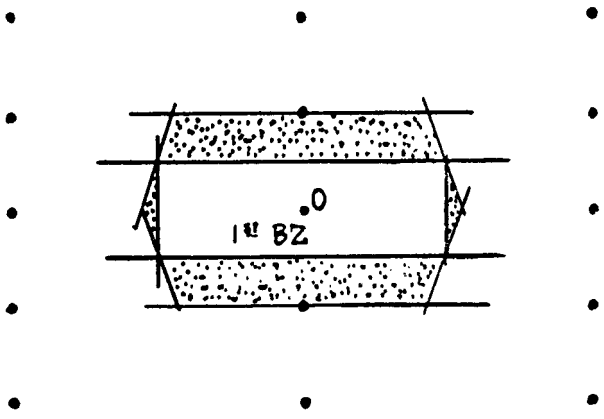
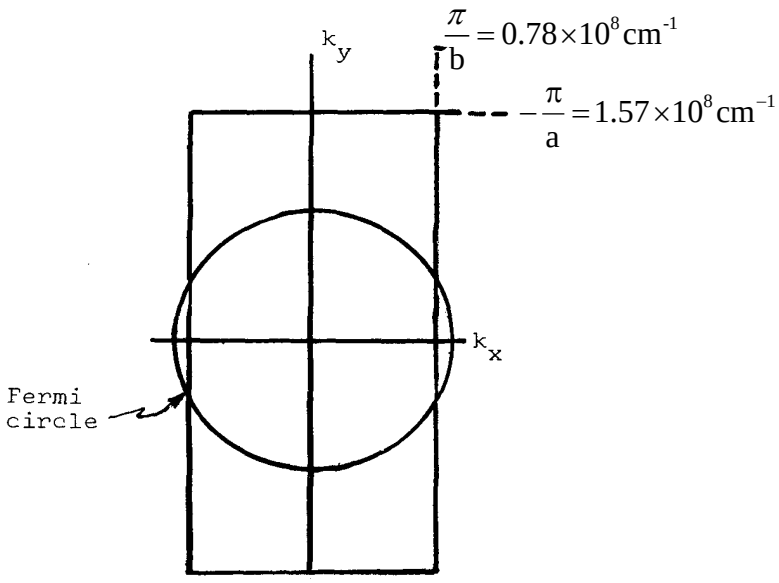
CHAPTER 9

1.



2a.



b.

$$N = 2 \times \frac{\pi k_F^2}{(2\pi/k)^2}$$

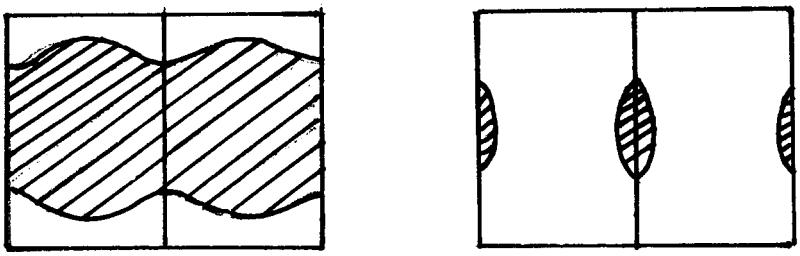
$$n = N/L^2 = k_F^2/2\pi$$

$$k_F = \sqrt{2\pi n}$$

$$n = \frac{1}{8} \times 10^{16}\ \text{els/cm}^2$$

$$k_F = 0.89 \times 10^8\ \text{cm}^{-1}$$

c.



3a. In the hcp structure there is one atom whose z coordinate is 0 and one at $\frac{1}{2}c$. The structure factor of the basis for $\underset{\sim}{G}_c = \frac{2\pi}{c}\,\hat{\underset{\sim}{z}}$ is

$$S_{\underset{\sim}{G}_c}(\text{basis}) = 1 + e^{-i\pi} = 1 - 1 = 0,$$

so that by the same argument as in Problem 9.4 the corresponding component $U_{\underset{\sim}{G}_c}$ of the crystal potential is zero.

