CHAPTER 12

1. We have $\underset{\sim}{S}_{\underset{\sim}{\rho}+\underset{\sim}{\delta}} = \underset{\sim}{S}_{\underset{\sim}{\rho}} e^{i\underset{\sim}{k}\cdot\underset{\sim}{\delta}}$. Thus

$$\frac{dS_{\underset{\sim}{\rho}}^{x}}{dt} = \left(\frac{2JS}{\hbar}\right)\left(6-\sum_{\delta} e^{i\underset{\sim}{k}\cdot\underset{\sim}{\delta}}\right) S_{\underset{\sim}{\rho}}^{y}$$

$$= \left(\frac{2JS}{\hbar}\right)\left[6-2\left(\cos k_x a + \cos k_y a + \cos k_z a\right)\right] S_{\underset{\sim}{\rho}}^{y};$$

$$\frac{dS_{\underset{\sim}{\rho}}^{y}}{dt} = -\left(\frac{2JS}{\hbar}\right)\left[6-2\left(\cos k_x a + \cos k_y a + \cos k_z a\right)\right] S_{\underset{\sim}{\rho}}^{x}.$$

These equations have a solution with time-dependence ~ exp(–iωt) if

$$\omega = \left(2JS/\hbar\right)\left(6 - 2\cos k_x a - 2\cos k_y a - 2\cos k_z a\right).$$

2. $U = \sum_{\underset{\sim}{k}} n_k\ \hbar\omega_k = \hbar\int d\omega\, D(\omega)\,\omega < n(\omega) >$. If $\omega = Ak^2$, then $d\omega/dk = 2Ak = 2\sqrt{A}\,\omega^{1/2}$, and

$$D(\omega) = \frac{4\pi}{8\pi^3}\frac{\omega}{A}\frac{1}{2\sqrt{A}\omega^{1/2}} = \frac{1}{4\pi^2}\frac{\omega^{1/2}}{A^{3/2}}.$$

Then

$$U = \frac{\hbar}{4\pi^2 A^{3/2}}\int d\omega\ \omega^{3/2}\frac{1}{e^{\beta\hbar\omega}-1}.$$

At low temps,

$$\int \simeq \frac{1}{(\hbar\beta)^{5/2}}\int_0^\infty dx\frac{x^{3/2}}{e^x-1} = \frac{1}{(\hbar\beta)^{5/2}}\underset{\text{gamma function}}{\Gamma}\left(\frac{5}{2}\right)\underset{\text{zeta function}}{\varsigma}\left(\frac{5}{2};1\right)$$

[See Dwight 860.39]

$$U \simeq 0.45\left(k_B T\right)^{5/2}/\pi^2 A^{3/2}\hbar^{3/2}$$
$$C = dU/dT \simeq 0.113\ k_B\left(k_B T/\hbar A\right)^{3/2}.$$

3.
$$M_A T = C\left(B - \mu M_B - \varepsilon M_A\right) \ \left(B = \text{applied field}\right)$$
$$M_B T = C\left(B - \varepsilon M_B - \mu M_A\right)$$

Non-trivial solution for B = 0 if

$$\begin{vmatrix} T + \varepsilon C & \mu C \\ \mu C & T + \varepsilon C \end{vmatrix} = 0; \ T_C = C\left(\mu - \varepsilon\right)$$

Now find $\chi = \left(M_A + M_B\right)/B$ at $T > T_C$:

$$MT = 2CH - CM\left(\varepsilon + \mu\right); \ \chi = \frac{2C}{T + C\left(\mu + \varepsilon\right)}$$
$$\therefore \theta/T_C = \left(\mu + \varepsilon\right)/\left(\mu - \varepsilon\right) .$$

4. The terms in $U_{e\ell} + U_c + U_K$ which involve e_{xx} are

$$\frac{1}{2} C_{11} e_{xx}^{\ 2} + C_{12} e_{xx}\left(e_{yy} + e_{zz}\right) + B_1 \alpha_1^{\ 2} e_{xx} .$$

Take $\partial/\partial e_{xx}$:

$$C_{11} e_{xx} + C_{12}\left(e_{yy} + e_{zz}\right) + B_1 \alpha_1^{\ 2} = 0, \text{ for minimum.}$$

Further:

$$C_{11} e_{yy} + C_{12}(e_{xx} + e_{zz}) + B_1 \alpha_2^{\ 2} = 0 .$$
$$C_{11} e_{zz} + C_{12}\left(e_{xx} + e_{yy}\right) + B_1 \alpha_3^{\ 2} = 0 .$$

Solve this set of equations for e_{xx}:

$$e_{xx} = B_1 \frac{C_{12} - \alpha_2^{\ 2}\left(C_{11} + 2C_{12}\right)}{\left(C_{11} - C_{12}\right)\left(C_{11} + 2C_{12}\right)} .$$

Similarly for e_{yy}, e_{zz}, and by identical method for e_{xy}, etc.

5a.

$$U(\theta) = K \sin^2\theta - B_a M_s \cos\theta$$

$$\simeq K\varphi^2 - B_a M_s \frac{1}{2}\varphi^2, \text{ for } \theta = \pi + \varphi$$

and expanding about small φ.

