A Computationally Efficient Mixed-Integer Linear Formulation for the Thermal Unit Commitment Problem

Miguel Carrión, *Student Member, IEEE*, and José M. Arroyo, *Senior Member, IEEE*

***Abstract*—This paper presents a new mixed-integer linear formulation for the unit commitment problem of thermal units. The formulation proposed requires fewer binary variables and constraints than previously reported models, yielding a significant computational saving. Furthermore, the modeling framework provided by the new formulation allows including a precise description of time-dependent startup costs and intertemporal constraints such as ramping limits and minimum up and down times. A commercially available mixed-integer linear programming algorithm has been applied to efficiently solve the unit commitment problem for practical large-scale cases. Simulation results back these conclusions.**

***Index Terms*—Mixed-integer linear programming (MILP), thermal generating units, unit commitment.**

NOMENCLATURE

Constants

$\mathrm{A_j}$	Coefficient of the piecewise linear production cost function of unit j.
$\mathrm{a_j, b_j, c_j}$	Coefficients of the quadratic production cost function of unit j.
$\mathrm{cc_j, hc_j, t_j^{cold}}$	Coefficients of the startup cost function of unit j.
$\mathrm{C_j}$	Shutdown cost of unit j.
$\mathrm{D(k)}$	Load demand in period k.
$\mathrm{DT_j}$	Minimum down time of unit j.
$\mathrm{F_{\ell j}}$	Slope of block ℓ of the piecewise linear production cost function of unit j.
$\mathrm{G_j}$	Number of periods unit j must be initially online due to its minimum up time constraint.
$\mathrm{K_j^t}$	Cost of the interval t of the stairwise startup cost function of unit j.
$\mathrm{L_j}$	Number of periods unit j must be initially offline due to its minimum down time constraint.
$\mathrm{ND_j}$	Number of intervals of the stairwise startup cost function of unit j.
$\mathrm{NL_j}$	Number of segments of the piecewise linear production cost function of unit j.
$\mathrm{\bar{P}_j}$	Capacity of unit j.
$\mathrm{\underline{P}_j}$	Minimum power output of unit j.
$\mathrm{R(k)}$	Spinning reserve requirement in period k.
$\mathrm{RD_j}$	Ramp-down limit of unit j.
$\mathrm{RU_j}$	Ramp-up limit of unit j.
$\mathrm{S_j(0)}$	Number of periods unit j has been offline prior to the first period of the time span (end of period 0).
$\mathrm{SD_j}$	Shutdown ramp limit of unit j.
$\mathrm{SU_j}$	Startup ramp limit of unit j.
T	Number of periods of the time span.
$\mathrm{T_{\ell j}}$	Upper limit of block ℓ of the piecewise linear production cost function of unit j.
$\mathrm{U_j^0}$	Number of periods unit j has been online prior to the first period of the time span (end of period 0).
$\mathrm{UT_j}$	Minimum up time of unit j.
$\mathrm{V_j(0)}$	Initial commitment state of unit j (1 if it is online, 0 otherwise).

Variables

$\mathrm{c_j^d(k)}$	Shutdown cost of unit j in period k.
$\mathrm{c_j^p(k)}$	Production cost of unit j in period k.
$\mathrm{c_j^u(k)}$	Startup cost of unit j in period k.
$\mathrm{p_j(k)}$	Power output of unit j in period k.
$\mathrm{\bar{p}_j(k)}$	Maximum available power output of unit j in period k.
$\mathrm{t_j^{off}(k)}$	Number of periods unit j has been offline prior to the startup in period k.
$\mathrm{v_j(k)}$	Binary variable that is equal to 1 if unit j is online in period k and 0 otherwise.
$\mathrm{\delta_\ell(j,k)}$	Power produced in block ℓ of the piecewise linear production cost function of unit j in period k.

Sets

J	Set of indexes of the generating units.
K	Set of indexes of the time periods.

Manuscript received November 15, 2005; revised January 31, 2006. This work was supported in part by the Ministry of Education and Science of Spain under Grant CICYT DPI2003-01362 and in part by the Junta de Comunidades de Castilla—La Mancha, Spain, under Grant PBI-05-053. Paper no. TPWRS-00717-2005.

The authors are with the Departamento de Ingeniería Eléctrica, Electrónica, Automática y Comunicaciones, E.T.S.I. Industriales, Universidad de Castilla—La Mancha, Ciudad Real E-13071, Spain (e-mail: miguel.carrion@uclm.es; josemanuel.arroyo@uclm.es).