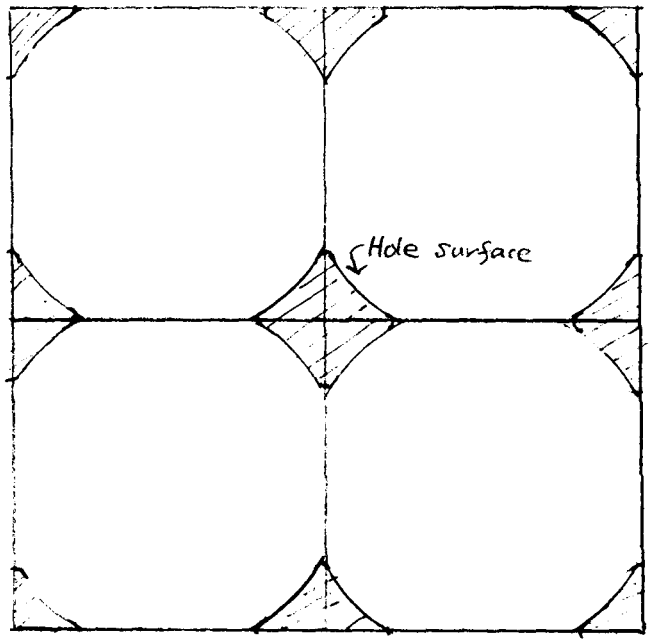
b. But for $U_{2\underset{\sim}{G}_c}$ the structure factor is

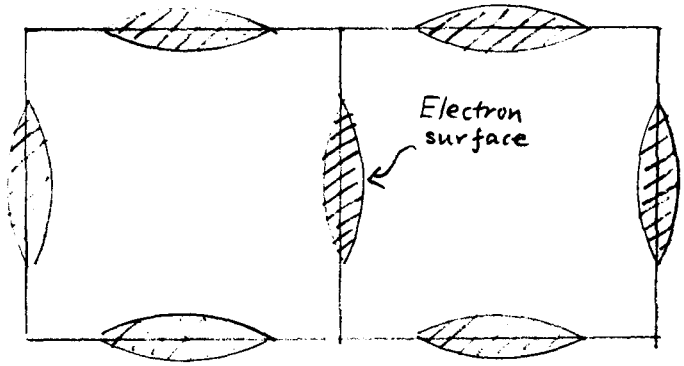
$$S_{2\underset{\sim}{G}_c}(\text{basis}) = 1 + e^{-i2\pi} = 2.$$

c. The two valence electrons can just fill the first BZ. All we need is an adequate energy gap at the zone boundary and for simple hex. there is no reason against a gap.

d. In hcp there will be no gap (at least in lowest order) on the top and bottom faces of the BZ, by the argument of part a.

4.





5a. $\hbar \dfrac{d\tilde{k}}{dt} = -\dfrac{e}{c}\tilde{v} \times \tilde{B}$;

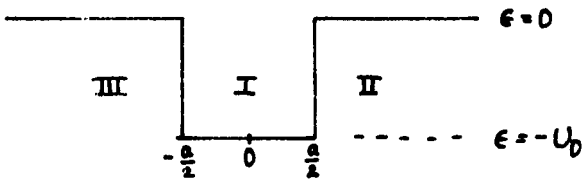
$$T = \frac{\hbar G c}{evB}$$

$$\simeq \frac{10^{-27}\,\text{erg sec})(2\times10^{8}\,\text{cm}^{-1})(3\times10^{10}\,\text{cm s}^{-1})}{(5\times10^{-10}\ \text{esu})(10^{8}\,\text{cm d}^{-1})(10^{3}\,\text{gauss})}$$

$$\simeq 1.2\times10^{-10}\ \text{sec}.$$

b. The electron moves in a direction normal to the Fermi surface -- more or less in a straight line if the Fermi surface is close to planar in the region of interest. The magnetic field puts a wiggle on the motion, but the field does not make the electron move in a helix, contrary to the behavior of a free electron.

6a.



Region I:

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - U_0\right)\psi = \varepsilon\psi$$

$$\psi = A \cos kx \;;\; \varepsilon = \frac{\hbar^2 k^2}{2m} - U_0 \quad (*)$$

Region II:

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = \varepsilon\psi$$

$$\psi = B\, e^{-qx} \;;\; \varepsilon = -\frac{\hbar^2 q^2}{2m} \qquad (*)$$

Boundary condition $\frac{1}{\psi}\frac{d\psi}{dx}$ continuous.

$$k \tan (ka/2) = q, \qquad (**)$$

with k and q related to ε as above.

b. The lazy way here is to show that the ε's in the equations marked (*) above are equal when k and q are connected by (**), with ε = –0.45 as read off Fig. 20. This is indeed so.

