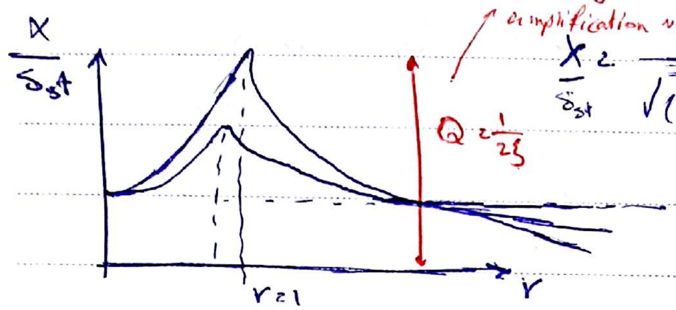
Subject: Fundamentals of vibrations by Kelly / problems in vibration shaum's outline

Year. Month. Date. ( )

Quality factor
amplification



$$\frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\frac{d}{d\omega}(x) = 0 \longrightarrow \frac{d}{dr}(x) = 0$$

$$\longrightarrow \frac{d}{dr}\left((1-r^2)^2 + (2\xi r)^2\right) = 0$$

$$\longrightarrow -2r(1-r^2) + (2\xi)(2\xi r) = 0 \longrightarrow r^2 - 1 + 2\xi^2 = 0 \longrightarrow \boxed{r = \sqrt{1-2\xi^2}} : Max$$

$$r = \sqrt{1-2\left(\frac{1}{2Q}\right)^2} = \sqrt{1-\frac{1}{2Q^2}}$$

$$r = 1 \longrightarrow Q = \left|\frac{X}{\delta_{st}}\right|_{r=1} = \frac{1}{2\xi}$$

$$r_{max} = \sqrt{1-2\xi^2} \longrightarrow \left|\frac{X}{\delta_{st}}\right|_{Max} = Q = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

در ماکس $\frac{X}{\delta_{st}}$ رو داریم

$\omega_n\sqrt{1-\xi^2}$

I) $\omega = \omega_n \longrightarrow$ resonance

تشدید (رزونانس) زمانی است که ما بیشترین دامنه را از سیستم بگیریم.

II) $\omega = \omega_d \longrightarrow$ resonance

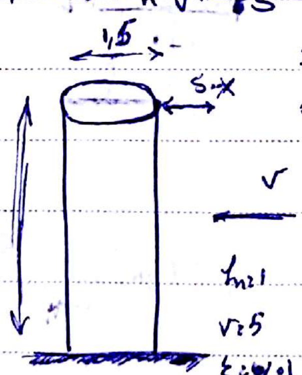
III) $\omega = \omega_n\sqrt{1-2\xi^2} \longrightarrow$ resonance

دقیق‌ترین تعریف

Strouhal No.

استروهال نامبر

مثال



$$0.2 = \frac{Df}{V} \longrightarrow f = \frac{0.2V}{D}$$

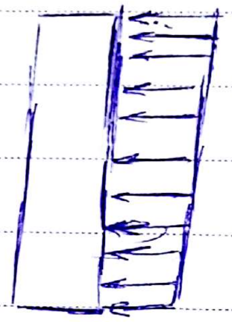
$f_n = 1$, $V = 5$, $\xi = 0.01$

$$\longrightarrow f = \frac{0.2 \times 5}{1.5} = 0.67 \longrightarrow r = \frac{\omega}{\omega_n} = \frac{2\pi f}{2\pi f_n} = \frac{0.67}{1} = 0.67$$

$$\frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{1}{\sqrt{(1-0.67^2)^2 + (2 \times 0.01 \times 0.67)^2}} = 1.8$$

if $V = 7 \longrightarrow f = \frac{0.2 \times 7}{1.5} = 0.93 \longrightarrow r = 0.93 \longrightarrow \frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-0.93^2)^2 + (2 \times 0.01 \times 0.93)^2}} = 7.3$

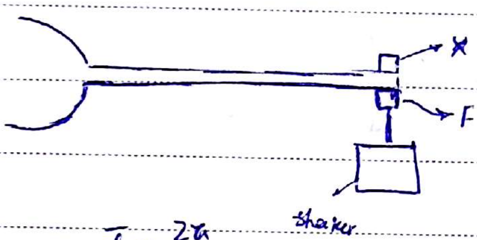
PAPCO



$w = C_D q S = 1.5 \times \frac{1}{2}\rho V^2 \times D = 1.5 \times \frac{1}{2} \times 1.229 \times 7^2 \times 1.5 = 225 \ N/m$

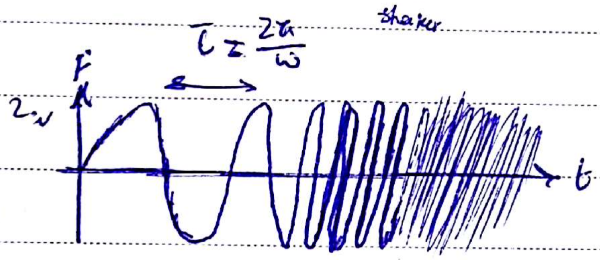
$\delta_{st} = \frac{w l^4}{8EI} = \frac{225 \times 20^4}{8 \times 7\times10^{10} \times 6\times10^{-6}} \simeq 0.05$

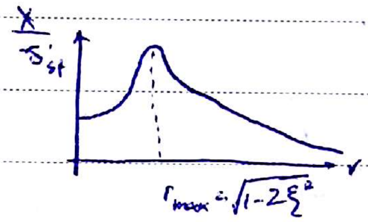
$\rightarrow x|_{V=7m} = 7.3 \times 0.05 = 0.35\ m$



$\varphi = \tan^{-1} \frac{2\xi r}{1-r^2}$ / $f = 25\ Hz$, $\varphi = 10°$, $\frac{X}{\delta_{st}} = 1.2$ فرض

$\rightarrow \xi = ?\ ,\ \omega_n = ?$





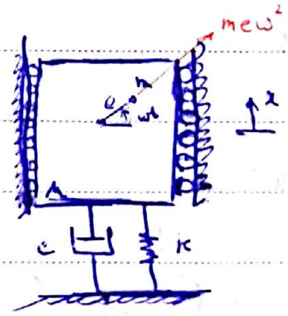
$\rightarrow 0.176 = \frac{2\xi r}{1-r^2} \rightarrow \frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \rightarrow \xi = ✓\ ,\ r = ✓$

$r = \frac{f}{f_n} \rightarrow f_n = ✓ \rightarrow Q = \frac{1}{2\xi\sqrt{1-\xi^2}}$

3.4 Rotating Unbalance

$M-m \rightarrow x$

$m \rightarrow x + e\sin\omega t$



- Displacement of non rotating mass $M-m$: x
- Displacement of rotating mass m : $x + e\sin\omega t$

$\rightarrow \sum F_y = ma \rightarrow (M-m)\ddot{x} + m\frac{d^2}{dt^2}(x + e\sin\omega t) = -kx - c\dot{x}$

$m\ddot{x} - me\omega^2\sin\omega t$

$\rightarrow M\ddot{x} + c\dot{x} + kx = \underbrace{me\omega^2}_{F_0}\sin\omega t$

