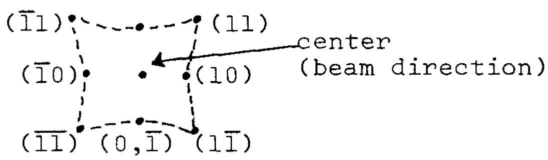
CHAPTER 17

1. (a) The interference condition for a linear lattice is a cos $\theta = n\lambda$. The values of θ that satisfy this condition each define a cone with axis parallel to the fiber axis and to the axis of the cylindrical film. Each cone intersects the film in a circle. When the film is flattened out, parallel lines result. (b) The intersection of a cone and a plane defines a conic section, here a hyperbola. (c) Let **a**, **b** be the primitive axes of a square lattice. The Laue equations (2.25) give $\mathbf{a} \bullet \Delta\mathbf{k} = 2\pi q$; $\mathbf{b} \bullet \Delta\mathbf{k} = 2\pi r$, where q, r are integers. Each equation defines a set of planes. The intersections of these planes gives a set of parallel lines, which play in diffraction from a two-dimensional structure the role played by reciprocal lattice points in diffraction from a three-dimensional structure. In the Ewald construction these lines intersect a sphere of radius $k = 2\pi/\lambda$ in a set of points. In two dimensions any wavelength (below some maximum) will give points; in three dimensions only special values of λ give points of intersection because one more Laue equation must be satisfied. The points correspond to the directions k' of the diffraction maxima. If the photographic plate is flat the diffraction pattern (2 dim.) will appear distorted.



Points near the direction of the incident beam are shown.

(d) The lattice of surface atoms in the (110) surface of an fcc crystal is simple rectangular. The long side of the rectangle in crystal (real) space is a short side in the reciprocal lattice. This explains the 90° rotation between (21a) and (21b).

2. With the trial function x exp (–ax), the normalization integral is $\int_0^\infty dx\ x^2\ \exp(-2ax) = 1/4a^3$. The kinetic energy operator applied to the trial function gives

$$-(\hbar^2/2m)d^2u/dx^2 = -(\hbar^2/2m)(a^2x - 2a)\exp(-ax)$$

while $Vu = eEx^2 \exp(-ax)$. The definite integrals that are needed have the form $\int_0^\infty dx\ x^n \exp(-ax) = n!/a^{n+1}$. The expectation value of the energy is $<\varepsilon> = (\hbar^2/2m)a^2 + (3eE/2a)$, which has an extremum with respect to the range parameter a when $d<\varepsilon>/da = (\hbar^2/2m)2a - 3eE/2a^2 = 0$, or $a^3 = 3eEm/2\hbar^2$. The value of $<\varepsilon>$ is a minimum at this value of a, so that

$$<\varepsilon>_{min} = (\hbar^2/2m)(3eEm/\hbar^2)^{2/3} + (3eE/2)(2\hbar^2/3eEm)^{1/3}$$
$$= (\hbar^2/2m)^{1/3}(3eE/2)^{2/3}(2^{-2/3} + 2^{1/3}),$$

where the last factor has the value 1.89 …. The Airy function is treated in Sec. 10.4 of the NBS Handbook of mathematical functions.

3. (a) $D(\varepsilon) = \frac{dN}{dk}\frac{dk}{d\varepsilon} = \frac{2}{(2\pi/L)^2}\frac{d(\pi k^2)}{dk}\frac{m}{\hbar^2 k} = \frac{m}{\pi\hbar^2}A$

where $A = L^2$.

Note: There are two flaws in the answer $m/\pi h^2$ quoted in the text. First, the area A is missing, meaning the quoted answer is a density per unit area. This should not be a major issue. Second, the h should be replaced by $\hbar$.

(b) $N = \frac{2}{(2\pi/L)^2}\cdot \pi k_F^2$ => $n_s = N/A = k_F^2/2\pi$

(c) $R_s = \frac{L}{W}\frac{m}{n_s e^2 \tau}$ where n_s is the 2D sheet density. For a square sample, $W=L$, so:

$R_s = \frac{2\pi m}{k_F^2 e^2 \tau}$ and using $\hbar k_F / m = v_F$:

$$R_s = \frac{2\pi\hbar}{k_F v_F e^2 \tau} = \frac{h}{e^2}\frac{1}{k_F \ell}$$