Digital Object Identifier 10.1109/TPWRS.2006.876672

I. INTRODUCTION

THE NEW competitive environment in power systems is demanding more efficient and accurate tools to support decisions for resource scheduling. The thermal unit commitment problem has been traditionally solved in centralized power systems to determine when to startup or shutdown thermal generating units and how to dispatch online generators to meet system demand and spinning reserve requirements while satisfying generation constraints (production limits, ramping limits, and minimum up and down times) over a specific short-term time span, so that the overall operation cost is minimized [1].

The generation scheduling problems solved by the independent system operator (ISO) in current electricity markets [2] are similar to the unit commitment problem in centralized noncompetitive power systems, as promoted by FERC's Standard Market Design [3]. The main conceptual difference between both problems is that, rather than minimizing operation costs, the ISO maximizes a measure of social welfare, which is a function of market participant bids and offers. Thus, the solution of the traditional, centralized unit commitment problem is relevant for the competitive power industry.

For several decades, this large-scale, mixed-integer, combinatorial, and nonlinear programming problem has been an active research topic because of potential savings in operation costs. As a consequence, several solution techniques have been proposed such as heuristics [4]–[6], dynamic programming [7]–[9], mixed-integer linear programming (MILP) [10], [11], Lagrangian relaxation [12]–[18], simulated annealing [19]–[21], and evolution-inspired approaches [22]–[26]. A recent extensive literature survey on unit commitment can be found in [27].

Among the aforementioned methodologies, Lagrangian relaxation is the most widely used approach because of its capability of solving large-scale problems. The main disadvantage of this method is that, due to the nonconvexities of the unit commitment problem, heuristic procedures are needed to find feasible solutions, which may be suboptimal.

In contrast, MILP guarantees convergence to the optimal solution in a finite number of steps [28] while providing a flexible and accurate modeling framework. In addition, during the search of the problem tree, information on the proximity to the optimal solution is available. Efficient mixed-integer linear software such as the branch-and-cut algorithm has been developed, and optimized commercial solvers with large-scale capabilities are currently available [29]–[31]. As a consequence, a great deal of attention has been paid to MILP-based approaches.

In [10], MILP was first applied to solve the unit commitment problem. The formulation in [10] was based on the definition of three sets of binary variables to, respectively, model the startup, shutdown, and on/off states for every unit and every time period. This mixed-integer linear formulation was extended in [32] to model the self-scheduling problem faced by a single generating unit in an electricity market. Nonconvex production costs, time-dependent startup costs, and intertemporal constraints such as ramping limits and minimum up and down times were accounted for at the expense of increasing the number of binary variables.

For realistic power systems comprising several tens of generators, the models of [10] and [32] require a large number of binary variables. Thus, the resulting MILP problems might be computationally intensive for state-of-the-art implementations of branch-and-cut algorithms [29]–[31] and current computing capabilities.

In [33], startup costs and minimum up and down times were formulated using linear expressions that required a single type of binary variables. However, the unit commitment model did not consider ramping limits and their influence on the spinning reserve constraints. In addition, shutdown costs were not modeled either.

The objective of this paper is to present an alternative mixed-integer linear formulation of the thermal unit commitment problem, hereinafter denoted by MILP-UC, requiring a single set of binary variables (one per unit and per period). Unlike previous MILP approaches [10], [32], the lower number of binary variables in MILP-UC yields a reduction in the number of nodes of the search tree used by the branch-and-cut algorithm, as well as a reduction in the number of constraints, thus decreasing the computing time required by available solvers [29]–[31] to tackle realistic cases. Moreover, MILP-UC accurately models thermal unit commitment states, intertemporal constraints, and time-dependent startup costs, thereby improving the modeling capabilities of [33].

The model proposed in this paper is also applicable to the scheduling problems arising in electricity markets such as market-clearing procedures solved by ISOs, self-scheduling problems solved by generating companies to derive bidding strategies, and market simulators used to analyze the behavior of market participants. Therefore, market agents can benefit from MILP-UC.

The main contributions of this paper are as follows.

1) A new formulation, MILP-UC, requiring fewer binary variables and constraints to accurately model the thermal unit commitment problem is presented in order to reduce the computational burden of existing MILP approaches.
2) Numerical experience is reported by solving a realistic application.

The remaining sections are outlined as follows. Section II provides a detailed description of the proposed MILP-UC formulation. This section includes a precise model of the physical and intertemporal constraints of the power generators. In Section III, numerical results are presented and discussed. In Section IV, some relevant conclusions are drawn. Finally, the data used in the numerical simulations are provided in the Appendix.

II. MILP-UC Formulation

The unit commitment problem can be formulated as [1]

$$\text{Minimize} \sum_{k\in K}\sum_{j\in J} c_j^p(k) + c_j^u(k) + c_j^d(k) \tag{1}$$

subject to

$$\sum_{j\in J} p_j(k) = D(k), \quad \forall k \in K \tag{2}$$

$$\sum_{j\in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K \tag{3}$$

$$p_j(k) \in \Pi_j(k), \quad \forall j \in J, \forall k \in K \tag{4}$$

where $\Pi_j(k)$ represents the region of feasible production of generating unit j in time period k.