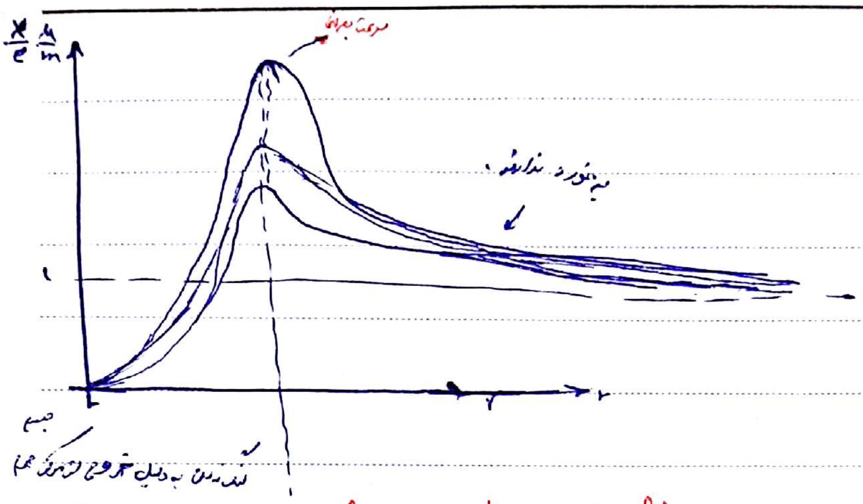
$\rightarrow X = \frac{me\omega^2}{\sqrt{(k - M\omega^2)^2 + (c\omega)^2}}\ ,\ \varphi = \tan^{-1}\frac{c\omega}{k - M\omega^2}$

$\xrightarrow[k = M\omega_n^2]{\omega_n = \sqrt{\frac{k}{M}}} X = \frac{\frac{me\omega^2}{k}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{me\omega^2 \times \frac{1}{M\omega_n^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{\frac{m}{M} e \cdot r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$

$\boxed{\frac{X}{e} \times \frac{M}{m} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}}$, $\tan\varphi = \frac{2\xi r}{1-r^2} \rightarrow \boxed{\varphi = \tan^{-1}\frac{2\xi r}{1-r^2}}$

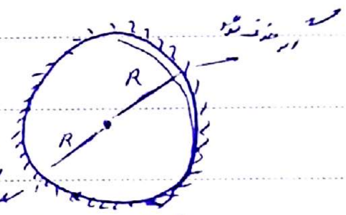
Subject:

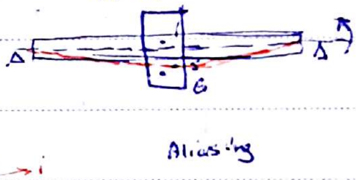
Year. Month. Date. ( )

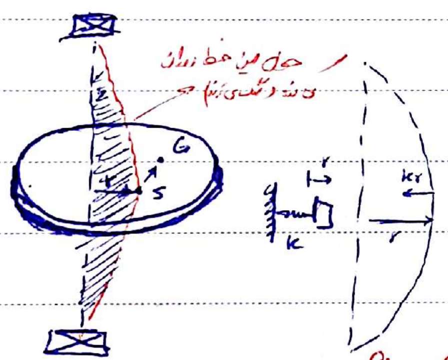


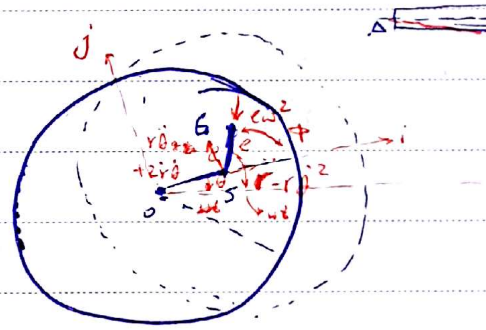
3.5. Whirling of Rotating Shafts

Defenition: Rotation of the plane made by bent shaft and the line of centers of bearing









$a_G = a_S + a_{G/S}$

$= [\ddot r - r\dot\theta^2 - e\omega^2\cos(\omega t-\theta)]\,\hat i$

$+ [r\ddot\theta + 2\dot r\dot\theta - e\omega^2\sin(\omega t-\theta)]\,\hat j$

$\rightarrow i: \; -kr = m(\ddot r - r\dot\theta^2 - e\omega^2\cos(\omega t-\theta))$

$j: \; 0 = m(r\ddot\theta + 2\dot r\dot\theta - e\omega^2\sin(\omega t-\theta))$

$\begin{cases}\sum F_i = m a_i \\ \sum F_j = m a_j\end{cases}$

Assuming viscous damping force to S, we have

$-kr - c\dot r = m[\ddot r - r\dot\theta^2 - e\omega^2\cos(\omega t-\theta)]$

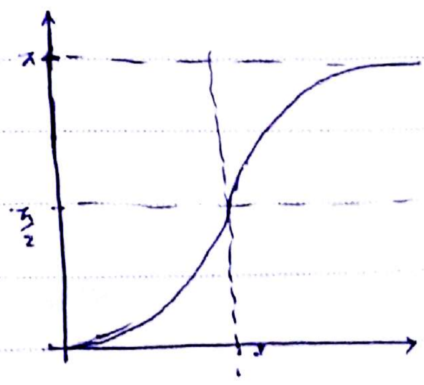
$-cr\dot\theta = m[r\ddot\theta + 2\dot r\dot\theta - e\omega^2\sin(\omega t-\theta)]$

$$\Rightarrow \begin{cases}\ddot r + \frac{c}{m}\dot r + \left(\frac{k}{m} - \dot\theta^2\right) r = e\omega^2\cos(\omega t-\theta) \\ r\ddot\theta + \left(\frac{c}{m}r + 2\dot r\right)\dot\theta = e\omega^2\sin(\omega t-\theta)\end{cases}$$

- Synchronous whirl

$\dot\theta = \omega$

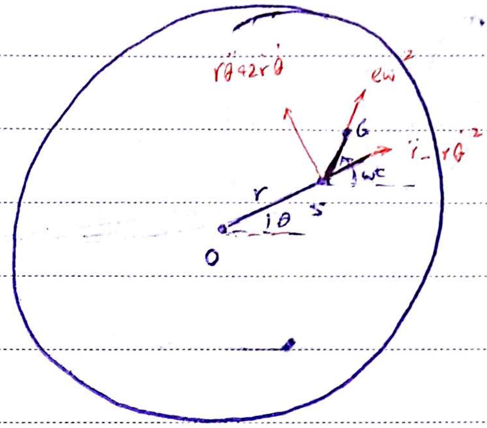
$\ddot\theta = \ddot r = \dot r = 0$ $\longrightarrow$ $\theta = \omega t - \phi$



Subject:

Year. Month. Date. ( )

$$\begin{cases} \left(\frac{k}{m} - w^2\right) r = e w^2 \cos\phi \\ \frac{c}{m} w r = e w^2 \sin\phi \end{cases}$$



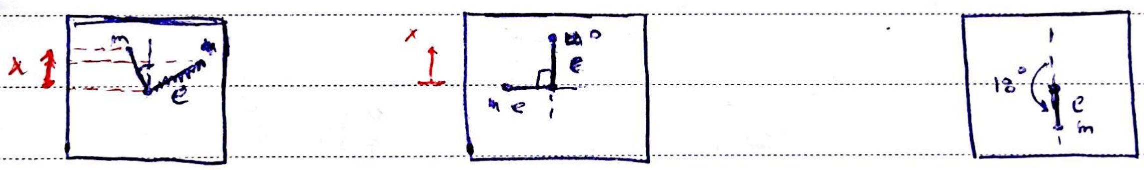
$$\begin{cases} \left[(k - m w^2)^2 + (c w)^2\right] r^2 = (m e w^2)^2 \\ \longrightarrow r = \frac{m e w^2}{\sqrt{(k - m w^2)^2 + (c w)^2}} \end{cases}$$

