

FUNDAMENTALS OF STRUCTURAL ANALYSIS

3rd Edition

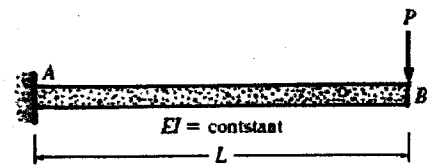
Kenneth M. Leet, Chia-Ming Uang, and Anne M. Gilbert

SOLUTIONS MANUAL

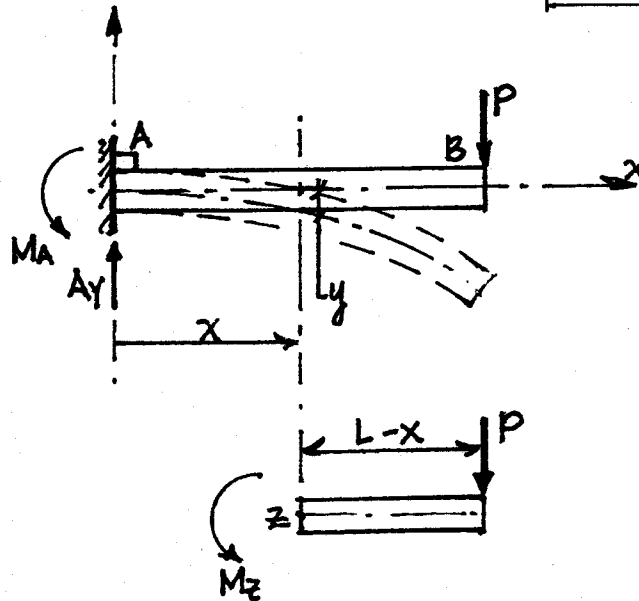
CHAPTER 9:

DEFLECTIONS OF BEAMS AND FRAMES

P9.1. Derive the equations for slope and deflection for the cantilever beam in Figure P9.1. Compute the slope and deflection at B. Express answer in terms of EI .



P9.1



$$M_z = P(L-x)$$

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} = -\frac{P(L-x)}{EI} = -\frac{PL}{EI} + \frac{Px}{EI}$$

(CONCAVE DOWN)

$$\theta = \frac{dy}{dx} = -\frac{PLx}{EI} + \frac{Px^2}{2EI} + C_1$$

$$y = -\frac{PLx^2}{2EI} + \frac{Px^3}{6EI} + C_1x + C_2$$

EVALUATE CONSTANTS:

$$x=0: \quad y=0: \quad C_2=0$$

$$x=0: \quad \theta = \frac{dy}{dx} = 0: \quad C_1=0$$

@ B $x=L$

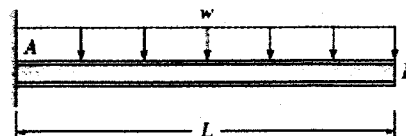
$$\theta_B = -\frac{PL^2}{EI} + \frac{PL^2}{2EI}$$

$$\underline{\underline{\theta_B = -PL^2/2EI}}$$

$$y = -\frac{PL^3}{2EI} + \frac{PL^3}{6EI}$$

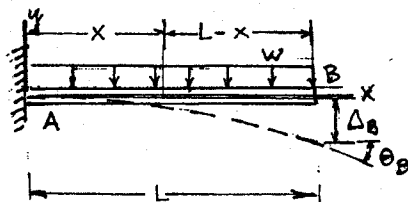
$$\underline{\underline{y = -PL^3/3EI}}$$

P9.2. Derive the equations for slope and deflection for the beam in Figure P9.2. Compare the deflection at B with the deflection at midspan.



P9.2

ANALYSIS BY DOUBLE INTEGRATION



$$\frac{w(L-x)^2}{2} = M$$

$$\frac{d^2 y}{dx^2} = \frac{M}{EI} = -\frac{w}{2EI} (L^2 - 2xL + x^2)$$

$$2EI \frac{d^2 y}{dx^2} = -wL^2 + 2wxL - wx^2$$

$$2EI \frac{dy}{dx} = -wL^2 x + \frac{2wLx^2}{2} - \frac{wx^3}{3} + C_1 \quad (1)$$

$$\text{AT } x=0, \frac{dy}{dx} = 0 \therefore C_1 = 0$$

$$2EI y = -\frac{wL^2 x^2}{2} + \frac{wLx^3}{3} - \frac{wx^4}{12} + C_2 \quad (2)$$

$$\text{AT } x=0, y = 0 \therefore C_2 = 0$$

COMPUTE Δ_B ; SET $x=L$ IN EQ(2)

$$\Delta_B = y = \frac{1}{2EI} \left(-\frac{wL^4}{2} + \frac{wL^4}{3} - \frac{wL^4}{12} \right)$$

$$\Delta_B = -\frac{wL^4}{8EI} \downarrow$$

COMPUTE θ_B ; SET $x=L$ IN EQ 1

$$\theta_B = \frac{1}{2EI} \left(-wL^3 + wL^3 - \frac{wL^3}{3} \right)$$

$$\theta_B = \frac{wL^3}{2EI} \left(-1 + 1 - \frac{1}{3} \right) = -\frac{wL^3}{6EI}$$

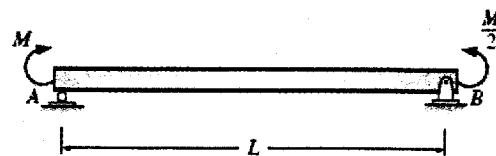
$$\theta_B = -\frac{wL^3}{6EI}$$

Δ at $L/2$: , SET $x=L/2$ IN EQ 2

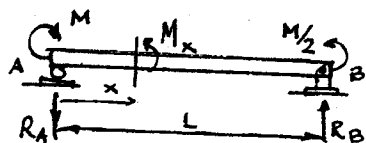
$$\Delta = -\frac{17wL^4}{384EI} \downarrow = -\frac{wL^4}{22.59}$$

$$\text{COMPARE } \frac{\Delta \text{ at } L/2}{\Delta \text{ at } B} = \frac{1/22.59}{1/8} = 0.35$$

P9.3. Derive the equations for slope and deflection for the beam in Figure P9.3. Compute the maximum deflection.
Hint: Maximum deflection occurs at point of zero slope.



P9.3



compute R_A : $\sum M_B = 0$

$$0 = M - M/2 - R_A L$$

$$R_A = M/2L$$

EVALUATE M_x : $\sum M$

$$M_x = M - R_A x = M - \frac{M}{2L} x$$

$$\frac{d^2 y}{dx^2} = \frac{M_x}{EI} = \left(M - \frac{M}{2L} x \right) \frac{1}{EI} \quad (1)$$

$$EI \frac{dy}{dx} = Mx - \frac{M}{2L} \frac{x^2}{2} + C_1 \quad (2)$$

$$EI y = \frac{Mx^2}{2} - \frac{Mx^3}{12L} + C_1 x + C_2 \quad (3)$$

Substi $y=0$ @ $x=0$ in Eq(3)

$$0 = 0 - 0 + 0 + C_2$$

$$\therefore C_2 = 0$$

Substi $y=0$ @ $x=L$ in Eq 3

$$0 = \frac{ML^2}{2} - \frac{ML^2}{12} + C_1 L$$

$$C_1 = -\frac{5}{12} ML$$

$$EI \frac{dy}{dx} = Mx - \frac{Mx^2}{4L} - \frac{5}{12} ML \quad (2a)$$

$$EI y = \frac{Mx^2}{2} - \frac{Mx^3}{12L} - \frac{5}{12} MLx \quad (3a)$$

COMPUTE Δ_{max} : SET $\frac{dy}{dx} = 0$ in Eq 2a
to locate position Δ_{max}

$$0 = Mx - \frac{Mx^2}{4L} - \frac{5}{12} ML$$

$$x^2 - 4Lx = -\frac{5}{12} L \quad (4L^2)$$

$$(x-2L)^2 = \frac{7}{3} L^2$$

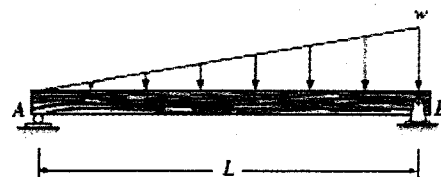
$$x = 0.4725L$$

SUBSTI $x = 0.4725L$ INTO (EQ. 3a)

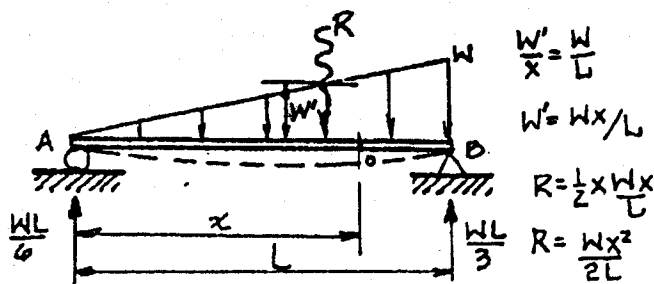
$$\Delta_{max} = y = \frac{M}{EI} \left[\frac{(0.4725L)^2}{2} - \frac{(0.4725L)^3}{12L} - \frac{5L(0.4725L)}{12} \right]$$

$$= \frac{-0.094 ML^2}{EI}$$

P9.4. Derive the equations for slope and deflection for the beam in Figure P9.4. Locate the point of maximum deflection and compute its magnitude.



P9.4



$$\begin{aligned} \frac{W'}{x} &= \frac{W}{L} \\ W' &= Wx/L \\ R &= \frac{1}{2}x \frac{Wx}{L} \\ R &= \frac{Wx^2}{2L} \end{aligned}$$

SEGMENT AB

$$\sum M_A = 0; \quad \frac{WL}{6}(x) - \frac{Wx^2}{2L}\left(\frac{x}{3}\right) - M = 0$$

$$M = \frac{WLx}{6} - \frac{Wx^3}{6L}$$

$$d^2y/dx^2 = M/EI$$

$$EI d^2y/dx^2 = WLx/6 - Wx^3/6L$$

$$EI dy/dx = WLx^2/12 - Wx^4/24L + C_1 \quad (1)$$

$$EI y = WLx^3/36 - Wx^5/120L + C_1x + C_2 \quad (2)$$

BOUNDARY CONDITIONS

B.C. 1 $x=0; y=0$; SUBST INTO (2) $C_2 = 0$

B.C. 2 $x=L; y=0$ SUBST. INTO (2) $C_1 = -\frac{7WL^3}{360}$

DET. δ_{MAX} ; $\theta = 0$, USING EQN (1)

$$0 = EI dy/dx = 0 = \frac{WLx^2}{12} - \frac{Wx^4}{24L} - \frac{7WL^3}{360}$$

TRIAL METHOD: GUESS $x = 0.5L$

$$0 = 0.0208 - 0.0026 - 0.0194 = -0.0012 \quad \text{CLOSE}$$

TRY $x = 0.52L$

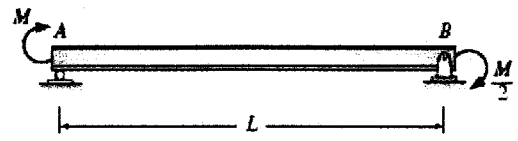
$$0 = 0.0225 - 0.003 - 0.0194 = +0.0001 \quad \text{CLOSE ENOUGH}$$

SUBSTITUTE $0.52L$ INTO EQN (2)

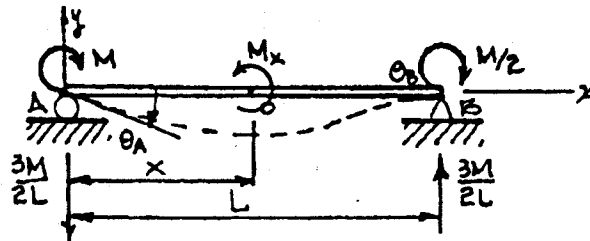
$$\delta_{MAX} = y_{MAX} = \frac{WL(0.52L)^3}{36EI} - \frac{W(0.52L)^5}{120LEI} - \frac{7WL^3(0.52L)}{360EI}$$

$$y_{MAX} = \frac{-0.0065WL^4}{EI}$$

P9.5. Establish the equations for slope and deflection for the beam in Figure P9.5. Evaluate the magnitude of the slope at each support. Express answer in terms of EI .



P9.5



SEGMENT AO

$$\sum M_o = 0; -\frac{3M}{2L}(x) + M - M_x = 0$$

$$M_x = M - \frac{3Mx}{2L}$$

$$d^2y/dx^2 = M_y/EI$$

$$EI d^2y/dx^2 = M - \frac{3Mx}{2L}$$

$$EI dy/dx = Mx - \frac{3Mx^2}{4L} + C_1 \quad (1)$$

$$EI y = \frac{Mx^2}{2} - \frac{Mx^3}{4L} + C_1x + C_2 \quad (2)$$

BOUNDARY CONDITIONS

B.C.1 $x=0; y=0$; SUBST. INTO (2)

$$C_2 = 0$$

B.C.2 $x=L; y=0$; SUBST. INTO (2)

$$C_1 = -\frac{ML}{4}$$

$$\theta = dy/dx = \frac{Mx}{EI} - \frac{3Mx^2}{4LEI} - \frac{ML}{4EI}$$

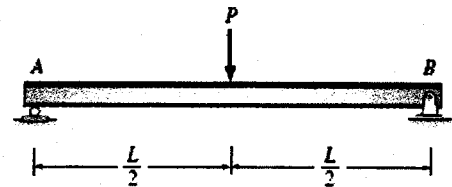
$\theta_A; x=0; \theta_A = 0 - 0 - \frac{ML}{4EI}$

$$\theta_A = -\frac{ML}{4EI}$$

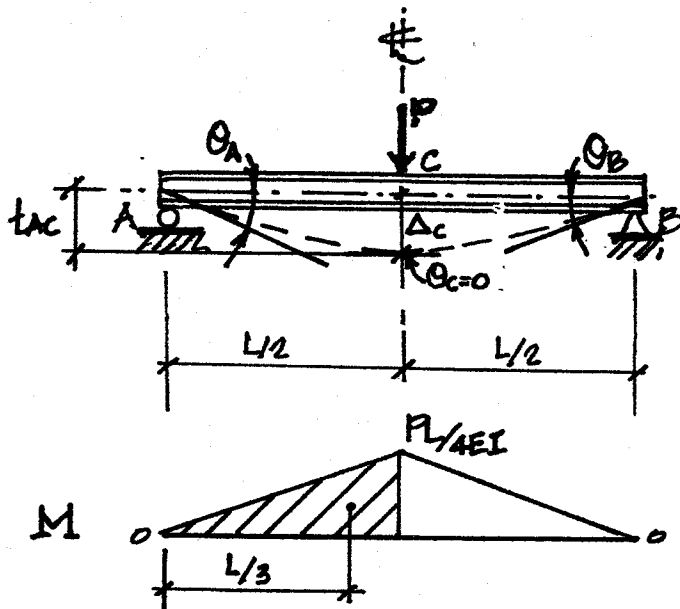
$\theta_B; x=L; \theta_B = \frac{ML}{EI} - \frac{3ML}{4EI} - \frac{ML}{4EI}$

$$\theta_B = 0$$

P9.6. Derive the equations for slope and deflection for the beam in Figure P9.6. Determine the slope at each support and the value of the deflection at midspan. (Hint: Take advantage of symmetry; slope is zero at midspan.)



P9.6



UNKNOWN θ_A
KNOWN $\theta_B = -\theta_A$ & $\theta_C = 0$

$$\theta_A = \theta_C + \Delta\theta_{AC}$$

$$\theta_C = 0$$

$$\Delta\theta_{AC} = \frac{PL}{4EI} \left(\frac{L}{2}\right) \frac{1}{2} = \frac{PL^2}{16EI}$$

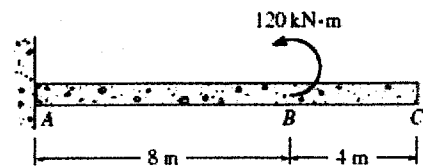
$$\theta_A = + \frac{PL^2}{16EI}$$

$$\theta_B = - \frac{PL^2}{16EI}$$

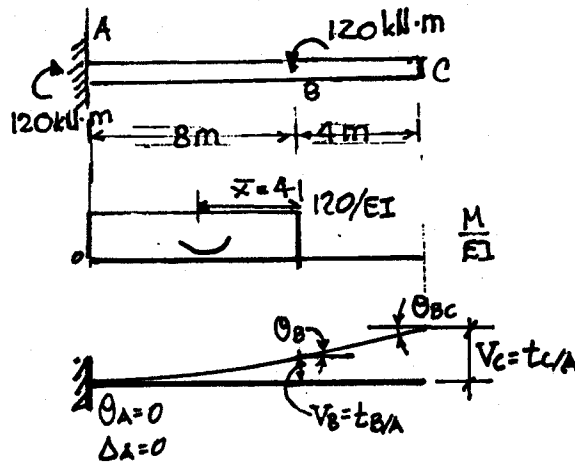
$$\Delta_c = t_{AC} = \frac{1}{2} \left(\frac{PL}{4EI} \right) \left(\frac{L}{2} \right) \left(\frac{L}{3} \right)$$

$$\Delta_c = \frac{PL^3}{48EI}$$

P9.7. Compute the slope and deflection at points B and C in Figure P9.7.



P9.7



BEAM STRAIGHT BETWEEN B-C

$$\theta_B = \theta_C = \theta_A + \Delta\theta_{A-B}$$

$$= 0 + \frac{120 \times 8}{EI}$$

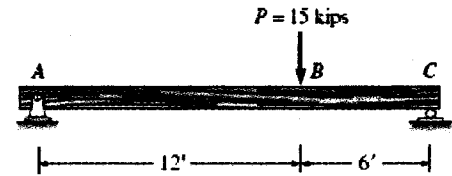
$$\theta_B = \theta_C = \frac{960}{EI}$$

$$V_B = t_{BA} = 8 \left(\frac{120}{EI} \right) 4 = \frac{3840}{EI} \uparrow$$

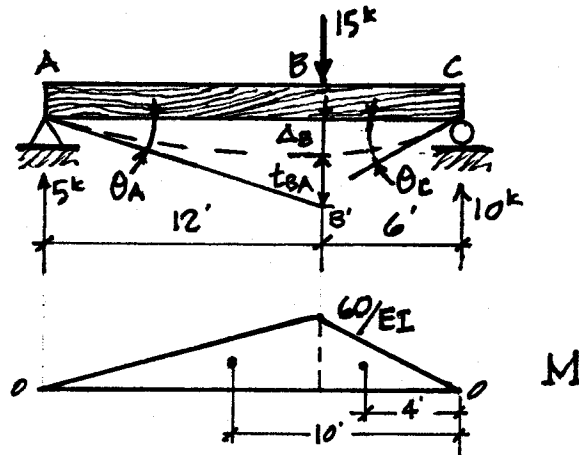
$$V_C = t_{CA} = 8 \left(\frac{120}{EI} \right) (4+4)$$

$$V_C = \frac{7680}{EI} \uparrow$$

P9.8. (a) Compute the slope at A and C and the deflection at B in Figure P9.8. (b) Locate and compute the magnitude of the maximum deflection.



