CHAPTER 20

1. $U = nE_I$.
The number of ways to pick n from N is N! / (N – n) !n!. The number of ways to put n into N' = n'! / (N' – n) !n!.

$$\text{Entropy } S = k_B\left(\log\frac{N!}{(N-n)!n!} + \log\frac{N'!}{(N'-n)!n!}\right).$$

$$\log\frac{N!}{(N-n)!n!} \simeq N\log N - (N-n)\log(N-n) - n\log n$$

$$\log\frac{N'!}{(N'-n)!n!} \simeq N'\log N' - (N'-n)\log(N'-n) - n\log n$$

$$\left(\frac{\partial F}{\partial n}\right)_T = \frac{\partial U}{\partial n} - T\frac{\partial S}{\partial n} = 0 \text{ in equilibrium; thus}$$

$$E_I = k_B T\left(\log\frac{N-n}{n} + \log\frac{N'-n}{n}\right)$$

$$= k_B T \log\frac{(N-n)(N'-n)}{n^2}. \quad \text{For } n << N, N',$$

$$E_I = k_B T \log\left(NN'/n^2\right); \quad n = \sqrt{NN'}\, e^{-E_I/2k_BT}.$$

2. From (2), $n = Ne^{-E_v/k_BT}$. For sodium, $N = 2.5 \times 10^{22}$ atoms/cm^3. Thus

$$n \simeq \left(2.5\times10^{22}\right)\left(4\times10^{-18}\right) \approx 10^5 \text{ per cm}^3.$$

3a. $\hbar\omega = \frac{13.6}{\varepsilon^2}\cdot\frac{3}{4}$ eV. From Table 10.3 we have ε (∞) = 2.25 for NaCl, whence $\hbar\omega = 2.0$ eV. The observed value quoted in Table 18.2 is 2.7 eV.

b. From Vol. I of Atomic energy levels, Circular 467 of the Nat. Bur. of Standards, 3p-3s = 16960 cm^{-1} = 2.1 eV.