

FUNDAMENTALS OF STRUCTURAL ANALYSIS

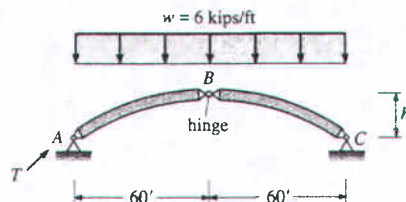
3rd Edition

Kenneth M. Leet, Chia-Ming Uang, and Anne M. Gilbert

SOLUTIONS MANUAL

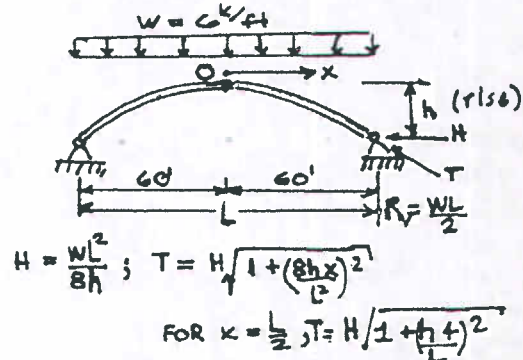
CHAPTER 7: ARCHES

P7.1. For the parabolic arch in Figure P7.1, plot the variation of the thrust T at support A for values of $h = 12, 24, 36, 48$, and 60 ft.



P7.1

PLOT THE VARIATION OF THRUST, T , AT SUPPORT A FOR VALUES OF $h = 12', 24', 36', 48'$ AND $60'$. (SEE §7.4 IN TEXT)



FOR $h = 12'$; $H = \frac{6(120)^2}{8 \times 12} = 900 \text{ kips}$

$T = 900 \sqrt{1 + \left(\frac{12 \times 4}{120}\right)^2} = \underline{969.33^k}$

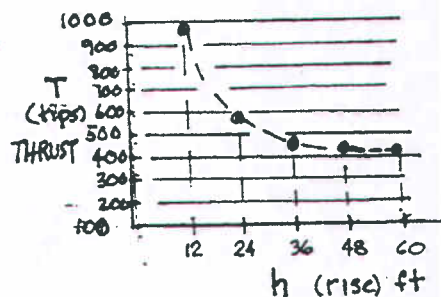
FOR $h = 24'$; $H = \frac{6(120)^2}{8 \times 24} = 450 \text{ kips}$

$T = 450 \sqrt{1 + \left(\frac{24 \times 4}{120}\right)^2} = \underline{576.28^k}$

FOR $h = 36'$ $H = \frac{6(120)^2}{8 \times 36} = 300 \text{ kips}$; and $T = \underline{468.6^k}$

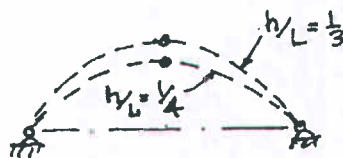
FOR $h = 48'$ $H = \frac{6(120)^2}{8 \times 48} = 225^k$; and $T = \underline{424.53^k}$

$h = 60'$ $H = \frac{6(120)^2}{8 \times 60} = 180^k$; and $T = \underline{402.5^k}$

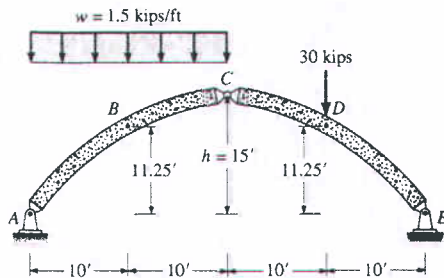


NOTE: FOR ECONOMY OF MATERIALS

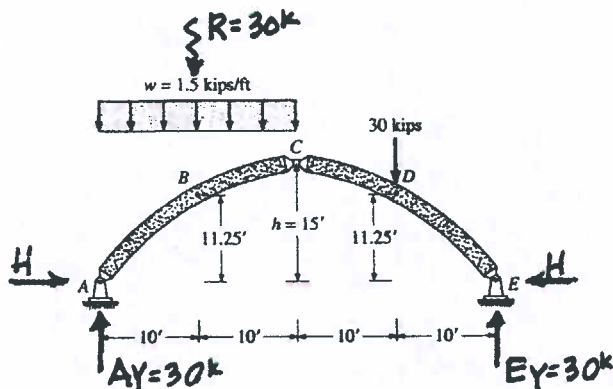
$\frac{h}{L}$ SHOULD BE IN THE RANGE OF $\frac{1}{4}$ TO $\frac{1}{3}$



P7.2. Compute the reactions at supports A and E of the three-hinged parabolic arch in Figure P7.2. Next compute the shear, axial load, and moment at points B and D, located at the quarter points.

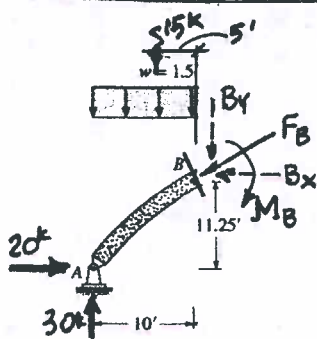


P7.2

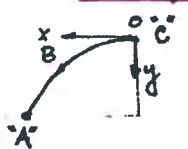


REACTIONS $A_y = E_y$ (SYMMETRY) = 30 kips
 $\sum M_C = 0$; (FREEBODY CE)
 $0 = +30 \times 10 - 30 \times 20 + H \times 15$
 $H = 20 \text{ kips} \leftarrow$

COMPUTE FORCES ON A NORMAL SECTION AT "B", FREEBODY OF "AB":



$$\begin{aligned} \sum F_x &= 20 - B_x = 0 \\ B_x &= 20 \text{ kips} \\ \sum F_y &= 0 = 30 - 15 - B_y \\ B_y &= 15 \text{ kips} \\ \sum M_B &= 0; 30(10) - 20(11.25) - 15(5) + M_B = 0 \\ &= 300 + 75 + 225 = M_B \\ M_B &= 0 \end{aligned}$$



Eq. of parabola:

$$\begin{aligned} y &= kx^2 \\ \text{At } x=20', y=15' \\ 15 &= k(20)^2 \\ k &= \frac{15}{400} = \frac{3}{80} \\ y &= \frac{3}{80}x^2 \\ \frac{dy}{dx} &= \frac{3x}{40} \end{aligned}$$

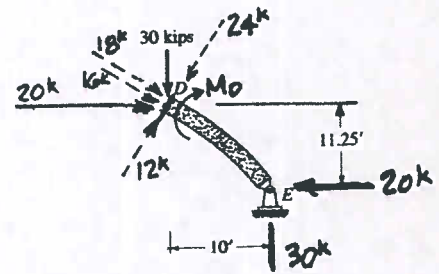
COMPUTE SLOPE AT B:

$$\begin{aligned} \text{SLOPE AT B: } \theta_B &= \frac{dy}{dx} = \frac{3x}{40}, x=10' \\ \frac{dy}{dx} &= \frac{3}{4} \end{aligned}$$

SINCE FORCE AT B HAS THE SAME SLOPE AS THE ARCH:
 IT IS AN AXIAL FORCE = 25 kips = $F_B = \sqrt{(20)^2 + (15)^2}$

CONCLUSION: ARCH IS STRESSED BY AN AXIAL FORCE. IE $M=0 \neq V=0$

COMPUTE FORCES ON A NORMAL SECTION AT "D", FREEBODY OF "DE":

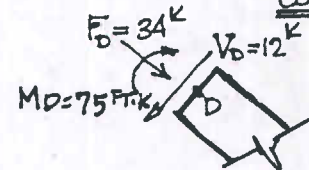


DETERMINE MOMENT AT D:

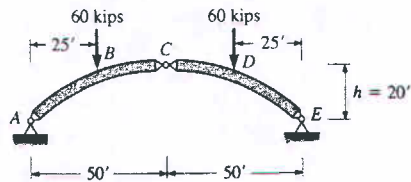
$$\begin{aligned} \sum M_D &= 0; 20(11.25) - 30(10) + M_D = 0 \\ M_D &= 75 \text{ kip} \cdot \text{ft} \end{aligned}$$

$$\text{SHEAR } V_D = 24 - 12 = 12 \text{ k}$$

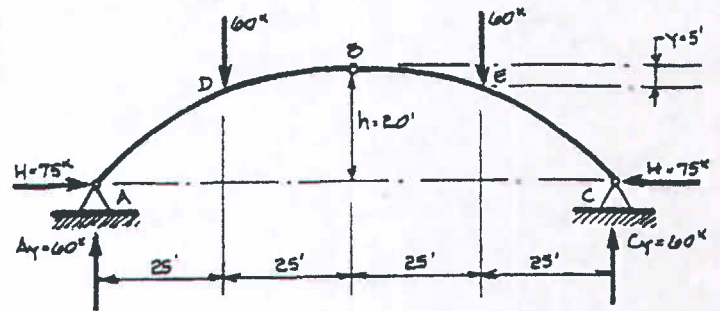
$$\text{AXIAL FORCE } F_D = 16 + 18 = 34 \text{ kips} \quad \text{COMPRESSION}$$



P7.3. The three-hinged parabolic arch in Figure P7.3 supports 60-kip loads at the quarter points. Determine the shear, axial load, and moment on sections an infinitesimal distance to the left and right of the loads. The equation for the arch axis is $y = 4hx^2/L^2$.

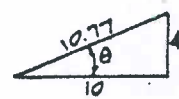


P7.3



$$y = \frac{4h}{L^2} x^2 \quad @ x = 25'; \quad y = \frac{4(20)(25)^2}{1000} = 5'$$

$$dy/dx = 8hx/L^2 \quad @ x = 25'; \quad dy/dx = \frac{8(20)(25)}{(100)^2} = 0.4$$

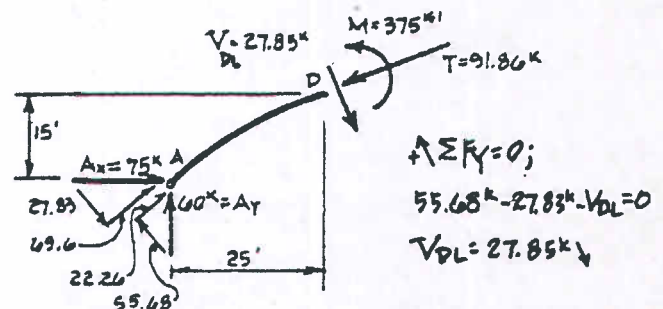


$$\sin \theta = 4/10.77 = 0.371$$

$$\cos \theta = 10/10.77 = 0.928$$

$$\sum M_0 = 0; \quad 60(50') - H(20) - 60(25) = 0 \quad \underline{H = 75 \text{ k}}$$

a.) dx BEFORE THE 1/4 POINT (LEFT OF "D")



$$\sum F_y = 0;$$

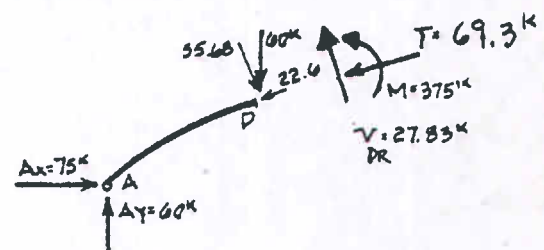
$$55.68 \text{ k} - 27.83 \text{ k} - V_{DL} = 0$$

$$V_{DL} = 27.85 \text{ k}$$

$$\sum M_0 = 0; \quad 60(25') - 75(15') - M = 0 \quad \underline{M = 375 \text{ k-ft}}$$

$$\sum F_x = 0; \quad 69.6 \text{ k} + 22.86 \text{ k} - T = 0 \quad \underline{T = 91.86 \text{ k}}$$

b.) dx AFTER THE 1/4 POINT (RIGHT OF "D")



ADD COMPONENTS OF THE 60 k LOAD @ D TO THE VALUES IN (a).

$$M = 375 \text{ k} \quad (\text{SAME})$$

$$T = 91.86 - 22.6$$

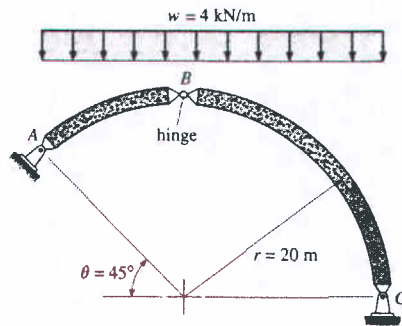
$$V_{DR} = 55.68 - 27.85$$

$$\underline{M = 375 \text{ k}}$$

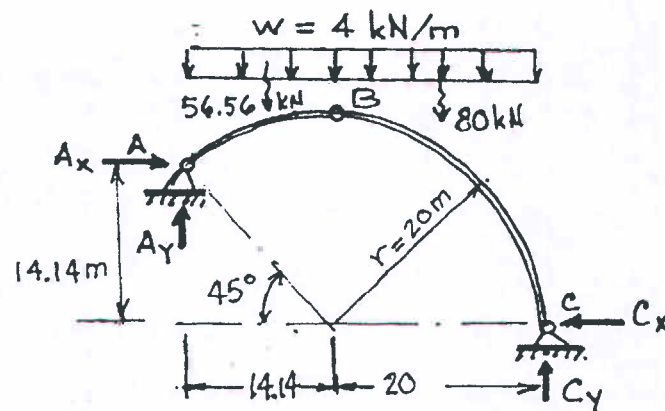
$$\underline{T = 69.3 \text{ k}}$$

$$\underline{V_{DR} = 27.83 \text{ k}}$$

P7.4. Determine the reactions at supports A and C of the three-hinged circular arch.



P7.4



DETERMINE REACTIONS:

EQUATION (1):

$$\sum M_C = 0; A_x(14.14) + A_y(34.14) - 56.56(27.07) - 80(10) = 0$$

EQUATION (2)

$\sum M_B$ FOR SEGMENT AB,

$$\sum M_B = 0; A_y(14.14) - A_x(20 - 14.14) - 56.56(7.07) = 0$$

SOLVE EQ'S (1) AND (2)

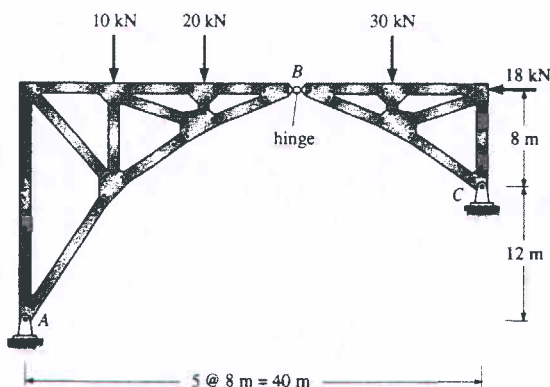
$$A_x = \underline{48.3 \text{ kN}} \rightarrow$$

$$A_y = \underline{48.3 \text{ kN}} \uparrow$$

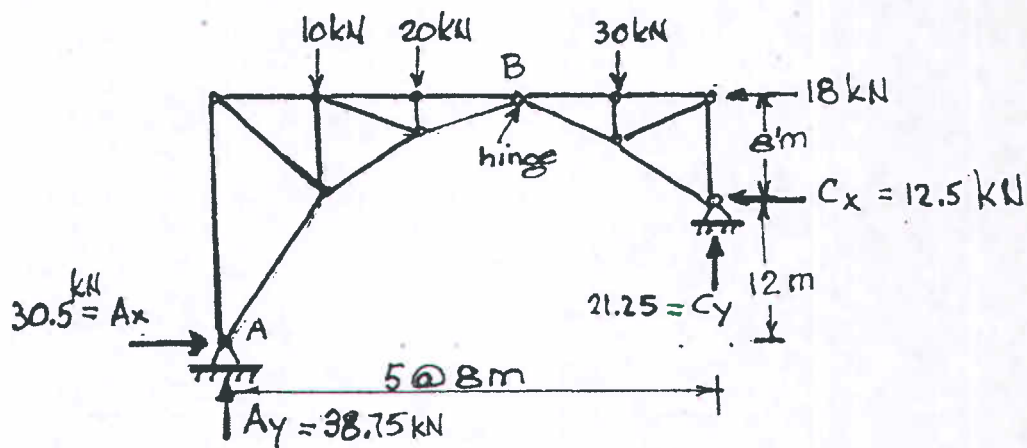
$$\sum F_x = 0; C_x = \underline{48.3 \text{ kN}} \leftarrow$$

$$\sum F_y = 0; C_y = 80 + 56.56 - 48.3 = \underline{88.26 \text{ kN}} \uparrow$$