P9.8



SLOPES @ A & C

$$\theta_A = t_{CA} / L$$

$$t_{CA} = \frac{1}{2EI} (60) (12' \cdot 10' + 6' \cdot 4') = \frac{4320}{EI}$$

$$\theta_A = \frac{4320/EI}{18'} = \underline{\underline{-240/EI \text{ (RADIAN)}}} \quad \triangle \theta_A$$

$$\theta_C = \theta_A + \Delta \theta_{AC}$$

$$= \frac{-240}{EI} + \frac{1}{2EI} (60) (18') = \underline{\underline{300/EI \text{ (RADIAN)}}}$$

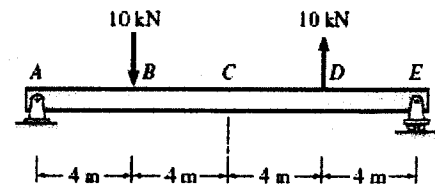
DEFLECTION @ B: $\Delta_B = BB' - t_{BA}$

$$BB' = \theta_A (12') = 2880/EI$$

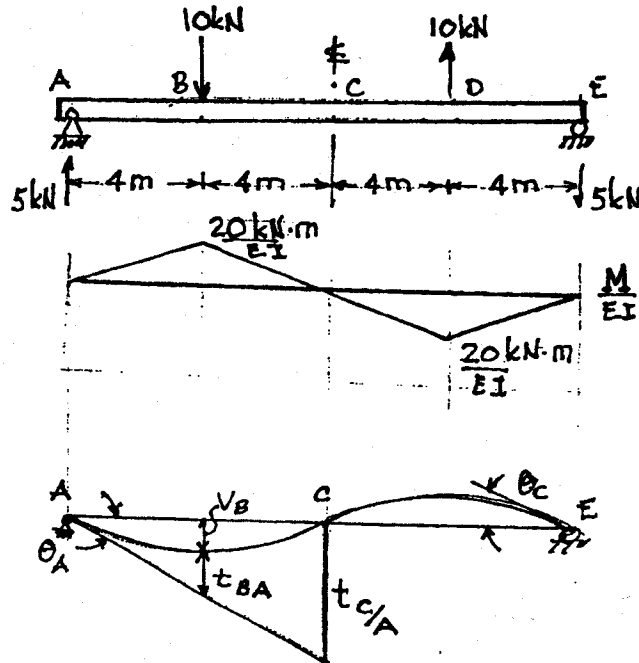
$$t_{BA} = \frac{1}{2} \frac{60}{EI} (12' \cdot 4') = 1440/EI$$

$$\Delta_B = (2880 - 1440)/EI = \underline{\underline{1440/EI}}$$

P9.9. Compute the slope at A and C and the deflection at B for the beam in Figure P9.9.



P9.9



$$\theta_A = \frac{t_{CA}}{8} = \frac{1}{8} \times \frac{1}{2} (8) \left(\frac{20}{EI} \right) 4 = \frac{40}{EI}$$

FROM SYMMETRY $\theta_C = \theta_A = \frac{40}{EI}$

$$\theta_B = \theta_A + \Delta\theta_{AB}$$

$$\theta_B = \frac{40}{EI} - \frac{1}{2} \left(\frac{4 \times 20}{EI} \right) = 0$$

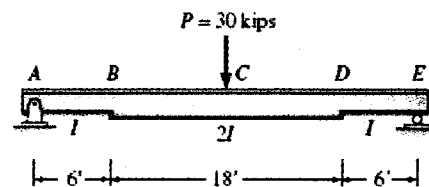
DEFL AT B

$$\Delta_B = 4\theta_A - t_{BA}$$

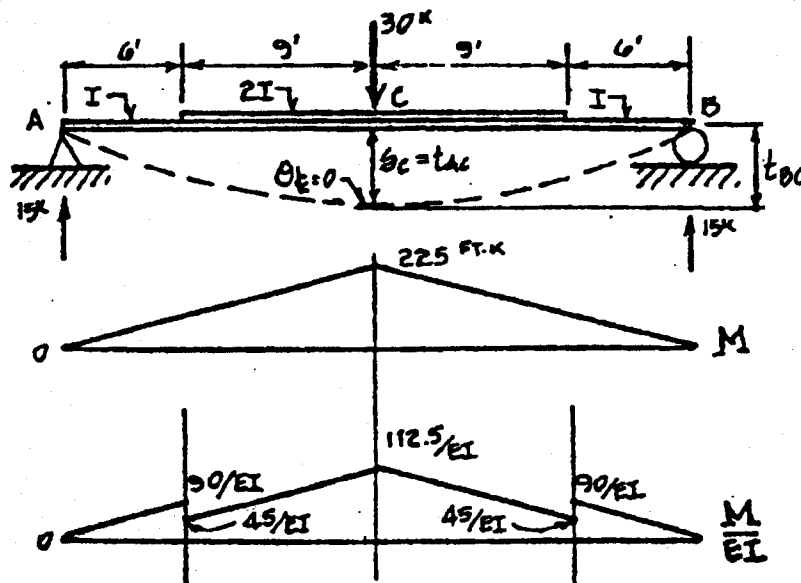
$$= 4 \times \frac{40}{EI} - \frac{1}{2} \left(\frac{4 \times 20}{EI} \right) \times \frac{4}{3}$$

$$= \frac{160}{EI} - \frac{53.3}{EI} = \frac{106.67}{EI} \downarrow$$

P9.10. (a) Compute the slope at A and the deflection at midspan in Figure P9.10. (b) If the deflection at midspan is not to exceed 1.2 in, what is the minimum required value of I ? $E = 29,000$ kips/in².



P9.10



$$\theta_A = \theta_c + \Delta\theta_{AC}$$

$$= 0 + \frac{1}{2} (6) \frac{90}{EI} + \left[\frac{45}{EI} + \frac{112.5}{EI} \right] \frac{1}{2} (9)$$

$$\theta_A = \frac{978.75}{EI}$$

$$\delta_c = t_{AC} = \frac{240}{EI} (4) + \frac{45}{EI} (9) \left[4 + \frac{9}{2} \right] + \frac{9 \left(\frac{112.5 - 45}{EI} \right)}{2} \left[6 + 9 \right]$$

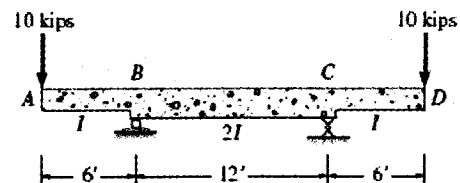
$$\delta_c = \frac{8977.5}{EI}$$

SET $\delta_c = 1.2$ in, THEN SOLVE FOR I

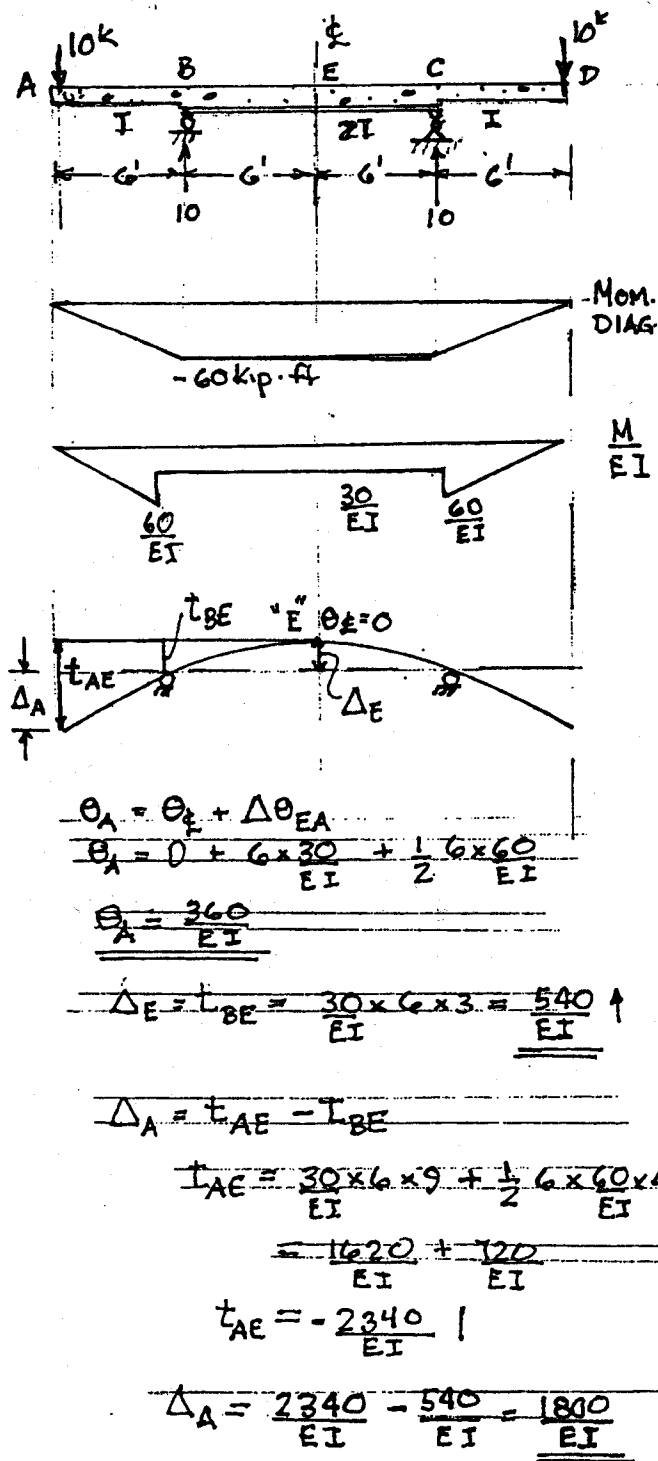
$$1.2 = \frac{8977.5 (1728)}{29000 I}$$

$$I = 445.78 \text{ in}^4$$

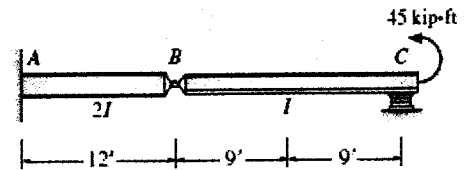
P9.11. (a) Find the slope and deflection at A in Figure P9.11. (b) Determine the location and the magnitude of the maximum deflection in span BC.



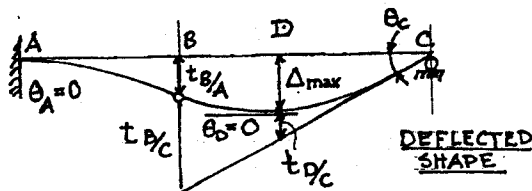
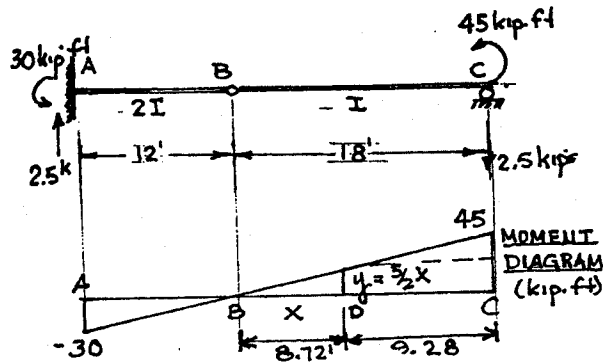
P9.11



P9.12. Compute the slopes of the beam in Figure P9.12 on each side of the hinge at B, the deflection of the hinge, and the maximum deflection in span BC. The elastomeric support at C acts as a roller.



P9.12



$$V_B = t_{B/A} = \frac{-30}{2EI} (12) \frac{1}{2} (8) = \underline{\underline{-\frac{720}{EI}}}$$

θ_B to LEFT of hinge

$$\theta_{B1} = \theta_A + \Delta\theta_{A-B}$$

$$= 0 + \left(\frac{1}{2}\right) \frac{-30}{2EI} (12) = \underline{\underline{-\frac{90}{EI}}}$$

$$\theta_C = \frac{t_{B/A} + t_{B/C}}{18}$$

$$\text{where } t_{B/C} = \frac{1}{2} (18) \left(\frac{45}{18}\right) \frac{1}{2} = \underline{\underline{-\frac{4860}{EI}}}$$

$$\theta_C = \frac{720/EI + 4860/EI}{18} = \underline{\underline{\frac{310}{EI}}}$$

θ_B to RIGHT of hinge

$$\theta_{BR} = \frac{310}{EI} - \frac{1}{2} \frac{18(45)}{EI} = \underline{\underline{-\frac{95}{EI}}}$$

POINT OF ZERO SLOPE IN SPAN BC (POINT D)

$$\frac{y}{45} = \frac{x}{18} \therefore y = \frac{5}{2}x$$

$$\theta_D = 0 = \theta_{BR} + \Delta\theta_{B/D}$$

$$0 = -\frac{95}{EI} + x \cdot \frac{5}{2} \left(\frac{1}{2}\right) \frac{1}{EI}$$

$$x^2 = \frac{95}{1.25} =$$

$$x = 8.72'$$

$$y = 21.8 \text{ kip-ft}$$

MAX DEFL IN SPAN BC

$$\Delta_{MAX} + t_{DC} = \theta_C (9.28)$$

$$t_{DC} = \frac{21.8}{EI} \left(\frac{9.28}{2}\right)^2 + \frac{1}{2} (9.28) \left(\frac{23.2}{EI}\right) \frac{2}{3} (9.28)$$

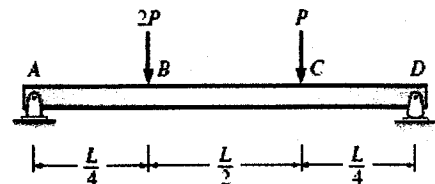
$$= \underline{\underline{\frac{1604.67}{EI}}}$$

$$\Delta_{MAX} = \theta_C (9.28) - t_{DC}$$

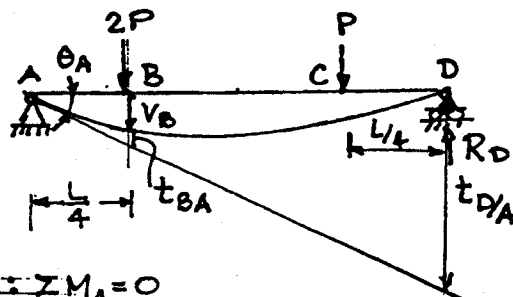
$$= \frac{310}{EI} (9.28) - \frac{1604.67}{EI}$$

$$\Delta_{MAX} = \underline{\underline{\frac{1272.13}{EI}}}$$

P9.13. Compute the slope at support A and the deflection at point B . Treat the rocker at D as a roller. Express the answer in terms of EI .



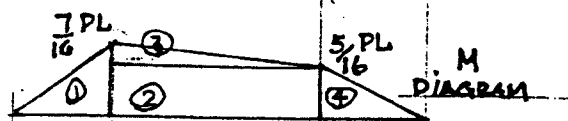
P9.13



$$R_D: \sum M_A = 0$$

$$\sum M_A = 2P\left(\frac{L}{4}\right) + P\left(\frac{3L}{4}\right) - R_D L$$

$$R_D = \frac{5P}{4}; R_A = 3P - \frac{5P}{4} = \frac{7P}{4}$$



$$t_{D/A} = \frac{1}{2} \times \frac{L}{4} \times \frac{7PL}{16EI} \left[\frac{3L}{4} + \frac{L}{12} \right] + \frac{5PL}{16EI} \times \frac{L}{2} \times \left[\frac{L}{2} \right]$$

$$+ \frac{2PL}{EI} \times \frac{1}{2} \times \frac{1}{2} \left[\frac{L}{4} + \frac{3L}{4} \right]$$

$$+ \frac{5PL}{EI} \times \frac{1}{4} \times \frac{1}{2} \left[\frac{2}{3} \times \frac{L}{4} \right]$$

$$t_{D/A} = \frac{114PL^3}{768EI}$$

$$\theta_A = \frac{t_{D/A}}{L} = \frac{114PL^2}{768EI} \quad \text{ANS.}$$

$$t_{BA} = \frac{1}{2} \times \frac{L}{4} \times \frac{7PL}{16EI} \left[\frac{L}{12} \right] = \frac{7PL^3}{1536EI}$$

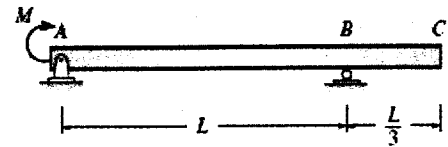
$$V_B = \theta_A \frac{L}{4} - t_{BA}$$

$$= \frac{114PL^2}{768EI} \times \frac{L}{4} - \frac{7PL^3}{1536EI}$$

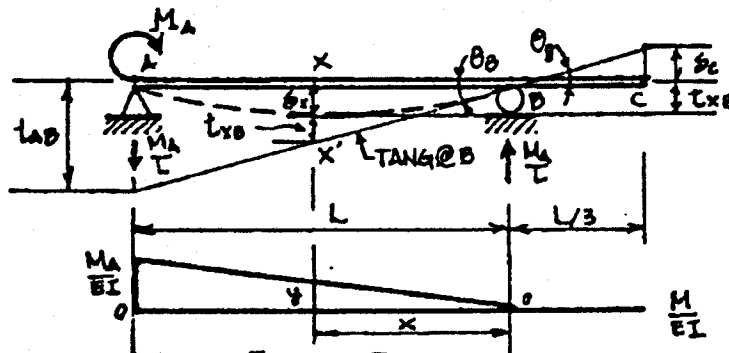
$$= \frac{57PL^3}{1536EI} - \frac{7PL^3}{1536EI}$$

$$V_B = \frac{50PL^3}{1536EI} \quad \text{ANS.}$$

P9.14. Determine the maximum deflection in span AB and the deflection of C in Figure P9.14. Express answers in terms of M , E , I , and L .