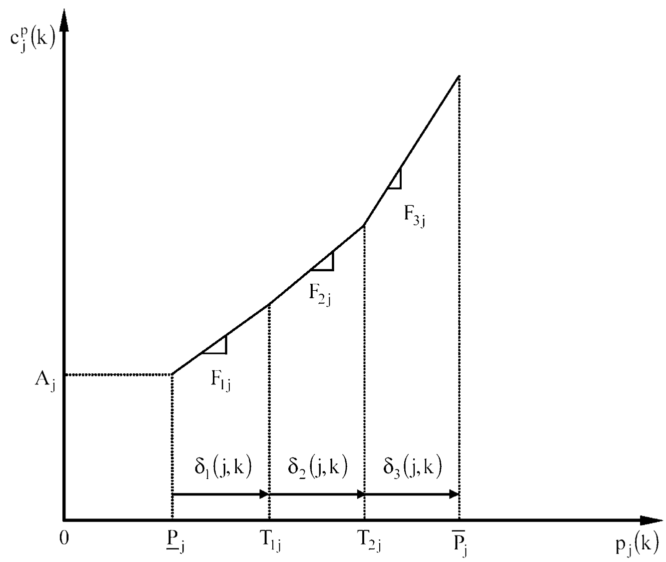

Fig. 1. Piecewise linear production cost.

The goal of the unit commitment problem is to minimize the total operation cost, which is defined as the sum of the production cost, the startup cost, and the shutdown cost (1). The production cost is typically expressed as a quadratic function of the power output, while the startup cost is usually modeled as a nonlinear (exponential) function of the offline time prior to the startup [1]. The block of constraints (2) represents power balances in all periods. Constraints (3) provide spinning reserve margins. The block of constraints (4) expresses in a compact way the operating constraints, for every time period, of every unit, e.g., generation limits, ramp rate limits, and minimum up and down times. Note that binary variables are used to model on/off decisions. Although network constraints and losses can be incorporated in the above formulation, for the sake of simplicity, we have opted to restrict our analysis to a one-bus system. For unit consistency, it should be noted that hourly time periods are considered.

Problem (1)–(4) is a mixed-integer and nonlinear optimization problem that is difficult to solve by standard nonlinear programming methods. Next, we describe an alternative mixed-integer linear formulation, MILP-UC, suitable for available MILP software [29]–[31].

A. Objective Function

The three components of the objective function (1) mentioned above are explained in the following.

1) Production Cost: The quadratic production cost function typically used in scheduling problems [1] can be formulated as

$$c_j^p(k) = a_j v_j(k) + b_j p_j(k) + c_j p_j^2(k), \forall j \in J, \forall k \in K. \quad (5)$$

As shown in Fig. 1, the cost function in (5) can be accurately approximated by a set of piecewise blocks [34]. For practical purposes, the piecewise linear function of Fig. 1 is indistinguishable from the nonlinear model if enough segments are used.

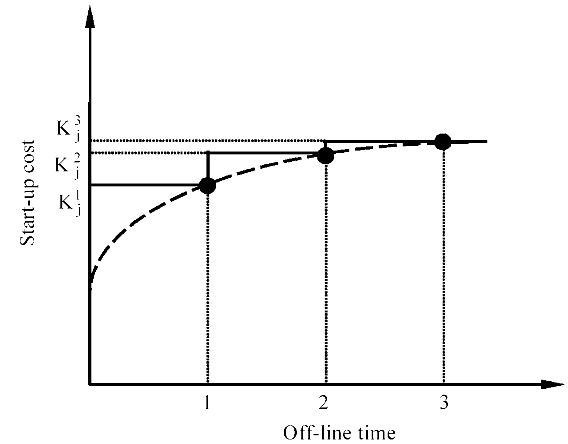

Fig. 2. Exponential, discrete, and stairwise startup cost functions.

The analytic representation of this linear approximation is

$$c_j^p(k) = A_j v_j(k) + \sum_{\ell=1}^{NL_j} F_{\ell j}\delta_\ell(j,k), \forall j \in J, \forall k \in K \quad (6)$$

$$p_j(k) = \sum_{\ell=1}^{NL_j} \delta_\ell(j,k) + \underline{P}_j v_j(k), \forall j \in J, \forall k \in K \quad (7)$$

$$\delta_1(j,k) \le T_{1j} - \underline{P}_j, \forall j \in J, \forall k \in K \quad (8)$$

$$\delta_\ell(j,k) \le T_{\ell j} - T_{\ell-1j}, \forall j \in J, \forall k \in K, \forall \ell = 2 \cdots NL_j - 1 \quad (9)$$

$$\delta_{NL_j}(j,k) \le \overline{P}_j - T_{NL_j-1j}, \forall j \in J, \forall k \in K \quad (10)$$

$$\delta_\ell(j,k) \ge 0, \forall j \in J, \forall k \in K, \forall \ell = 1 \cdots NL_j \quad (11)$$

where $A_j = a_j + b_j\underline{P}_j + c_j\underline{P}_j^2$.