P7.5. Compute the support reactions for the arch in Figure P7.5. (Hint: You will need two moment equations: Consider the entire free body for one, and a free body of the portion of truss to either the left or right of the hinge at B.)



P7.5



$$\sum M_A = 0 = 10 \times 8 + 20 \times 16 + 30 \times 32 - 18 \times 20 - C_y \times 40 - C_x \times 12 \quad (\text{EQN 1})$$

$$\sum M_B \text{ (FBD TO RIGHT OF HINGE)} = 0$$

$$0 = 30 \times 8 - C_y \times 16 + C_x \times 8 \quad (\text{EQN 2})$$

+ Solving Eq's 1 & 2 GIVES $C_y = 21.25 \text{ kN} \uparrow$; $C_x = 12.5 \text{ kN} \leftarrow$

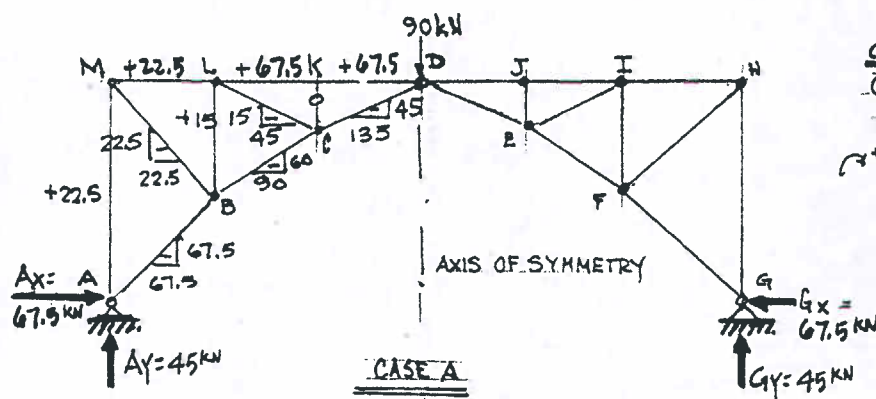
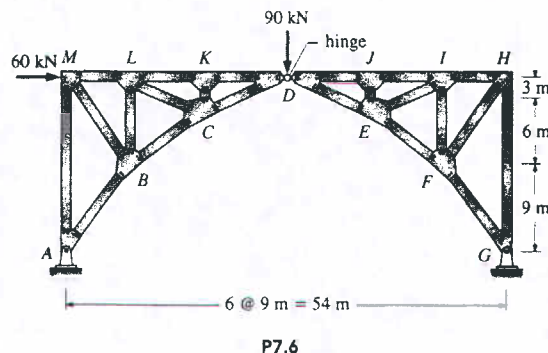
$$\sum F_x = 0 = A_x - 18 - 12.5 \therefore A_x = 30.5 \text{ kN} \rightarrow$$

$$\sum F_y = 0 = 21.25 - 60 + A_y \therefore A_y = 38.75 \text{ kN} \uparrow$$

P7.6. (a) Determine the reactions and all bar forces of the three-hinged, trussed arch P7.6 for the following cases.

Case A: Only the 90-kN force at joint D acts. Case B: Both the 90-kN and 60-kN forces at joints D and M act.

(b) Determine the maximum axial force in the arch in Case B.



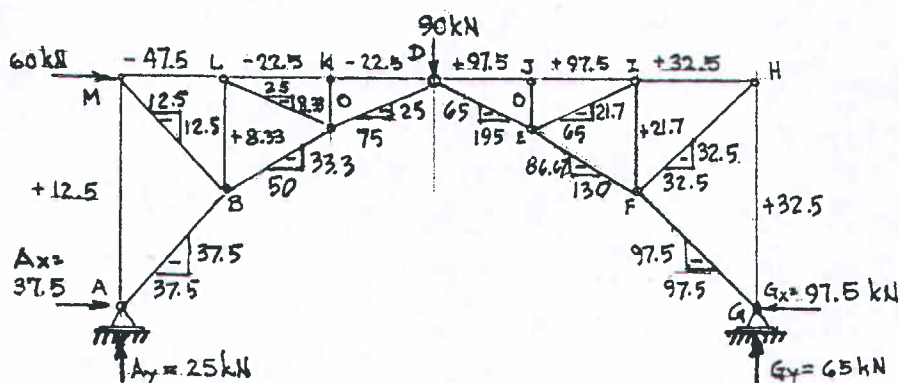
CASE A:

COMPUTE G_x FREE BODY TO RIGHT OF HINGE AT D:

$$\sum M_D = 0 = 45 \times 27 + G_x \times 18 = 0$$

$$G_x = 67.5 \text{ kN} \leftarrow$$

$$A_x = G_x = 67.5 \text{ kN} \rightarrow$$



CASE B:

MAX. AXIAL FORCE (kN)

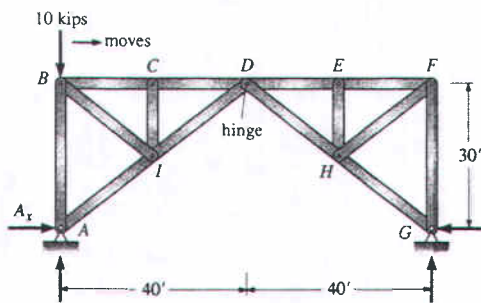
$$F_{DE} = \sqrt{65^2 + 195^2} = 205.55 \text{ kN}$$

$$F_{EF} = \sqrt{86.67^2 + 130^2} = 156.21 \text{ kN}$$

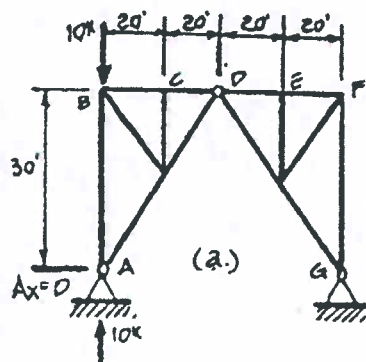
$$F_{FG} = \sqrt{97.5^2 + 97.5^2} = 137.88 \text{ kN}$$

NOTICE: IN A TRUSSED ARCH THE AXIAL FORCE IN THE ARCH RIB INCREASES AS WE MOVE UPWARD TO THE CROWN AT THE TOP HINGE.

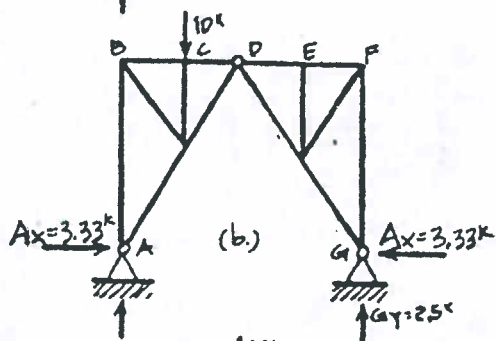
P7.7. (a) In Figure P7.7 compute the horizontal reaction A_x at support A for a 10-kip load at joint B. (b) Repeat the computation if the 10-kip load is also located at joints C and D respectively.