P9.14



$$\theta_B = \frac{\delta_B}{L} = \left[\frac{1}{2} L \frac{M_A}{EI} \frac{1}{3} \right] \frac{1}{L}$$

$$\delta_C = \theta_B \left[\frac{L}{3} \right]$$

$$\theta_A = \theta_B + \Delta\theta_{BA} = \frac{M_A L}{6EI} - \left[\frac{1}{2} \frac{M_A}{EI} \right] L$$

LOCATE δ_{MAX} WHERE $\theta = 0$

$$\theta_x = 0 = \theta_B - \frac{1}{2} x y$$

$$\text{WHERE } \frac{y}{x} = \left[\frac{M_A}{EI} \right] / L$$

$$0 = \frac{M_A L}{6EI} - \frac{1}{2} x \frac{M_A}{EI}$$

$$\delta_{MAX} = \delta_x = x x' - \frac{1}{2} x^2 \theta_B = \theta_B x - \frac{1}{2} x^2 \theta_B$$

$$= \frac{M_A L}{6EI} (0.577L) - \left[\frac{1}{2} (0.577L) \right] \left(\frac{M_A}{EI} \right) \left[\frac{0.577L}{3} \right]$$

$$\delta_{MAX} = \frac{385 M_A L^2}{6EI}$$

$$\theta_B = \frac{M_A L}{6EI}$$

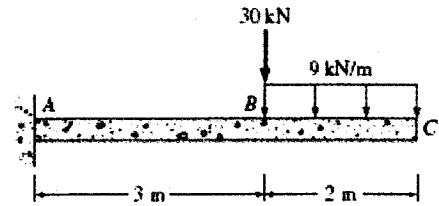
$$\delta_C = \frac{M_A L^2}{18EI}$$

$$\theta_A = -\frac{M_A L}{3EI}$$

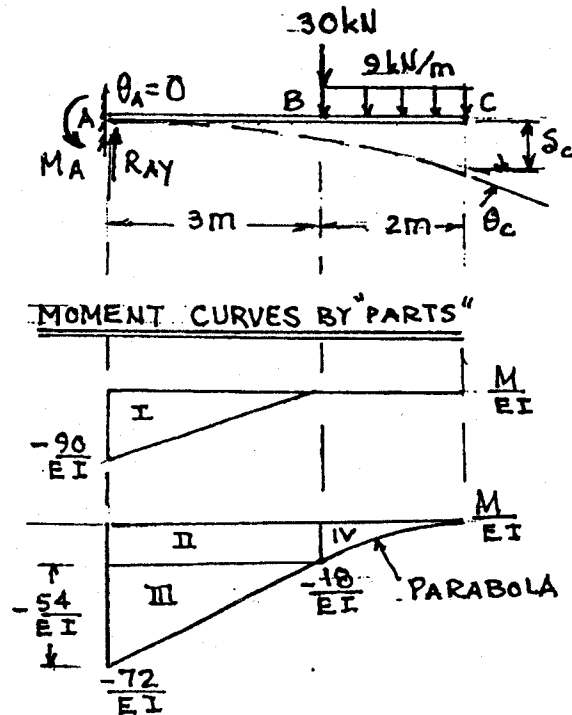
$$y = \frac{MAX}{EI L}$$

$$x = 0.577L$$

P9.15. Determine the slope and deflection of point C in Figure P9.15. *Hint: Draw moment curves by parts.*



P9.15



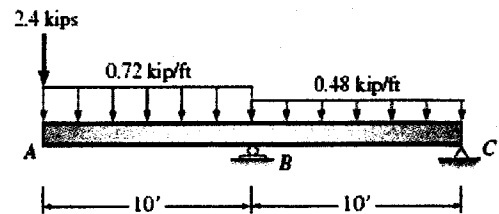
$$\theta_C = \theta_A + \Delta\theta_{AC}$$

$$\begin{aligned}\theta_C &= 0 + \left[\frac{1}{2}(3)\left(-\frac{90}{EI}\right) + \left(-\frac{18}{EI}\right) \times 3 \right. \\ &\quad \left. + \frac{1}{2}(3)\left(-\frac{54}{EI}\right) + \frac{1}{3}(2)\left(-\frac{18}{EI}\right) \right] \\ \theta_C &= -\frac{282}{EI}\end{aligned}$$

$$\delta_C = t_{C/A} = \sum A \cdot \bar{x}$$

$$\begin{aligned}\delta_C &= \left[-\frac{135}{EI} \right] (4) + \left[\frac{54}{EI} \times (3.5) \right] \\ &\quad + \left[-\frac{81}{EI} \times 4 \right] + \left[-\frac{12}{EI} \times \frac{3}{2} \right] \\ \delta_C &= -\frac{1071}{EI}\end{aligned}$$

P9.16. The roof beam of a building is subjected to the loading shown in Figure P9.16. If a $\frac{3}{8}$ -in. deflection is permitted at the cantilever end before the ceiling and roofing materials would be damaged, calculate the required moment of inertia for the beam. Use $E = 29,000$ ksi.



P9.16

MOMENT AREA BY PARTS:

$$\sum M_C = 0; -2.4^k(20') + B_Y(10') = 0$$

$$B_Y = 4.8^k \uparrow$$

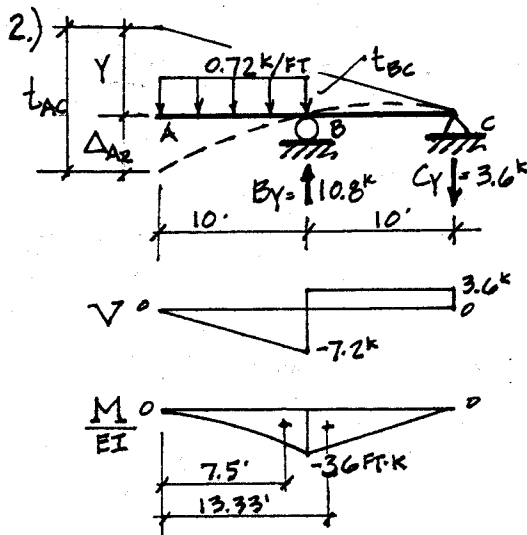
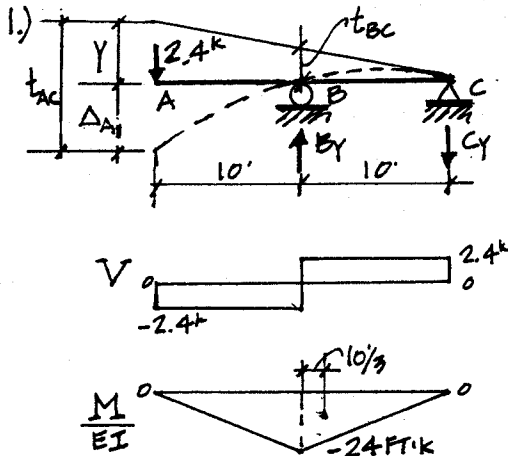
$$\uparrow \sum F_Y = 0; -2.4^k + 4.8^k - C_Y = 0$$

$$C_Y = 2.4^k \downarrow$$

$$\Delta_{A_1} = t_{AC} - Y = \frac{-2.4(20')(10')}{2EI} + \frac{800}{EI} = \frac{-1600}{EI}$$

WHERE

$$Y = t_{BC} \left(\frac{20'}{10'} \right) = \frac{-2.4^k(10') \left(\frac{10'}{3} \right) \frac{20'}{10'}}{2EI} = \frac{-800}{EI}$$



$$\Delta_{A_2} = t_{AC} - Y$$

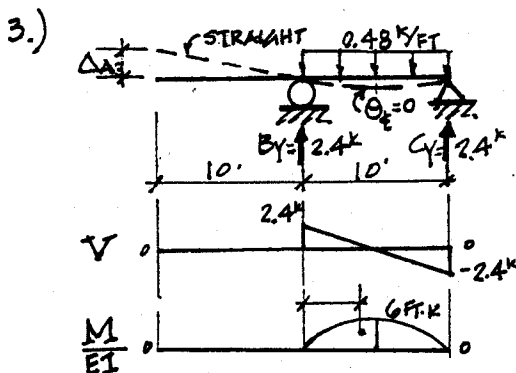
WHERE

$$Y = t_{BC} \left(\frac{20'}{10'} \right) = \frac{-3.6(10') \left(\frac{10'}{3} \right) \left(\frac{20'}{10'} \right)}{2EI} = \frac{-1200}{EI}$$

AND

$$t_{AC} = \frac{(-3.6)(10') \left(\frac{13.33'}{3} \right)}{2EI} + \frac{1(-3.6)(10') \left(7.5' \right)}{3EI} = \frac{3299.4}{EI}$$

$$\Delta_{A_2} = \frac{-2099.4}{EI}$$



$$\Delta_{A_3} = \theta_B(10') = 200/EI$$

WHERE

$$\theta_B = \theta_C + \Delta \theta_B$$

$$= 0 + \frac{2(6 \text{ ft-k})(5')}{3EI} = \frac{20}{EI}$$

TOTAL DEFLECTION $\Delta_A = \Delta_{A_1} + \Delta_{A_2} + \Delta_{A_3}$

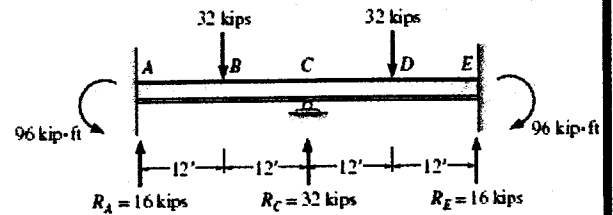
$$\Delta_A = \frac{-1600}{EI} + \frac{-2099.4}{EI} + \frac{200}{EI}$$

$$= \frac{-3499.4}{EI}$$

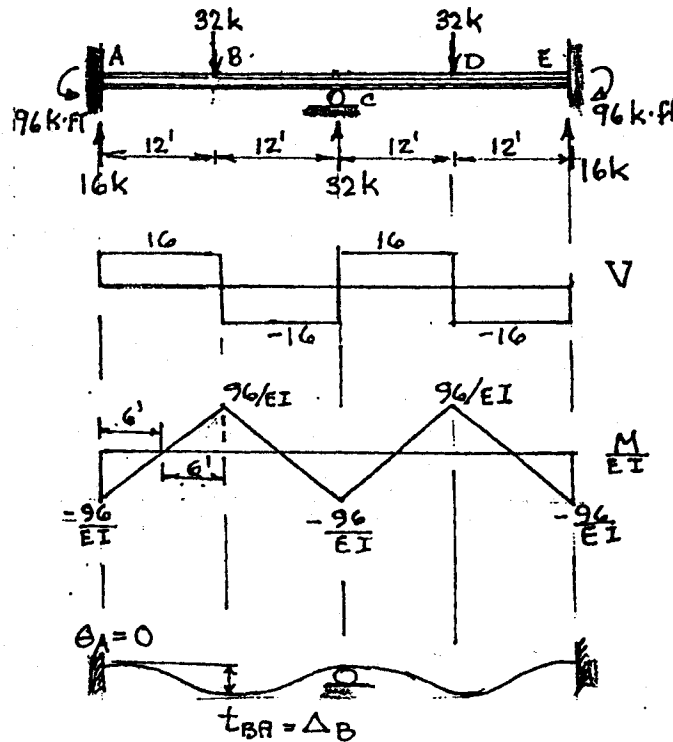
FOR $\Delta_A \text{ MAX.} = \frac{3}{8} \text{ IN.}$ AND $E = 29,000 \text{ KSI}$

$$\underline{I_{\text{REQUIRED}} = \frac{3,499.4 (1728 \text{ in}^3/\text{ft}^3)}{29,000 \text{ ksi} \left(\frac{3}{8} \text{ in} \right)} = 556 \text{ in}^4}$$

P9.17. Using the moment-area method, compute the slope and deflection under the 32-kip load at B. Reactions are given. $I = 510 \text{ in}^4$ and $E = 29,000 \text{ kips/in}^2$. Sketch the deflected shape.

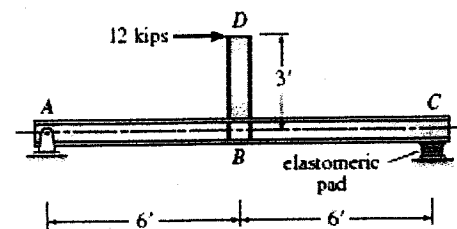


P9.17

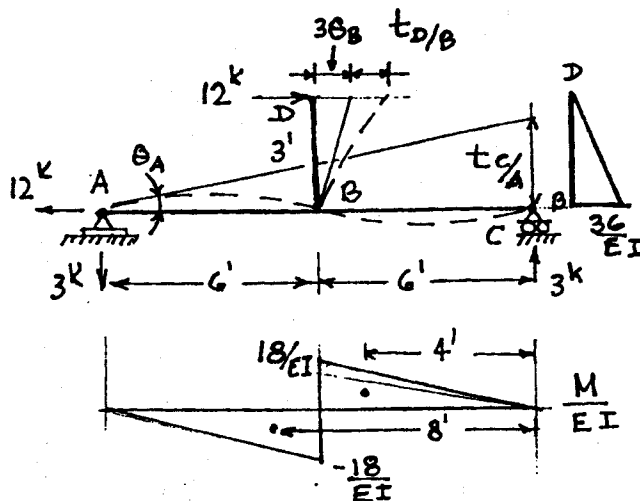


$$\begin{aligned}\Delta_B &= t_{BA} = \frac{1}{2} \left(\frac{-96}{EI} \right) \times 6 \left[10 \right] + \frac{1}{2} \times 6 \left(\frac{96}{EI} \right) 2' \\ \Delta_B &= -\frac{2304}{EI} \\ &= \frac{2304 \text{ k} \cdot \text{ft}^3 (1728 \text{ in}^3/\text{ft}^3)}{29,000 \text{ k/in}^2 \cdot 510 \text{ in}^4} \\ \Delta_B &= \underline{\underline{0.269 \text{ in}}}\end{aligned}$$

P9.18. Compute the deflection of points B and D in Figure P9.18. The elastomeric pad at C acts as a roller.



P9.18



$$t_{CA} = \frac{6}{2} \left(\frac{-18}{EI} \right) 8 + \frac{6}{2} \left(\frac{18}{EI} \right) 4$$

$$t_{CA} = -\frac{216}{EI}$$

$$\theta_A = \frac{t_{CA}}{12} = \frac{18}{EI}$$

$$\theta_B = \theta_A + \Delta\theta_{AB}$$

$$\theta_B = \frac{18}{EI} + \frac{1}{2} 6 \left(\frac{-18}{EI} \right) = -\frac{36}{EI}$$

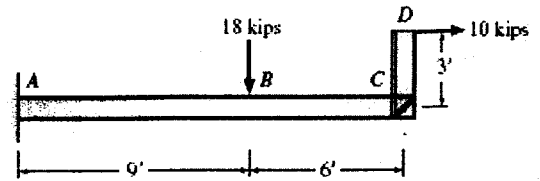
$$S_{DH} = 3\theta_B + t_{D/B}$$

$$S_{DH} = 3 \left(\frac{36}{EI} \right) + \frac{1}{2} \frac{36}{EI} (3)(2)$$

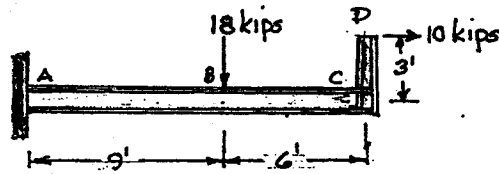
$$S_D = 108/EI + 108/EI = \underline{\underline{216/EI}}$$

$$S_B = 0 \quad \text{ANTISYMMETRICAL DEFORMATION OF BM ABC}$$

P9.19. Compute the slope and the vertical deflection at point C and the horizontal displacement at point D. $I_{AC} = 800 \text{ in}^4$, $I_{CD} = 120 \text{ in}^4$, and $E = 29,000 \text{ kips/in}^2$.

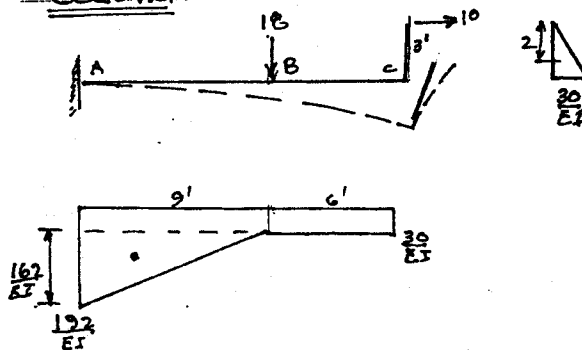


P9.19



P 9.19

SOLUTION



$$\Delta_{DV} = \Delta_{CV} = t_{C/A}$$

$$= \frac{15 \times 30 \times 7.5}{EI} + \frac{1}{2} \times \frac{162}{EI} \times 9 [12]$$

$$= \frac{12,123}{EI} \text{ kip} \cdot \text{ft}^3$$

$$\Delta_{CV} = \Delta_{DV} = \frac{12,123 (1728)}{29,000 \times 800}$$

$$= 0.903 \text{ in} \downarrow$$

$$\theta_C = \Delta \theta_{AC} = \text{AREA } \frac{M}{EI} \text{ at } C$$

$$= 0 + \frac{30}{EI} \times 15 + \frac{162 \times 9}{EI} \times \frac{1}{2}$$

$$= \frac{450}{EI} + \frac{729}{EI}$$

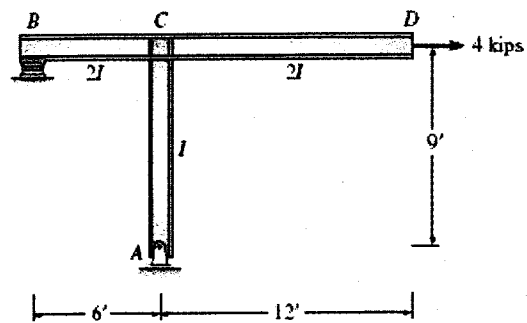
$$= \frac{1179}{EI} = \frac{1179 (144)}{29,000 (800)} = .00732 \text{ RADIAN}$$

$$\Delta_{DH} = \theta_C (3') + t_{DC}$$

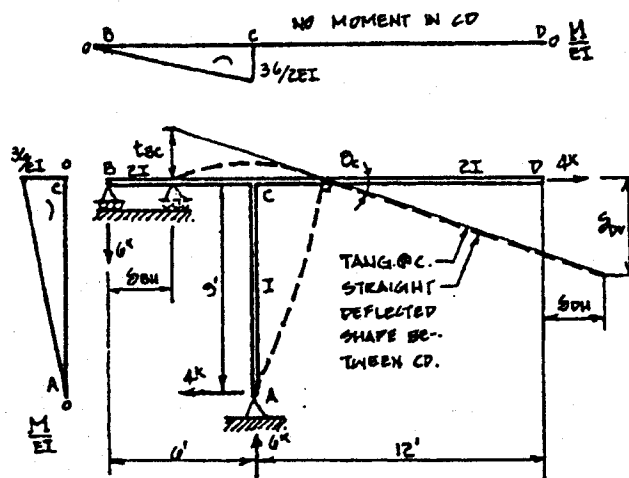
$$= .00732 (36) + \frac{1}{2} \times \frac{30 (3) (2) 1728}{29,000 \times 120}$$

$$= .264 + .045 = \underline{\underline{0.309 \text{ in} \rightarrow}}$$

P9.20. The moment of inertia of the girder in Figure P9.20 is twice that of the column. If the vertical deflection at D is not to exceed 1 in and if the horizontal deflection at C is not to exceed 0.5 in, what is the minimum required value of the moment of inertia? $E = 29,000$ kips/in². The elastomeric pad at B is equivalent to a roller.



P9.20



$$\delta_D = 12 \theta_C = 12 (t_{BC}/6)$$

$$= 2 \left[\frac{1}{2} (6) \frac{36}{EI} (6) \right]$$

$$\delta_D = \frac{432}{EI}$$

$$\text{LET } \delta_D = 1"$$

$$1 = \frac{432 (1728)}{29000 (I)}$$

$$I = 25.74 \text{ in}^4$$

$$\Delta_{XC} = \frac{9}{12} \delta_D + t_{AC} = 0.5" \text{ MAX.}$$

$$= \frac{9}{12} \left(\frac{432}{EI} \right) + \frac{1}{2} \left(\frac{36}{EI} \right) (9) (6)$$

$$= 1296/EI$$

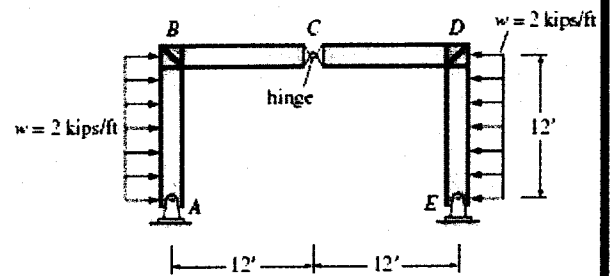
$$0.5" = \frac{1296 (1728)}{29000 I}$$

$$I_{\text{MIN}} = 154.4 \text{ in}^4$$

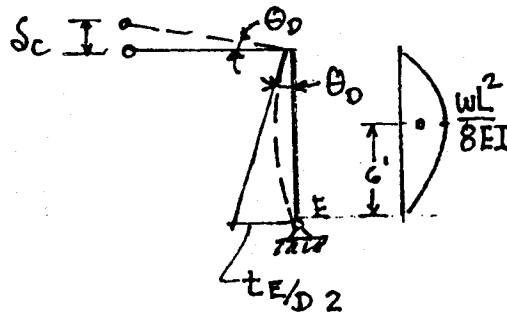
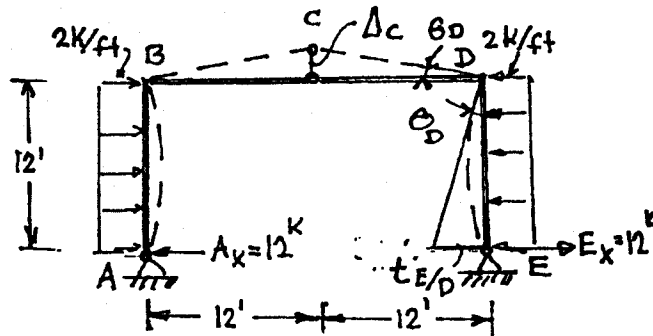
CONTROL

$$\therefore \text{ GIRDER } 2I = \underline{\underline{308.9 \text{ in}^4}}$$

P9.21. Compute the vertical displacement of the hinge at C in Figure P9.21. EI is constant.