$$\tan\phi = \frac{c w}{k - m w^2}$$

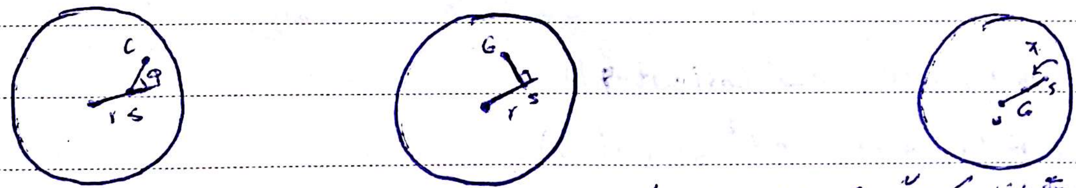
$$\longrightarrow \frac{r}{e} = \frac{\left(\frac{w}{w_n}\right)^2}{\sqrt{\left(1 - \left(\frac{w}{w_n}\right)^2\right)^2 + \left(2\xi\left(\frac{w}{w_n}\right)\right)^2}} \longrightarrow \frac{r}{e} = \frac{r'^2}{\sqrt{(1 - r'^2)^2 + (2\xi r')^2}}, \quad r' = \frac{w}{w_n}$$

$$\phi = \tan^{-1} \frac{2\xi r}{1 - r'^2}$$

a) $w \ll w_n$ b) $w = w_n$ c) $w \gg w_n$



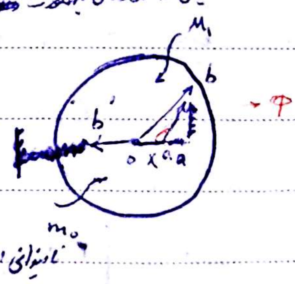
: نامیزانی



: whirling

نامیزانی که همزمان بصورت معادل در یک جای گذاریم تا بعد بتوانیم در جهت درست ترکیب جمع و نامیزانی را تشخیص میدهیم.

- Balancing



$oa + ab = ob \rightarrow ab = ob - oa$

$\downarrow$ $m_0 + m_1$, m_0

$$|ob'| = \frac{m_0}{m_1} oa$$

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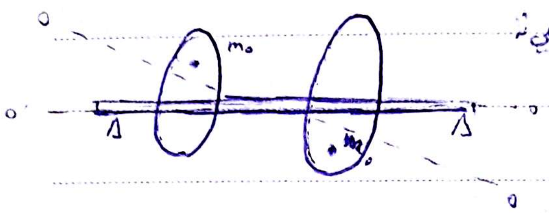
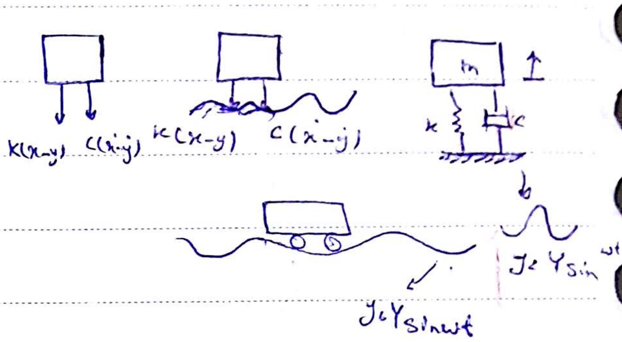
Scanned by CamScanner



تعادل اشکالی که در دو صفحه هستند

3.5. Base Excitation



$$\sum F_x = m\ddot{x} \longrightarrow m\ddot{x} = -k(x-y) - c(\dot{x}-\dot{y})$$

$$\longrightarrow m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$\longrightarrow z = x - y \longrightarrow \ddot{x} = \ddot{z} + \ddot{y}$$

$$\longrightarrow m\ddot{z} + c\dot{z} + kz = -m\ddot{y} = m\omega^2 Y \sin\omega t \quad \text{I}$$

$$\longrightarrow z = Z\sin(\omega t - \phi) \longrightarrow F_0$$

$$Z = \frac{m\omega^2 Y}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad , \quad \phi = \tan^{-1}\frac{c\omega}{k - m\omega^2}$$

حالت کلی‌تر

$$y = Y e^{i\omega t}$$

$$z = Z e^{i(\omega t - \phi)} = (Z e^{-i\phi}) e^{i\omega t}$$

$$x = X e^{i(\omega t - \psi)} = (X e^{-i\psi}) e^{i\omega t}$$

$$\text{I} \longrightarrow -m\omega^2 (Z e^{-i\phi}) \cancel{e^{i\omega t}} + ci\omega Z(e^{-i\phi}) \cancel{e^{i\omega t}} + kZe^{-i\phi} \cancel{e^{i\omega t}} = m\omega^2 Y \cancel{e^{i\omega t}}$$

$$\longrightarrow Ze^{-i\phi} = \frac{m\omega^2 Y}{(k - m\omega^2) + ic\omega}$$

$$* \; x = z + y = (Ze^{-i\phi}) e^{i\omega t} + Y e^{i\omega t} = (Ze^{-i\phi} + Y) e^{i\omega t}$$

$$X e^{-i\psi} e^{i\omega t} = \left(\frac{m\omega^2 y}{(k - m\omega^2) + ic\omega} + Y\right) e^{i\omega t}$$

Subject:

Year. Month. Date. ( )

$$Xe^{-i\psi} = \left(\frac{k+ic\omega}{(k-m\omega^2)+ic\omega}\right)Y \rightarrow \frac{X}{Y}e^{-i\psi} = \frac{k+ic\omega}{k-m\omega^2+ic\omega}\times\frac{k-m\omega^2-ic\omega}{k-m\omega^2-ic\omega}$$

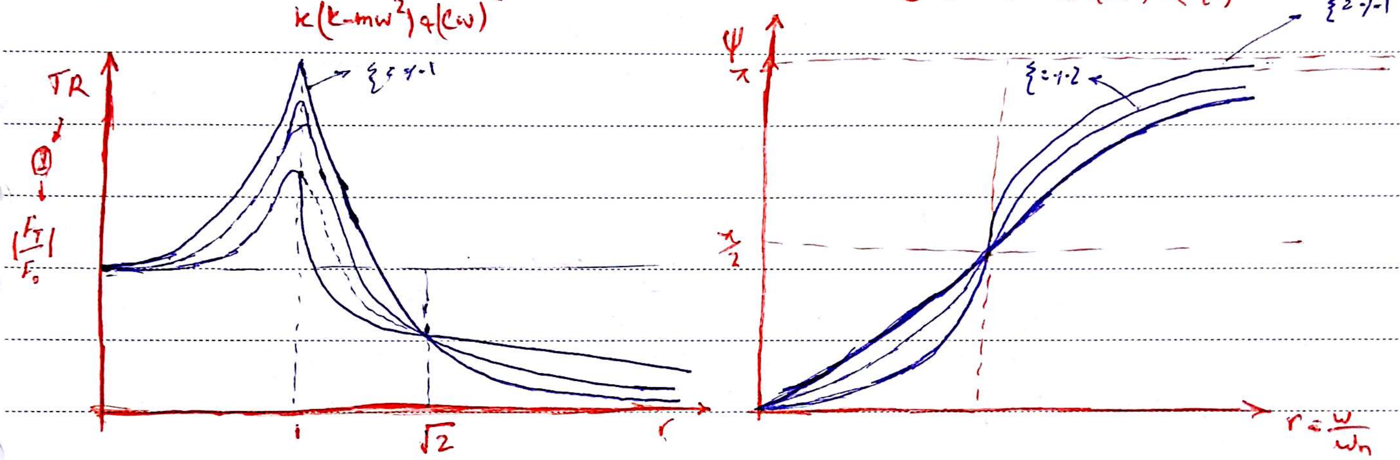
(2) انواع تحریک ها: ۱) نیرو ۲) جابجایی ← از طریق پایه