P7.7



(a) $A_x = 0$; NO ARCH ACTION



(b) ENTIRE ARCH

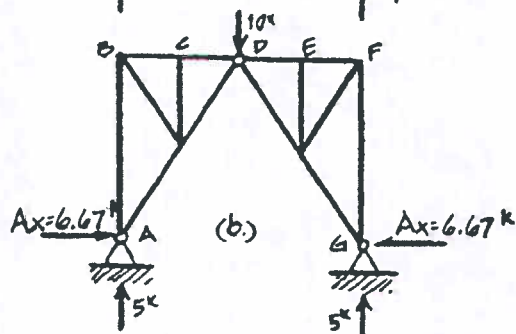
$$\sum \vec{M}_A = 0; \quad 10^k(20') - G_y(80') = 0$$

$$G_y = 2.5^k$$

FBD TO THE RIGHT OF D

$$\sum \vec{M}_D = 0; \quad A_x(30') - 2.5^k(40') = 0$$

$$A_x = 3.33^k$$

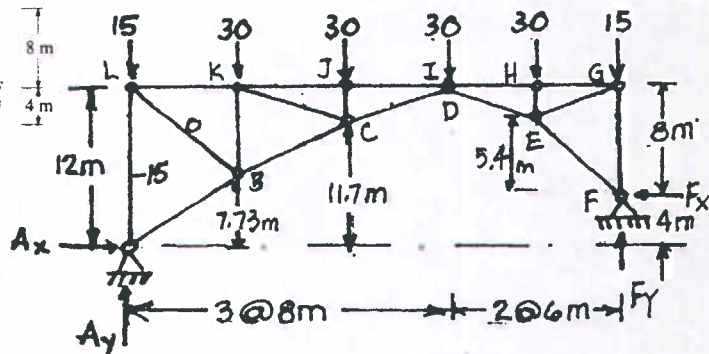
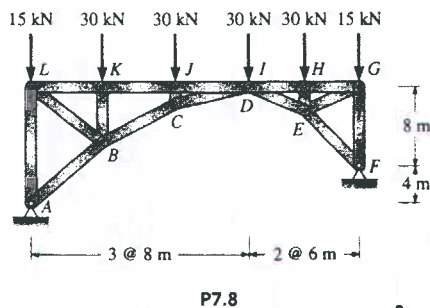


(b) F.B.D. TO THE RIGHT OF D

$$\sum \vec{M}_D = 0 \quad 5^k(40') - A_x(30') = 0$$

$$A_x = 6.67^k$$

P7.8. For the arch rib to be funicular for the dead loads shown, establish the elevation of the lower chord joints B, C, and E.



REACTIONS

$$\sum M_A = 0 = 30(8+16+24+30) + 15 \times 36 - F_y 36 - 4 F_x \quad (1)$$

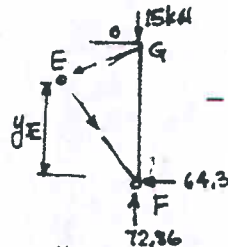
$$\sum M_D = 0 = 30 \times 6 + 15 \times 12 + F_x 8 - F_y (12) = 0 \quad (2)$$

SOLVING EQ'S 1 & 2 $F_x = \underline{64.3 \text{ kN}}$, $F_y = \underline{72.86 \text{ kN}}$

$$\sum F_x = 0: A_x = F_x = \underline{64.3 \text{ kN}}$$

$$\sum F_y = 0: A_y = \underline{77.14 \text{ kN}}$$

AT JOINT E

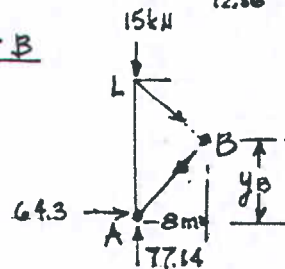


$$\sum M_E = 0;$$

$$- (72.86 - 15) 6 + 64.3 y_E = 0$$

$$y_E = \underline{5.4 \text{ m}}$$

JOINT B

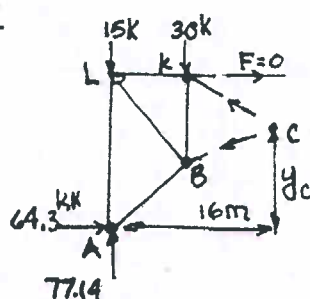


$$\sum M_B = 0;$$

$$77.14(8) - 64.3(y_B) - 15(8) = 0$$

$$y_B = \underline{7.73 \text{ m}}$$

JOINT C

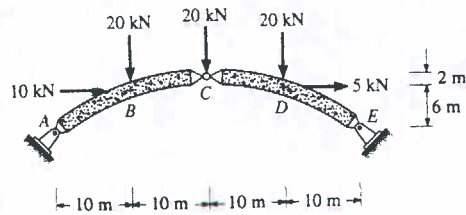


$$\sum M_C = 0;$$

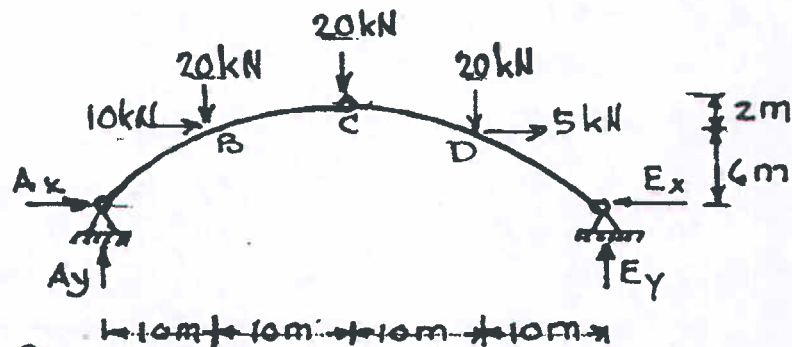
$$(77.14 - 15) 16 - (64.3) y_C - 30(8) = 0$$

$$y_C = \underline{11.7 \text{ m}}$$

P7.9. Determine the reactions at supports A and E of the three-hinged arch in Figure P7.9.



P7.9



$$\sum M_A = 0: 20(10 + 20 + 30) + 6(10 + 5) - 40E_y = 0$$

$$E_y = \underline{\underline{32.25 \text{ kN}}}$$

$$\sum M_C = 0; \text{ RIGHT } 20 \times 10 - 5 \times 2 - \underset{32.25}{E_y} 20 + E_x 8 = 0$$

$$E_x = \underline{\underline{56.875 \text{ kN}}}$$

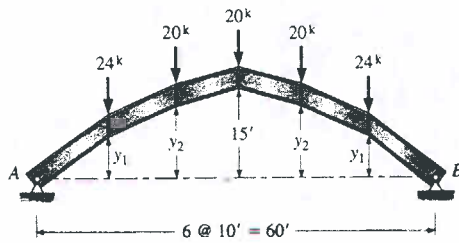
$$\sum F_x = 0; A_x + 10 + 5 - 56.875 = 0$$

