CHAPTER 16

1. $\frac{e^2}{r} \cdot \frac{x}{r} = eE$; $ex = r^3 E = p$; $\alpha = p/E = r^3 = a_H{}^3$.

2. $E_i = E_0 - \frac{4\pi}{3} P = 0$ inside a conducting sphere. Thus $p = \frac{4\pi}{3} a^3 P = a^3 E_0$, and $\alpha = p/E_0 = a^3$.

3. Because the normal component of **D** is continuous across a boundary, $E_{air} = \varepsilon E_{diel}$, where $E_{air} = 4\pi Q/A$, with Q the charge on the boundary. The potential drop between the two plates is $E_{air}\ qd + E_{diel}\ d = E_{air}\ d\left(q + \frac{1}{\varepsilon}\right)$. For a plate of area A, the capacitance is

$$C = \frac{A}{4\pi d\left(q + \frac{1}{\varepsilon}\right)} .$$

It is useful to define an effective dielectric constant by

$$\frac{1}{\varepsilon_{eff}} = \frac{1}{\varepsilon} + q .$$

If $\varepsilon = \infty$, then $\varepsilon_{eff} = 1/q$. We cannot have a higher effective dielectric constant than 1/q. For $q = 10^{-3}$, $\varepsilon_{eff} = 10^3$.

4. The potential drop between the plates is $E_1\ d + E_2\ qd$. The charge density

$$\frac{Q}{A} = \frac{D_1}{4\pi} = \frac{\varepsilon E_1}{4\pi} = \frac{i\sigma}{\omega} E_2 , \qquad \text{(CGS)}$$

by comparison of the way σ and ε enter the Maxwell equation for curl H. Thus

$$E_1 + \frac{4\pi i\sigma}{\varepsilon\omega} E_2 \ ; \ V = E_2 d\left(\frac{4\pi i\sigma}{\varepsilon\omega} + q\right) ;$$

$Q = \frac{\sigma A i}{\omega} E_2$; and thus $C \equiv \frac{Q}{V} = \frac{A}{4\pi d\left(\frac{1}{\varepsilon} - \frac{i\omega q}{4\pi\sigma}\right)}$,

and $\varepsilon_{eff} = (1+q)\frac{\varepsilon}{1-(i\omega\varepsilon q/4\pi\sigma)}$.

5a. $E_{int} = E_0 - \frac{4\pi}{3} P = E_0 - \frac{4\pi}{3} \chi^{E}{}_{int}$.

$$E_{int} = \frac{E_0}{1 + \frac{4\pi}{3}\chi} \quad .$$

b. $P = \chi \ E_{int} = \frac{\chi}{1 + \frac{4\pi}{3}\chi} E_0$.

6. E = $2P_1/a^3$. $P_2 = \alpha E = 2\alpha P_1/a^3$. This has solution $p_1 = p_2$ 0 if $2\alpha = a^3$; $\alpha = \frac{1}{2} a^3$.

7 (a). One condition is, from (43),

$$\gamma\left(T_C - T_0\right) - \left|g_4\right| P_s^{\ 2} + g_6 P_s^{\ 4} = 0 \ .$$

The other condition is

$$\frac{1}{2}\gamma\left(T_c - T_0\right) P_s^{\ 2} - \frac{1}{4}\left|g_4\right| P_s^{\ 4} + \frac{1}{6} g_6 P_s^{\ 6} = 0 \ .$$

Thus

$$-\left|g_4\right| P_s^{\ 2} + g_6 P_s^{\ 4} = -\frac{1}{2}\left|g_4\right| P_s^{\ 2} + \frac{1}{3} g_6 P_s^{\ 4} \ ;$$

$$\frac{2}{3} g_6 P_s^{\ 2} = \frac{1}{2}\left|g_4\right| \ ; \ \ P_s^{\ 2} = \frac{3}{4}\frac{\left|g_4\right|}{g_6} \ .$$

(b) From the first line of part (a),

$$\gamma\left(T_c - T_0\right) = \frac{3}{4}\frac{\left|g_4\right|^2}{g_6} - \frac{9}{16}\frac{\left|g_4\right|^2}{g_6} = \frac{3}{16}\frac{g_4^{\ 2}}{g_6} \ .$$

8. In an electric field the equilibrium condition becomes $-E + \gamma\left(T - T_c\right)P + g_4 P^3 = 0$, where the term in g_6 is neglected for a second-order transition. Now let $P = P_s + \Delta P$. If we retain only linear terms in ΔP, then $-E + \gamma\left(T - T_c\right)\Delta P + g_4 3P_s^{\ 2}\Delta P = 0$, with use of (40). Further, we can eliminate $P_s^{\ 2}$ because $P_s^{\ 2} = \left(\gamma/g_4\right)\left(T_c - T\right)$. Thus $\Delta P/E = 1/2\gamma\left(T_c - T\right)$.

9 a. $|\leftarrow a \rightarrow|$ $\qquad \cos\frac{\pi}{a}(na)$

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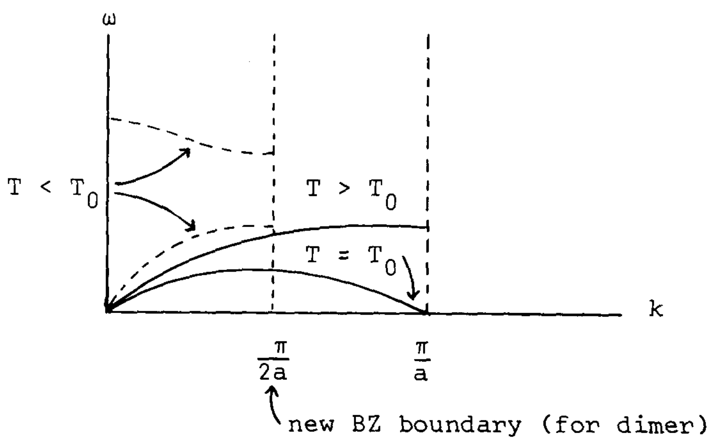
b. $|\leftarrow 2a \rightarrow|$

$\cdot\cdot \qquad \cdot\cdot \qquad \cdot\cdot$

Deforms to new stable structure of dimers, with lattice constant 2 × (former constant).

c.



10. The induced dipole moment on the atom at the origin is $p = \alpha E$, where the electric field is that of all other dipoles: $E = \left(2/a^3\right)\sum p_n = \left(4p/a^3\right)\left(\sum_n{}^{-3}\right)$; the sum is over positive integers. We assume all dipole moments equal to p. The self-consistency condition is that $p = \alpha(4p/a^3)\,(\Sigma n^{-3})$, which has the solution $p = 0$ unless $\alpha \geq (a^3/4)\,(1/\Sigma n^{-3})$. The value of the summation is 1.202; it is the zeta function $\zeta(3)$.