Chapter 25

1. (a) The capacitance of the system is

$$C = \frac{q}{\Delta V} = \frac{70\,\text{pC}}{20\,\text{V}} = 3.5\,\text{pF}.$$

(b) The capacitance is independent of q; it is still 3.5 pF.

(c) The potential difference becomes

$$\Delta V = \frac{q}{C} = \frac{200\,\text{pC}}{3.5\,\text{pF}} = 57\,\text{V}.$$

2. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery. Thus,

$$q = (25 \times 10^{-6}\text{ F})(120\text{ V}) = 3.0 \times 10^{-3}\text{ C}.$$

3. **THINK** The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/d$, where A is the area of each plate and d is the plate separation.

EXPRESS Since the plates are circular, the plate area is $A = \pi R^2$, where R is the radius of a plate. The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates.

ANALYZE (a) Substituting the values given, the capacitance is

$$C = \frac{\varepsilon_0 \pi R^2}{d} = \frac{\left(8.85\times10^{-12}\text{ F/m}\right)\pi\left(8.2\times10^{-2}\text{ m}\right)^2}{1.3\times10^{-3}\text{m}} = 1.44\times10^{-10}\text{F} = 144\text{ pF}.$$

(b) Similarly, the charge on the plate when $V = 120$ V is

$$q = (1.44 \times 10^{-10}\text{ F})(120\text{ V}) = 1.73 \times 10^{-8}\text{ C} = 17.3\text{ nC}.$$

LEARN Capacitance depends only on geometric factors, namely, the plate area and plate separation.

4. (a) We use Eq. 25-17:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\text{mm})(38.0\,\text{mm})}{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(40.0\,\text{mm}-38.0\,\text{mm})} = 84.5\ \text{pF}.$$

(b) Let the area required be A. Then $C = \varepsilon_0 A/(b - a)$, or

$$A = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\,\text{pF})(40.0\,\text{mm}-38.0\,\text{mm})}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 191\,\text{cm}^2.$$

5. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 \quad\Rightarrow\quad R' = 2^{1/3}R.$$

The new capacitance is

$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3} R = 5.04\pi\varepsilon_0 R.$$

With R = 2.00 mm, we obtain $C = 5.04\pi\left(8.85\times10^{-12}\ \text{F/m}\right)\left(2.00\times10^{-3}\text{m}\right) = 2.80\times10^{-13}\,\text{F}$.

6. (a) We use $C = A\varepsilon_0/d$. The distance between the plates is

$$d = \frac{A\varepsilon_0}{C} = \frac{\left(1.00\,\text{m}^2\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{1.00\,\text{F}} = 8.85\times10^{-12}\ \text{m}.$$

(b) Since d is much less than the size of an atom (~ 10^{-10} m), this capacitor cannot be constructed.

7. For a given potential difference V, the charge on the surface of the plate is

$$q = Ne = (nAd)e$$

where d is the depth from which the electrons come in the plate, and n is the density of conduction electrons. The charge collected on the plate is related to the capacitance and the potential difference by $q = CV$ (Eq. 25-1). Combining the two expressions leads to

$$\frac{C}{A} = ne\frac{d}{V}.$$

With $d/V = d_s/V_s = 5.0\times10^{-14}$ m/V and $n = 8.49\times10^{28}/\text{m}^3$ (see, for example, Sample Problem 25.01 — "Charging the plates in a parallel-plate capacitor"), we obtain

$$\frac{C}{A} = (8.49\times10^{28}\ /\ \text{m}^3)(1.6\times10^{-19}\ \text{C})(5.0\times10-14\ \text{m/V}) = 6.79\times10^{-4}\ \text{F/m}^2.$$

8. The equivalent capacitance is given by $C_{eq} = q/V$, where q is the total charge on all the capacitors and V is the potential difference across any one of them. For N identical capacitors in parallel, $C_{eq} = NC$, where C is the capacitance of one of them. Thus, $NC = q/V$ and

$$N = \frac{q}{VC} = \frac{1.00\,\text{C}}{(110\,\text{V})\,(1.00\times10^{-6}\,\text{F})} = 9.09\times10^{3}.$$

9. The charge that passes through meter A is

$$q = C_{eq}V = 3CV = 3(25.0\,\mu\text{F})(4200\,\text{V}) = 0.315\,\text{C}.$$

10. The equivalent capacitance is

$$C_{eq} = C_3 + \frac{C_1C_2}{C_1+C_2} = 4.00\,\mu\text{F} + \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}} = 7.33\,\mu\text{F}.$$