For minimum near $\varphi = 0$ we need $K > \frac{1}{2} B_a M_s$. Thus at $B_a = 2K/M_s$ the magnetization reverses direction (we assume the magnetization reverses uniformly!).

b. If we neglect the magnetic energy of the bidomain particle, the energies of the single and bidomain particles will be roughly equal when

$$M_s^2 d^3 \approx \sigma_w d^2 \text{ ; or } d_c \approx \sigma_w / M_s^2.$$

For Co the wall energy will be higher than for iron roughly in the ratio of the (anisotropy constant $K_1)^{1/2}$, or $\sqrt{10}$. Thus $\sigma_w \approx 3 \text{ ergs/cm}^2$. For Co, M_s = 1400 (at room temperature), so $M_s^2 \approx 2\times10^6 \text{ erg/cm}^3$. We have $dc \approx 3/2\times10^6 \approx 1.10^{-6}$ cm, or $\simeq 100\,\mathring{A}$, as the critical size. The estimate is <u>very rough</u> (the wall thickness is d_c; the mag. en. is handled crudely).

6. Use the units of Eq. (9), and expand

$$\tanh\frac{m}{t} = \frac{m}{t} - \frac{1}{3}\frac{m^3}{t^3} + \cdots \text{ . [Dwight 657.3]}$$

Then (9) becomes $m \simeq \frac{m}{t} - \frac{m^3}{3t^3} + \cdots$;

$$3\left(t^3 - t^2\right) \simeq m^2 \text{ ; } m^2 \simeq 3t^2(1-t) \text{ ,}$$

but 1 – t is proportional to T_c – T, so that $m \propto \sqrt{T_c - T}$ for T just below T_c.

7. The maximum demagnetization field in a Néel wall is $-4\pi M_s$, and the maximum self-energy density is $\frac{1}{2}(4\pi M_s)M_s$. In a wall of thickness Na, where <u>a</u> is the lattice constant, the demagnetization contribution to the surface energy is $\sigma_{demag} \approx 2\pi M_s^2 Na$. The total wall energy, exchange + demag, is $\sigma_w \approx \left(\pi^2 JS^2/Na^2\right) + \left(2\pi M_s^2 Na\right)$, by use of (56). The minimum is at

$$\partial\sigma_w/\partial N = 0 = -\pi^2 JS^2/N^2a^2 + 2\pi M_s{}^2 a\text{, or}$$
$$N = \left(\frac{1}{2}\pi JS^2/M_s{}^2a^3\right)^{1/2},$$

and is given by

$$\sigma_w \approx \pi M_s S(2\pi J/a)^{1/2} \approx (10)(10^3)(10^{-4}/10^{-8})^{1/2} \approx 10 \text{ erg/cm}^2,$$

which is larger than (59) for iron. (According to Table 8.1 of the book by R. M. White and T. H. Geballe, the Bloch wall thickness in Permalloy is 16 times that in iron; this large value of δ favors the changeover to Néel walls in thin films.)

8. (a) Consider the resistance of the up and down spins separately.
Magnetizations parallel:

$$R_{\uparrow\uparrow}(up) = \sigma_p^{-1}(L/A) + \sigma_p^{-1}(L/A) = 2\sigma_p^{-1}(L/A)$$
$$R_{\uparrow\uparrow}(down) = \sigma_a^{-1}(L/A) + \sigma_a^{-1}(L/A) = 2\sigma_a^{-1}(L/A)$$

These resistances add in parallel:

$$R_{\uparrow\uparrow} = R_{\uparrow\uparrow}(down)R_{\uparrow\uparrow}(up)/[R_{\uparrow\uparrow}(down) + R_{\uparrow\uparrow}(up)] = 2(L/A)/(\sigma_a + \sigma_p)$$

Magnetizations antiparallel:

$$R_{\uparrow\downarrow}(up) = \sigma_p^{-1}(L/A) + \sigma_a^{-1}(L/A)$$
$$R_{\uparrow\downarrow}(down) = \sigma_a^{-1}(L/A) + \sigma_p^{-1}(L/A) = R_{\uparrow\downarrow}(up)$$

These (equal) resistances add in parallel :

$$R_{\uparrow\downarrow} = R_{\uparrow\downarrow}(up)/2 = (L/A)(\sigma_a^{-1} + \sigma_p^{-1})/2$$

The GMRR is then:

$$GMRR = R_{\uparrow\downarrow}/R_{\uparrow\uparrow} - 1 = (\sigma_a^{-1} + \sigma_p^{-1})(\sigma_a + \sigma_p)/4 - 1$$
$$= (\sigma_a/\sigma_p + \sigma_p/\sigma_a - 2)/4$$

(b) For the $\uparrow\downarrow$ magnetization configuration, an electron of a given spin direction must always go through a region where it is antiparallel to the magnetization. If $\sigma_a \to 0$, then the conductance is blocked and the resistance $R_{\uparrow\downarrow}$ is infinite.