P9.21



$$t_{ED} = A\bar{x} = \frac{2}{3}(12)\frac{wL^2}{8EI}(6)$$

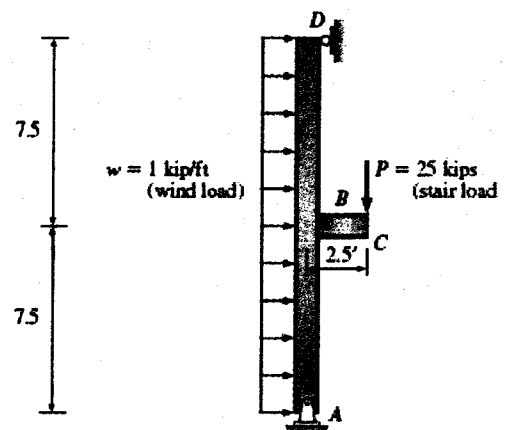
$$t_{ED} = \frac{8 \times 2 (12)^2 \times 6}{8EI} = \frac{1728}{EI}$$

$$\theta_D = \frac{t_{E/D}}{12} = \frac{1728}{12EI} = \frac{144}{EI}$$

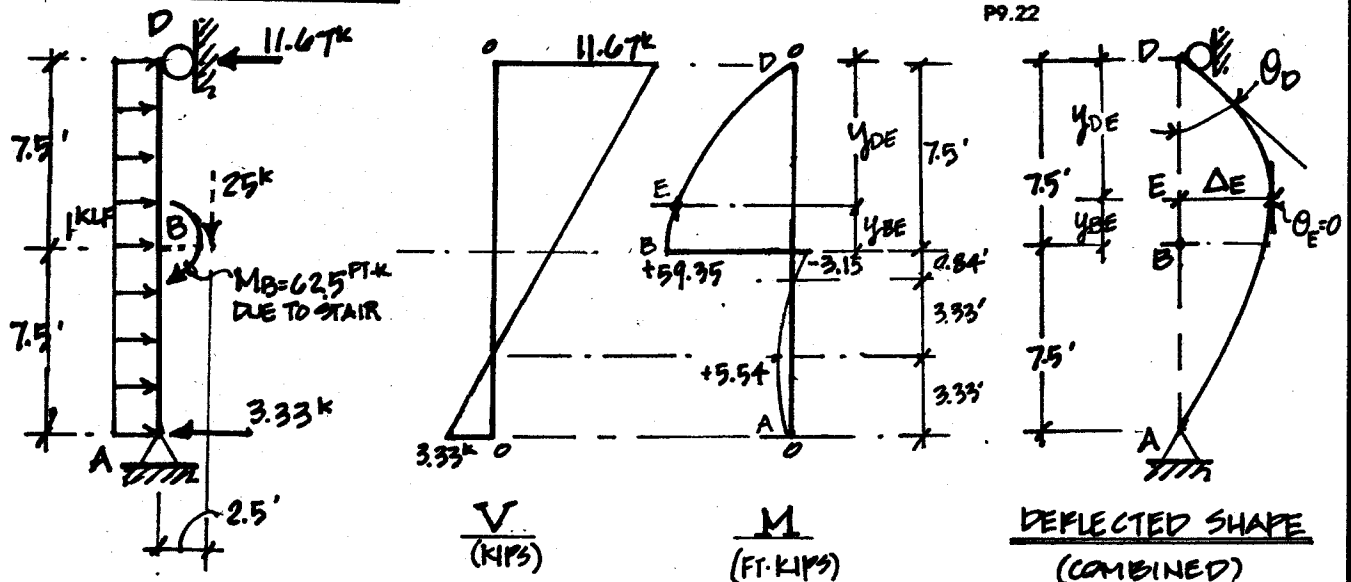
$$\Delta_C = 12\theta_D = 12\left(\frac{144}{EI}\right) = \frac{1728}{EI}$$

P9.22. The loading acting on a column that supports a stair and exterior veneer is shown in Figure P9.22. Determine the required moment of inertia for the column such that the maximum lateral deflection does not exceed $\frac{1}{4}$ in, a criterion set by the veneer manufacturer. Use $E = 29,000$ kips/in².

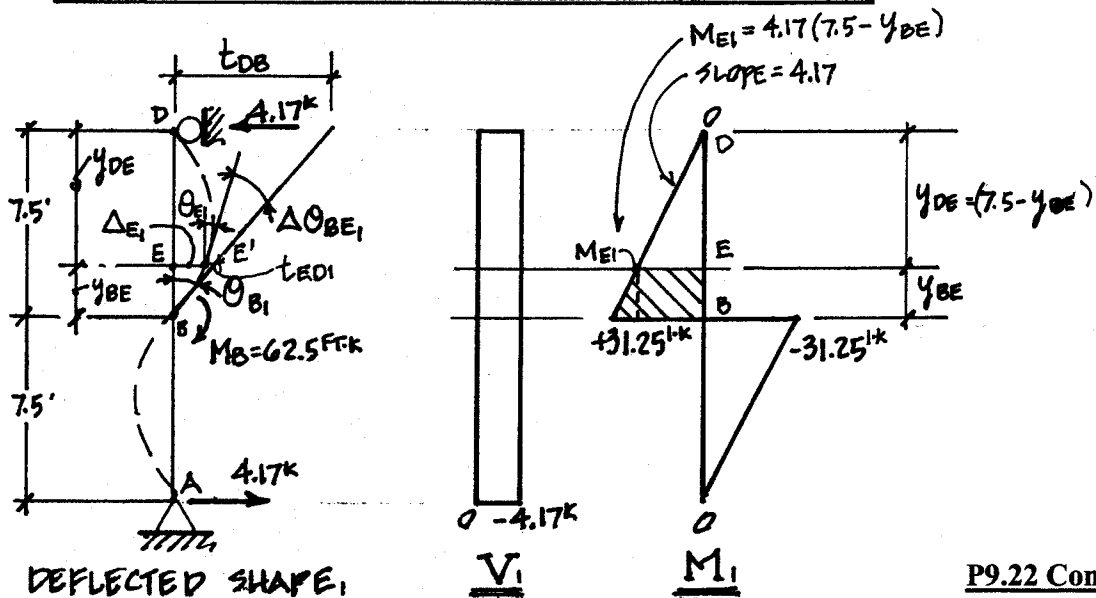
MAX. LATERAL DEFLECTION OCCURS WHERE $\theta_E = 0$, WHICH OCCURS BETWEEN MAX. POSITIVE DEFLECTIONS CAUSED BY STAIR & WIND LOADS INDEPENDENTLY.



ENTIRE STRUCTURE:

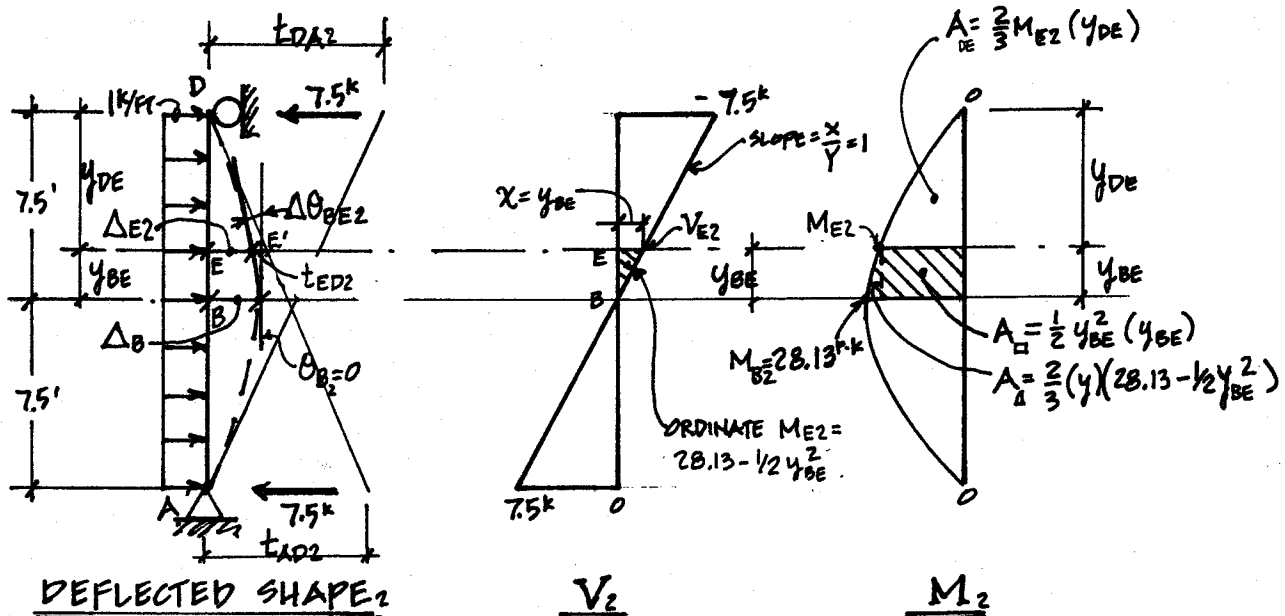


1. MOMENT-AREA BY PARTS: STAIR LOAD ONLY:



P9.22 Continued

2. MOMENT-AREA: WIND LOAD ONLY:



Δ_{MAX} OCCURS @ $\theta_E = 0$ OF THE COMBINED DEFLECTED SHAPES

$$\theta_E = \theta_{E1} + \theta_{E2} = [\theta_{B1} - \Delta\theta_{BE1}] + [\theta_{B2}^0 - \Delta\theta_{BE2}] = 0$$

$$\theta_{B1} = \frac{t_{DB}}{L=7.5'} = \frac{1/2(31.25/EI)(7.5')(2/3(7.5'))}{7.5'} = \frac{585.94}{7.5EI} = \frac{78.13}{EI}$$

$$\Delta\theta_{BE1} = \frac{(31.25 - y_{BE}(4.17))y_{BE}}{2EI} + \frac{(31.25 - y_{BE}(4.17))y_{BE}}{EI} = \frac{46.88y_{BE}}{EI} - \frac{6.25y_{BE}^2}{EI}$$

$$\Delta\theta_{BE2} = \frac{2}{3}y_{BE}(28.13 - 1/2y_{BE}^2) + \frac{1}{2}y_{BE}^3 = \frac{18.753y_{BE}}{EI} - \frac{0.17y_{BE}^3}{EI}$$

$$\theta_E = \frac{78.13}{EI} - \frac{46.88y_{BE}}{EI} + \frac{6.25y_{BE}^2}{EI} - \frac{18.75y_{BE}}{EI} + \frac{0.17y_{BE}^3}{EI} = \frac{78.13 - 65.63y_{BE} + 6.25y_{BE}^2 + 0.17y_{BE}^3}{EI} = 0$$

DETERMINE y_{BE} FOR $\theta_E = 0$

$$\left. \begin{array}{l} y_{BE} = 1.5' \quad \theta_E = -5.68/EI \\ y_{BE} = 1.3' \quad \theta_E = +3.75/EI \\ y_{BE} = 1.35' \quad \theta_E = +1.39/EI \\ y_{BE} = 1.37' \quad \theta_E = +0.325/EI \end{array} \right\} \therefore y_{BE} \text{ BETW. } 1.5' \text{ \& } 1.3'$$

$$\boxed{y_{BE} = 1.378'}$$

$$\boxed{y_{DE} = 7.5' - y_{BE} = 6.12'}$$

P9.22 Continued

DETERMINE $\Delta_E = \Delta_{E1} + \Delta_{E2}$

$$\Delta_{E1} = EE'_1 - t_{ED1}$$

$$\text{WHERE } EE'_1 = \theta_{D1} \cdot y_{DE} =$$

$$\theta_{D1} = \frac{t_{AD1}}{L} = \left[\frac{31.25}{2EI} \right] 7.5' (10'-5') / 15' = 39.1/EI$$

$$t_{ED1} = \frac{1}{2} M_{E1} (y_{DE}) \left[\frac{1}{3} (y_{DE}) \right] = \left[\frac{1}{2} \left(\frac{25.52}{EI} \right) (6.12') \right] \left[\frac{6.12'}{3} \right] = 159.3/EI$$

$$M_{E1} = \text{ORDINATE ON } M_1 \text{ CURVE} = 4.17 (7.5 - y_{BE}) = 25.52/EI$$

$$\text{THUS } \Delta_{E1} = \frac{39.1 (6.12')}{EI} - 159.3 = \boxed{\frac{80}{EI}}$$

$$\Delta_{E2} = EE'_2 - t_{ED2}$$

$$\text{WHERE } EE'_2 = \theta_{D2} \cdot y_{DE}$$

$$\theta_{D2} = \frac{t_{AD2}}{L} = \frac{2}{3} \left(\frac{28.13}{EI} \right) \frac{(15' \times 7.5')}{(15')} = \frac{140.65}{EI}$$

$$EE'_2 = (140.65/EI) 6.12' = \frac{860.8}{EI}$$

$$t_{ED2} = \left(\frac{2}{3} M_{E2} \cdot y_{DE} \right) \frac{1}{3} y_{DE}$$

$$M_{E2} = 28.13 - \frac{1}{2} y_{BE}^2 = 27.18/EI$$

$$t_{ED2} = \frac{2}{3} (27.18) (6.12') \frac{1}{3} (6.12') = \frac{226.2}{EI}$$

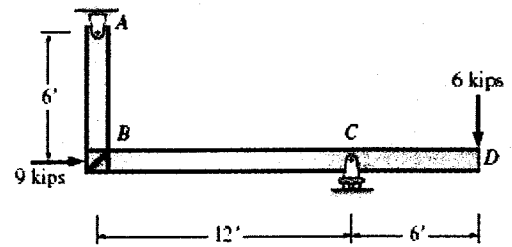
$$\text{THUS } \Delta_{E2} = \frac{860.8 - 226.2}{EI} = \boxed{\frac{634.7}{EI}}$$

$$\Delta_E = \Delta_{E1} + \Delta_{E2} = \frac{80 + 634.7}{EI} = \boxed{\frac{714.7}{EI}}$$

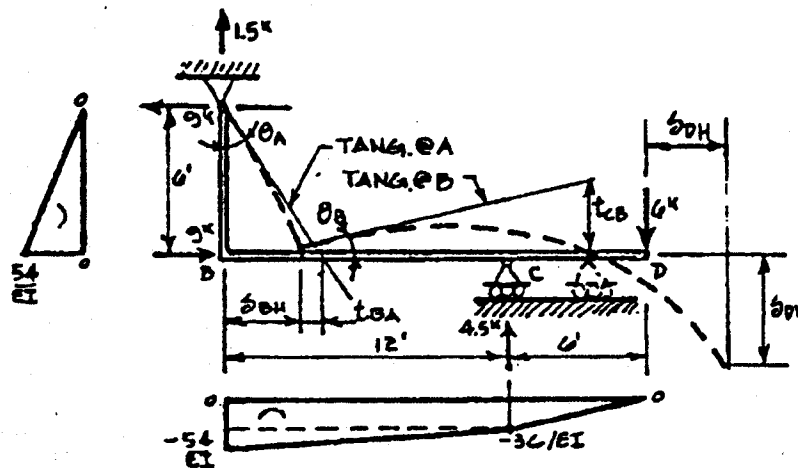
DETERMINE $I_{REQ'D}$ FOR Δ_E MAX. DEFLECTION = $1/4''$

$$I_{REQ'D} = \frac{714.7 (1728)}{29000 (1/4'')} = \boxed{170.4 \text{ in}^4}$$

P9.23. Compute the slope at A and the horizontal and vertical components of deflection at point D in Figure P9.23.



P9.23



$$\theta_B = t_{CB}/12 = \frac{1}{12} \left[\frac{36}{EI} (12)(6) + \frac{1}{2} \frac{18}{EI} 12(8) \right]$$

$$\theta_B = \frac{288}{EI}$$

$$\theta_A = \theta_B + \Delta\theta_{BA}$$

$$= \frac{288}{EI} + \frac{1}{2}(6) \frac{36}{EI}$$

$$\theta_A = \frac{450}{EI}$$

$$\delta_{DH} = \delta_{BH} = 6\theta_A - t_{BA}$$

$$= 6 \left(\frac{450}{EI} \right) - \left[\frac{1}{2}(6) \frac{54}{EI} (2) \right]$$

$$\delta_{DH} = \frac{2376}{EI}$$

$$\delta_{DV} = 6\theta_C + t_{DC}$$

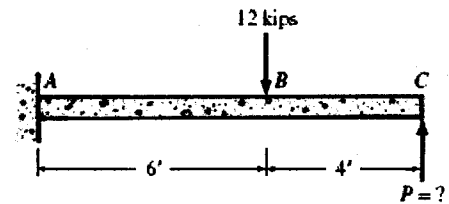
$$\theta_C = \theta_B + \Delta\theta_{BC} = \frac{288}{EI} - \frac{36}{EI} (12) - \frac{1}{2} (12) \frac{18}{EI}$$

$$\theta_C = -\frac{252}{EI}$$

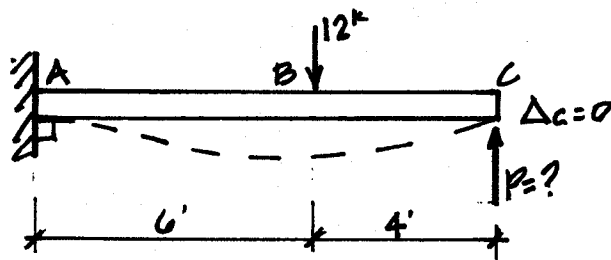
$$\delta_{DV} = 6 \left(\frac{252}{EI} \right) + \frac{1}{2}(6) \frac{36}{EI} (4)$$

$$\delta_{DV} = \frac{944}{EI}$$

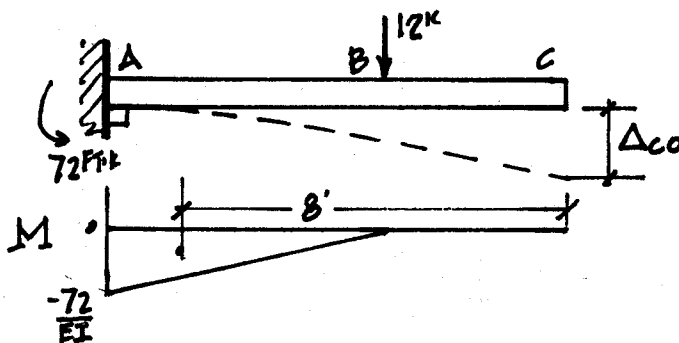
P9.24. What value of force P is required at C in Figure P9.24 if the vertical deflection at C is to be zero?