2) Startup Cost: The dashed line in Fig. 2 shows a typical exponential startup cost function [1]. Since the time span has been discretized into hourly periods, the startup cost is also a discrete function, as shown in Fig. 2 with blackened circles. The discrete startup cost can be asymptotically approximated by a stairwise function (solid line in Fig. 2), which is more accurate as the number of intervals increases.

A mixed-integer linear formulation for the stairwise startup cost was proposed in [33]

$$c_j^u(k) \ge K_j^t \left[v_j(k) - \sum_{n=1}^{t} v_j(k-n) \right], \quad \forall j \in J, \forall k \in K$$
$$\forall t = 1 \cdots ND_j \quad (12)$$

$$c_j^u(k) \ge 0, \quad \forall j \in J, \forall k \in K. \quad (13)$$

Note that (12) and (13) only depend on the binary variables associated with the on/off state of generating units, $v_j(k)$.

3) Shutdown Cost: A constant shutdown cost C_j is incurred if unit j is brought offline due to the waste of fuel [1]. Previously reported formulations made use of an extra binary variable associated with the shutdown state [10], [11].

Constraints (14) and (15), however, show an alternative equivalent formulation for the shutdown cost using only binary variables $v_j(k)$

$$c_j^d(k) \ge C_j[v_j(k-1) - v_j(k)], \quad \forall j \in J, \forall k \in K \quad (14)$$

$$c_j^d(k) \ge 0, \quad \forall j \in J, \forall k \in K. \quad (15)$$

B. Thermal Constraints

The proposed formulation of the set of constraints related to every generating unit over the time span, $\Pi_j(k)$, is described next.

1) Generation Limits and Ramping Constraints: The generation limits of each unit for each period are set as follows:

$$\underline{P}_j v_j(k) \leq p_j(k) \leq \bar{p}_j(k), \quad \forall j \in J, \forall k \in K \tag{16}$$
$$0 \leq \bar{p}_j(k) \leq \overline{P}_j v_j(k), \quad \forall j \in J, \forall k \in K. \tag{17}$$

Constraints (16) bound the generation by the minimum power output and the maximum available power output of unit j in period k, $\bar{p}_j(k)$, which is a nonnegative variable bounded above by the unit capacity (17). Note that if unit j is offline in period k, i.e., $v_j(k) = 0$, both $p_j(k)$ and $\bar{p}_j(k)$ are equal to 0.

Variables $\bar{p}_j(k)$ are also constrained by ramp-up and startup ramp rates (18), as well as by shutdown ramp rates (19)

$$\begin{aligned}\bar{p}_j(k) \leq\; & p_j(k-1) + RU_j v_j(k-1) \\ & + SU_j[v_j(k) - v_j(k-1)] \\ & + \overline{P}_j[1 - v_j(k)], \quad \forall j \in J, \forall k \in K\end{aligned} \tag{18}$$

$$\begin{aligned}\bar{p}_j(k) \leq\; & \overline{P}_j v_j(k+1), \\ & + SD_j[v_j(k) - v_j(k+1)], \forall j \in J, \quad \forall k = 1 \cdots T-1.\end{aligned} \tag{19}$$

Furthermore, ramp-down limits are imposed on the power output

$$\begin{aligned}p_j(k-1) - p_j(k) \leq\; & RD_j v_j(k) \\ & + SD_j[v_j(k-1) - v_j(k)] \\ & + \overline{P}_j[1 - v_j(k-1)], \forall j \in J, \forall k \in K.\end{aligned} \tag{20}$$

The above formulation (16)–(20) extends that presented in [33] by precisely modeling the spinning reserve contribution of each unit in each period, which can be easily computed as the difference between $\bar{p}_j(k)$ and $p_j(k)$. Note that spinning reserve constraints (3) include variables $\bar{p}_j(k)$ and therefore meet the ramping limitations, yielding an accurate representation of the actual operation of generating units.

Again, constraints (16)–(20) only include binary variables $v_j(k)$ as opposed to the equivalent model provided in [32], which required extra binary variables.