$$A_x = \underline{\underline{41.875 \text{ kN}}}$$

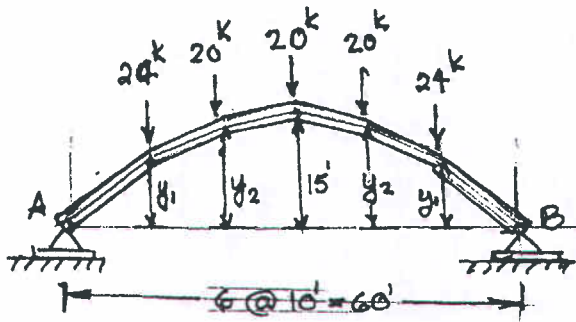
$$\sum F_y = 0; A_y - 60 + 32.25 = 0$$

$$A_y = \underline{\underline{27.75 \text{ kN}}}$$

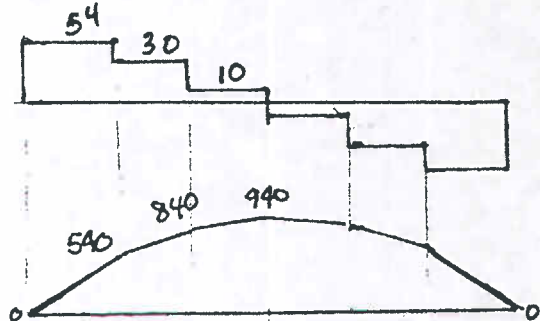
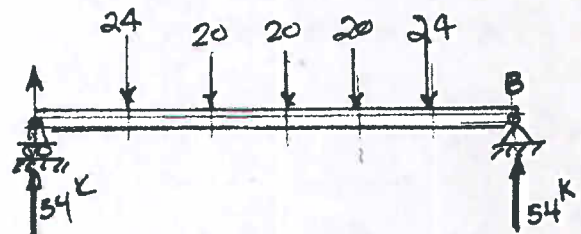
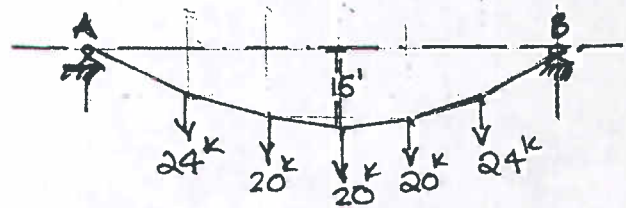
P7.10. Establish the funicular arch for the system of loads in Figure P7.10.



P7.10



USE CABLE THEORY



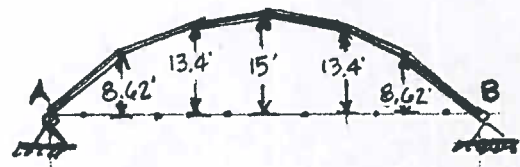
$$Hy = M$$

$$\text{at } H \cdot 15 = 940$$

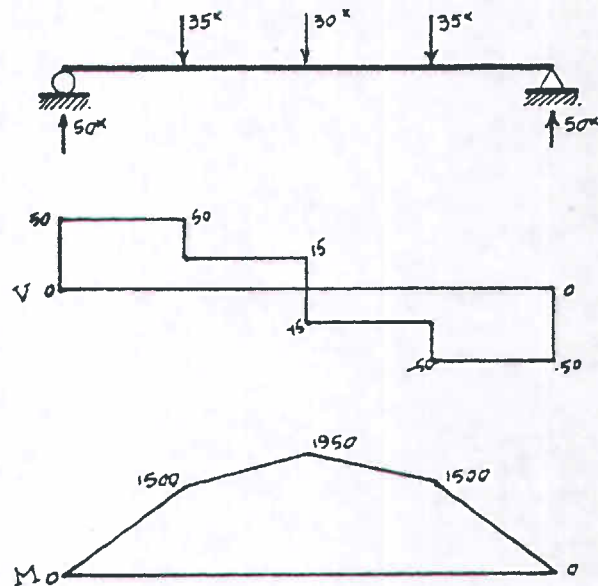
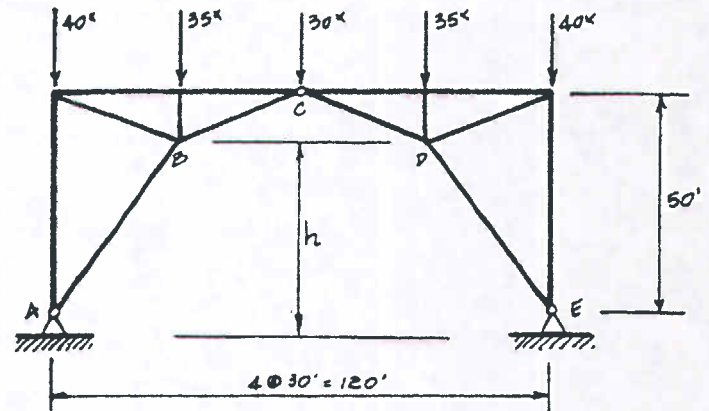
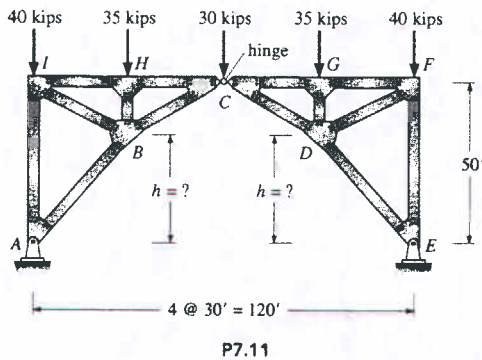
$$H = 62.67$$

$$\text{COMPUTE } y_1 \quad Hy_1 = 540 \quad \therefore y_1 = 8.62 \text{ ft}$$

$$\text{COMPUTE } y_2 \quad Hy_2 = 840 \quad y_2 = 13.40 \text{ ft}$$



P7.11. If the arch rib *ABCDE* in Figure P7.11 is to be funicular for the dead loads shown at the top joints, establish the elevation of the lower chord joints at *B* and *D*.



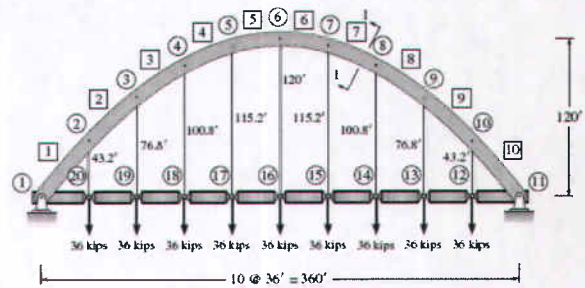
COMPUTE *H* $H \cdot Y_E = M$
 $H \cdot 50 = 1950$
 $H = 39 \text{ k}$

@ *B* AND *D* $H \cdot h = M$
 $39h = 1500$
 $h = 38.46'$

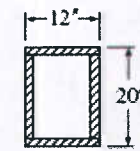
P7.12. Computer study of a two-hinged arch. The objective is to establish the difference in response of a parabolic arch to (1) uniformly distributed loads and (2) a single concentrated load.

(a) The arch in Figure P7.12 supports a roadway consisting of simply supported beams connected to the arch by high-strength cables with area $A = 2 \text{ in}^2$ and $E = 26,000 \text{ ksi}$. (Each cable transmits a dead load from the beams of 36 kips to the arch.) Determine the reactions; the axial force, shear, and moment at each joint of the arch; and the joint displacements. Plot the deflected shape. Represent the arch by a series of straight segments between joints. The arch has a constant cross section with $A = 24 \text{ in}^2$, $I = 2654 \text{ in}^4$, and $E = 29,000 \text{ ksi}$.

(b) Repeat the analysis of the arch if a single 48-kips vertical load acts downward at joint 18. Again, determine all the forces acting at each joint of the arch, the joint displacements, etc. and compare results with those in (a). Briefly describe the difference in behavior.



