CHAPTER 18

1. *Carbon nanotube band structure.*

(a) $\mathbf{b}_i \cdot \mathbf{a}_j = 2\pi\delta_{ij}$ => $\mathbf{b}_1 = (-\frac{2\pi}{a}, \frac{2\pi}{\sqrt{3}a})$, $\mathbf{b}_2 = (\frac{2\pi}{a}, \frac{2\pi}{\sqrt{3}a})$.

(b) The angle between **K** and $\mathbf{b}_1$is 30^o ; A right triangle is formed in the first BZ with two sides of length K and $b_1/2$. Now $b_1 = \frac{4\pi}{\sqrt{3}a}$, so:

$K = (b_1/2)/\cos(30^o) = 4\pi/3a$.

(c) Quantization of k along x: $k_x(na)=2\pi j=$ $k_x = 2\pi j/na$.

Assume $n = 3i$, where i is an integer. Then: $k_x = K(j/2i)$. For $j = 2i$, $k_x=K$. Then

$\Delta\mathbf{K} = k_y\hat{j}$ and there is a massless subband.

(d) For $n = 10$, $k_x = 2\pi\, j/10a = K(3j/20)$. The closest k comes to K is for $j = 7$, where $\Delta k_x = K/20$. Then:

$$\varepsilon_{11} = 2\hbar v_F(4\pi/3a)/10 = 1.8 \text{ eV}.$$

The next closest is for $j = 6$, where $\Delta k_x = K/10$, twice the previous one. Therefore:

$\varepsilon_{22} = 2\varepsilon_{11}$.

(e) For the lowest subband: $|\Delta\mathbf{k}|^2 = (K/20)^2 + k_y^2$, so:

$$\varepsilon^2 = [(\hbar K/20v_F)v_F^2]^2 + (\hbar k_y v_F)^2$$

This is of the desired form, with $m^* = \hbar K/20v_F$.

$m^*/m = \hbar K/20mv_F = 0.12$.

2. *Filling subbands*

$\varepsilon(n_x, n_y) = \frac{\hbar^2\pi^2}{2mW^2}(n_x^2 + n_y^2)$ => States are filled up to $\varepsilon(2,2) = \frac{\hbar^2\pi^2}{2mW^2}(8)$

(1,1) subband: $\frac{\hbar^2 k_{1,1}^2}{2m} = \frac{\hbar^2\pi^2}{2mW^2}(8-2)$ => $k_{1,1} = \frac{\sqrt{6}\pi}{W}$ => $n_{1,1} = \frac{2}{\pi}k_{1,1} = \frac{2\sqrt{6}}{W}$

(2,1) subband: $\frac{\hbar^2 k_{2,1}^2}{2m} = \frac{\hbar^2\pi^2}{2mW^2}(8-5)$ => $k_{2,1} = \frac{\sqrt{3}\pi}{W}$ => $n_{2,1} = \frac{2}{\pi}k_{2,1} = \frac{2\sqrt{3}}{W}$

(2,1) subband: same.

$$n = \frac{2\sqrt{6}}{W} + \frac{4\sqrt{3}}{W} \quad = \quad 5.9 \text{ x } 10^8 \text{ /m.}$$

3. *Breit-Wigner form of a transmission resonance*

(a) $\cos(\delta\varphi) \cong 1 - \delta\varphi^2/2 \qquad ; |r_i| = \sqrt{1-|t_i|^2} \cong 1 - \frac{1}{2}|t_i|^2 - \frac{1}{8}|t_i|^4$

The denominator of (29) is then:

$1 + (1-|t_1|^2)(1-|t_2|^2) - 2(1-\frac{1}{2}|t_1|^2 - \frac{1}{8}|t_1|^4)(1-\frac{1}{2}|t_2|^2 - \frac{1}{8}|t_2|^4)(1-\frac{1}{2}\delta\varphi^2)$

$= \frac{1}{4}(|t_1|^4 + |t_2|^4) + \frac{1}{2}|t_1|^2|t_2|^2 + \delta\varphi^2 = \frac{1}{4}(|t_1|^2 + |t_2|^2)^2 + \delta\varphi^2$

$$\Im = \frac{4|t_1|^2|t_2|^2}{(|t_1|^2 + |t_2|^2)^2 + 4\delta\varphi^2}.$$

(b) $\delta\varphi = 2L\delta k$ and $\delta k/\delta\varepsilon = \Delta k/\Delta\varepsilon = (\pi/L)/\Delta\varepsilon$. Combining:

$\delta\varphi = (2L)(\pi/L)\delta\varepsilon/\Delta\varepsilon \quad => \quad \delta\varphi/2\pi = \delta\varepsilon/\Delta\varepsilon$

(c) Combining:

$$\Im = \frac{4|t_1|^2|t_2|^2 \, (\Delta\varepsilon/2\pi)^2}{(\Delta\varepsilon/2\pi)^2(|t_1|^2 + |t_2|^2)^2 + 4\delta\varepsilon^2} \quad \text{which is (33).}$$