2) Minimum Up and Down Time Constraints: In [32], minimum up and down time constraints were first formulated as mixed-integer linear expressions relying on binary variables associated with the startup, shutdown, and on/off states of generating units. Next, an equivalent mixed-integer linear formulation based only on binary variables $v_j(k)$ is presented.

The new expressions for the minimum up time constraints are as follows:

$$\sum_{k=1}^{G_j} [1 - v_j(k)] = 0, \quad \forall j \in J \tag{21}$$

$$\sum_{n=k}^{k+UT_j-1} v_j(n) \geq UT_j[v_j(k) - v_j(k-1)] \quad \forall j \in J, \forall k = G_j + 1 \cdots T - UT_j + 1 \tag{22}$$

$$\sum_{n=k}^{T} \{v_j(n) - [v_j(k) - v_j(k-1)]\} \geq 0 \quad \forall j \in J, \forall k = T - UT_j + 2 \cdots T \tag{23}$$

where G_j is the number of initial periods during which unit j must be online. G_j is mathematically expressed as $G_j = \mathrm{Min}\{T, [UT_j - U_j^0]V_j(0)\}$.

Constraints (21) are related to the initial status of the units as defined by G_j. Constraints (22) are used for the subsequent periods to satisfy the minimum up time constraint during all the possible sets of consecutive periods of size UT_j. Constraints (23) model the final $UT_j - 1$ periods in which if unit j is started up, it remains online until the end of the time span.

Analogously, minimum down time constraints are formulated as follows:

$$\sum_{k=1}^{L_j} v_j(k) = 0, \quad \forall j \in J \tag{24}$$

$$\sum_{n=k}^{k+DT_j-1} [1 - v_j(n)] \geq DT_j[v_j(k-1) - v_j(k)] \quad \forall j \in J, \forall k = L_j + 1 \cdots T - DT_j + 1 \tag{25}$$

$$\sum_{n=k}^{T} \{1 - v_j(n) - [v_j(k-1) - v_j(k)]\} \geq 0 \quad \forall j \in J, \forall k = T - DT_j + 2 \cdots T \tag{26}$$

where L_j is the number of initial periods during which unit j must be offline. L_j is mathematically expressed as $L_j = \mathrm{Min}\{T, [DT_j - S_j(0)][1 - V_j(0)]\}$.

Note that the respective replacement of $v_j(k), DT_j$, and $S_j(0)$ by $1 - v_j(k), UT_j$ and U_j^0 in (24)–(26) yields (21)–(23).

In summary, problem (1)–(3) and (6)–(26) constitutes the proposed MILP-UC, which only requires a single type of binary variables, namely, $v_j(k)$, unlike other MILP formulations [10], [32] based on the definition of extra binary variables. Moreover, MILP-UC also outperforms the model of [33] by allowing for an accurate model of ramping limits and individual contributions to the spinning reserve requirement.

III. Numerical Results

The proposed formulation has been applied to solve a real-size case study based on the ten-unit system of [22], which has been replicated ten times so that the case study comprises 100 units. The load demand has been accordingly multiplied by 10. A spinning reserve requirement of 10% of the load demand has to be met in each of the 24 hourly periods in which the time span is divided. For quick reference, the data of the original ten-unit system of [22] can be found in the Appendix.

Quadratic production costs have been linearized through a piecewise linear approximation with four segments. Time-dependent startup costs have been modeled by a 15-interval stairwise linear function. Once the solution from MILP-UC is obtained, a quadratic-programming-based economic dispatch is run to facilitate the assessment of the results.

The model has been implemented on a Dell PowerEdge 6600 with two processors at 1.60 GHz and 2 GB of RAM memory using CPLEX 9.0 [30] to solve MILP-UC and MINOS 5.51 [35]

TABLE I
COMPARISON OF TOTAL OPERATION COSTS

Approach	Total operation cost ($)
LR [22]	5657277
GA [22]	5627437
EP [23]	5623885
LRGA [37]	5613127
PL [6]	5608440
ELR [18]	5605678
MILP-UC	5605189

TABLE II
COMPARISON OF COMPUTATIONAL DIMENSION AND RESULTS

	MILP-UC	MILP-3	MILP-3R
# of binary variables	2400	7200	4800
# of continuous variables	19201	19201	21601
# of constraints	69489	74349	74349
# of non-zero elements in the constraint matrix	389161	414741	414741
Computing time (s)	123	324	409
Total operation cost ($)	5605189	5612129	5606877

for the quadratic economic dispatch. Both solvers have been called from GAMS [36]. In CPLEX, an optimality parameter can be specified to decide whether to find the optimal solution or to quickly obtain a suboptimal solution. In this case study, the execution of CPLEX was stopped when the value of the objective function was within 0.5% of the optimal solution. With this stopping criterion, MILP-UC required 123 s to achieve a solution with a total operation cost of $5605189.

