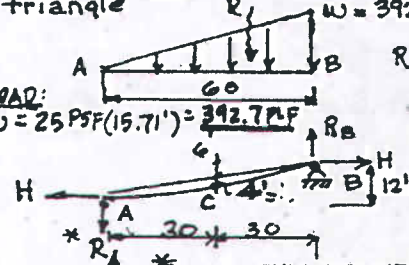


EACH CABLE SUPPORTS $\frac{1}{24}$ OF AREA.
Assume area is triangular.

$$\text{circumference} = 2\pi R = 2\pi 60 = 377.4 \text{ ft}$$

$$\text{base of triangle} = \frac{1}{24} 377 = 15.71 \text{ ft}$$

$$\begin{aligned} \text{LOAD: } W &= 25 \text{ PSF}(15.71') = 392.7 \text{ PLF} \\ W &= 392.7 \text{ lb/ft} \\ R &= \frac{1}{2} 60(392.7) = 11.781 \text{ kips} \end{aligned}$$



* NOTE: THE WEIGHT OF TENSION RING MUST BE EQUAL TO $24R_A = 11.78 \text{ kips}$

$$\Sigma M_B = 0: -R_A 60 + H 12 - \frac{392.7 \times 60(2)}{2} = 0$$

$$\Sigma M_C = 0: -R_A 30 + H 2 - \frac{392.7 \times 30 \times 10}{2} = 0$$

FBD TO LEFT OF C.

SOLVING ABOVE TWO EQ'S

$$H = 22,089 \text{ lb} = 22.1 \text{ kips}$$

$$R_A = 490.9 \text{ lb} = 0.491 \text{ kips}$$

$$\Sigma F_y = 0: -0.491 - 11.781 + R_B = 0$$

$$R_B = 12.272 \text{ kips}$$

MAX. TENSILE FORCE AT B

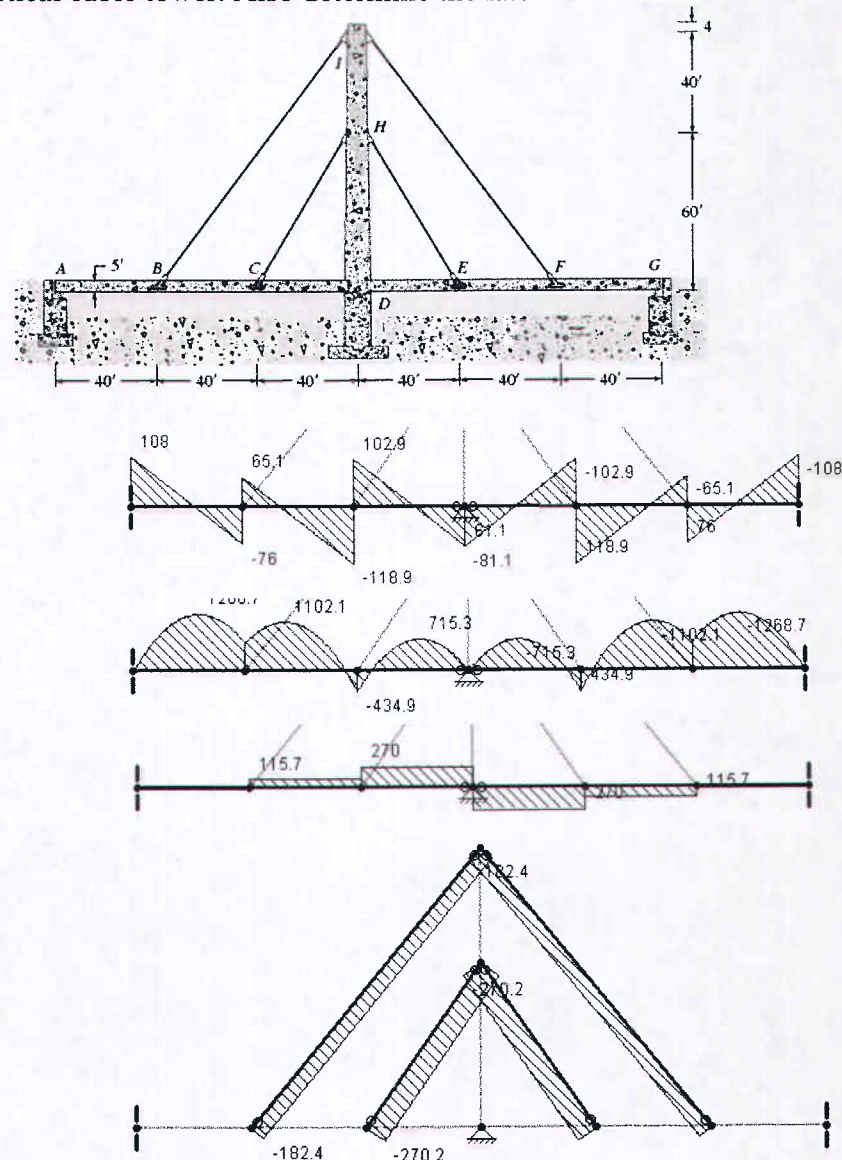
$$T = \sqrt{H^2 + R_B^2} = 25.28 \text{ kips}$$