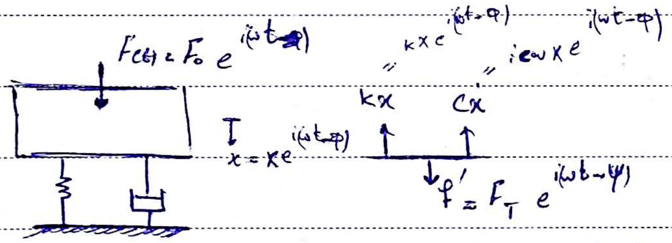
راسه

$$\left|\frac{X}{Y}\right| = \sqrt{\frac{k^2+(c\omega)^2}{(k-m\omega^2)^2+(c\omega)^2}} \rightarrow \left|\frac{X}{Y}\right| = \sqrt{\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}}$$

$$\psi = \tan^{-1}\frac{mc\omega^3}{k(k-m\omega^2)+(c\omega)^2}$$

$$\text{Transmissibility (TR)} = \sqrt{\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}}$$





$A = a+ib$

$|A| = \sqrt{a^2+b^2}$

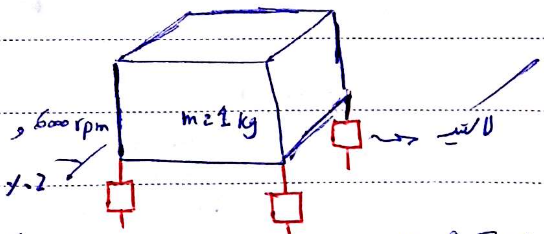
$$F_T e^{i(\omega t-\varphi-\psi)} = kXe^{i(\omega t-\varphi)} + ic\omega Xe^{i(\omega t-\varphi)}$$

$$\rightarrow |F_T| = \sqrt{(kX)^2+(c\omega X)^2} = X\sqrt{k^2+(c\omega)^2}, \quad X = \frac{F_0}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}}$$

$$TR = \left|\frac{F_T}{F_0}\right| = \sqrt{\frac{k^2+(c\omega)^2}{(k-m\omega^2)^2+(c\omega)^2}}$$

Base Excite شبیه (I) ← * قطعا سؤال امتحانی است *

3.6. Vibration Isolation



پیچ جای لاستیک میخورد → $4 * M8$, $k = \frac{EA}{L}$, $l = 0.2$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2\times10^{4}\times\frac{\pi}{4}\times(8\times10^{-3})^{2}\times4}{0.02\times1}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 7136\ Hz$$

باید کاری کنیم که r از $\sqrt{2}$ بیشتر شود

با عث می شود ارتعاش هم منتقل شده کم شود

$$r = \frac{\omega}{\omega_n} = \frac{\frac{6000}{60}}{7136} = 0.014$$

← در نمودار بالا چپ نشان داده شده

ارتعاش زیادی منتقل می کند

$TR < 0.1 \rightarrow TR = \sqrt{\dfrac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}}$ حال اگر همان واکنش را به کار ببریم (با $\xi = 0.05$)

$\rightarrow \sqrt{\dfrac{1+(2\times 0.05 r)^2}{(1-r^2)^2+(2\times 0.05\times r)^2}} = 0.1 \rightsquigarrow r = 3.5$

$\rightsquigarrow r = \dfrac{100}{f_n} = 3.5 \rightsquigarrow f_n = 29\,Hz \longrightarrow f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{k\times 4}{m}} \Rightarrow 29 = \dfrac{1}{2\pi}\sqrt{\dfrac{4k}{1}} \rightarrow k = 8300\ N/m$

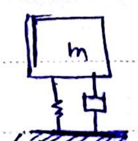
Ex: An engine of mass 100 kg is supported an springs of total stiffness 700 $\frac{kN}{m}$ and has an unbalanced rotating element with force of 350 N at a speed of 3000 rpm. Assuming a damping factor of $\xi = 0.2$, determine
a) its amplitude of motion due to the unbalance
b) the transmissibility
c) the transmitted force

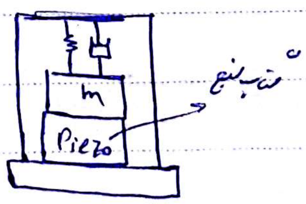
a) $f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{700\times 10^3}{100}} = 13.32\ Hz$

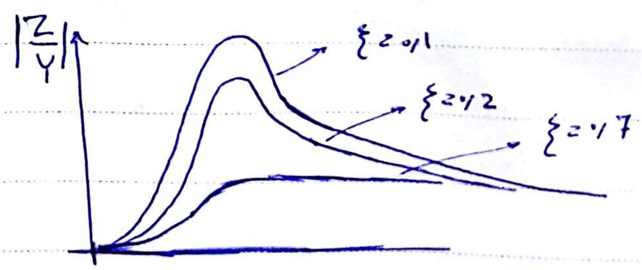
$$X = \frac{F_0/k}{\sqrt{(1-r^2)^2+(2\xi r)^2}} = \frac{\frac{350}{700\times 10^3}}{\sqrt{\left(1-\left(\frac{50}{13.32}\right)^2\right)^2+\left(2\times 0.2\times\frac{50}{13.32}\right)^2}} = 0.0638\ m$$

b) $TR = \sqrt{\dfrac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}} = 0.137$

c) $TR = \left|\dfrac{F_0}{F_T}\right| \rightsquigarrow F_T = F_0 \cdot TR = 350\times 0.137 = 47.9\ N$



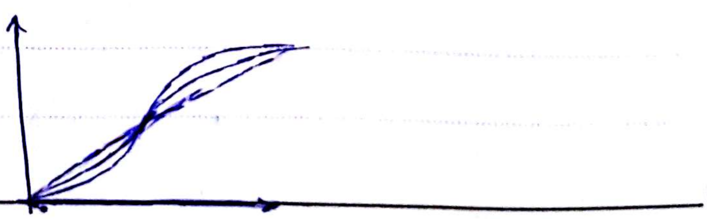




$y = Y\,\cancel{\text{start}}\,e^{i\omega t}$

$$Z = \frac{m\omega^2 Y}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}}$$

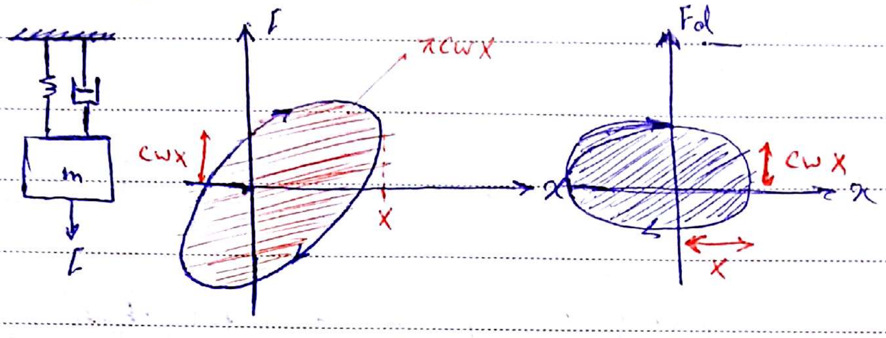
$\phi = \tan^{-1}\dfrac{c\omega}{k-m\omega^2} = \tan^{-1}\dfrac{2\xi r}{1-r^2}$

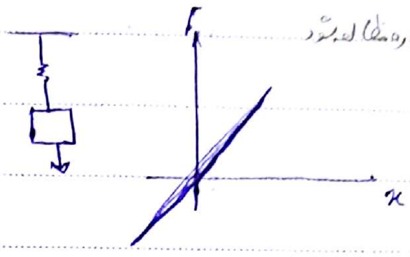


Subject:
Year. Month. Date. ( )

3.7. Equivalent Viscous Damping

- Energy Dissipated by Viscous damping





$W = \oint F_d \, dx \qquad F_d = c\dot{x} \rightarrow W_d = \oint c\dot{x}\, dx \quad , \dot{x} = \frac{dx}{dt} \rightarrow dx = \dot{x}\, dt$

$\begin{cases} x = X \sin(\omega t - \varphi) \\ \dot{x} = \omega X \cos(\omega t - \varphi) \end{cases} \longrightarrow W_d = \oint c\dot{x}^2 dt = \int_0^{\frac{2\pi}{\omega}} (\omega X \cos(\omega t - \varphi))^2 dt$

$= c\omega^2 X^2 \int_0^{\frac{2\pi}{\omega}} \cos^2(\omega t - \varphi)\, dt$

$\longrightarrow W_d = \pi c \omega X^2 = 2\xi \pi k X^2$

در واقعیت این چنین نیست و به w ربطی ندارد.