P9.24



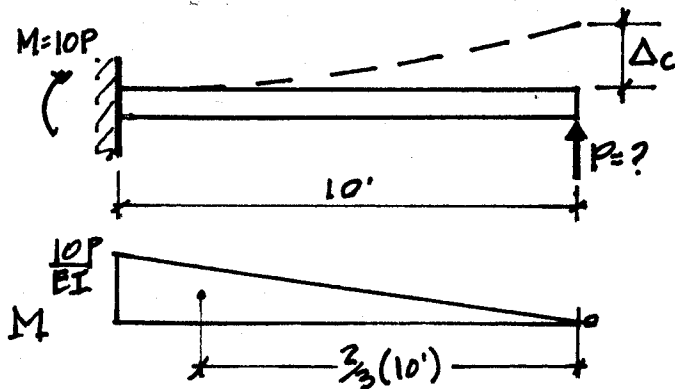
Δ_{CO} DUE TO 12 KIP LOAD:



$$\Delta_{CO} = t_{CA} = \frac{1}{2}(6') \frac{72}{EI} (8')$$

$$\Delta_{CO} = \frac{1728}{EI}$$

P REQUIRED FOR $\Delta_c = 0$:



$$\Delta_c = t_{CA} = \frac{1}{2}(10') \frac{10P}{EI} \left(\frac{3}{3} 10'\right) = (333.3/EI)P$$

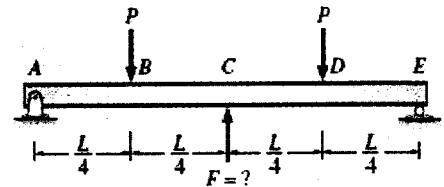
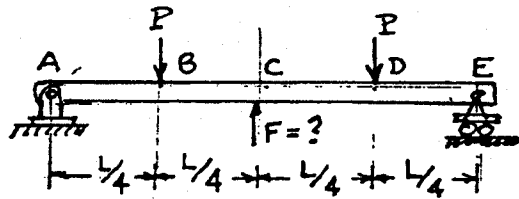
$$\text{IF } \Delta_c = 0; \text{ THEN } \Delta_{CO} = \Delta_c$$

$$1728/EI = (333.3/EI)P$$

$$\underline{\underline{P = 5.184 \text{ KIPS}}}$$

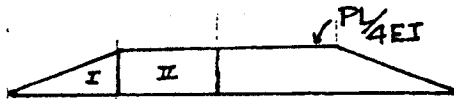
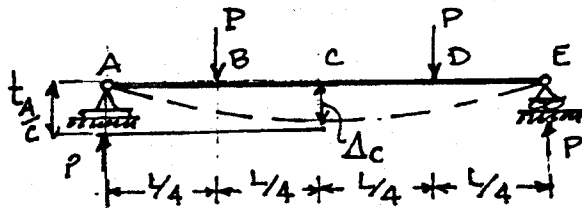
P9.25. If the vertical deflection of the beam at midspan (i.e., point C) is to be zero, determine the magnitude of force F . EI is constant. Express F in terms of P and EI .

EVALUATE F IF $\Delta_c = 0$



P9.25

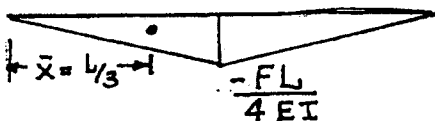
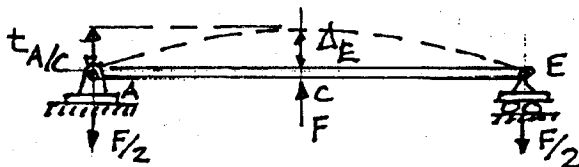
Δ_c AT C DUE TO LOADS P AT B AND D



$$\Delta_c = t_{A/C} = \frac{PL}{4EI} \cdot \frac{L}{4} \left(\frac{1}{2} \right) \left[\frac{2}{3} \times \frac{L}{4} \right] + \frac{PL}{4EI} \times \frac{L}{4} \left(\frac{1}{4} + \frac{1}{8} \right)$$

$$\Delta_c = \frac{11PL^3}{384EI}$$

Δ_c DUE TO F



$$\Delta_c = t_{A/C} = A\bar{x} = \frac{1}{2} \times \frac{L}{2} \left(\frac{-FL}{4EI} \right) \frac{L}{3}$$

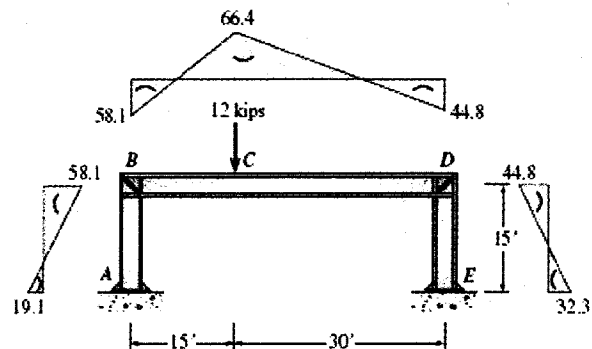
$$\Delta_c = -\frac{FL^3}{48EI}$$

$$\text{IF } \Delta_c = 0; \Delta_c = \Delta_c$$

$$\frac{11PL^3}{384EI} = \frac{FL^3}{48EI}$$

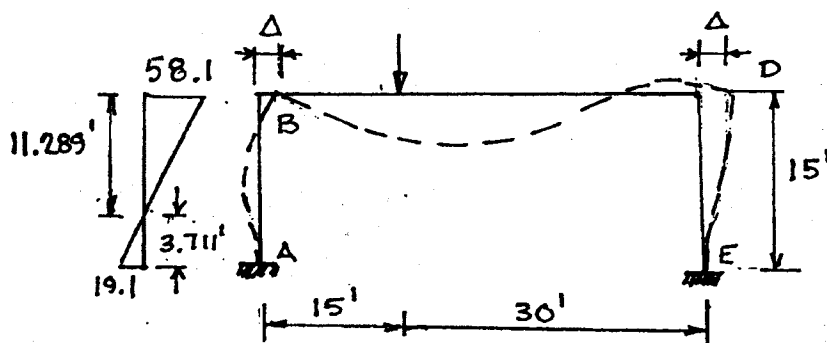
$$F = \frac{11}{8}P = 1.375P \quad \text{ANS.}$$

P9.26. Compute the horizontal displacement of joint B in Figure P9.26. The moment diagram produced by the 12-kip load is given. The base of the columns at points A and E may be treated as fixed supports. *Hint:* Begin by sketching the deflected shape, using the moment diagrams to establish the curvature of members. Moments in units of kip-ft.

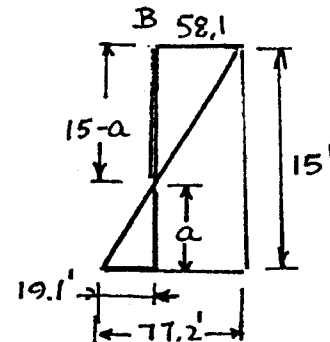


P9.26

CAN CONSIDER EITHER LEG. Will consider AB



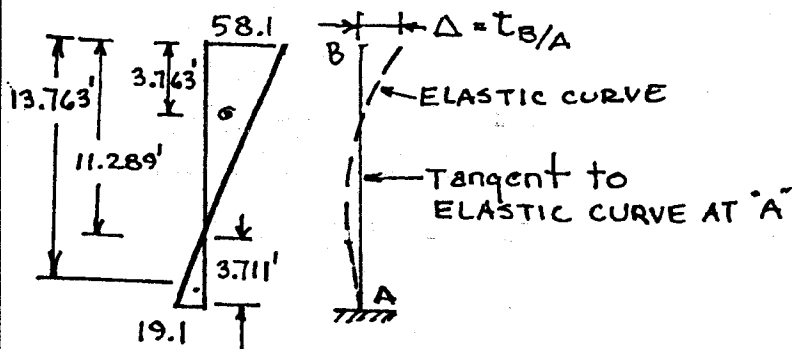
MEMBER AB
MOMENT DIAGRAM



$$\frac{15}{77.2} = \frac{a}{19.1}$$

$$\underline{a = 3.711'}$$

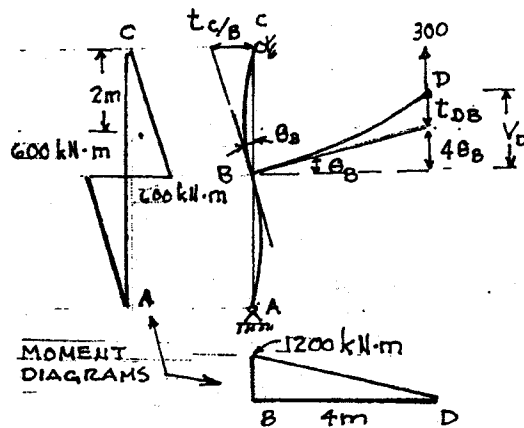
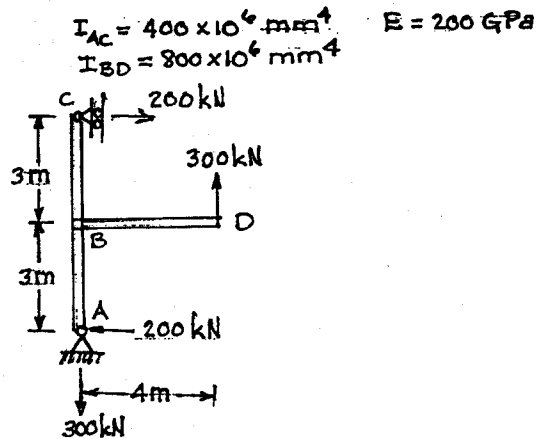
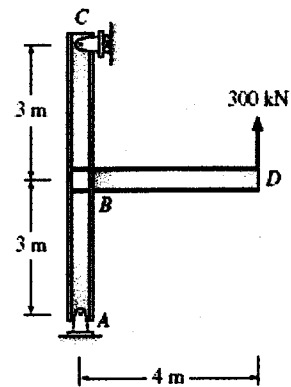
$$15 - a = 11.289'$$



$$\Delta = t_{BA} = \frac{1}{2} \frac{19.1(3.711)13.763}{EI} - \frac{1}{2} \frac{58.1(11.289)(11.289)}{3EI}$$

$$\Delta = \frac{746.3}{EI}$$

P9.27. Compute the rotation at B and the vertical deflection at D . Given: $E = 200 \text{ GPa}$, $I_{AC} = 400 \times 10^6 \text{ mm}^4$, and $I_{BD} = 800 \times 10^6 \text{ mm}^4$.



$$\theta_B = \frac{t_{CB}}{3\text{m}} = \frac{1}{2} \times \frac{600 \times 10^3 \times 3 [2]}{EI_{AC}(3)} = \frac{1800 \times 10^3}{EI_{AC}(3)}$$

$$\theta_B = \frac{1800 \times 10^3}{200 \times 10^9 (400 \times 10^{-6})(3)}$$

$$\theta_B = \underline{\underline{0.0075 \text{ RADIANS}}}$$

$$V_D = (4\text{m}) \theta_B + t_{DB}$$

$$\text{where } t_{DB} = \frac{1}{2} \times \frac{1200 \times 10^3}{EI_{BD}} (4) \left(\frac{8}{3} \right)$$

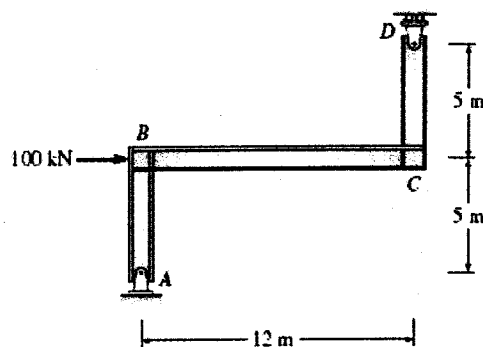
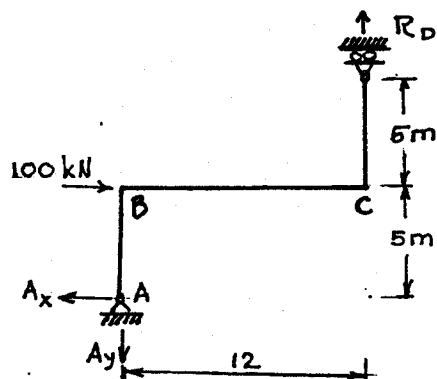
$$= \frac{6400 \times 10^3}{EI_{BD}}$$

$$= \frac{6400 \times 10^3}{200 \times 800 \times 10^3} = \underline{\underline{0.04 \text{ m}}}$$

$$V_D = 0.0075(4) + 0.04 = 0.07 \text{ m}$$

or 7 cm

P9.28. The frame shown in Figure P9.28 is loaded by a horizontal load at B. Compute the horizontal displacements at B and D using the moment-area method. For all members $E = 200 \text{ GPa}$ and $I = 500 \times 10^6 \text{ mm}^4$.

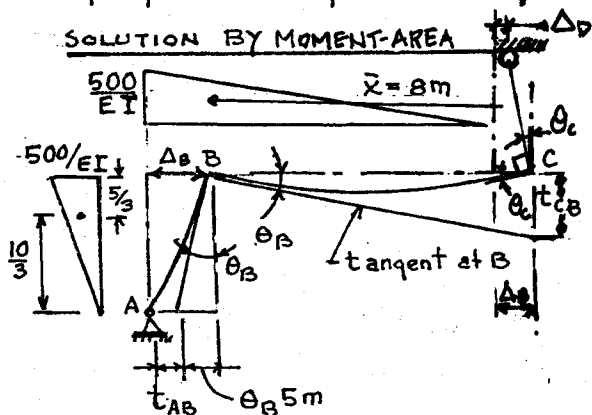


P9.28

REACTIONS

$$\begin{aligned} \rightarrow \sum F_x = 0: & 100 - A_x = 0 \\ & A_x = 100 \text{ kN} \leftarrow \\ + \sum M_A = 0: & (5)(100 \text{ kN}) - R_D(12) = 0 \\ & R_D = 500/12 = 41.67 \text{ kN} \\ \uparrow \sum F_y = 0: & A_y = 41.67 \text{ kN} \downarrow \end{aligned}$$

SOLUTION BY MOMENT-AREA



COMPUTE Δ_B

$$t_{CB} = \frac{500}{EI} \times \frac{12 \text{ m}}{2} \times 8 \text{ m} = \frac{24,000}{EI}$$

$$\theta_B = t_{CB}/12 = \frac{2,000}{EI} \text{ kN} \cdot \text{m}^2$$

$$t_{AB} = \frac{500}{EI} \times \frac{5 \text{ m}}{2} \times \frac{10 \text{ m}}{3} = \frac{4166.67}{EI}$$

$$\Delta_B = t_{AB} + \theta_B 5 \text{ m}$$

$$= \frac{4166.67}{EI} + \frac{2,000}{EI} (5)$$

$$\Delta_B = \frac{14,166.67 \text{ kN} \cdot \text{m}^3}{EI}$$

$$\begin{aligned} &= \frac{14,166.67 \text{ kN} \cdot \text{m}^3}{200 \times 10^6 \text{ kN/m}^2 \times 500 \times 10^6 \text{ m}^4} \\ &= 0.142 \text{ m} = \underline{\underline{14.2 \text{ cm}}} \rightarrow \end{aligned}$$

COMPUTE Δ_D :

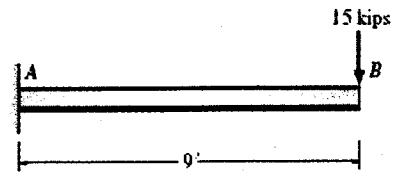
$$\Delta_D = \Delta_B - \theta_C(5')$$

$$\theta_C = -\frac{2000}{EI} + \frac{500}{2EI} (12) = \frac{1000}{EI}$$

$$\Delta_D = \frac{14,167}{EI} - \frac{1000(5)}{EI}$$

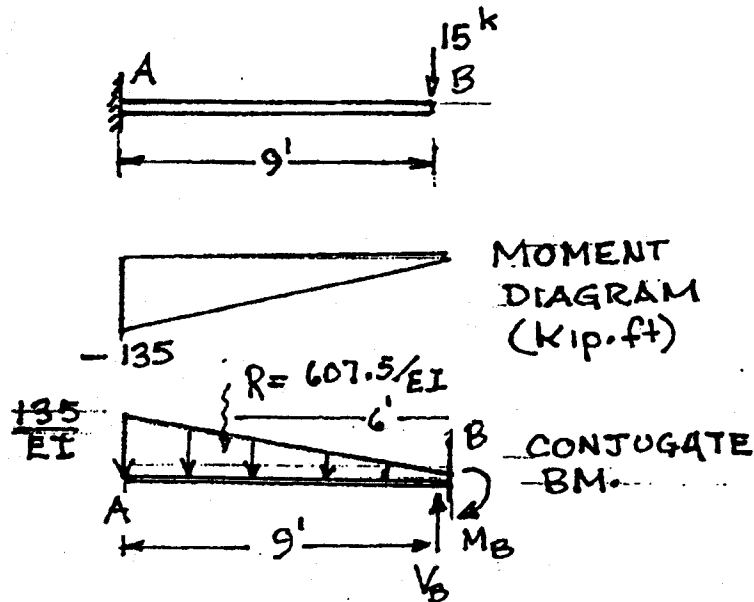
$$= 0.092 \text{ m} = \underline{\underline{9.2 \text{ cm}}} \rightarrow$$

P9.29. Compute the slope and deflection at point B of the cantilever beam in Figure P9.29. EI is constant.



P9.29

CONJUGATE BEAM ANALYSIS



$\theta_B = \text{Shear at } B$

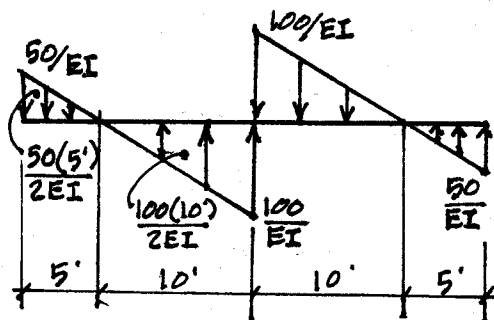
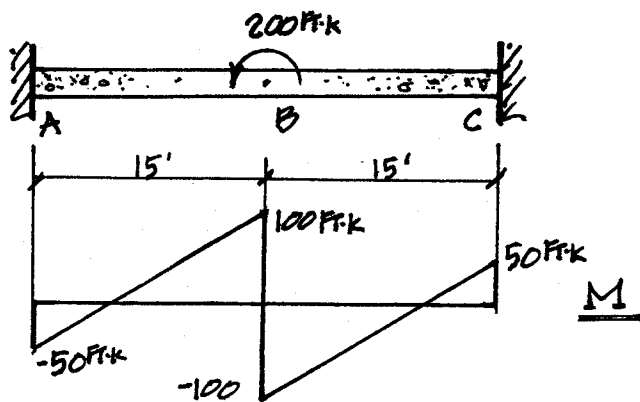
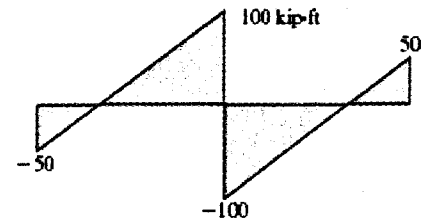
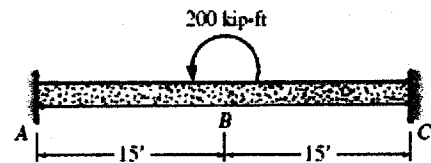
$$+\uparrow \sum F_y = 0 : V_B - \frac{135 \times 9}{EI \times 2} = 0$$

$$\theta_B = V_B = \underline{\underline{\frac{607.5}{EI}}}$$

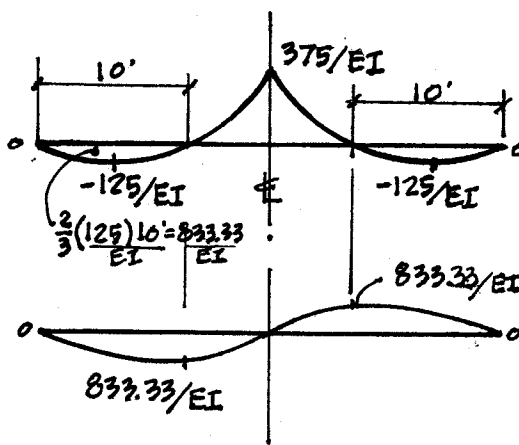
$\Delta_B = \text{Moment at } B$

$$\Delta_B = \frac{-607.5 (6)}{EI} = \underline{\underline{\frac{-3645}{EI}}}$$

P9.30. The moment diagram of a fix-ended beam with an external moment of 200 kip-ft applied at midspan is shown in Figure P9.30. Determine the maximum vertical deflection and maximum slope and their locations.