The quality of the solution found by MILP-UC has been assessed through the comparison with the results achieved by previously reported methods: Lagrangian relaxation (LR) [22], genetic algorithms (GA) [22], evolutionary programming (EP) [23], a hybrid of Lagrangian relaxation and genetic algorithms (LRGA) [37], priority list (PL) [6], and an enhanced adaptive Lagrangian relaxation (ELR) [18]. As can be seen in Table I, the superiority of MILP-UC over the other approaches is substantiated by the achievement of a better total operation cost.

The computational performance of MILP-UC has been assessed with two equivalent MILP formulations denoted as MILP-3 and MILP-3R, respectively.

MILP-3 includes two additional sets of binary variables for the startup and shutdown processes [10], [32]. The logic of startups and shutdowns is enforced in MILP-3 through additional equality constraints relating the three sets of binary variables [10], [32]. The new equality constraints in MILP-3 allow relaxing the integrality of any one of the three types of binary variables, since these constraints guarantee that such variables will always take on the value 0 or 1 when the other two are binary. MILP-UC has also been tested against the relaxed version of MILP-3, referred to as MILP-3R, in which one type of binary variables has been defined as a set of real variables belonging to the interval [0, 1].

Table II summarizes the computational dimension and the results for the three MILP-based approaches when an optimality parameter of 0.5% is specified. The imposition of the same stopping criterion to the three methods yields similar solutions in terms of total operation cost, but MILP-UC outperforms MILP-3 and MILP-3R through the reduction in the computing time by factors of 2.63 and 3.33, respectively, thus showing the computational advantage of eliminating the variables related to startups and shutdowns. For illustration purposes, the production schedule found by MILP-UC is shown in Table III.

The performance of the three MILP formulations in terms of objective function value has been analyzed by allowing the branch-and-cut algorithm to run for a maximum of 900 s, which has been considered the limit for a moderate computing time for this type of short-term scheduling problem. Fig. 3 shows the evolution with computing time of the best solution found by each MILP formulation. The top figure plots the evolution over the first 500 s, whereas the solutions found within computing times ranging from 500 to 900 s are depicted in the bottom figure.

MILP-UC (solid line) starts the search process from a better initial solution with respect to MILP-3 (dotted line) and MILP-3R (dashed line). Moreover, MILP-UC converges faster (123 s) to a solution within a 0.5% optimality tolerance in comparison to MILP-3 and MILP-3R, which, respectively, require 324 and 409 s to reach the same level of solution quality (see Table II). It is remarkable that MILP-3R is not capable of achieving significant improvement over the first solution satisfying the 0.5% optimality tolerance (see Table II), thus yielding a worse overall performance. In addition, although a slight improvement of 0.004% over MILP-UC is achieved by MILP-3 in the interval ranging between 543 and 738 s, MILP-3 gets stuck at that solution for the remaining time span analyzed. In contrast, MILP-UC is able to further reduce the optimality gap down to 0.29% by achieving a better solution of $5602253 after 738 s. This solution represents a 0.05% improvement upon the solution previously found by MILP-UC within a 0.5% optimality gap (see Table II). In addition, this solution outperforms the best solutions attained by MILP-3 and MILP-3R by 0.03% and 0.06%, respectively.

Finally, in order to assess the influence of the problem size on the computational performance, MILP-UC, MILP-3, and MILP-3R have also been applied to the original ten-unit system [22] and to systems comprising up to 90 units that were created in the same way as the 100-unit case. Table IV shows the computing times required by each method to find a solution satisfying a 0.5% optimality tolerance.

As can be seen in Table IV, the three approaches generally require higher computing times as the number of units increases. However, several aspects are worth mentioning.

1) An increase ratio in computing time cannot be clearly determined. Computing times of MILP algorithms are dependent on many factors, especially on the problem structure. This fact may lead to somewhat unexpected results, such as the computing time taken by MILP-UC for the 100-unit case being less than the computing times required for the 80- and 90-unit cases. Similar behavior is observed in MILP-3R for the 50- and 60-unit cases.
2) The advantage of MILP-3R over MILP-3 cannot be inferred from the solution times provided in Table IV since MILP-3R outperforms MILP-3 for the 30-, 60-, 70-, and 80-unit cases, but MILP-3 is superior for the 90- and 100-unit cases. Therefore, relaxing the integrality of one set of binary variables with the subsequent change in the search

TABLE III
PRODUCTION SCHEDULE BY MILP-UC (MW)

