CHAPTER 13

1. Consider a coil which when empty has resistance R_0 and inductance L_0. The impedance is $Z_0 = R_0 - i\omega L_0$. When the coil is filled with material of permeability $\mu = 1 + 4\pi\chi$ the impedance is $Z = R_0 - i\omega L_0(1+4\pi\chi) = R_0 - i\omega L_0(1+4\pi\chi' + 4\pi i\chi'')$, or

$$Z = \underbrace{R_0 + 4\pi\omega\chi'' L_0}_{R} - i\omega \underbrace{L_0(1+4\pi\chi')}_{L} .$$

2a. $\dfrac{d\tilde{F}}{dt} = \dfrac{dF_x}{dt}\hat{\tilde{x}} + F_x \dfrac{d\hat{\tilde{x}}}{dt} + \cdots .$

$$= \left(\frac{d\tilde{F}}{dt}\right)_R + \left(F_x \frac{d\hat{\tilde{x}}}{dt} + F_y \frac{d\hat{\tilde{y}}}{dt} + F_z \frac{d\hat{\tilde{z}}}{dt}\right) .$$

Now

$$\frac{d\hat{\tilde{x}}}{dt} = (\tilde{\Omega} \times \hat{\tilde{x}}) \; ; \; \frac{d\hat{\tilde{y}}}{dt} = (\tilde{\Omega} \times \hat{\tilde{y}}) \; ; \; \frac{d\hat{\tilde{z}}}{dt} = (\tilde{\Omega} \times \hat{\tilde{z}}) .$$

$$F_x \frac{d\hat{\tilde{x}}}{dt} + \cdots = \tilde{\Omega} \times \tilde{F} .$$

b. $\dfrac{d\tilde{M}}{dt} = \gamma \tilde{M} \times \tilde{B}$; $\left(\dfrac{d\tilde{M}}{dt}\right)_R + \tilde{\Omega} \times \tilde{M} = \gamma \tilde{M} \times \tilde{B}$.

$$\left(\frac{d\tilde{M}}{dt}\right)_R = \gamma \tilde{M} \times \left(\tilde{B} + \frac{\tilde{\Omega}}{\gamma}\right) .$$

c. With $\tilde{\Omega} = -\gamma B_0 \hat{\tilde{z}}$ we have

$$\left(\frac{d\tilde{M}}{dt}\right)_R = \gamma \tilde{M} \times B_1 \hat{\tilde{x}} ,$$

so that $\tilde{M}$ precesses about $\hat{\tilde{x}}$ with a frequency $\omega = \gamma B_1$. The time $t_{1/2}$ to give $t_{1/2}\omega = \pi$ is $t_{1/2} = \pi/\gamma B_1$.

d. The field $\tilde{B}_1$ rotates in the xy plane with frequency $\Omega = \gamma B_0$.

3a. $< B_i^{\ 2} > \ = \left(\frac{a}{N}\right)^2 \sum_j \sum_k < I_j^{\ z} I_k^{\ z} >$, where for $I = \frac{1}{2}$ we have $< I_j^{\ z} I_k^{\ z} > = \frac{1}{4}\delta_{jk}$. Thus

$$< B_i^{\ 2} > \ = \left(\frac{a}{N}\right)^2 \frac{1}{4}\sum_{jk}\delta_{jk} = \frac{a^2}{4N} .$$

b.
$$< B_i^{\ 4} > = \left(\frac{a}{N}\right)^4 \sum_{jk\ell m} < I_j^{\ z} I_k^{\ z} I_\ell^{\ z} I_m^{\ z} > .$$

Now

$$< I_j^{\ z} I_k^{\ z} I_\ell^{\ z} I_m^{\ z} >= \frac{1}{16}[\delta_{jk}\delta_{k\ell}\delta_{\ell m} + \delta_{jk}\delta_{\ell m}$$
$$+\delta_{j\ell}\delta_{km} + \delta_{jm}\delta_{k\ell}], \quad \text{and}$$
$$< B_i^{\ 4} > = \left(\frac{a}{N}\right)^4 \frac{1}{16}[N + 3N^2] \simeq \left(\frac{a}{N}\right)^4 \frac{3N^2}{16} .$$

4. For small θ, we have $U_K \simeq K\theta^2$. Now the magnetic energy density $U_M = -BM\cos\theta \simeq -BM + \frac{1}{2}BM\theta^2$, so that with proper choice of the zero of energy the anisotropy energy is equivalent to a field

$$B_A = 2K/M$$

along the z axis. This is valid for $\theta << 1$. For a sphere the demagnetizing field is parallel to M and exerts no torque on the spin system. Thus $B_0 + B_A$ is the effective field.

5. We may rewrite (48) with appropriate changes in M, and with $B_{anisotropy} = 0$. Thus

$$-i\omega M_A^{\ +} = -i\gamma_A \left(M_A^{\ +}\lambda|M_B| + M_B^{\ +}\lambda|M_A|\right) ;$$
$$-i\omega M_B^{\ +} = i\gamma_B \left(M_B^{\ +}\lambda|M_A| + M_A^{\ +}\lambda|M_B|\right) .$$

The secular equation is

$$\begin{vmatrix} \gamma_A\lambda|M_B| - \omega & \gamma_A\lambda|M_A| \\ -\gamma_B\lambda|M_B| & \gamma_B\lambda|M_A| - \omega \end{vmatrix} = 0 ,$$

or

$$\omega^2 - \omega\left(\gamma_A \lambda \left|M_B\right| - \gamma_B \lambda \left|M_A\right|\right) = 0 \ .$$

One root is $\omega_0 = 0$; this is the uniform mode. The other root is

$$\omega_0 = \lambda\left(\gamma_A \left|M_B\right| - \gamma_B \left|M_A\right|\right) = 0;$$

this is the exchange mode.