CONJUGATE BEAM
LOADED BY M/EI



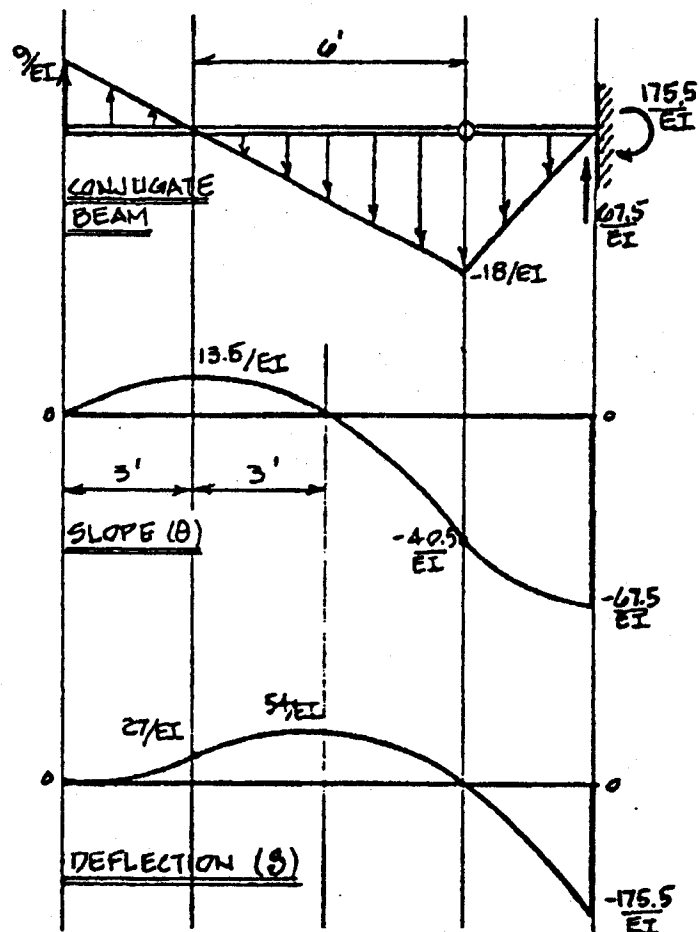
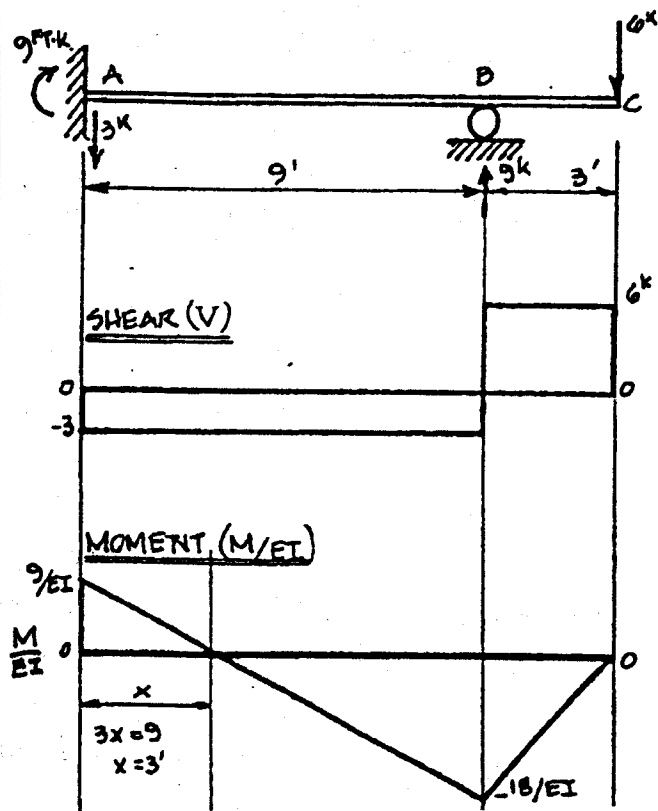
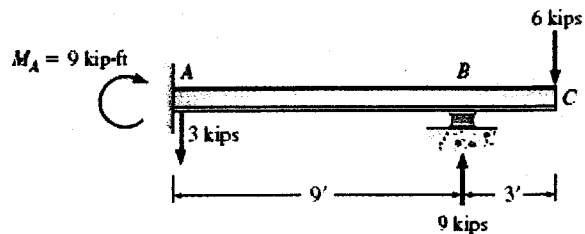
$V(\theta)$

$M(\Delta)$

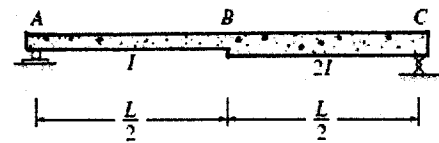
$\theta_{MAX} = 375/EI$ @ MIDSPAN @ B

$\Delta_{MAX} = 833.33/EI$ @ 10 FT FROM BEAM ENDS

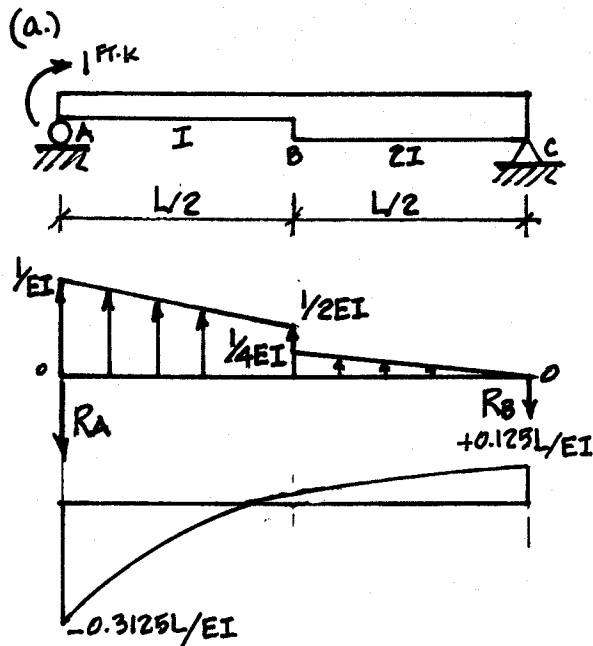
P9.31. Compute the slope and deflection at point C and the maximum deflection between A and B for the beam in Figure P9.31. The reactions are given, and EI is constant. The elastomeric pad at B is equivalent to a roller.



P9.32. Determine the flexural stiffness of the beam in Figure P9.32 (see Example 9.16 for criteria) for (a) moment applied at A, and (b) moment applied at C. E is constant.



P9.32



APPLY $M = 1 \text{ Ft} \cdot \text{K} @ A$

CONJUGATE BEAM LOADED BY M/EI

$$\sum M_B = 0;$$

$$R_A = \frac{1}{EI} \left((0.5)^2 (0.75) + (0.5)^3 (0.833) + 0.25 (0.5)^2 (0.33) \right) =$$

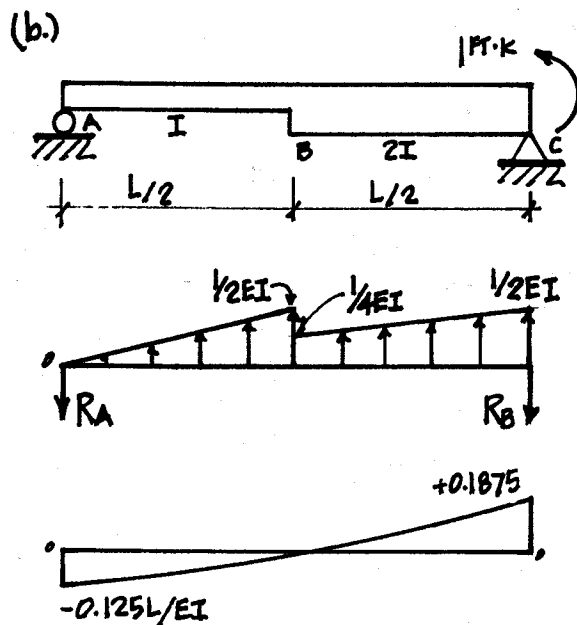
$$R_A = \underline{0.3125L/EI}$$

$$R_B = \frac{0.4375L}{EI} - R_A = \underline{0.125L/EI}$$

FLEXURAL STIFFNESS K

$$\frac{K}{1 \text{ RAD}} = \frac{1 \text{ Ft} \cdot \text{K}}{0.3125L/EI}$$

$$\underline{K = 3.20 EI/L \text{ FOR } M @ A}$$



CONJUGATE BEAM LOADED BY M/EI

$$\sum M_B = 0;$$

$$R_A = \frac{1}{EI} \left((0.5)^3 (0.44) + \frac{0.25 (0.5)^2}{2} + \frac{0.25 (0.5)^3}{3} \right) =$$

$$R_A = \underline{0.125L/EI}$$

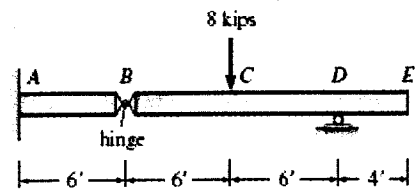
$$R_B = 0.3125L/EI - R_A = \underline{0.1875L/EI}$$

FLEXURAL STIFFNESS K

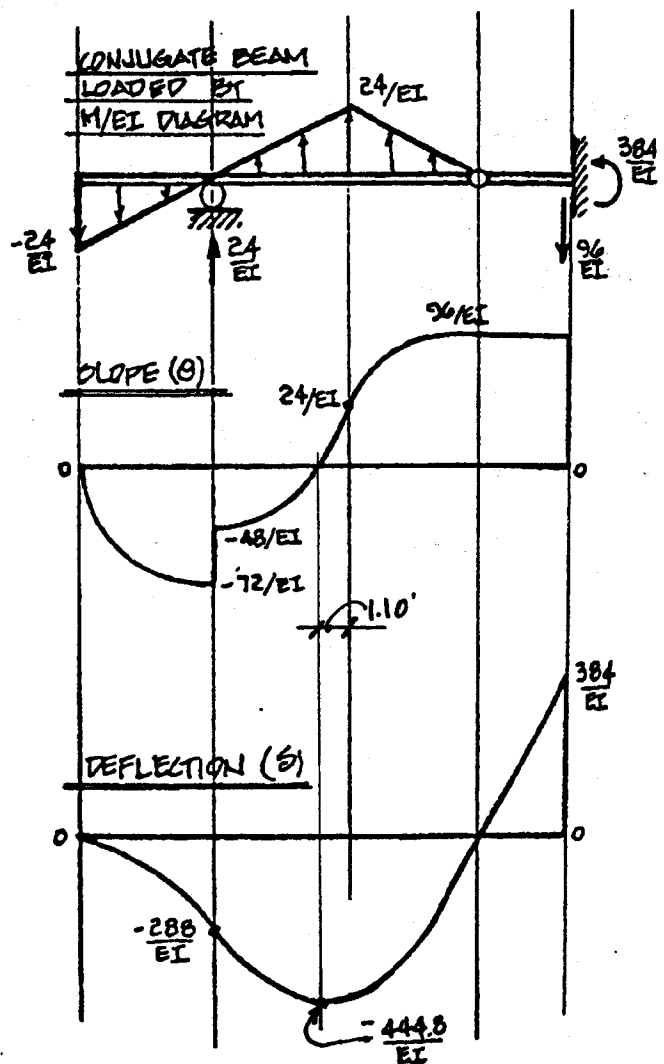
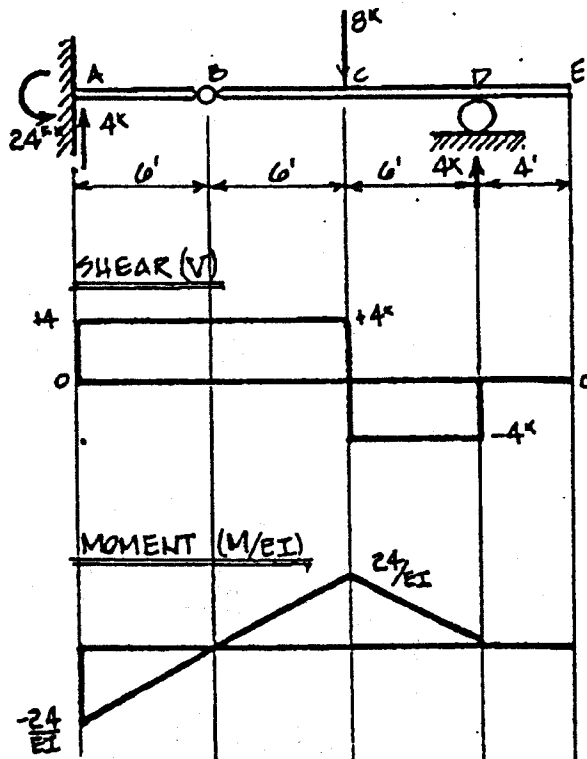
$$\frac{K}{1 \text{ RAD}} = \frac{1 \text{ Ft} \cdot \text{K}}{0.1875L/EI}$$

$$\underline{K = 5.33 EI/L \text{ FOR } M @ B}$$

P9.33. Using the conjugate beam method, compute the maximum deflection in span BD of the beam in Figure P9.33 and the slope on each side of the hinge.

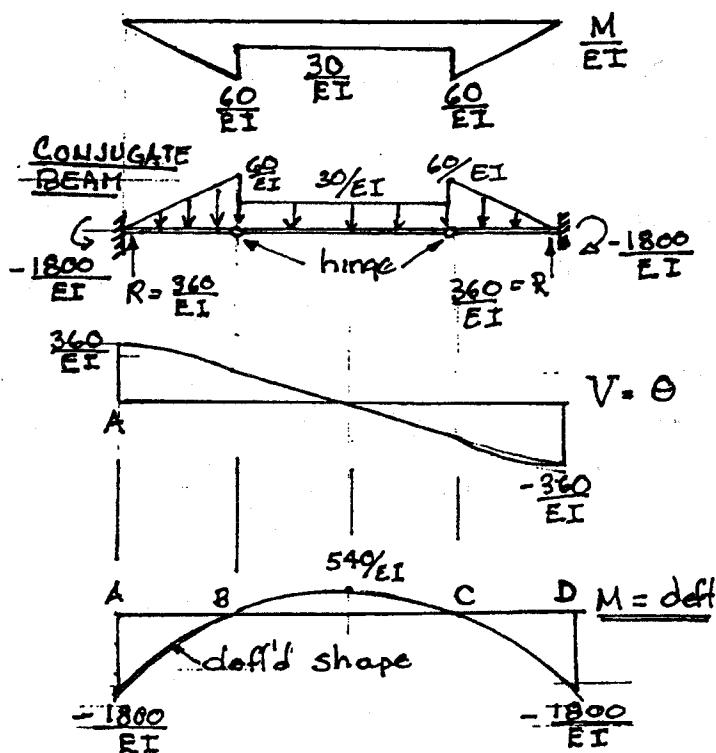


P9.33



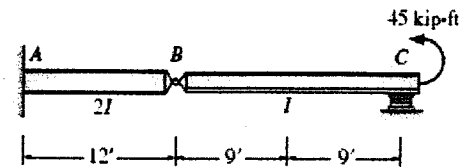
MOMENT
DIAGRAM

-60 kip-ft

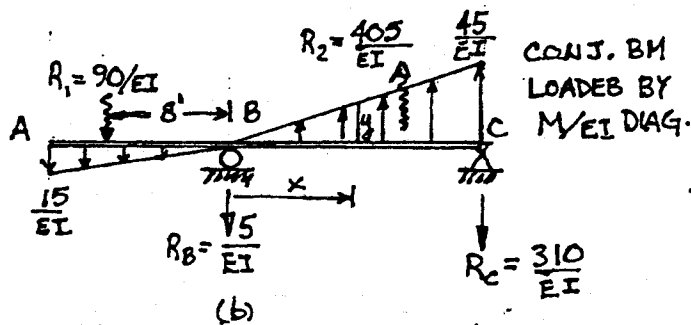
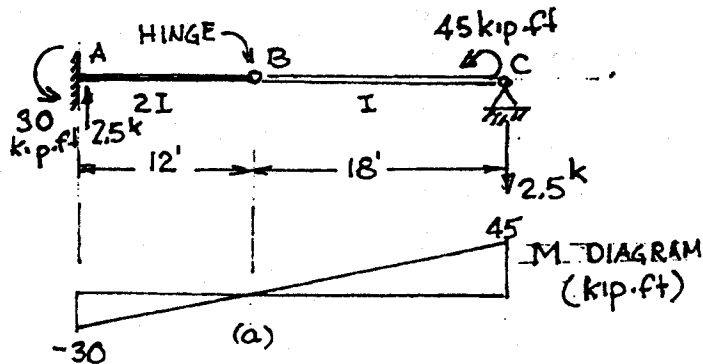


SUMMARY: $\Delta_A = \frac{-1800}{EI}$ $\Delta_L = \frac{540}{EI}$
 $\theta_A = \frac{360}{EI}$

P9.35. Solve Problem P9.12 by the conjugate beam method.