4. *Barriers in series and Ohm's law*

(a)

$$\frac{1}{\Im} = \frac{1-|r_1|^2|r_2|^2}{|t_1|^2|t_2|^2} = 1 + \frac{1-|r_1|^2|r_2|^2 - |t_1|^2|t_2|^2}{|t_1|^2|t_2|^2} = 1 + \frac{1-(1-|t_1|^2)|r_2|^2 - (1-|r_1|^2)|t_2|^2}{|t_1|^2|t_2|^2}$$

$$= 1 + \frac{1-(|r_2|^2 + |t_2|^2) + |t_1|^2|r_2|^2 + |r_1|^2|t_2|^2}{|t_1|^2|t_2|^2} = 1 + \frac{|r_2|^2}{|t_2|^2} + \frac{|r_1|^2}{|t_1|^2} \quad \text{which gives (36).}$$

(b) $\sigma_{1D} = \frac{n_{1D}e^2\tau}{m} = \frac{2k_F e^2\tau}{\pi m}$, and $\frac{\hbar k_F}{m} = v_F \quad => \quad \sigma_{1D} = \frac{2v_F e^2\tau}{\hbar\pi} = \frac{2e^2(2v_F\tau)}{h}$

But: $\ell_B = v_F\tau_B = 2v_F\tau \quad => \quad \sigma_{1D} = \frac{2e^2\ell_B}{h}$.

5. *Energies of a spherical quantum dot*

(a) $\int_A \mathbf{E}\cdot d\mathbf{a} = Q_{encl}/\varepsilon\varepsilon_o \quad => \quad E = q/4\pi\varepsilon\varepsilon_o r^2$ Integrating from inner to outer shell:

$$V = \int_R^{R+d} \frac{q\,dr}{4\pi\varepsilon\varepsilon_o r^2} = \frac{q}{4\pi\varepsilon\varepsilon_o}\left(\frac{1}{R} - \frac{1}{R+d}\right) = \frac{q}{4\pi\varepsilon\varepsilon_o}\frac{d}{R(R+d)}$$

$$C=\frac{q}{V}=4\pi\varepsilon\varepsilon_o\frac{R(R+d)}{d} \quad \text{and therefore} \quad U=\frac{e^2}{C}=\frac{e^2}{4\pi\varepsilon\varepsilon_o}\frac{d}{R(R+d)}.$$

(b) For $d << R$, $\quad C\cong 4\pi\varepsilon\varepsilon_o\frac{R^2}{d}=\varepsilon\varepsilon_o\frac{A}{d}$.

(c) For $d >> R$, $\quad U=\frac{e^2}{4\pi\varepsilon\varepsilon_o R}$. Also $\varepsilon_{0,0}=\frac{\hbar^2\pi^2}{2m^*R^2}$ => $\quad \frac{U}{\varepsilon_{0,0}}=\frac{e^2}{4\pi\varepsilon\varepsilon_o R}\cdot\frac{2m^*R^2}{\hbar^2\pi^2}$

$$\frac{U}{\varepsilon_{0,0}}=\frac{e^2}{4\pi\varepsilon\varepsilon_o R}\cdot\frac{2m^*R^2}{\hbar^2\pi^2}=\frac{e^2}{4\pi\varepsilon\varepsilon_o}\cdot\frac{2m^*R}{\hbar^2\pi^2}=\frac{2}{\pi^2}\frac{R}{a_B^*}$$

6. *Thermal properties in 1D*

(a) $\quad D(\omega)=\frac{2K}{2\pi/L}\frac{1}{v}=\frac{L}{\pi v}$

$$U_{tot}=\int_0^{\omega_D}\frac{d\omega D(\omega)\hbar\omega}{\exp(\hbar\omega/k_BT)-1}\cong\frac{\hbar L}{\pi v}\int_0^{\infty}\frac{\omega d\omega}{\exp(\hbar\omega/k_BT)-1} = \frac{\hbar L}{\pi v}\left(\frac{k_BT}{\hbar}\right)^2\int_0^{\infty}\frac{xdx}{\exp(x)-1}$$

Obtaining value from table of integrals:

$$U_{tot}=\frac{Lk_B^2T^2}{\hbar\pi v}\frac{\pi^2}{6} = \frac{\pi^2Lk_B^2T^2}{3hv}$$

$$C_V=\partial U_{tot}/\partial T\big|_V == \frac{2\pi^2Lk_B^2T}{3hv}$$

(b) The heat flow to the right out of reservoir 1 is given by:

$$J_R=\int_0^{\infty}\frac{D_R(\omega)}{L}\cdot v\cdot\frac{d\omega\hbar\omega}{\exp(\hbar\omega/k_BT_1)-1}\Im=\frac{\hbar\Im}{2\pi}\int_0^{\infty}\frac{\omega d\omega}{\exp(\hbar\omega/k_BT_1)-1} = \frac{\hbar\Im}{2\pi}\left(\frac{k_BT_1}{\hbar}\right)^2\frac{\pi^2}{6}=\frac{\pi^2k_B^2T_1^2}{6h}\Im$$

and similarly for J_L. The difference is:

$$J_R-J_L=\frac{\pi^2k_B^2\Im}{6h}\left(T_1^2-T_2^2\right)$$

Let $T_1=T+\Delta T$, $T_2=T$ => $\left(T_1^2-T_2^2\right)\approx 2T\Delta T$ for small ΔT.

=> $\quad J_R-J_L=\frac{\pi^2k_B^2\Im}{3h}\Delta T$ which gives (78).