11. The equivalent capacitance is

$$C_{eq} = \frac{(C_1+C_2)C_3}{C_1+C_2+C_3} = \frac{(10.0\,\mu\text{F}+5.00\,\mu\text{F})(4.00\,\mu\text{F})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}+4.00\,\mu\text{F}} = 3.16\,\mu\text{F}.$$

12. The two 6.0 μF capacitors are in parallel and are consequently equivalent to $C_{eq} = 12\ \mu\text{F}$. Thus, the total charge stored (before the squeezing) is

$$q_{total} = C_{eq}V = (12\,\mu\text{F})\,(10.0\,\text{V}) = 120\ \mu\text{C}.$$

(a) and (b) As a result of the squeezing, one of the capacitors is now 12 μF (due to the inverse proportionality between C and d in Eq. 25-9), which represents an increase of $6.0\ \mu\text{F}$ and thus a charge increase of

$$\Delta q_{total} = \Delta C_{eq}V = (6.0\,\mu\text{F})\,(10.0\,\text{V}) = 60\ \mu\text{C}.$$

13. **THINK** Charge remains conserved when a fully charged capacitor is connected to an uncharged capacitor.

EXPRESS The charge initially on the charged capacitor is given by $q = C_1V_0$, where C_1 = 100 pF is the capacitance and V_0 = 50 V is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1V$, where V = 35 V is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where C_2 is the capacitance of the second capacitor.

ANALYZE Substituting C_1V_0 for q and C_1V for q_1, we obtain $q_2 = C_1(V_0 - V)$. The potential difference across the second capacitor is also V, so the capacitance of the second capacitor is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V} C_1 = \frac{50\,\text{V} - 35\,\text{V}}{35\,\text{V}}(100\,\text{pF}) = 42.86\text{ pF} \approx 43\,\text{pF}.$$

LEARN Capacitors in parallel have the same potential difference. To verify charge conservation explicitly, we note that the initial charge on the first capacitor is $q = C_1V_0 = (100\text{ pF})(50\text{ V}) = 5000\text{ pC}.$ After the connection, the charges on each capacitor are

$$q_1 = C_1V = (100\text{ pF})(35\text{ V}) = 3500\text{ pC}$$
$$q_2 = C_2V = (42.86\text{ pF})(35\text{ V}) = 1500\text{ pC}.$$

Indeed, $q = q_1 + q_2$.

14. (a) The potential difference across C_1 is V_1 = 10.0 V. Thus,

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\text{ V}) = 1.00 \times 10^{-4}\text{ C}.$$

(b) Let C = 10.0 μF. We first consider the three-capacitor combination consisting of C_2 and its two closest neighbors, each of capacitance C. The equivalent capacitance of this combination is

$$C_{\text{eq}} = C + \frac{C_2C}{C + C_2} = 1.50\ C.$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C + C_{\text{eq}}} = \frac{CV_1}{C + 1.50\ C} = 0.40V_1.$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage difference across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2V_2 = (10.0\,\mu\text{F})\left(\frac{10.0\,\text{V}}{5}\right) = 2.00\times10^{-5}\,\text{C}.$$

15. (a) First, the equivalent capacitance of the two 4.00 μF capacitors connected in series is given by 4.00 μF/2 = 2.00 μF. This combination is then connected in parallel with two other 2.00-μF capacitors (one on each side), resulting in an equivalent capacitance C = 3(2.00 μF) = 6.00 μF. This is now seen to be in series with another combination, which

consists of the two 3.0-μF capacitors connected in parallel (which are themselves equivalent to $C' = 2(3.00\ \mu\text{F}) = 6.00\ \mu\text{F}$). Thus, the equivalent capacitance of the circuit is

$$C_{\text{eq}} = \frac{CC'}{C+C'} = \frac{(6.00\,\mu\text{F})\,(6.00\,\mu\text{F})}{6.00\,\mu\text{F}+6.00\,\mu\text{F}} = 3.00\,\mu\text{F}.$$

(b) Let $V = 20.0$ V be the potential difference supplied by the battery. Then

$$q = C_{\text{eq}}V = (3.00\ \mu\text{F})(20.0\ \text{V}) = 6.00 \times 10^{-5}\ \text{C}.$$

(c) The potential difference across C_1 is given by

$$V_1 = \frac{CV}{C+C'} = \frac{(6.00\,\mu\text{F})\,(20.0\,\text{V})}{6.00\,\mu\text{F}+6.00\,\mu\text{F}} = 10.0\,\text{V}.$$

(d) The charge carried by C_1 is $q_1 = C_1V_1 = (3.00\ \mu\text{F})(10.0\ \text{V}) = 3.00 \times 10^{-5}$ C.

