CHAPTER 8

1a. $E_d = 13.60\ \text{eV} \times \frac{m^*}{m} \times \frac{1}{\varepsilon^2} \simeq 6.3 \times 10^{-4}\ \text{eV}$

b. $r = a_H \times \varepsilon \times \frac{m}{m^*} \simeq 6 \times 10^{-6}\,\text{cm}$

c. Overlap will be significant at a concentration

$$N = \frac{1}{\frac{4\pi}{3} r^3} \approx 10^{15}\,\text{atoms cm}^{-3}$$

2a. From Eq. (53), $n \simeq (n_0 N_d)^{1/2} e^{-E_d/2k_BT}$, in an approximation not too good for the present example.

$$n_0 \equiv 2\left(\frac{m^* k_B T}{2\pi\hbar^2}\right)^{3/2} \approx 4 \times 10^{13}\ \text{cm}^{-3};$$

$$\frac{E_d}{2k_BT} \simeq 1.45;\ \ e^{-1.45} \simeq 0.23\ .$$

$$n \simeq 0.46 \times 10^{13}\ \text{electrons cm}^{-3}.$$

b. $R_H = -\frac{1}{nec} \simeq -1.3 \times 10^{-14}$ CGS units

3. The electron contribution to the transverse current is

$$j_y(e) \simeq ne\mu_e\left(\frac{\mu_e B}{c} E_x + E_y\right);$$

for the holes $j_y(h) \simeq ne\mu_h\left(\frac{-\mu_n B}{c} E_x + E_y\right)$.

Here we have used

$$\omega_{ce}\tau_e = \frac{\mu_e B}{c}\ \text{for electrons;} \qquad \omega_{ch}\tau_h = \frac{\mu_h B}{c}\ \text{for holes.}$$

The total transverse (y-direction) current is

$$0 = (ne\mu_e{}^2 - pe\mu_h{}^2)(B/c)E_x + (ne\mu_e + pe\mu_h)E_y\,, \qquad (*)$$

and to the same order the total current in the x-direction is

$$j_x = (pe\mu_h + ne\mu_e)E_x\,.$$

Because (*) gives

$$E_y = E_x B \frac{p\mu_h{}^2 - n\mu_e{}^2}{p\mu_h + n\mu_e} \cdot \frac{1}{c} \ ,$$

we have for the Hall constant

$$R_H = \frac{E_y}{j_x B} = \frac{1}{ec} \cdot \frac{p\mu_h{}^2 - n\mu_e{}^2}{(p\mu_h + n\mu_e)^2} \ .$$

4. The velocity components are $v_x = \hbar k_x / m_t;\ v_y = \hbar k_y / m_t;\ v_z = \hbar k_z / m_\ell$. The equation of motion in k space is $\hbar\, dk/dt = -(e/c)\, v \times B$. Let B lie parallel to the k_x axis; then $dk_x / dt = 0;\ dk_y / dt = -\omega_\ell k_z;\ \omega_\ell \equiv eB/m_\ell c;\ dk_z / dt = \omega_t k_y;\ \omega_t \equiv eB/m_t c$. We differentiate with respect to time to obtain $d^2k_y / dt^2 = -\omega_\ell\, dk_z / dt$; on substitution for dk_z/dt we have $d^2k_y / dt^2 + \omega_\ell \omega_t k_y = 0$, the equation of motion of a simple harmonic oscillator of natural frequency

$$\omega_0 = (\omega_\ell \omega_t)^{1/2} = eB/(m_\ell m_t)^{1/2} c.$$

5. Define $Q_e \equiv eB\tau_e / m_e c;\ Q_h = eB\tau_h / m_h c$. In the strong field limit $Q >> 1$ the magnetoconductivity tensor (6.64) reduces to

$$\underset{\approx}{\sigma} \simeq \frac{ne^2\tau_e}{m_e} \begin{pmatrix} Q_e^{-2} & -Q_e^{-1} & 0 \\ Q_e^{-1} & Q_e^{-2} & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{pe^2\tau_h}{m_h} \begin{pmatrix} Q_h^{-2} & Q_h^{-1} & 0 \\ -Q_h^{-1} & Q_h^{-2} & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

We can write nec Q_e/B for $ne^2\tau_e / m_e$ and pec Q_h/B for $pe^2\tau_h / m_h$. The strong field limit for σ_{yx} follows directly. The Hall field is obtained when we set

$$j_y = 0 = \frac{ec}{H} \left[(n - p)\, E_x + \left(\frac{n}{Q_e} + \frac{p}{Q_h} \right) E_y \right] .$$

The current density in the x direction is

$$j_x = \frac{ec}{B} \left[\left(\frac{n}{Q_e} + \frac{p}{Q_h} \right) E_x - (n - p)\, E_y \right] ;$$

using the Hall field for the standard geometry, we have

$$j_x = \frac{ec}{H} \left[\left(\frac{n}{Q_e} + \frac{p}{Q_h} \right) + \frac{(n-p)^2}{\left(\dfrac{n}{Q_e} + \dfrac{p}{Q_h} \right)} \right] E_x \ .$$