$\dot{x} = \omega X \cos(\omega t - \varphi) = \pm \omega X \sqrt{1 - \sin^2(\omega t - \varphi)} = \pm \omega \sqrt{X^2 - X^2 \sin^2(\omega t - \varphi)} = \pm \omega \sqrt{X^2 - x^2}$

$F_d = c\dot{x} = \pm c\omega \sqrt{X^2 - x^2} \longrightarrow \left(\frac{F_d}{c\omega X}\right)^2 = 1 - \left(\frac{x}{X}\right)^2 \longrightarrow \left(\frac{F_d}{c\omega X}\right)^2 + \left(\frac{x}{X}\right)^2 = 1$

دوران بیضی به علت k می باشد

میرایی سازه ای

$W_d = \alpha X^2 \longrightarrow \alpha = \pi c \omega \longrightarrow C_{eq} = \frac{\alpha}{\pi \omega}$ Structural (Hysteretic) Damping. مثل انرژی تلف شده در واقعیت

- Body moving with moderate speed ($3 - 20 \, m/s$) in fluids such as water or air:

$F_d = \pm a \dot{x}^2$

$x = X \cos \omega t \longrightarrow \dot{x} = -X\omega \sin \omega t \longrightarrow \frac{dx}{d(\omega t)} = -X \sin \omega t$

$W_d = 2\int_{-X}^{X} a\dot{x}^2 dx = 2 a \omega^2 X^3 \int_0^{\pi} \sin^3 \omega t \, d(\omega t)$

$W_d = \pi c \omega X^2 \longrightarrow \pi C_{eq} \omega X^2 = \frac{8}{3} a \omega^2 X^3 \longrightarrow C_{eq} = \frac{8}{3\pi} a \omega X$

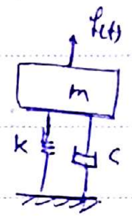
Subject:

Year. Month. Date. ( )

- Coullomb damping

$$W_d = 4F_d X \longrightarrow \pi C_{eq} \omega X^2 = 4F_d X \rightsquigarrow C_{eq} = \frac{4F_d}{\pi \omega X}$$

$$\rightarrow m\ddot{x} + \left(\frac{4F_d}{\pi \omega X}\right)\dot{x} + kx = f(t) \longrightarrow X = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + \left(\frac{4\omega F_d}{\pi \omega X}\right)^2}}$$

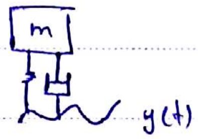


3.8. Multifrequency Excitation

$$f(t) = \sum_{i=1}^{n} F_i \sin(\omega_i t + \Psi_i)$$

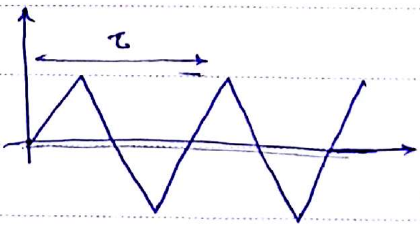
$$* \; x(t) = \sum_{i=1}^{N} X_i \sin(\omega_i t + \Psi_i - \varphi_i) \quad , * \; X_i = \frac{\frac{F_i}{k}}{\sqrt{(1-r_i)^2 + (2\xi r_i)^2}} \quad , * \; \varphi_i = \tan^{-1} \frac{2\xi r_i}{1 - r_i^2}$$

$$y(t) = \sum_{i=1}^{N} Y_i \sin(\omega_i t + \Psi_i)$$



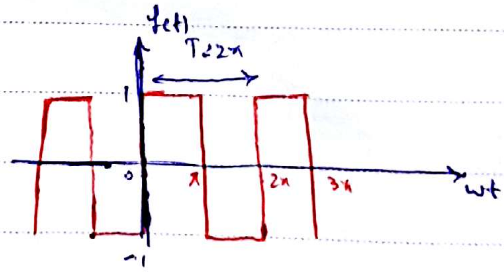
$$x(t) = \sum_{i=1}^{N} X_i \sin(\omega_i t + \Psi_i - \varphi_i) \quad , \quad X_i = Y_i \sqrt{\frac{1 + (2\xi r_i)^2}{(1 - r_i^2)^2 + (2\xi r_i)^2}}$$

$$f(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty} (a_i \cos \omega_i t + b_i \sin \omega_i t) \quad , \; \omega_1 = \frac{2\pi}{T} \; , \; \omega_n = n\omega_1$$



$$a_i = \frac{2}{T} \int_0^T f(t) \cos \omega_i t \, dt \quad , \quad b_i = \frac{2}{T} \int_0^T f(t) \sin \omega_i t \, dt$$

$$x(t) = \sum_{i=1}^{N} X_i \sin(\omega_i t - \varphi)$$



Example: $f(t) = \frac{a_0}{2} + \sum_{i=1}^{N} (a_i \cos \omega_i t + b_i \sin \omega_i t)$

$$a_i = \frac{2}{2\pi} \int_0^{2\pi} f(t) \cos \omega_i t \, dt = \frac{1}{\pi} \int_0^{\frac{\pi}{\omega}} 1 \times \cos \omega_i t \, dt - \int_{\frac{\pi}{\omega}}^{\frac{2\pi}{\omega}} 1 \times \cos \omega_i t \, dt = 0$$

Subject:

Year. Month. Date. ( )

$$b_1 = \frac{2}{2\pi}\int_0^{2\pi/\omega} f(t)\,\sin\omega_1 t\,dt = \frac{1}{\pi}\left(\int_0^{\pi/\omega} \sin\omega_1 t\,dt - \int_{\pi/\omega}^{2\pi/\omega} \sin\omega_1 t\,dt\right)$$

$$b_1 = \frac{1}{\pi}\left(\int_0^{\pi/\omega} \sin t\,dt - \int_{\pi/\omega}^{2\pi/\omega} \sin t\,dt\right) = \frac{4}{\pi}$$

$$b_2 = b_4 = b_6 = \cdots = 0$$

$$b_3 = \frac{1}{\pi}\left(\int_0^{\pi/\omega} \sin(\omega_3 t)\,dt - \int_{\pi/\omega}^{2\pi/\omega} \sin(\omega_3 t)\,dt\right) = \frac{4}{3\pi}$$

$$f(t) = \frac{4}{\pi}\left(\sin\omega_1 t + \frac{1}{3}\sin 3\omega_1 t + \frac{1}{5}\sin 5\omega_1 t + \cdots\right)$$

$$x(t) = \frac{4}{\pi}\left[\frac{\sin(\omega_1 t - \varphi)}{\sqrt{(k - m\omega_1)^2 + (c\omega_1)^2}} + \frac{\frac{1}{3}\sin(3\omega_1 t - \varphi)}{\sqrt{(k - m(3\omega_1))^2 + (3c\omega_1)^2}} + \cdots\right]$$

$$\varphi_1 = \tan^{-1}\frac{c\omega_1}{k - m\omega_1}, \quad \varphi_2 = \tan^{-1}\frac{3c\omega_1}{k - m(3\omega_1)^2}$$

CH 4. Transient vibration

4.1. Introduction

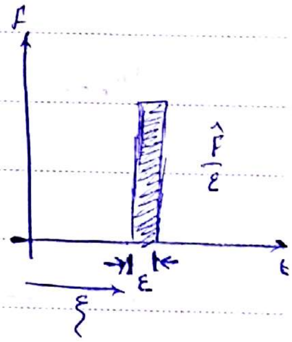
4.2. Impulse excitation:

- unit impulse or delta function

$$\delta(t-\xi)=0 \quad ; \quad t\neq\xi$$

$$\int_0^\infty \delta(t-\xi)\,dt=1 \qquad 0<\xi<\infty$$

$$\therefore \int_0^\infty f(t)\,\delta(t-\xi)\,dt = f(\xi) \qquad 0<\xi<\infty$$



- Impulse

$\hat F = \int F(t)\,dt \to$ unit impulse $\quad \lim_{\varepsilon\to 0}\hat F = 1$

- $F = ma = m\frac{dv}{dt} \to F\,dt = m\,dv \to \hat F = mV$

I.C. $\quad x(0)=x_0 \quad , \quad \dot x(0)=\dot x_0$

$$x(t) = \frac{\dot x_0}{\omega_n}\sin\omega_n t + x_0\cos\omega_n t$$

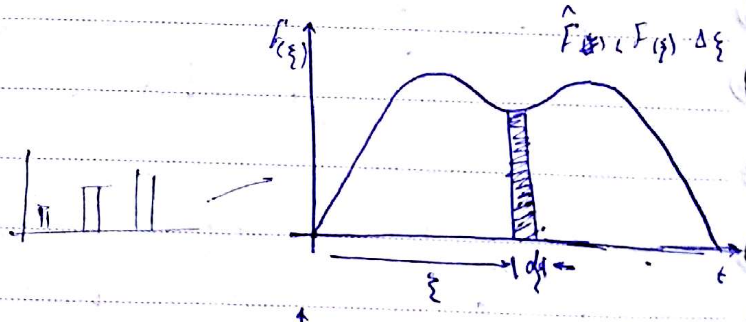
$x_0=0 \quad \dot x_0 = \frac{\hat F}{m} \to x(t)=\frac{\hat F}{m\omega_n}\sin\omega_n t = \hat F\,h(t)$

$\to h(t)=\frac{1}{m\omega_n}\sin\omega_n t$ is Impulse Response function (IRF)

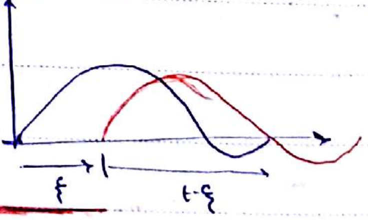
if $\zeta\neq 0$ $\quad x(t)=\frac{\dot x_0}{\omega_d}e^{-\zeta\omega_n t}\sin\omega_d t = \frac{\hat F}{m\omega_d}e^{-\zeta\omega_n t}\sin\omega_d t = \hat F\,h(t)$

where $h(t)=\frac{1}{m\omega_d}e^{-\zeta\omega_n t}\sin\omega_d t$

4.3. Arbitrary Excitation



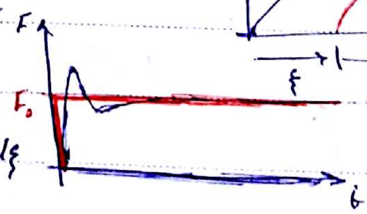
$$x(t)=\int_0^t F(\xi)\,h(t-\xi)\,d\xi$$



Example: step excitation of an SDOF system.

$h(t)=\frac{1}{m\omega_n}\sin\omega_n t$

$x(t)=\int_0^t F(\xi)\,h(t-\xi)\,d\xi = \int_0^t F_0\times\frac{1}{m\omega_n}\sin(\omega_n(t-\xi))\,d\xi$



Subject :

Year . Month . Date . ( )

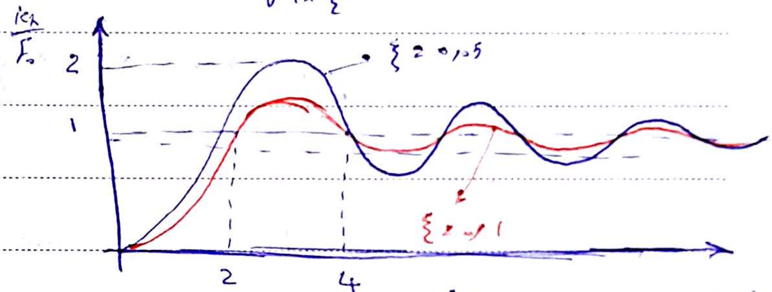
$$= \frac{F_0}{m\omega_n}\int_0^t \sin(\omega_n(t-\xi))d\xi = \frac{F_0}{m\omega_n^2}(1-\cos\omega_n t) = \frac{F_0}{k}(1-\cos\omega_n t)$$

if $\xi \neq 0$: $\quad h(t) = \frac{1}{m\omega_d} e^{-\xi\omega_n t}\sin\omega_d t$

$$\to x(t) = \int_0^t F(\xi)\, h(t-\xi)\, d\xi = \int_0^t F_0 \frac{1}{m\omega_d} e^{-\xi\omega_n(t-\xi)} \sin(\omega_d(t-\xi))\, d\xi$$

$$= \frac{F_0}{m\omega_d}\int_0^t e^{-\xi\omega_n(t-\xi)}\sin(\omega_d(t-\xi))\,d\xi = \frac{F_0}{k}\left[1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cos(\omega_d t-\psi)\right]$$

$$\psi = \tan^{-1}\frac{\xi}{\sqrt{1-\xi^2}}$$

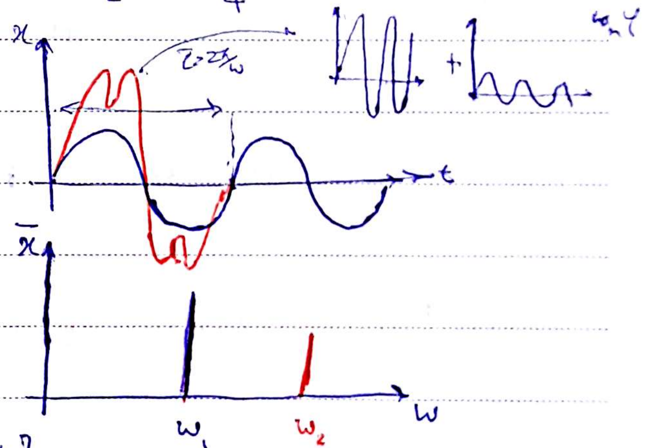


_ Base excitation

$$\ddot{z} + 2\xi\omega_n\dot{z} + \omega_n^2 z = -\ddot{y}$$

Replace $\frac{F}{m}$ by $-\ddot{y}$:

$$z = -\frac{1}{\omega_n}\int_0^t \ddot{y}(\xi)\sin(\omega_n(t-\xi))\,d\xi$$

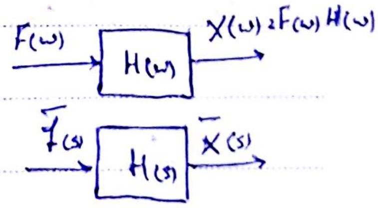


4.4. Laplace Transform

_ $f(t)$, $t>0$ & $\mathcal{L}[f(t)]$

تبدیل فوریه : $\bar{x}(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt \quad \to \quad \mathcal{L}[f(t)] = \bar{f}(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt$