7a. $\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c S}$, where $S = \pi k_F^2$, with $k_F = 0.75 \times 10^8$ cm^{-1} from Table 6.1, for potassium. Thus

$$\Delta\left(\frac{1}{H}\right) \simeq \frac{2}{137\, k_F^{\,2}\, e} \simeq 0.55 \times 10^{-8}\ G^{-1}.$$

b.

$$\omega_c R = v_F \; ; \quad R = \frac{v_F mc}{eB} = \frac{\hbar k_F c}{eB}$$

$$\simeq 0.5\times 10^{-3} \text{ cm}$$

$$\pi R^2 \simeq 0.7\times 10^{-6} \text{ cm}^2 .$$

8. Write (17) as $H = H_0 + H_1$, where $H_1 = (\hbar/m)\,\underset{\sim}{k}\cdot\underset{\sim}{p}$. Then (18) is an eigenfunction of H_0 with the eigenvalue $\varepsilon_n(0) + \hbar^2 k^2/2m$. In this representation the diagonal matrix element of H_1 is equal to $(\hbar/m)\int dV\; u_0(\underset{\sim}{r})\,\underset{\sim}{k}\cdot\underset{\sim}{p}\,U_0(\underset{\sim}{r})$. In a cubic crystal $U_0(\underset{\sim}{r})$ will be even or odd with respect to the inversion operation $\underset{\sim}{r} \to -\underset{\sim}{r}$, but $\underset{\sim}{p}$ is an odd operator. It follows that the diagonal matrix element vanishes, and there is no first-order correction to the energy. The function $U_{\underset{\sim}{k}}(\underset{\sim}{r})$ to first order in H_1 is

$$U_{\underset{\sim}{k}}(\underset{\sim}{r}) = U_0(\underset{\sim}{r}) \; + \sum_j{}' \frac{< j0 \,|\, H_1 \,|\, n0 >}{\varepsilon_n(0) - \varepsilon_j(0)},$$

and the energy to second order is

$$\varepsilon_n(\underset{\sim}{k}) = \varepsilon_n(0) + (\hbar k)^2/2m + (\hbar/m)^2 \sum_j{}' \frac{|< n0 \,|\, \underset{\sim}{k}\cdot\underset{\sim}{p} \,|\, j0 >|^2}{\varepsilon_n(0) - \varepsilon_j(0)}.$$

The effective mass ratio is the coefficient of $\hbar^2 k^2/2m$, or

$$\frac{m}{m^*} = 1 + \frac{2}{m} \sum_j{}' \frac{|< n0 \,|\, \underset{\sim}{p} \,|\, j0 >|^2}{\varepsilon_n(0) - \varepsilon_j(0)}.$$

9a.

$$\int dV \;\; w^*(r - r_n)\, w(r - r_m)$$

$$= N^{-1} \sum_k \sum_{k'} e^{ik'\cdot r_n} \; e^{-ik\cdot r_m} \int dV\, \psi^*_{k'}(r)\psi_k(r)$$

$$= N^{-1} \sum_k e^{ik\cdot(r_n - r_m)}$$

where the summation is zero unless n = m, when it is equal to N.

b. $w(x - x_n) = N^{-1/2} U_0(x) \sum_k e^{ik(x - x_n)}$. The summation is

equal to

$$\sum_{P} e^{i2\pi p(x-x_n)/Na} \simeq \int_{-N/2}^{N/2} e^{i2\pi p(x-x_n)/Na}\, dp$$

$$= \frac{e^{i\pi(x-x_n)/a} - e^{-i\pi(x-x_n)/a}}{i2\pi(x-x_n)/Na} = \frac{\sin[\pi(x-x_n)/a]}{\pi(x-x_n)/Na},$$

whence

$$w(x-x_n) = N^{1/2}\, u_0(x) \frac{\sin[\pi(x-x_n)/a]}{\pi(x-x_n)/a}.$$

10a. $j_y = \sigma_0 (Q^{-1} E_x + sE_y) = 0$ in the Hall geometry, whence $E_y = -E_x/sQ$.

b. We have $j_x = \sigma_0 (Q^{-2} E_x - Q^{-1} E_y)$, and with our result for E_y it follows that

$$j_x = \sigma_0 (Q^{-2} + s^{-1} Q^{-2}) E_x,$$

whence $\rho = E_x / j_x = (Q^2/\sigma_0) \frac{s}{s+1}$.