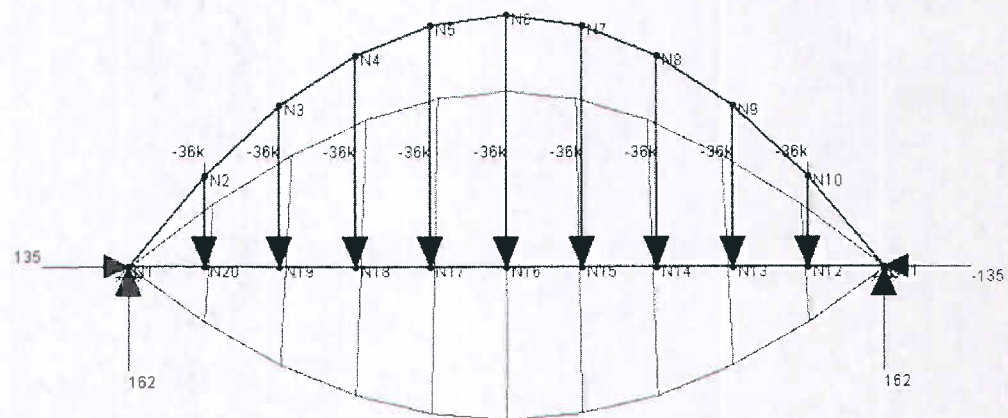
P7.12



Section 1-1

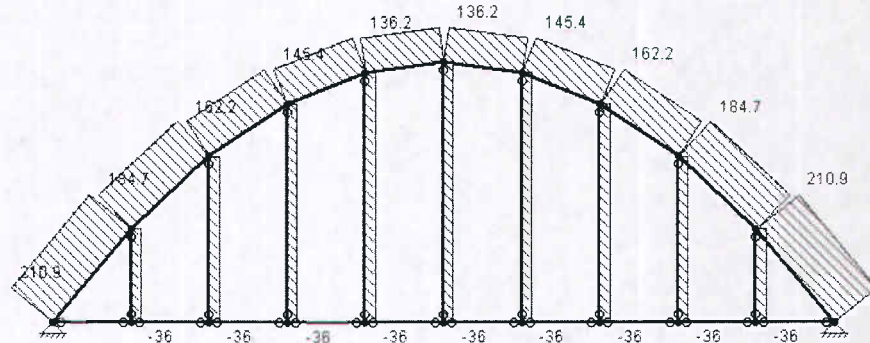
(a) Uniformly distributed dead load only

Reactions (kips),
Deflections (in),
and Deflected
Shape

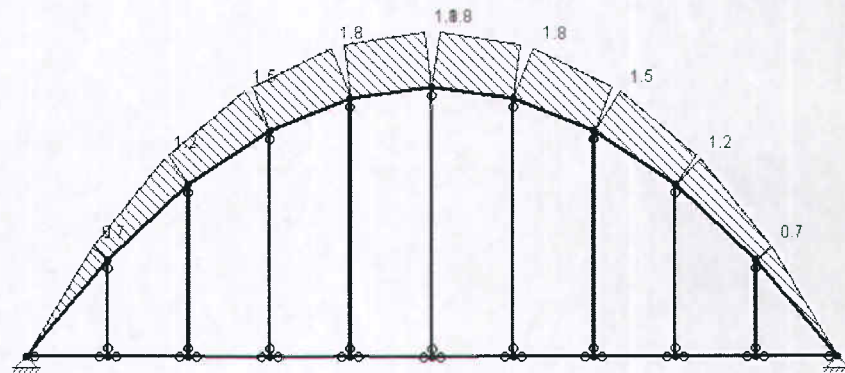


Joint Deflections			
	Joint Label	X [in]	Y [in]
1	N1	0	0
2	N2	.108	-.356
3	N3	.154	-.635
4	N4	.139	-.832
5	N5	.081	-.947
6	N6	0	-.985
7	N7	-.081	-.947
8	N8	-.139	-.832
9	N9	-.154	-.635
10	N10	-.108	-.356
11	N11	0	0
12	N12	0	-.715
13	N13	0	-1.273
14	N14	0	-1.669
15	N15	0	-1.904
16	N16	0	-1.982
17	N17	0	-1.904
18	N18	0	-1.669
19	N19	0	-1.273
20	N20	0	-.715

Axial Force
Diagram (kips)



Moment Diagram
(kip-ft)



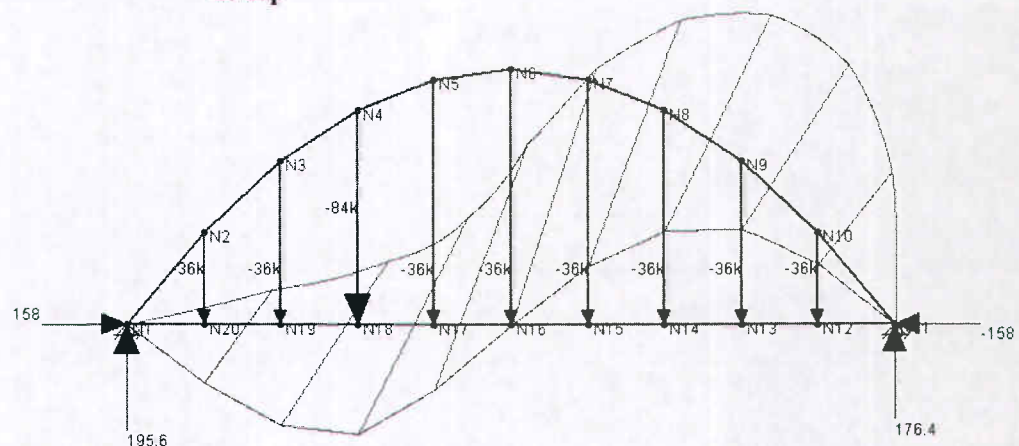
Observation

A continuous uniformly distributed load, acting on a parabolic arch will produce a resultant axial compression force acting at the centroid of all cross sections. While the axial force, and the direct stresses it produces, will shorten the arch, the changes in length of the arch axis and the vertical deflection of points along the arch axis will typically be small if the magnitude of the direct stresses are in the elastic range of the material. In the case of the arch in Fig. P7.12, the nine equally spaced loads of 36 kips approximate a uniformly distributed load that will produce a uniform compressive stress on all cross sections of the arch (see the plot of axial forces and moments on page 7-12B).

The magnitude of joint deflections is listed in the table on page 7.12A. At the level of the bridge deck, the vertical deflection of the floor system will vary with the length of the cable. In practice, the deflection of the floor system due to the dead weight of the bridge can be eliminated by shortening the length of the vertical cables that support the floor system (joints 12 to 20). This will eliminate the appearance of a sagging roadway. For example, if the vertical deflections of the floor system at joints 12 through 20 produced by the 36 kip loads is to be eliminated, the cables can be shortened by the displacements tabulated in column Y in the table of joint deflections on Page 7-12A. These deflections range from approximately 3/4 inches near the supports to nearly 2 inches at midspan where the cables are longest.

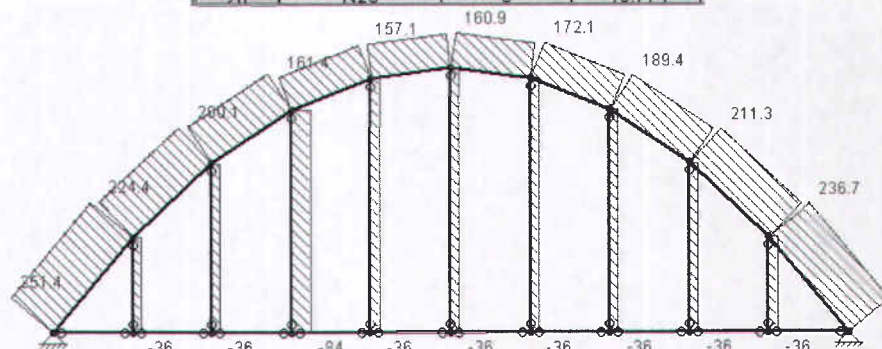
(b) Uniformly distributed dead load + 48-kip live load