$$\text{Area} = \frac{25.28 \text{ kips}}{110 \text{ KSI}} = 0.23 \text{ in}^2$$

P6.14. Computer study of a cable-stayed bridge. The deck and tower making up the two-span, cable-stayed bridge in Figure P6.14 are constructed of reinforced concrete. The cross-section of the bridge is constant with an area of 15 ft^2 and a moment of inertia of 19 ft^4 . The dead weight of the girders is 4 kips/ft . In addition the girders are to be designed to support a live load of 0.6 kips/ft that is to be positioned to maximize the design forces in individual members. The vertical cable tower, located at the center support, has a cross-sectional area of 24 ft^2 and a moment of inertia of 128 ft^4 . Four cables, each with an area of 13 in^2 and an effective modulus of elasticity of $26,000 \text{ kips/in}^2$, are used to support the deck at the third points of each 120 ft span. The modulus of elasticity of the concrete is $5,000 \text{ kips/in}^2$. The cable reaction may be assumed to be applied to the underside of the roadway. Members have been detailed such that the support at D acts as a simple support for both the tower and the roadway girders.

(a) Analyze the structure for full live and dead loads on both spans, that is, establish the shear, moment, and axial load diagrams for the girders, the forces in the cables, and the maximum deflection of the girders.

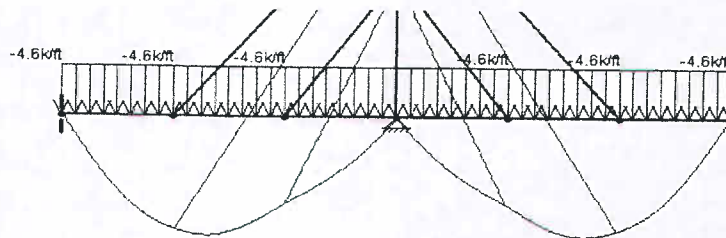
(b) With the dead load on both spans and the live load on the left span $ABCD$, determine the shear, moment, and axial load diagrams for both spans, the axial force in the cables, and the shear, moment, and axial load in the vertical cable tower. Also determine the lateral deflection of the cable tower.



P6.14 Continued
To Next Page

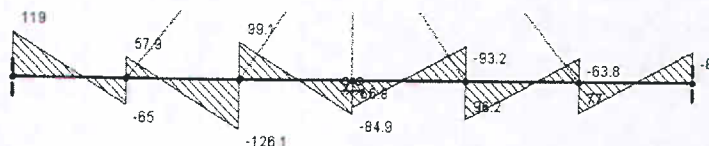
P6.14(a) Continued

Deflection
(Max. girder
deflection
= 1.17 in.)

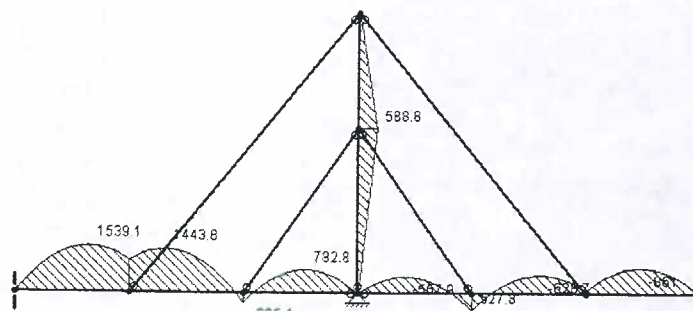


(b)

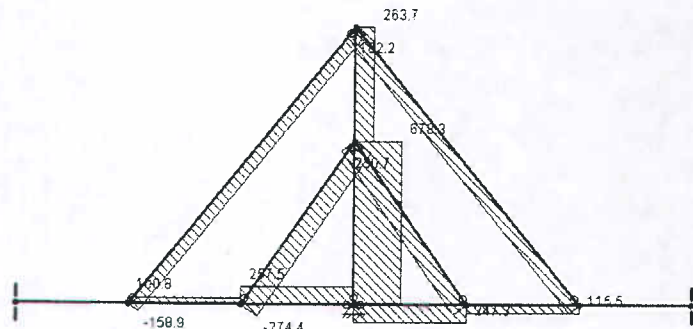
Shear
(kips)



Moment
(kip-ft)



Axial Force
(kips)



**Lateral
Deflection of
Tower (= 0.63
in. at joint I)**