P9.12



$$R_1 = \frac{12 \times 15}{2 \times EI} = \frac{90}{EI}$$

$$R_2 = \frac{18 \times 45}{2 \times EI} = \frac{405}{EI}$$

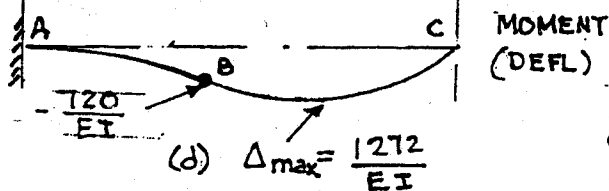
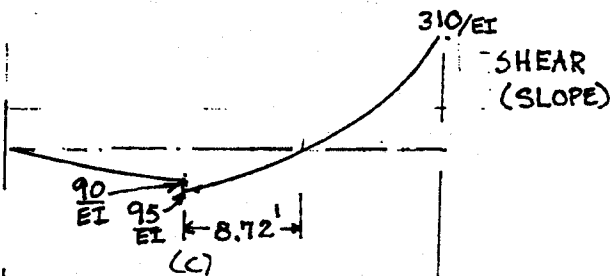
REACTIONS: $\sum M_B = 0$

$$0 = R_C(18) - \frac{405}{EI}(12) - \frac{90}{EI}(2)$$

$$R_C = \frac{310}{EI}$$

$$\sum F_y = 0 = -\frac{90}{EI} + R_B + \frac{405}{EI} - \frac{310}{EI}$$

$$R_B = -\frac{5}{EI}$$



LOCATE POINT OF MAX DEFL IN SPAN BC (SEE b)

(SHEAR = 0)

$$\frac{x}{18} = \frac{y/EI}{45/EI} \text{ (SIMILAR TRIANGLES)}$$

$$y = \frac{45x}{18} = \frac{5x}{2}$$

$$\Delta \text{SHEAR} = \text{AREA}$$

$$95 = \frac{1}{2} \times \left(\frac{5}{2}x\right)$$

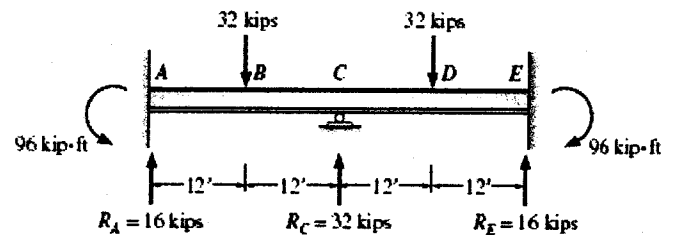
$$x = 8.72' \therefore y = \frac{5}{2}(8.72) = 21.8$$

$$\text{MAX DEFL} = M @ x = 8.72'$$

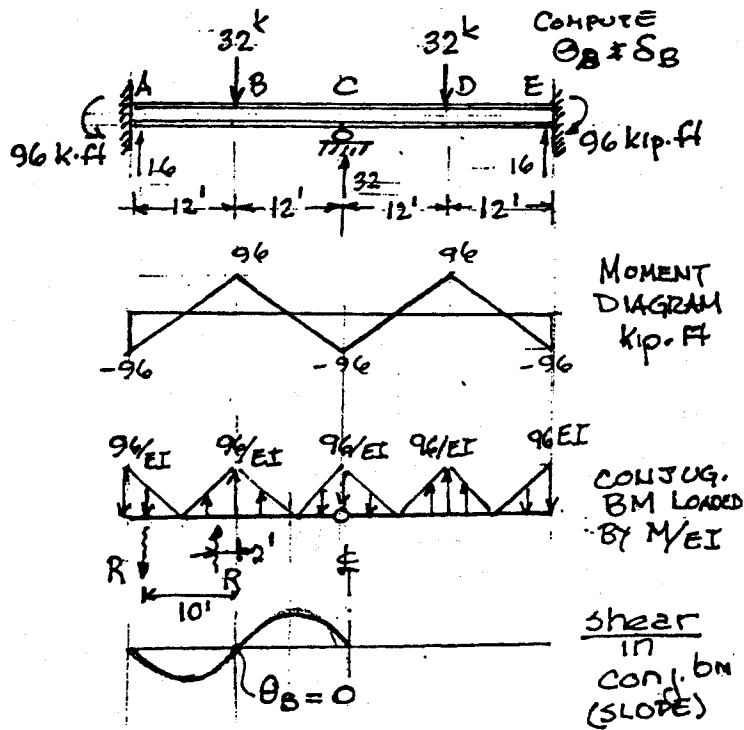
$$= \frac{-90}{EI}(8 + 8.72) - \frac{5}{EI}(8.72) + \frac{8.72(21.8)}{2} \frac{8.72}{3} = M$$

$$\Delta_{\text{MAX}} = M / \text{DEFL} = \frac{1272}{EI}$$

P9.36. Solve Problem P9.17 by the conjugate beam method.



P9.17



M at B = DEFLECTION

$$R = \frac{1}{2} \times 6 \times \frac{96}{EI} = \frac{288}{EI}$$

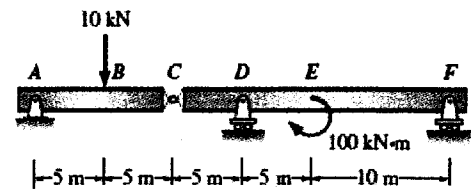
$$M_B = -\frac{288}{EI} \times 10 + \frac{288}{EI} \times 2$$

$$= -\frac{2304}{EI} \downarrow$$

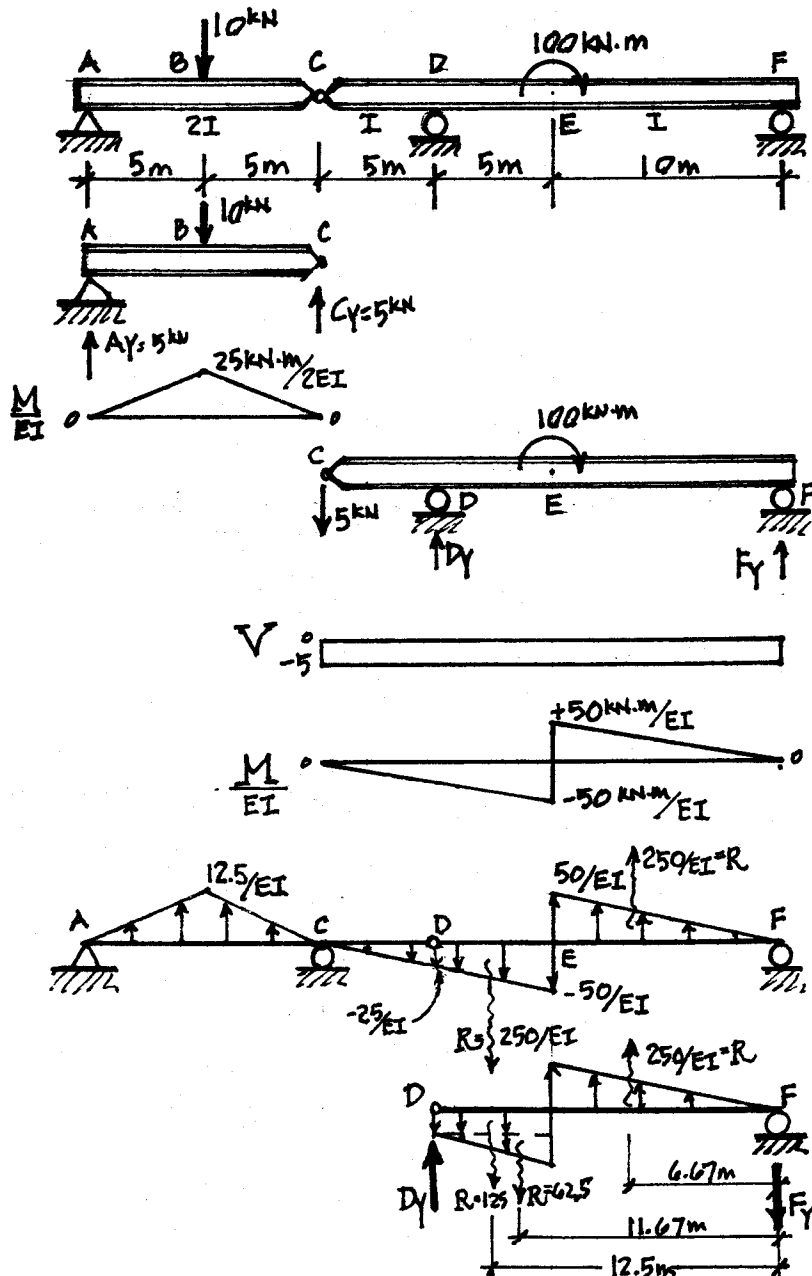
$$\Delta_B = M_B = -\frac{2304 \times 1728}{29,000 \times 510}$$

$$\Delta_B = -0.269 \text{ in } \downarrow$$

P9.37. For the beam shown in Figure P9.37, use the conjugate beam method to compute the vertical deflection and the rotation to the left and the right of the hinge at C. Given: $E = 200 \text{ GPa}$, $I_{AC} = 100 \times 10^6 \text{ mm}^4$, and $I_{CF} = 50 \times 10^6 \text{ mm}^4$.



P9.37



$$I_{CF} = 50 (10^6) \text{ mm}^4 = I$$

$$I_{AC} = 2I$$

SEGMENT "CF"

$$\sum M_F = 0;$$

$$-5^{\text{kN}}(20\text{m}) + 100^{\text{kN}\cdot\text{m}} + D_Y(15\text{m}) = 0$$

$$D_Y = 0$$

$$F_Y = 5^{\text{kN}} \uparrow$$

SEGMENT "DE"

$$\sum M_F = 0;$$

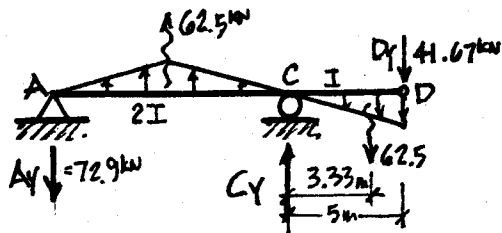
$$-125(12.5) - 6.25(11.67) + 250(6.67) + D_Y(15\text{m}) = 0$$

$$D_Y = 41.67 \text{ kN} \uparrow$$

$$\uparrow \sum F_Y = 0; \quad F_Y = 104.17^{\text{kN}} \downarrow$$

P9.37 Continued

P9.37 Continued



SEGMENT "AC"

$$\sum M_A = 0;$$

$$-62.5(5) + 62.5(13.33) + 41.67(15) - C_y(10) = 0$$

$$C_y = 114.57 \text{ kN} \uparrow$$

ROTATIONS @ 'C'

USE SEGMENT "DF" FOR SLOPE RIGHT OF "C":

$$\theta_R = \theta_{R'} = (250 - 250 - 104.17)/EI = 104.17/EI_{CF}$$

$$= 104.17 / 200(100 \times 10^6)$$

$$\theta_L = \theta_{L'} = C_y - V_R = 114.57 - 104.17 = 10.4/EI_{AC}$$

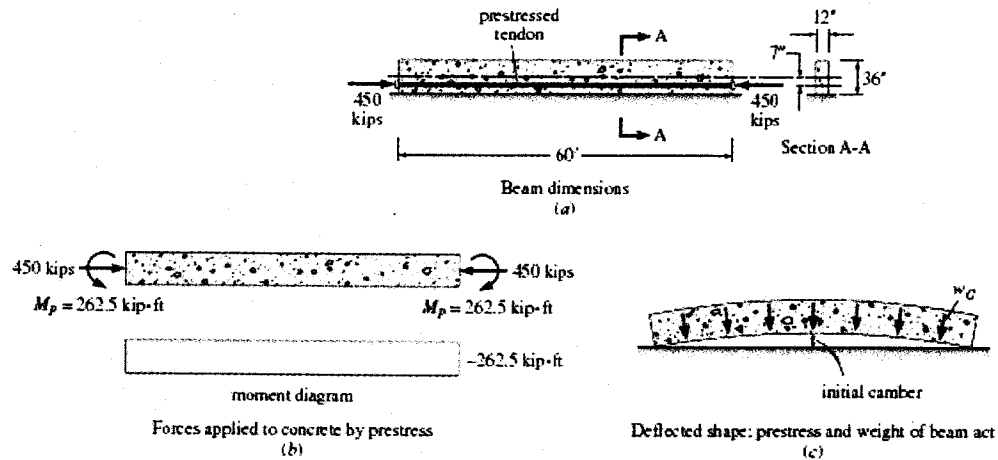
DEFLECTION @ HINGE 'C' = MOMENT IN CONJUGATE BM:

USE SEGMENT "AC"

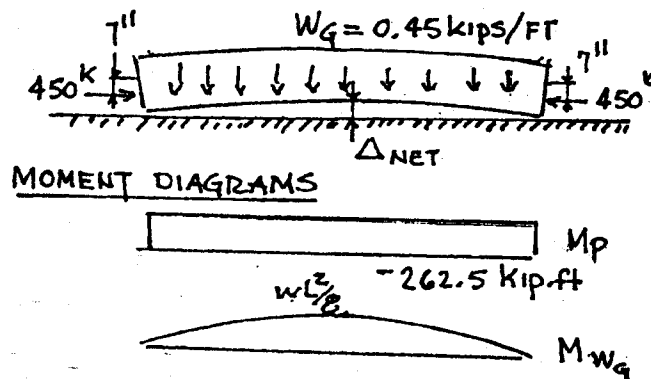
$$\Delta_c = M_c = (62.5(3.33) + 41.67(5))/EI = -416.5/EI_{CF} \downarrow$$

$$= -416.5 \text{ kN}\cdot\text{m} \left(\frac{1000 \text{ mm/m}}{(200 \times 10^6 \text{ kN/m}^2)(50 \times 10^{-6} \text{ m}^4)} \right) = -41.65 \text{ mm} \downarrow$$

P9.38. The reinforced concrete girder shown in Figure P9.38a is prestressed by a steel cable that induces a compression force of 450 kips with an eccentricity of 7 in. The external effect of the prestressing is to apply an axial force of 450 kips and equal end moment $M_p = 262.5$ kip-ft at the ends of the girder (Figure P9.38b). The axial force causes the beam to shorten but produces no bending deflections. The end moments M_p bend the beam upward (Figure P9.38c) so that the entire weight of the beam is supported at the ends, and the member acts as a simply supported beam. As the beam bends upward, the weight of the beam acts as a uniform load to produce downward deflection. Determine the initial camber of the beam at midspan immediately after the cable is tensioned. *Note:* Over time the initial deflection will increase due to creep by a factor of approximately 100 to 200 percent. The deflection at midspan due to the two end moments equals $ML^2/(8EI)$. Given: $I = 46,656$ in⁴, $A = 432$ in², beam weight $w_g = 0.45$ kip/ft, and $E = 5000$ kips/in².



P9.38



DEFL (PRESTRESS)

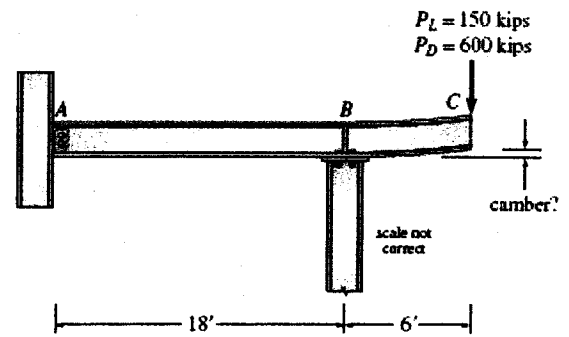
$$\Delta_p = \frac{ML^2}{8EI} = \frac{262.5(60')^2}{8 \times 5,000 \times 46,656} = 0.875" \uparrow$$

DEFL (WEIGHT OF BM)

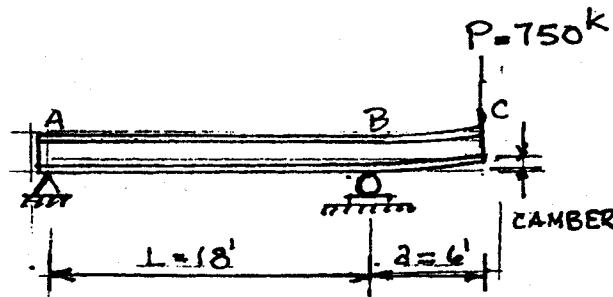
$$\Delta_d = \frac{5wL^4}{384EI} = \frac{5(0.45)(60')^4}{384 \times 5,000 \times 46,656} = 0.563" \downarrow$$

$$\underline{\underline{NET DEFL}} = 0.875 - 0.563 = \underline{\underline{.312" \uparrow}}$$

P9.39. Because of poor foundation conditions, a 30-in deep steel beam with a cantilever is used to support an exterior building column that carries a dead load of 600 kips and a live load of 150 kips (Figure P9.39). What is the magnitude of the initial camber that should be induced at point C, the tip of the cantilever, to eliminate the deflection produced by the total load? Neglect the beam's weight. Given: $I = 46,656 \text{ in}^4$ and $E_s = 30,000 \text{ ksi}$. See case 5 in Table 9.1 for the deflection equation. The clip angle connection at A may be treated as a pin and the cap plate support at B as a roller.

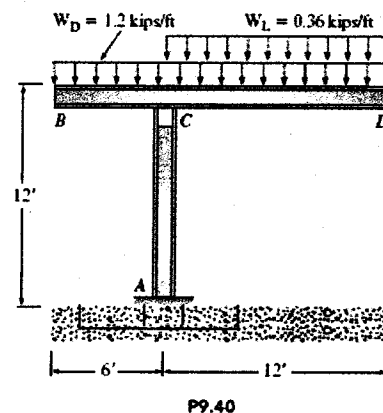


P9.39

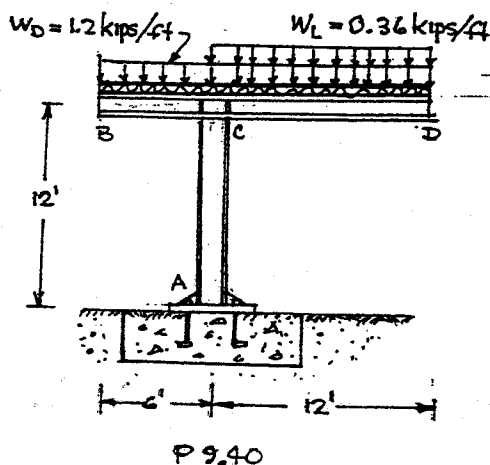


$$\begin{aligned} \text{CAMBER} = \Delta_C &= \frac{Pa^2(L+a)}{3EI} \\ &= \frac{750^k (6 \times 12)^2 (18+6) 12}{3 \times 30,000 \times 46,656} \\ &= \underline{\underline{0.27 \text{ in UPWARD}}} \end{aligned}$$

P9.40. The rigid jointed steel frame with a fixed base at support A has to carry both the dead and live loads shown in Figure P9.40. Both the column and the girder are constructed from the same size members. What is the minimum required moment of inertia of the frame members in the vertical deflection at D produced by these loads cannot exceed 0.5 in.? Use $E = 29,000$ ksi.