Reactions (kips),
Deflections (in),
and Deflected
Shape

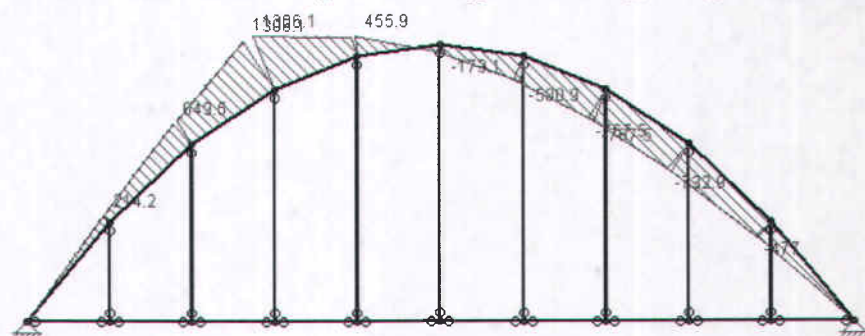


Joint Deflections			
	Joint Label	X [in]	Y [in]
1	N1	0	0
2	N2	48.118	-40.416
3	N3	75.823	-70.379
4	N4	78.774	-75.074
5	N5	66.504	-44.69
6	N6	60.323	.919
7	N7	65.773	42.555
8	N8	75.672	67.613
9	N9	76.051	68.436
10	N10	52.439	43.4
11	N11	0	0
12	N12	0	43.041
13	N13	0	67.798
14	N14	0	66.775
15	N15	0	41.598
16	N16	0	-.078
17	N17	0	-45.647
18	N18	0	-77.028
19	N19	0	-71.017
20	N20	0	-40.774

Axial Force
Diagram (kips)



Moment Diagram
(kip-ft)

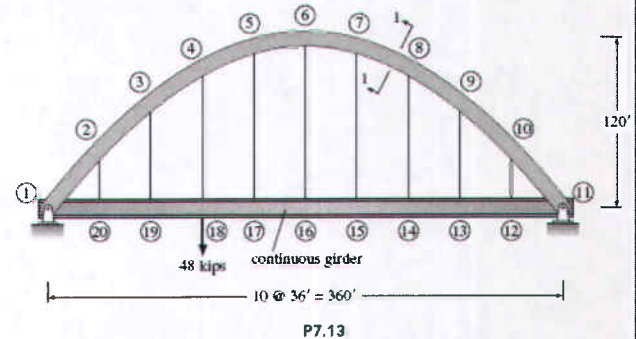


Observation:

The addition of the 48-kip concentrated load at joint 18 will produce large values of bending moment in the arch and large deflections (over 6 feet at some joints) in both the vertical and horizontal directions. Such large deflections are obviously not acceptable and would result in damage to the bridge and its deck as well as produce discomfort for people driving over the excessively flexible bridge. To reduce deflections, the designer has several choices:

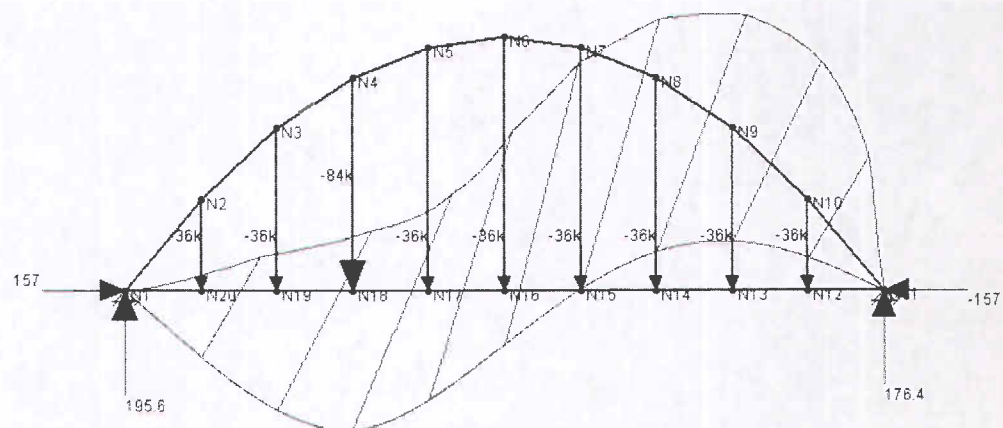
1. A deep continuous bridge girder can be used (see Fig. P7.13). The effect of a deep stiff continuous girder is to spread the concentrated load longitudinally to adjacent vertical cables, thereby applying a more uniformly distributed load to the arch. The effectiveness of this strengthening scheme is demonstrated in P7.13.
2. The arch can be stiffened by incorporating a truss. See, for example, page 240 of the textbook; the highway bridge is composed of an arch, which also forms the lower chord of a truss composed of diagonal members and the girders of the roadway. The arch carries most of the dead load and the deep, stiff trussed-arch carries most of the forces produced by the vehicles passing over the bridge. The Bayonne Bridge, one of the longest in the world, is also stiffened by a truss (see page 602 of the text).

P7.13. Computer study of arch with a continuous floor girder. Repeat part (b) in problem P7.12 if a continuous girder with $A = 102.5 \text{ in}^2$ and $I = 40,087 \text{ in}^4$, as shown in Figure P7.13, is provided to support the floor system. For both the girder and the arch, determine all forces acting on the arch joints as well as the joint displacements. Discuss the results of your study of P7.12 and P7.13 with particular emphasis on the magnitude of the forces and displacements produced by the 48-kip load.



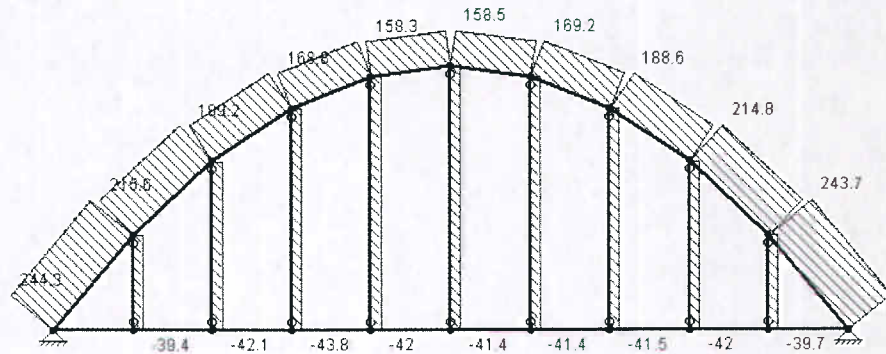
Uniformly distributed dead load + 48-kip live load

Reactions (kips),
Deflections (in),
and Deflected
Shape

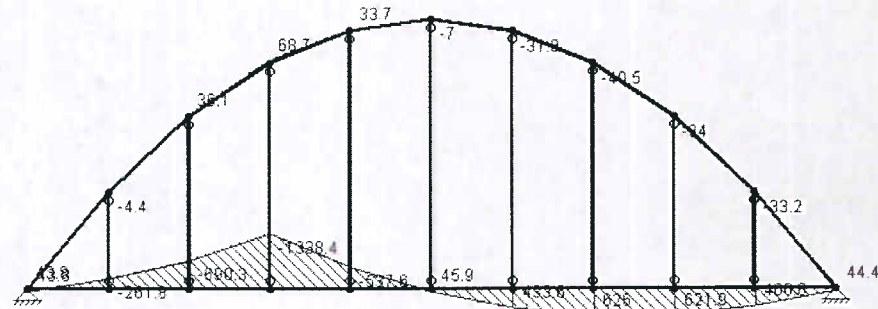


Joint Deflections			
	Joint Label	X [in]	Y [in]
1	N1	0	0
2	N2	2.505	-2.396
3	N3	4.021	-4.288
4	N4	4.233	-4.861
5	N5	3.564	-3.493
6	N6	3.144	-1.089
7	N7	3.351	1.212
8	N8	3.804	2.651
9	N9	3.777	2.865
10	N10	2.553	1.82
11	N11	0	0
12	N12	0	1.424
13	N13	0	2.12
14	N14	0	1.687
15	N15	0	.111
16	N16	0	-2.237
17	N17	0	-4.611
18	N18	0	-5.879
19	N19	0	-5.034
20	N20	0	-2.789

Axial Force
Diagram (kips)