$\mathcal{L}[f'(t)] = s\bar{f}(s) - f(0)$, $\mathcal{L}[f''(t)] = s^2\bar{f}(s) - s f(0) - f'(0)$



$$m\ddot{x} + c\dot{x} + kx = f(t) \to \mathcal{L}[m\ddot{x} + c\dot{x} + kx] = \mathcal{L}[f(t)]$$

$$\to m[s^2\bar{x}(s) - s x(0) - \dot{x}(0)] + c[s\bar{x}(s) - x(0)] + k\bar{x} = \bar{f}(s)$$

$$\to \bar{x}(s)[ms^2 + cs + k] = \bar{f}(s) + ms\, x(0) + m\dot{x}(0) + c\, x(0)$$

$$\to \bar{x}(s) = \frac{\bar{f}(s)}{ms^2+cs+k} + \frac{ms\,x(0) + m\dot{x}(0) + c\,x(0)}{ms^2+cs+k}$$

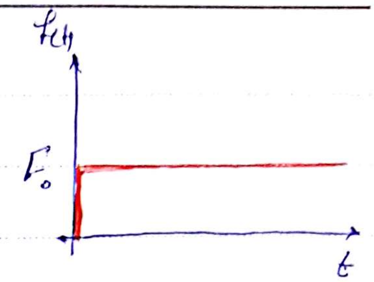
$$\to x(t) = \mathcal{L}^{-1}[\bar{x}(s)]$$

13

Subject:

Year. Month. Date. ( )

Example: $x_0 = 0$, $\dot{x}_0 = 0$, $\zeta = 0$

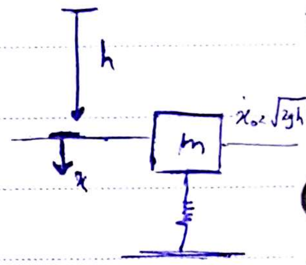


$f(t) = F_0 \longrightarrow \mathcal{L}[F_0] = \frac{F_0}{s}$

$\longrightarrow \bar{x}(s) = \frac{F_0/s}{ms^2+k} = \frac{F_0}{m}\frac{1}{s(s^2+\omega_n^2)} \Rightarrow \frac{F_0}{m\omega_n^2}\left[\frac{1}{s} + \frac{-s}{s^2+\omega_n^2}\right] *$

$* = \frac{1}{s(s^2+\omega_n^2)} = \frac{A}{s} + \frac{B}{s^2+\omega_n^2} = \frac{As^2+A\omega_n^2+Bs}{s(s^2+\omega_n^2)}$ $\nearrow \boxed{A = 1/\omega_n^2}$ $\searrow As^2+Bs = 0 \rightarrow \boxed{B = \frac{-s}{\omega_n^2}}$

$\longrightarrow x(t) = \mathcal{L}^{-1}[\bar{x}(s)] = \frac{F_0}{m\omega_n^2}(1-\cos\omega_n t)$



Example: $m\ddot{x} + kx = mg$ $x(0) = 0$ $\dot{x}(0) = \sqrt{2gh}$

$\longrightarrow \ddot{x} + \omega_n^2 x = g \longrightarrow [s^2\bar{x}(s) - sx(0) - \dot{x}(0)] + \omega_n^2\bar{x}(s) = \frac{g}{s}$

$\longrightarrow \bar{x}(s) = \frac{\sqrt{2gh}}{s^2+\omega_n^2} + \frac{g}{s(s^2+\omega_n^2)} \longrightarrow x(t) = \frac{\sqrt{2gh}}{\omega_n}\sin\omega_n t + \frac{g}{\omega_n^2}(1-\cos\omega_n t)$

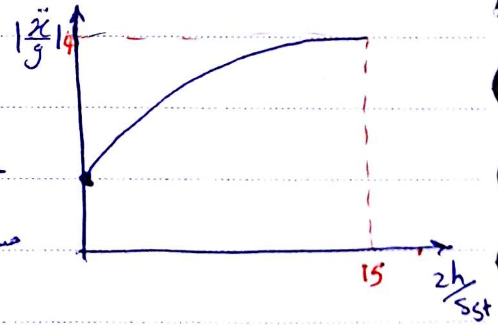
$= \sqrt{\frac{2gh}{\omega_n^2} + \left(\frac{g}{\omega_n^2}\right)^2}\sin(\omega_n t - \phi) + \frac{g}{\omega_n^2}$

$\dot{x}(t) = \omega_n\sqrt{\frac{2gh}{\omega_n^2} + \left(\frac{g}{\omega_n^2}\right)^2}\cos(\omega_n t - \phi)$

$\ddot{x}(t) = -\omega_n^2\sqrt{\frac{2gh}{\omega_n^2} + \left(\frac{g}{\omega_n^2}\right)^2}\sin(\omega_n t - \phi) = -g\sqrt{\frac{2h\omega_n^2}{g}+1}\sin(\omega_n t - \phi)$

$\frac{g}{\omega_n^2} = \frac{gm}{k} = \delta_{st}$

$\longrightarrow \ddot{x}(t) = -g\sqrt{\frac{2h}{\delta_{st}}+1}\sin(\omega_n t - \phi) \longrightarrow \frac{\ddot{x}}{g} = \sqrt{\frac{2h}{\delta_{st}}+1}$



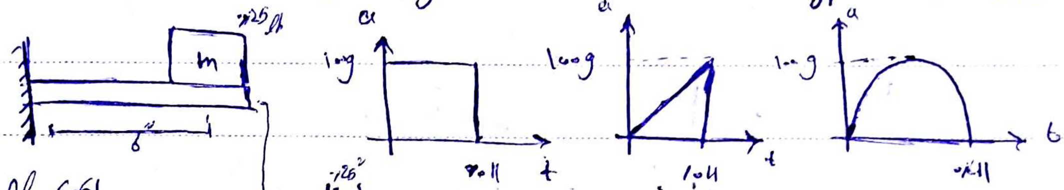
$\frac{2h}{\delta_{st}} = 15 \longrightarrow h = \frac{15\,\delta_{st}}{2} \longrightarrow$ اگر $\delta_{st} = 1mm$ باشد، در $7mm$ شتاب $4g$ حس می‌شد

چون δ_{st} کوچک و در حد میلیمتر هست، هنگام پرتاب از ارتفاعات خیلی کوچک، شتاب زیادی را حس می‌شد.

Subject

Date

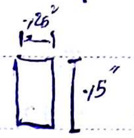
Example: A transformer is mounted on a Cantilever bracket, as shown. The bracket is mounted within a Container that will be dropped from a low-flying helicopter. the Container may land on different types of sand or dirt, which will generate different types of shock pulses.



Al 6061

$S_y = 33\,ksi$

$\rho = 0.1\ \frac{lb}{in^3}$



Sol: $F = ma$

$\sigma = \frac{Mc}{I}$

$m_{eff} = \frac{33}{140} \times \rho \times abcl = 0.017$

$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\frac{3EI}{l^3}}{.125+0.017}} = \frac{1}{2\pi}\sqrt{\frac{\frac{3\times10.3\times10^6\times\frac{1}{12}\times .125\times .5^3}{6^3}}{.1267}}$

$\simeq 104\ Hz$

$f_p = \frac{1}{\tau_p} = \frac{1}{2\times 0.011} = 45.4\ Hz$

$R = \frac{f_n}{f_p} = \frac{104}{45.4} \simeq 2.03$

$a_{out} = \begin{cases} 2\times100 = 200\,g & ; \text{Rectangular} \\ 1.6\times100 = 166\,g & ; \text{half-sine} \\ 1.2\times100 = 120\,g & ; \text{saw tooth} \end{cases}$

$\sigma = \frac{Mc}{I} = \frac{Fl \times .5/2}{\frac{1}{12}\times .125\times .5^3} = \frac{m a_{out} \times 1.5}{\frac{1}{12}\times .125\times .5^3}$ ($m = .1267$)

$154\,a_{out} = \begin{cases} 30758\ psi \\ 24640\ psi \\ 18480\ psi \end{cases} \longrightarrow S.F. = \begin{cases} \frac{33000}{30758} = 1.07 \\ 1.34 \\ 1.7 \end{cases}$