P9.40



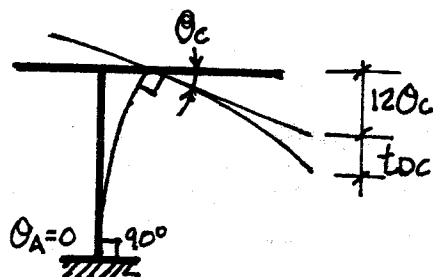
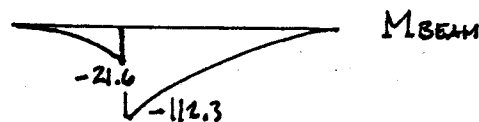
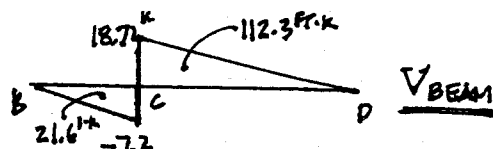
P9.40

$$\sum M_A = 0; (1.2 \text{ k/ft})(18)(3) + (0.36 \text{ k/ft})(12)^2/2 - M_A = 0$$

$$M_A = 90.72 \text{ Ft}\cdot\text{K} \uparrow$$

$$+\uparrow \sum F_y = 0; (-1.2 \text{ k/ft})(18) + (-0.36 \text{ k/ft})(12) + A_y = 0$$

$$A_y = 25.92 \text{ K} \uparrow$$



$$\theta_C = \theta_A + \Delta \theta_{AC}$$

$$= 0 + (90.72/EI)(12') = 1088.6/EI$$

$$t_{DC} = -\frac{112.3(12')(9')}{3} = 4042.8/EI$$

$$\Delta D_y = 12\theta_C + t_{DC} = 17,106.5/EI$$

$$I_{REQ'D} = \frac{17,106.5 (1728 \text{ in}^3/\text{ft}^3)}{29,000 \text{ ksi} (112'')}$$

$$I_{REQ'D} = 2038.6 \text{ in}^4$$

P9.41. Computer study of the behavior of multistory building frames. The object of this study is to examine the behavior of building frames fabricated with two common types of connections. When open interior spaces and future flexibility of use are prime considerations, building frames can be constructed with *rigid joints* usually fabricated by welding. Rigid joints (see Figure P9.41b) are expensive to fabricate and now cost in the range of \$700 to \$850 depending on the size of members. Since the ability of a welded frame to resist lateral loads depends on the bending stiffness of the beams and columns, heavy members may be required when lateral loads are large or when lateral deflections must be limited. Alternately, frames can be constructed less expensively by connecting the webs of beams to columns by angles or plates, called *shear connections*, which currently cost about \$80 each (Figure P9.41c). If *shear connections* are used, diagonal bracing, which forms a deep vertical truss with the attached columns and floor beams, is typically required to provide lateral stability (unless floors can be connected to stiff shear walls constructed of reinforced masonry or concrete).

Properties of Members

In this study all members are constructed of steel with $E = 29,000$ kips/in².

All beams: $I = 300$ in⁴ and $A = 10$ in²

All columns: $I = 170$ in⁴ and $A = 12$ in²

Diagonal bracing using 2.5 in square hollow structural tubes (Case 3 only—see dashed lines in Figure P9.41a), $A = 3.11$ in², $I = 3.58$ in⁴

Using the RISA-2D computer program, analyze the structural frames for gravity and wind loads in the following three cases.

Case 1 Unbraced Frame with Rigid Connections

- Analyze the frame for the loads shown in Figure P9.41a. Determine the forces and displacements at 7 sections along the axis of each member. Use the computer program to plot shear and moment diagrams.
- Determine if the relative lateral displacement between adjacent floors exceeds $3/8$ in—a limit specified to prevent cracking of the exterior façade.
- Using the computer program, plot the deflected shape of the frame.
- Note the difference between the magnitudes of the vertical and lateral displacements of joints 4 and 9. What are your conclusions?

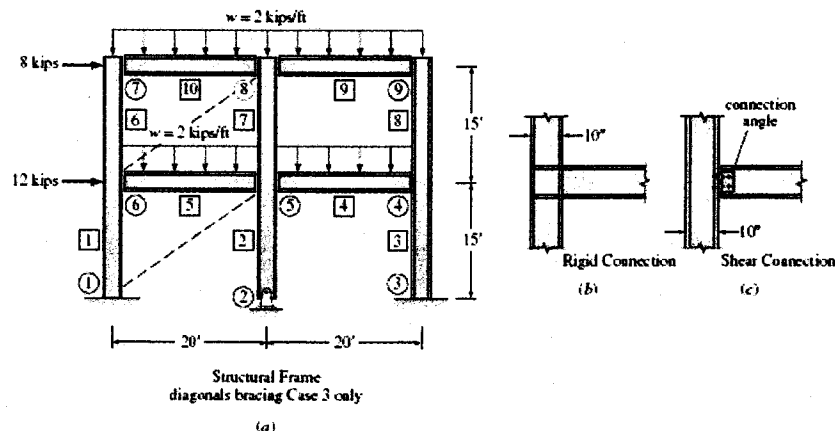
Case 2 Unbraced Frame with Shear Connections

- Repeat steps a, b, and c in Case 1, assuming that the shear connections act as hinges, that is, can transmit shear and axial load, but no moments.
- What do you conclude about the unbraced frame's resistance to lateral displacements?

Case 3 Braced Frame with Shear Connections

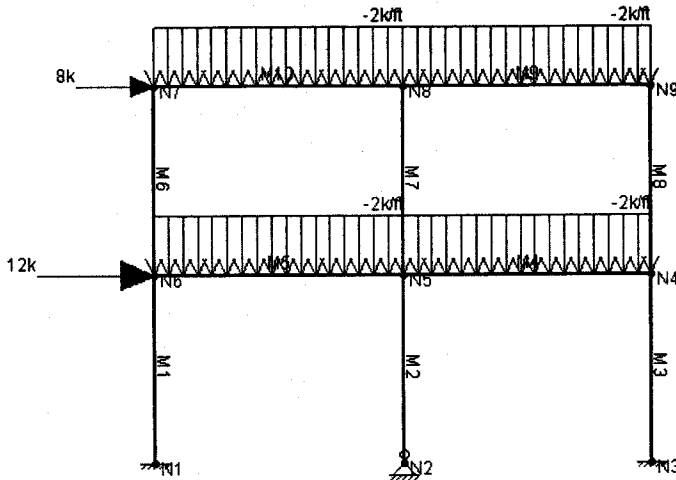
As in case 2, all beams are connected to columns with shear connectors, but diagonal bracing is added to form a vertical truss with floor beams and columns (see dashed lines in Figure P9.41a).

- Repeat steps a, b, and c in Case 1.
- Compute the lateral deflections of the frame if the area and moment of inertia of the diagonal members are *doubled*. Compare results to the original lighter bracing in (a) to establish the effectiveness of heavier bracing.
- Make up a table comparing lateral displacements of joints 4 and 9 for the three cases. Discuss briefly the results of this study.



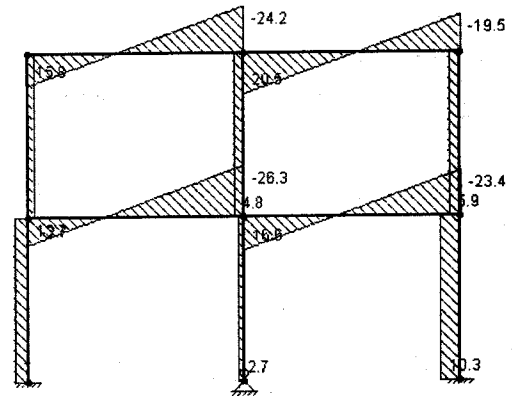
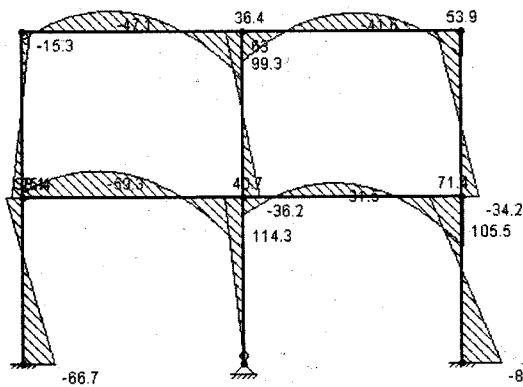
P9.41 SOLUTION FOLLOWS

Case 1 Unbraced Frame with Rigid Joints

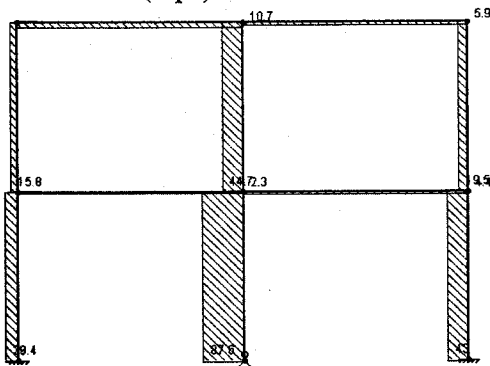


(a)
Moment Diagram (kip-ft)

Shear Diagram (kips)



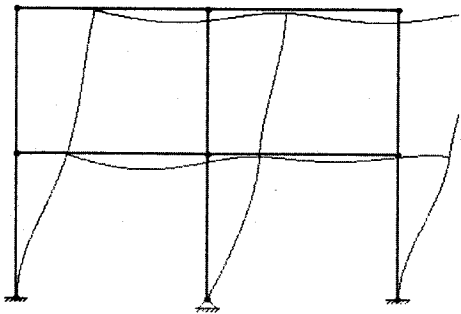
Axial Force (kips)



- (b)
Relative Displacement between 1st and 2nd floors = $1.25 - 0 = 1.25$ in. > $3/8$ in. (NG)
Relative Displacement between 2nd and 3rd floors = $1.90 - 1.25 = 0.65$ in. > $3/8$ in. (NG)

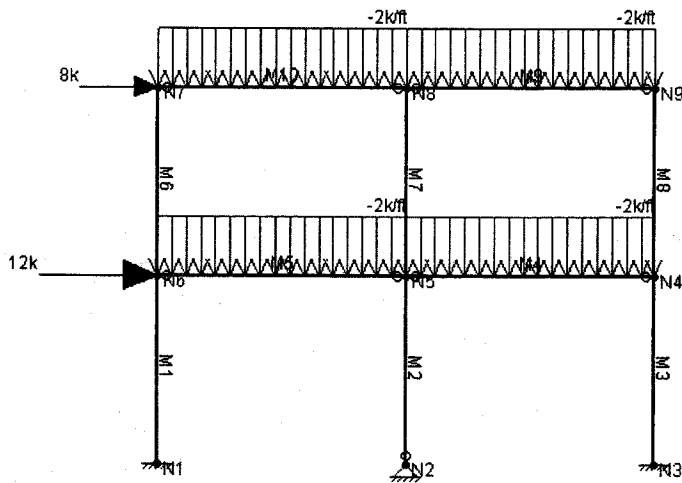
P9.41 CONTINUED (CASE 1 & CASE 2)

(c). (CASE 1 CONTINUED)

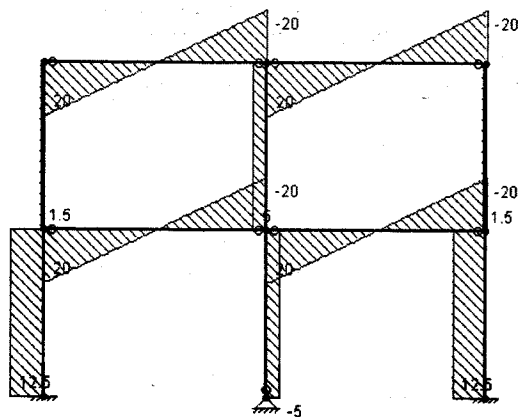


Joint Deflections				
Joint Label	X (in)	Y (in)	Rotation	
N1	0	0	0	
N2	0	0	0	
N3	0	0	0	
N4	1.247	-.022	-2.572e-3	
N5	1.25	-.045	-9.978e-4	
N6	1.252	-.015	-6.258e-3	
N7	1.913	-.023	-4.051e-3	
N8	1.904	-.068	-9.606e-4	
N9	1.899	-.032	1.759e-3	

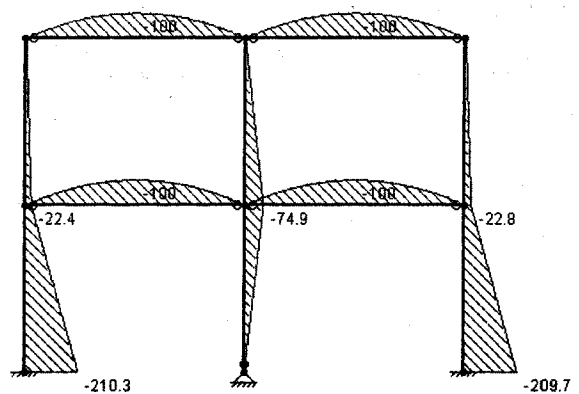
Case 2 Unbraced Frame with Shear Joints



(a)
Moment Diagram (kip-ft)



Shear Diagram (kips)

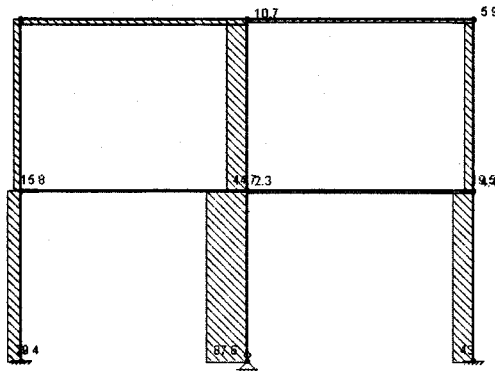


P9.41 CONTINUED

P9.41 CONTINUED (CASE 2)

CASE 2(Q.) CONTINUED

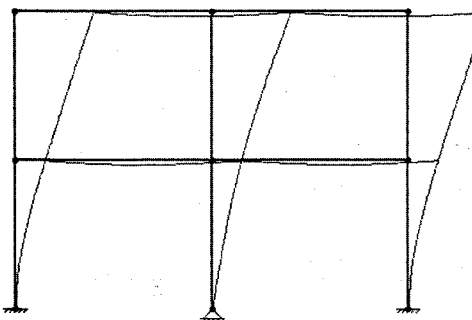
Axial Force (kips)



Relative Displacement between 1st and 2nd floors = $5.81 - 0 = 5.81$ in. $> 3/8$ in. (NG)

Relative Displacement between 2nd and 3rd floors = $15.58 - 5.81 = 9.77$ in. $> 3/8$ in. (NG)

Deflected Shape



Joint Deflections				
Joint Label	X [in]	Y [in]	Rotation	
N1	0	0	0	
N2	0	0	0	
N3	0	0	0	
N4	5.812	-.021	-5.093e-2	
N5	5.821	-.041	-4.327e-2	
N6	5.822	-.021	-5.097e-2	
N7	15.584	-.031	-5.587e-2	
N8	15.578	-.062	-5.968e-2	
N9	15.578	-.031	-5.591e-2	

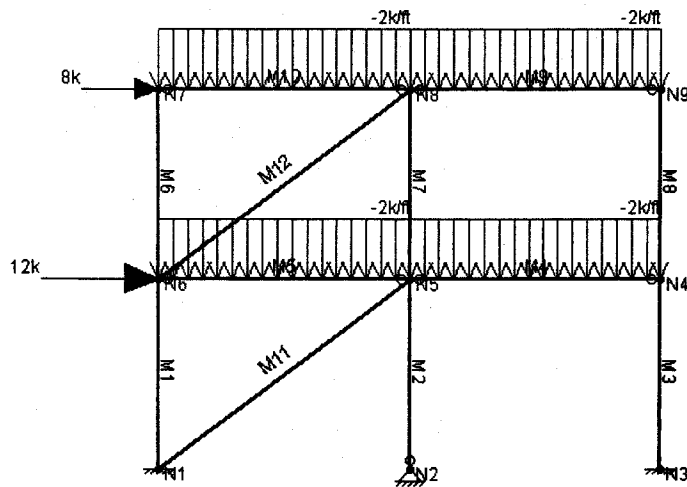
(b)

The lateral displacements are significantly increased if shear connections are used.

P9.41 CONTINUED

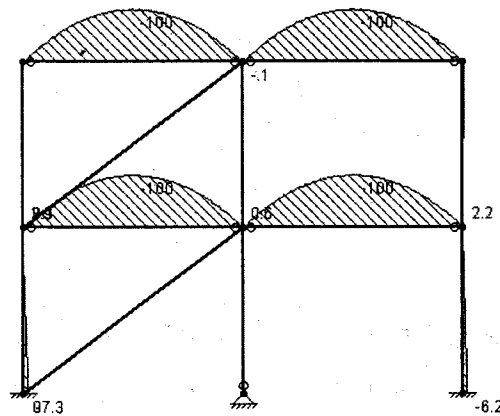
P9.41 CONTINUED (CASE 3)

Case 3 Braced Frame with Shear Joints

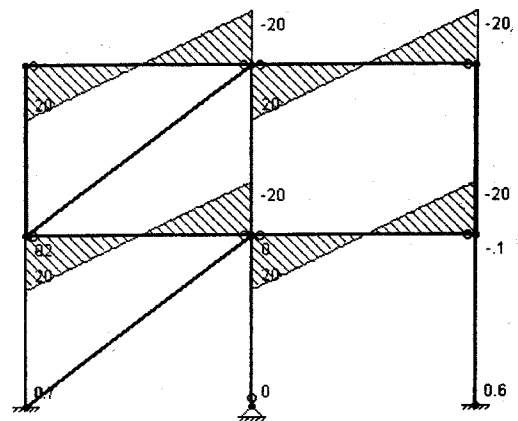


(a)

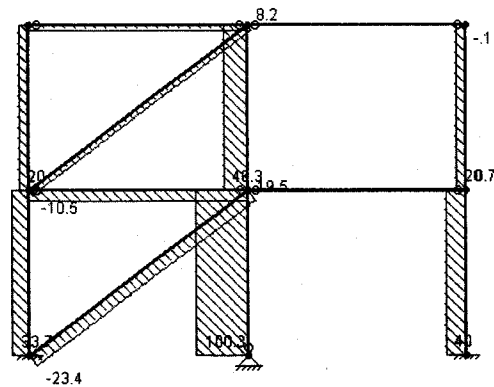
Moment Diagram (k-ft)



Shear Diagram (kips)



Axial Force Diagram (kips)



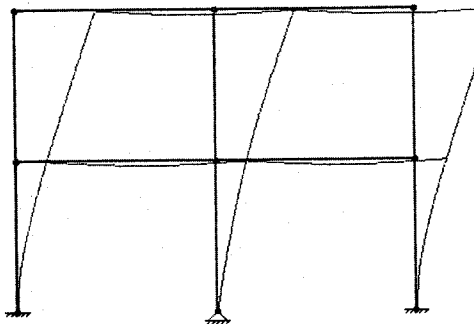
P9.41 CONTINUED

P9.41 Continued – Case 3

Relative Displacement between 1st and 2nd floors = $0.136 - 0 = 0.136$ in. > $3/8$ in. (OK)

Relative Displacement between 2nd and 3rd floors = $0.24 - 0.136 = 0.104$ in. > $3/8$ in. (OK)

Deflected Shape



Joint Deflections				
Joint Label	X (in)	Y (in)	Rotation	
N1	0	0	0	
N2	0	0	0	
N3	0	0	0	
N4	.136	-.021	-8.938e-4	
N5	.136	-.052	-6.674e-4	
N6	.152	-.017	-9.443e-4	
N7	.246	-.028	-3.121e-4	
N8	.24	-.076	-5.222e-4	
N9	.24	-.031	-4.212e-4	

(b)

Joint Deflections				
Joint Label	X (in)	Y (in)	Rotation (rad)	
N1	0	0	0	
N2	0	0	0	
N3	0	0	0	
N4	.089	-.021	-6.163e-4	
N5	.089	-.052	-4.757e-4	
N6	.105	-.018	-6.651e-4	
N7	.177	-.028	-2.66e-4	
N8	.17	-.076	-4.315e-4	
N9	.17	-.031	-3.73e-4	

Doubling the area and moment of inertia of the braces reduces the lateral displacements by 35% and 29% at the 2nd and 3rd floors, respectively.

(c)

Case	Lateral Displacement (in.)	
	Joint 4	Joint 9
1	1.247	1.899
2	5.812	15.578
3	0.136	0.240

For the given member sizes, the rigid frame (Case 1) is not stiff enough to limit the relative lateral displacements between floors to within $3/8$ in. Without rigid joints between beams and columns for a frame—as in Case 2—the lateral displacements are increased significantly. One efficient and practical way to increase the lateral stiffness of a frame with simple framing is to use diagonal braces, as in Case 3.