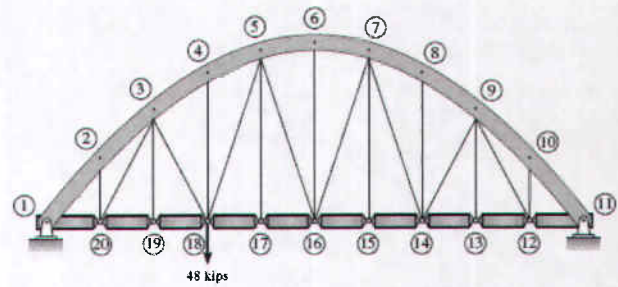


Moment Diagram
(kip-ft)



The structure is composed of a deep, stiff, continuous girder, which is supported by pin supports at each end as well as by vertical cables that transmit roadway loads to the supporting arch. Loads applied to the roadway have two paths to the end supports. They can be transmitted directly by the continuous girder to the supports or they can be carried upward to the arch by the cables and then transmitted through the arch to the end supports. Since all vertical cables carry a portion of the live load applied to the stiff continuous girder, the distribution of forces to the arch is more uniform, which in turn creates a more uniform stress distribution on the arch's cross section. As a result, the arch undergoes less deformation and deflections of the floor system are significantly reduced. Photo 3.2 in Chapter 3, (page 82 of the text) shows the end details of the Barracks Bridge over the Mississippi River—a 909 ft-long, 280 ft high arch-continuous beam system that uses a 12'-5" deep continuous I-shaped plate girder supported by 17 hangers.

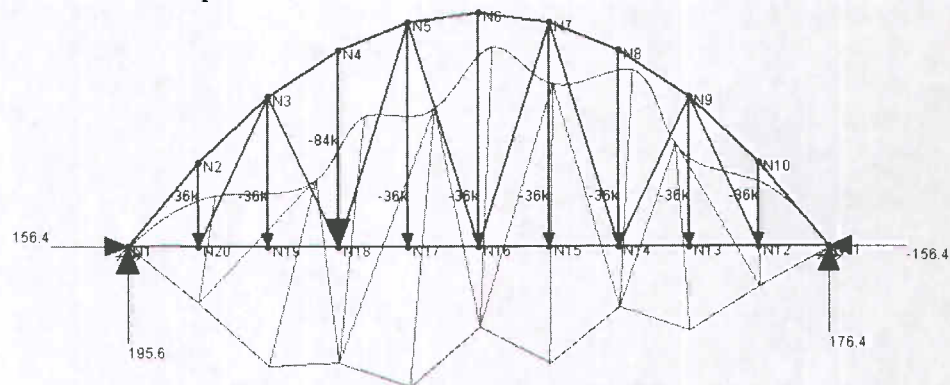
P7.14. To reduce the vertical displacement of the roadway floor system of the arch (shown in P7.12, part *b*) produced by the 48-kip load at joint 18, diagonal cables of 2-in. in diameter are added as shown in Figure P7.14. For this configuration, determine the vertical displacement of all the floor system joints. Compare the results of this analysis with part *b* of P7.12 by plotting to scale the vertical deflections of all joints along the roadway from joints 1 to 11. Properties of the diagonal cables are the same as those of the vertical cables.



P7.14

Uniformly distributed dead load + 48-kip live load

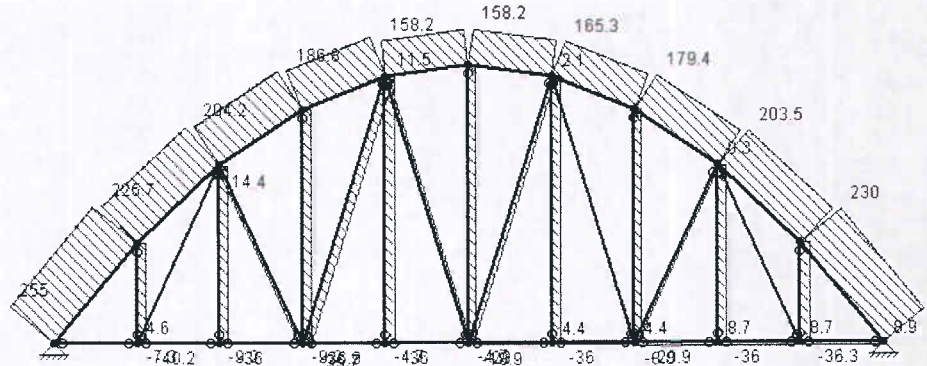
Reactions (kips),
Deflections (in),
and Deflected
Shape



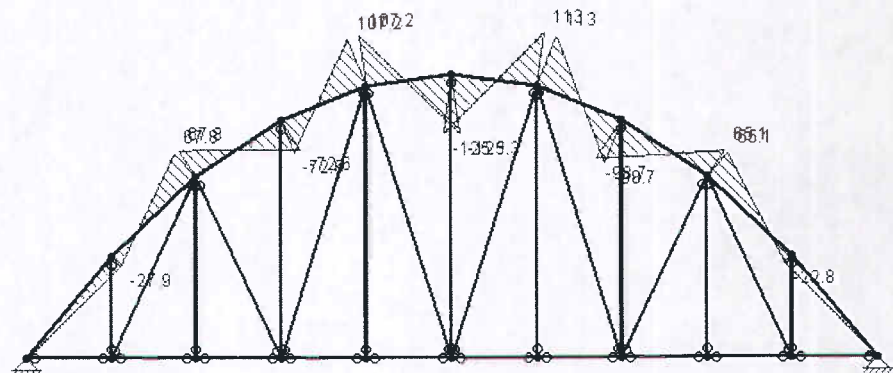
Joint Deflections

	Joint Label	X [in]	Y [in]
1	N1	0	0
2	N2	.286	-.561
3	N3	.814	-1.406
4	N4	.443	-1.126
5	N5	.442	-1.457
6	N6	.224	-.578
7	N7	.062	-1.048
8	N8	.23	-.331
9	N9	-.239	-.792
10	N10	-.029	-.314
11	N11	0	0
12	N12	.001	-.676
13	N13	.003	-1.43
14	N14	.004	-1.028
15	N15	.004	-2.005
16	N16	.005	-1.38
17	N17	.004	-2.414
18	N18	.004	-1.984
19	N19	.002	-2.044
20	N20	.001	-.961

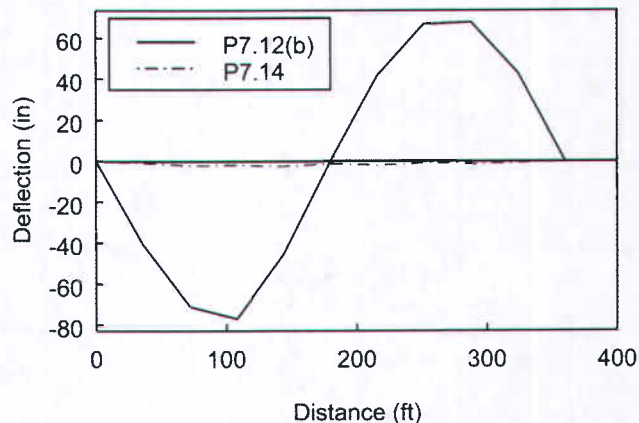
Axial Force
Diagram (kips)



Moment Diagram
(kip-ft)



Comparison of
roadway vertical
deflections



An alternate modification to reduce deflections of the floor system is to add diagonal cables extending from the floor system to the arch as shown in Fig P7.14. The addition of diagonal cables produces a very effective stiffening system. The diagonal cables have two effects. At alternate joints, floor load is distributed to three points along the arch axis, thereby reducing joint deflections significantly in both the horizontal and vertical directions. Also the diagonal cables reduce the lateral displacement of the arch, producing a more uniform distribution of stress on the arch's cross section. For older bridges constructed at a time when highway loads were much smaller, the addition of cables is an economical way to increase the capacity of a bridge and also reduce deflections and vibrations produced by moving traffic.