Units	Periods																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1-10	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0
11-19	245.0	295.0	390.0	451.5	408.0	443.0	441.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	349.0	299.0	399.0	455.0	455.0	455.0	455.0	364.5	367.8
20	245.0	295.0	390.0	451.5	408.0	443.0	441.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	349.0	299.0	399.0	455.0	455.0	455.0	455.0	364.5	0.0
21	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0
22	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0
23	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
24	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0
26-27	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	0.0
29-30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
31	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	0.0
32-34	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
35	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
36-38	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
39	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
40	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
41	0.0	0.0	25.0	25.0	25.0	25.0	25.0	30.0	103.5	162.0	162.0	162.0	162.0	122.5	62.0	25.0	25.0	25.0	41.0	162.0	104.0	0.0	0.0	0.0
42	0.0	0.0	25.0	25.0	25.0	25.0	25.0	30.0	103.5	162.0	162.0	162.0	162.0	122.5	62.0	25.0	25.0	25.0	41.0	162.0	104.0	27.5	0.0	0.0
43-44	0.0	0.0	0.0	25.0	25.0	25.0	25.0	30.0	103.5	162.0	162.0	162.0	162.0	122.5	62.0	25.0	25.0	25.0	41.0	162.0	104.0	27.5	0.0	0.0
45-47	0.0	0.0	0.0	25.0	25.0	25.0	25.0	30.0	103.5	162.0	162.0	162.0	162.0	122.5	62.0	25.0	25.0	25.0	41.0	162.0	104.0	0.0	0.0	0.0
48	0.0	0.0	0.0	0.0	25.0	25.0	25.0	30.0	103.5	162.0	162.0	162.0	162.0	122.5	62.0	25.0	25.0	25.0	41.0	162.0	104.0	0.0	0.0	0.0
49	0.0	0.0	0.0	0.0	0.0	0.0	0.0	30.0	103.5	162.0	162.0	162.0	162.0	122.5	62.0	25.0	25.0	25.0	41.0	162.0	104.0	0.0	0.0	0.0
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	30.0	103.5	162.0	162.0	162.0	162.0	122.5	62.0	25.0	25.0	25.0	41.0	162.0	104.0	27.5	0.0	0.0
51-52	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	20.0	20.0	0.0	0.0	0.0	0.0	43.5	20.0	20.0	0.0	0.0
53-58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	20.0	0.0	0.0	0.0	0.0	0.0	43.5	20.0	20.0	0.0	0.0
59	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	20.0	20.0	0.0	0.0	0.0	20.0	43.5	20.0	0.0	0.0	0.0
60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	35.0	75.0	80.0	35.0	20.0	0.0	0.0	0.0	0.0	0.0	43.5	20.0	20.0	0.0	0.0
61-62	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
63	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
64-65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
66-68	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
69	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	0.0
70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	0.0	0.0
71-76	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	45.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
77	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	45.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
78	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	45.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
79	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	45.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	0.0	0.0	0.0
80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	45.0	10.0	10.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
81	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
82-87	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
88-90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
91	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	10.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
92-95	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
96-97	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
98	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0
99	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

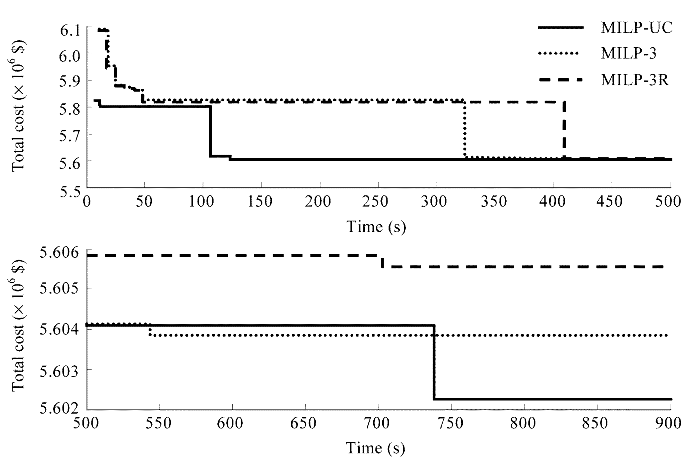

Fig. 3. Evolution of the best solution found by the MILP-based approaches.

tree does not necessarily lead to a faster convergence for a specific optimality gap.

TABLE IV
INFLUENCE OF PROBLEM SIZE ON COMPUTING TIMES (S)

# of units	MILP-UC	MILP-3	MILP-3R
10	0.97	2.09	2.56
20	4.49	16.17	16.04
30	11.77	63.83	44.11
40	19.53	76.23	76.09
50	25.65	119.68	118.95
60	37.83	171.20	106.04
70	50.71	171.24	161.85
80	133.88	216.38	203.97
90	167.73	231.97	349.43
100	123.00	324.00	409.00

3) Irrespective of the problem size, MILP-UC reaches a solution satisfying the specified optimality gap in a shorter time than MILP-3 and MILP-3R. The reduction factors with respect to MILP-3 range between 1.62 and 5.42. Analogously, MILP-UC improves the computing times required by MILP-3R with reduction factors between 2.61 and 4.64.