(e) The potential difference across C_2 is given by $V_2 = V - V_1 = 20.0\ \text{V} - 10.0\ \text{V} = 10.0$ V.

(f) The charge carried by C_2 is $q_2 = C_2V_2 = (2.00\ \mu\text{F})(10.0\ \text{V}) = 2.00 \times 10^{-5}$ C.

(g) Since this voltage difference V_2 is divided equally between C_3 and the other 4.00-μF capacitors connected in series with it, the voltage difference across C_3 is given by $V_3 = V_2/2 = 10.0\ \text{V}/2 = 5.00$ V.

(h) Thus, $q_3 = C_3V_3 = (4.00\ \mu\text{F})(5.00\ \text{V}) = 2.00 \times 10^{-5}$ C.

16. We determine each capacitance from the slope of the appropriate line in the graph. Thus, $C_1 = (12\ \mu\text{C})/(2.0\ \text{V}) = 6.0\ \mu\text{F}$. Similarly, $C_2 = 4.0\ \mu\text{F}$ and $C_3 = 2.0\ \mu\text{F}$. The total equivalent capacitance is given by

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2+C_3} = \frac{C_1+C_2+C_3}{C_1(C_2+C_3)},$$

or

$$C_{123} = \frac{C_1(C_2+C_3)}{C_1+C_2+C_3} = \frac{(6.0\ \mu\text{F})(4.0\ \mu\text{F}+2.0\ \mu\text{F})}{6.0\ \mu\text{F}+4.0\ \mu\text{F}+2.0\ \mu\text{F}} = \frac{36}{12}\ \mu\text{F} = 3.0\ \mu\text{F}.$$

This implies that the charge on capacitor 1 is $q_1 = (3.0\ \mu\text{F})(6.0\ \text{V}) = 18\ \mu\text{C}$. The voltage across capacitor 1 is therefore $V_1 = (18\ \mu\text{C})/(6.0\ \mu\text{F}) = 3.0$ V. From the discussion in section 25-4, we conclude that the voltage across capacitor 2 must be 6.0 V – 3.0 V = 3.0 V. Consequently, the charge on capacitor 2 is $(4.0\ \mu\text{F})(3.0\ \text{V}) = 12\ \mu\text{C}$.

17. (a) and (b) The original potential difference V_1 across C_1 is

$$V_1 = \frac{C_{eq}V}{C_1+C_2} = \frac{(3.16\,\mu\text{F})(100.0\,\text{V})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}} = 21.1\,\text{V}.$$

Thus ΔV_1 = 100.0 V – 21.1 V = 78.9 V and

$$\Delta q_1 = C_1\Delta V_1 = (10.0\ \mu\text{F})(78.9\ \text{V}) = 7.89 \times 10^{-4}\ \text{C}.$$

18. We note that the voltage across C_3 is V_3 = (12 V – 2 V – 5 V) = 5 V. Thus, its charge is $q_3 = C_3 V_3$ = 4 μC.

(a) Therefore, since C_1, C_2 and C_3 are in series (so they have the same charge), then

$$C_1 = \frac{4\ \mu\text{C}}{2\ \text{V}} = 2.0\ \mu\text{F}\ .$$

(b) Similarly, C_2 = 4/5 = 0.80 μF.

19. (a) and (b) We note that the charge on C_3 is q_3 = 12 μC – 8.0 μC = 4.0 μC. Since the charge on C_4 is q_4 = 8.0 μC, then the voltage across it is q_4/C_4 = 2.0 V. Consequently, the voltage V_3 across C_3 is 2.0 V $\Rightarrow C_3 = q_3/V_3$ = 2.0 μF.

Now C_3 and C_4 are in parallel and are thus equivalent to 6 μF capacitor which would then be in series with C_2; thus, Eq 25-20 leads to an equivalence of 2.0 μF which is to be thought of as being in series with the unknown C_1. We know that the total effective capacitance of the circuit (in the sense of what the battery "sees" when it is hooked up) is (12 μC)/V_{battery} = 4 μF/3. Using Eq 25-20 again, we find

$$\frac{1}{2\ \mu\text{F}} + \frac{1}{C_1} = \frac{3}{4\ \mu\text{F}} \quad\Rightarrow\quad C_1 = 4.0\ \mu\text{F}\ .$$

20. For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of (n – 1) identical single capacitors connected in parallel. Each capacitor has surface area A and plate separation d so its capacitance is given by $C_0 = \varepsilon_0 A/d$. Thus, the total capacitance of the combination is

$$C=(n-1)C_0=\frac{(n-1)\varepsilon_0 A}{d}=\frac{(8-1)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.25\times10^{-4}\ \text{m}^2)}{3.40\times10^{-3}\ \text{m}}=2.28\times10^{-12}\,\text{F}.$$

21. **THINK** After the switches are closed, the potential differences across the capacitors are the same and they are connected in parallel.