Subject

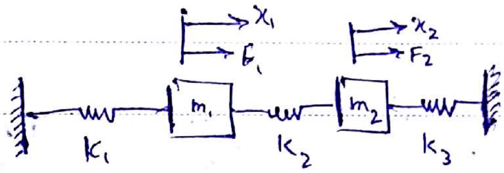
Date

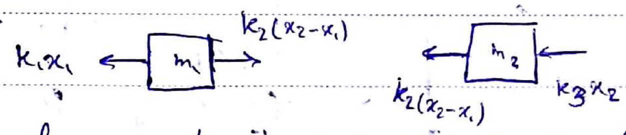
CHAPTER 5. 2DOF Systems

5.1. Introduction

5.2. Normal mode Analysis

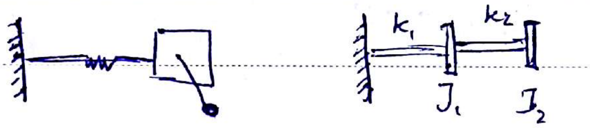
Example:





$$\begin{cases} m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2-x_1) + f_1 \\ m_2\ddot{x}_2 = -k_2(x_2-x_1) - k_3x_2 + f_2 \end{cases} \longrightarrow \begin{cases} m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = f_1 \\ m_2\ddot{x}_2 - k_2x_1 + (k_2+k_3)x_2 = f_2 \end{cases}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \longrightarrow \underset{\sim}{M}\,\underset{\sim}{\ddot{x}} + \underset{\sim}{K}\,\underset{\sim}{x} = \underset{\sim}{f}$$



- Free vibrations

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} x_1 = X_1 e^{i\omega t} \longrightarrow \dot{x}_1 = i\omega X_1 e^{i\omega t} \longrightarrow \ddot{x}_1 = -\omega^2 X_1 e^{i\omega t} \\ x_2 = X_2 e^{i\omega t} \longrightarrow \dot{x}_2 = i\omega X_2 e^{i\omega t} \longrightarrow \ddot{x}_2 = -\omega^2 X_2 e^{i\omega t} \end{cases}$$

$$-\omega^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \cancel{e^{i\omega t}} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \cancel{e^{i\omega t}} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\left(\begin{bmatrix} -\omega^2 m_1 & 0 \\ 0 & -\omega^2 m_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \right) \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \longrightarrow \begin{bmatrix} -m_1\omega^2+(k_1+k_2) & -k_2 \\ -k_2 & -m_2\omega^2+(k_2+k_3) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

روش کرامر

$$X_1 = \frac{\begin{vmatrix} 0 & -k_2 \\ 0 & -m_2\omega^2+(k_2+k_3) \end{vmatrix}}{\begin{vmatrix} -m_1\omega^2+(k_1+k_2) & -k_2 \\ -k_2 & -m_2\omega^2+(k_2+k_3) \end{vmatrix}} \qquad X_2 = \frac{\begin{vmatrix} -m_1\omega^2+(k_1+k_2) & 0 \\ -k_2 & 0 \end{vmatrix}}{\begin{vmatrix} \quad \end{vmatrix}}$$

چون صورت صفر است جواب های صفری دهد. درصورتی جواب های غیر صفری دهد که مخرج هم صفر شود و $\frac{0}{0}$ جواب شود که ممکن است جواب غیرصفر نیز داشته باشد.

Subject
Date

$$\rightsquigarrow \begin{vmatrix} -m_1\omega^2+(k_1+k_2) & -k_2 \\ -k_2 & -m_2\omega^2+(k_2+k_3) \end{vmatrix} = 0$$

فرض می کنیم $k_1 = k_2 = k_3 = k$ ، $m_1 = m_2 = m$

$$\rightsquigarrow \begin{vmatrix} -m\omega^2+2k & -k \\ -k & -m\omega^2+2k \end{vmatrix} = 0 \longrightarrow \begin{vmatrix} -\lambda m+2k & -k \\ -k & -\lambda m+2k \end{vmatrix} = 0 \quad \text{و} \quad \lambda = \omega^2$$

چون نیروی خارجی نداریم و حرکت هارمونیک است.

$$\rightsquigarrow \lambda^2 + \frac{3k}{m}\lambda \overset{-4}{\cancel{+2}}\left(\frac{k}{m}\right)^2 = 0 \longrightarrow \lambda_1 = \overset{1}{\cancel{}}\frac{k}{m} \;,\; \lambda_2 = \overset{3}{\cancel{2}}\frac{k}{m}$$

فرکانس طبیعی

$$\hookrightarrow (-\lambda m+2k)^2 = k^2$$

فرکانسی که کمتر است زودتر به آن می رسد و به آن فرکانس پایه می گویند.

$$\begin{bmatrix} -\lambda m+2k & -\lambda \\ -k & -\lambda m+2k \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \longrightarrow \lambda_1 = \frac{k}{m} \longrightarrow \begin{bmatrix} -k+2k & -k \\ -k & -k+2k \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \longrightarrow \begin{cases} kX_1 - kX_2 = 0 \\ kX_1 - kX_2 = 0 \end{cases}$$

یک معادله است. $\xrightarrow{\text{if } x_2=1} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \rightarrow \Phi_1$

Normal Mode
~~Shape~~
mode shape

$$\lambda = \lambda_2 = 3\frac{k}{m} \longrightarrow \begin{bmatrix} -3k+2k & -k \\ -k & -3k+2k \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \longrightarrow \begin{cases} -kx_1 - kx_2 = 0 \\ -kX_1 - kx_2 = 0 \end{cases} \rightarrow \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

Φ_2

$$\rightsquigarrow \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = C_1\Phi_1 + C_2\Phi_2 = \underline{A_1 e^{i\omega_1 t}}\Phi_1 + \underline{A_2 e^{i\omega_2 t}}\Phi_2$$

ادامه در Powerpoint موجود در FTP

(دو تا پاندول متصل به یک فنر)

$$\begin{bmatrix} ml^2 & 0 \\ 0 & ml^2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} -mgl+ka^2 & ka^2 \\ ka^2 & mgl-ka^2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$-\omega^2 \begin{bmatrix} ml^2 & 0 \\ 0 & ml^2 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} \cancel{\cos\omega t} + \begin{bmatrix} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} \cancel{\cos\omega t} = 0$$

$$\begin{bmatrix} -\omega^2 ml^2 + (ka^2 - mgl) & ka^2 \\ ka^2 & -\omega^2 ml^2 + mgl - ka^2 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Subject

Date

$$\begin{cases} J_1\ddot{\theta}_1 = -k_{t_1}\theta_1 + k_{t_2}(\theta_2-\theta_1) \\ J_2\ddot{\theta}_2 = -k_{t_2}(\theta_2-\theta_1) \end{cases} \longrightarrow \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} k_{t_1}+k_{t_2} & -k_{t_2} \\ -k_{t_2} & k_{t_2} \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

مثال از حالت پیچشی در اسلایدها

$$\begin{cases} \theta_1 = \Theta_1 e^{i\omega t} \\ \theta_2 = \Theta_2 e^{i\omega t} \end{cases} \longrightarrow \begin{cases} \ddot{\theta}_1 = -\omega^2\Theta_1 e^{i\omega t} \\ \ddot{\theta}_2 = -\omega^2\Theta_2 e^{i\omega t} \end{cases} \longrightarrow \begin{bmatrix} -\omega^2 J_1 + (k_{t_1}+k_{t_2}) & -k_{t_2} \\ -k_{t_2} & -\omega^2 J_2 + k_{t_2} \end{bmatrix}\begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Forced harmonic vibration

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}\sin\omega t \longrightarrow \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} = -\omega^2\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}\sin\omega t \longrightarrow \begin{bmatrix} -m_1\omega^2 & 0 \\ 0 & -m_2\omega^2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{bmatrix} & \\ & \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix}$$