TABLE A
SYSTEM DATA I

Units	$\overline{P}$ (MW)	$\underline{P}$ (MW)	UT (h)	DT (h)	inistate (h)
1	455	150	8	8	8
2	455	150	8	8	8
3	130	20	5	5	-5
4	130	20	5	5	-5
5	162	25	6	6	-6
6	80	20	3	3	-3
7	85	25	3	3	-3
8	55	10	1	1	-1
9	55	10	1	1	-1
10	55	10	1	1	-1

TABLE B
SYSTEM DATA II

Units	a ($/h)	b ($/MWh)	c ($/MW^{2}h)	hc ($/h)	cc ($/h)	t^{cold} (h)
1	1000	16.19	0.00048	4500	9000	5
2	970	17.26	0.00031	5000	10000	5
3	700	16.60	0.00200	550	1100	4
4	680	16.50	0.00211	560	1120	4
5	450	19.70	0.00398	900	1800	4
6	370	22.26	0.00712	170	340	2
7	480	27.74	0.00079	260	520	2
8	660	25.92	0.00413	30	60	0
9	665	27.27	0.00222	30	60	0
10	670	27.79	0.00173	30	60	0

TABLE C
LOAD DEMAND

Hour	1	2	3	4	5	6
Demand (MW)	700	750	850	950	1000	1100
Hour	7	8	9	10	11	12
Demand (MW)	1150	1200	1300	1400	1450	1500
Hour	13	14	15	16	17	18
Demand (MW)	1400	1300	1200	1050	1000	1100
Hour	19	20	21	22	23	24
Demand (MW)	1200	1400	1300	1100	900	800

It should finally be emphasized that although MILP-UC, like other MILP-based approaches, finds difficulties in determining that the optimal solution to large-scale cases has been reached, the quality of the near-optimal solutions attained in moderate times justifies its practical applicability.

IV. CONCLUSION

This paper has presented a computationally efficient mixed-integer linear formulation for the unit commitment problem of thermal units. The salient feature of the proposed approach is the requirement of a single type of binary variables to accurately model intertemporal constraints, individual contributions to spinning reserve, and time-dependent startup costs. The reduction in the computational burden decreases the computing time required by available commercial software to solve the problem. The proposed model has been successfully tested on a realistic case study. Numerical results have revealed the accurate and computationally efficient performance of the new formulation. Finally, although the formulation has been used to solve the unit commitment problem in traditional centralized power systems, it is straightforwardly applicable to the new scheduling problems arising in electricity markets.

APPENDIX

The data of the ten-unit system of [22] are provided in Tables A–C.

The last column of Table A lists the number of hours each unit has been online $(+)$ or offline $(-)$ prior to the first period of the time span.

Finally, note that the startup cost is defined as follows:

$$c_j^u(k) = \begin{cases} hc_j, & \text{if } t_j^{off}(k) \le t_j^{cold} + DT_j, \\ cc_j, & \text{if } t_j^{off}(k) > t_j^{cold} + DT_j, \end{cases} \quad \begin{matrix} \forall j \in J, \\ \forall k \in K \end{matrix} \tag{A1}$$

which is modeled as a stairwise function with coefficients

$$K_j^t = \begin{cases} hc_j, & \text{if } t = 1 \cdots t_j^{cold} + DT_j, \quad \forall j \in J \\ cc_j, & \text{if } t = t_j^{cold} + DT_j + 1 \cdots ND_j. \end{cases} \tag{A2}$$

ACKNOWLEDGMENT

The authors would like to thank Prof. A. J. Conejo and Dr. N. Alguacil for their relevant comments.

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Miguel Carrión (S'04) received the Ingeniero Industrial degree from the Universidad de Castilla—La Mancha, Ciudad Real, Spain, in 2003. He is currently working toward the Ph.D. degree at the Universidad de Castilla—La Mancha.

His research interests are in the fields of power systems economics, stochastic programming, and electricity markets.

José M. Arroyo (S'96–M'01–SM'06) received the Ingeniero Industrial degree from the Universidad de Málaga, Málaga, Spain, in 1995 and the Ph.D. degree in power systems operations planning from the Universidad de Castilla—La Mancha, Ciudad Real, Spain, in 2000.

From June 2003 through July 2004, he held a Richard H. Tomlinson Postdoctoral Fellowship at the Department of Electrical and Computer Engineering of McGill University, Montreal, QC, Canada. He is currently an Associate Professor of Electrical Engineering at the Universidad de Castilla—La Mancha. His research interests include operations, planning, and economics of power systems, as well as optimization and parallel computation.