EXPRESS The potential difference from a to b is given by $V_{ab} = Q/C_{eq}$, where Q is the net charge on the combination and C_{eq} is the equivalent capacitance.

ANALYZE (a) The equivalent capacitance is $C_{eq} = C_1 + C_2 = 4.0 \times 10^{-6}$ F. The total charge on the combination is the net charge on either pair of connected plates. The initial charge on capacitor 1 is

$$q_1 = C_1 V = \left(1.0\times10^{-6}\ \text{F}\right)\left(100\ \text{V}\right) = 1.0\times10^{-4}\ \text{C}$$

and the initial charge on capacitor 2 is

$$q_2 = C_2 V = \left(3.0\times10^{-6}\ \text{F}\right)\left(100\ \text{V}\right) = 3.0\times10^{-4}\ \text{C}.$$

With opposite polarities, the net charge on the combination is

$$Q = 3.0 \times 10^{-4}\ \text{C} - 1.0 \times 10^{-4}\ \text{C} = 2.0 \times 10^{-4}\ \text{C}.$$

The potential difference is

$$V_{ab} = \frac{Q}{C_{eq}} = \frac{2.0\times10^{-4}\ \text{C}}{4.0\times10^{-6}\ \text{F}} = 50\ \text{V}.$$

(b) The charge on capacitor 1 is now $q_1' = C_1 V_{ab} = (1.0 \times 10^{-6}\ \text{F})(50\ \text{V}) = 5.0 \times 10^{-5}\ \text{C}$.

(c) The charge on capacitor 2 is now $q_2' = C_2 V_{ab} = (3.0 \times 10^{-6}\ \text{F})(50\ \text{V}) = 1.5 \times 10^{-4}\ \text{C}$.

LEARN The potential difference $V_{ab} = 50$ V is half of the original V (= 100 V), so the final charges on the capacitors are also halved.

22. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1 V_{bat} = 100\ \mu$C, and q_1, q_2 and q_3 are the charges on C_1, C_2 and C_3 after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2 + q_3.$$

Since the parallel pair C_2 and C_3 are identical, it is clear that $q_2 = q_3$. They are in parallel with C_1 so that $V_1=V_3$, or

$$\frac{q_1}{C_1} = \frac{q_3}{C_3}$$

which leads to $q_1 = q_3/2$. Therefore,

$$Q = (q_3/2) + q_3 + q_3 = 5q_3/2$$

which yields $q_3 = 2Q/5 = 2(100\ \mu\text{C})/5 = 40\ \mu\text{C}$ and consequently $q_1 = q_3/2 = 20\ \mu$C.

23. We note that the total equivalent capacitance is $C_{123} = [(C_3)^{-1} + (C_1 + C_2)^{-1}]^{-1} = 6\ \mu$F.

(a) Thus, the charge that passed point *a* is $C_{123}\, V_{\text{batt}} = (6\ \mu\text{F})(12\ \text{V}) = 72\ \mu\text{C}$. Dividing this by the value $e = 1.60 \times 10^{-19}$ C gives the number of electrons: 4.5×10^{14}, which travel to the left, toward the positive terminal of the battery.

(b) The equivalent capacitance of the parallel pair is $C_{12} = C_1 + C_2 = 12\ \mu$F. Thus, the voltage across the pair (which is the same as the voltage across C_1 and C_2 individually) is

$$\frac{72\ \mu\text{C}}{12\ \mu\text{F}} = 6\ \text{V}.$$

Thus, the charge on C_1 is

$$q_1 = (4\ \mu\text{F})(6\ \text{V}) = 24\ \mu\text{C},$$

and dividing this by *e* gives $N_1 = q_1/e = 1.5\times10^{14}$, the number of electrons that have passed (upward) through point *b*.

(c) Similarly, the charge on C_2 is $q_2 = (8\ \mu\text{F})(6\ \text{V}) = 48\ \mu\text{C}$, and dividing this by *e* gives $N_2 = q_2/e = 3.0\times10^{14}$, the number of electrons which have passed (upward) through point *c*.

(d) Finally, since C_3 is in series with the battery, its charge is the same charge that passed through the battery (the same as passed through the switch). Thus, 4.5×10^{14} electrons passed rightward though point *d*. By leaving the rightmost plate of C_3, that plate is then the positive plate of the fully charged capacitor, making its leftmost plate (the one closest to the negative terminal of the battery) the negative plate, as it should be.

(e) As stated in (b), the electrons travel up through point *b*.

(f) As stated in (c), the electrons travel up through point *c*.

24. Using Equation 25-14, the capacitances are

$$C_1 = \frac{2\pi\varepsilon_0 L_1}{\ln(b_1/a_1)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.050\ \text{m})}{\ln(15\ \text{mm}/5.0\ \text{mm})} = 2.53\ \text{pF}$$

$$C_2 = \frac{2\pi\varepsilon_0 L_2}{\ln(b_2/a_2)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.090\ \text{m})}{\ln(10\ \text{mm}/2.5\ \text{mm})} = 3.61\ \text{pF}\ .$$

Initially, the total equivalent capacitance is

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1C_2} \Rightarrow C_{12} = \frac{C_1C_2}{C_1 + C_2} = \frac{(2.53\ \text{pF})(3.61\ \text{pF})}{2.53\ \text{pF} + 3.61\ \text{pF}} = 1.49\ \text{pF}\,,$$

and the charge on the positive plate of each one is (1.49 pF)(10 V) = 14.9 pC. Next, capacitor 2 is modified as described in the problem, with the effect that

$$C_2' = \frac{2\pi\varepsilon_0 L_2}{\ln(b_2'/a_2)} = \frac{2\pi(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.090\ \mathrm{m})}{\ln(25\ \mathrm{mm}/2.5\ \mathrm{mm})} = 2.17\ \mathrm{pF}\ .$$

The new total equivalent capacitance is

$$C_{12}' = \frac{C_1 C_2'}{C_1 + C_2'} = \frac{(2.53\ \mathrm{pF})(2.17\ \mathrm{pF})}{2.53\ \mathrm{pF} + 2.17\ \mathrm{pF}} = 1.17\ \mathrm{pF}$$

and the new charge on the positive plate of each one is (1.17 pF)(10 V) = 11.7 pC. Thus we see that the charge transferred from the battery (considered in absolute value) as a result of the modification is 14.9 pC – 11.7 pC = 3.2 pC.

(a) This charge, divided by e gives the number of electrons that pass point P. Thus,

$$N = \frac{3.2\times10^{-12}\,\mathrm{C}}{1.6\times10^{-19}\,\mathrm{C}} = 2.0\times10^{7}\ .$$

(b) These electrons move rightward in the figure (that is, away from the battery) since the positive plates (the ones closest to point P) of the capacitors have suffered a *decrease* in their positive charges. The usual reason for a metal plate to be positive is that it has more protons than electrons. Thus, in this problem some electrons have “returned” to the positive plates (making them less positive).

25. Equation 23-14 applies to each of these capacitors. Bearing in mind that $\sigma = q/A$, we find the total charge to be

$$q_{\text{total}} = q_1 + q_2 = \sigma_1 A_1 + \sigma_2 A_2 = \varepsilon_0 E_1 A_1 + \varepsilon_0 E_2 A_2 = 3.6\ \mathrm{pC}$$

where we have been careful to convert cm^2 to m^2 by dividing by 10^4.

26. Initially the capacitors C_1, C_2, and C_3 form a combination equivalent to a single capacitor which we denote C_{123}. This obeys the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)}\ .$$

Hence, using $q = C_{123}V$ and the fact that $q = q_1 = C_1 V_1$, we arrive at

$$V_1 = \frac{q_1}{C_1} = \frac{q}{C_1} = \frac{C_{123}}{C_1}V = \frac{C_2 + C_3}{C_1 + C_2 + C_3}V\ .$$

(a) As $C_3 \to \infty$ this expression becomes $V_1 = V$. Since the problem states that V_1 approaches 10 volts in this limit, so we conclude V = 10 V.

(b) and (c) At $C_3 = 0$, the graph indicates V_1 = 2.0 V. The above expression consequently implies $C_1 = 4C_2$. Next we note that the graph shows that, at C_3 = 6.0 μF, the voltage across C_1 is exactly half of the battery voltage. Thus,

$$\frac{1}{2} = \frac{C_2 + 6.0\ \mu\text{F}}{C_1 + C_2 + 6.0\ \mu\text{F}} = \frac{C_2 + 6.0\ \mu\text{F}}{4C_2 + C_2 + 6.0\ \mu\text{F}}$$

which leads to C_2 = 2.0 μF. We conclude, too, that C_1 = 8.0 μF.

27. (a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1C_3V}{C_1+C_3} = \frac{(1.00\mu\text{F})\,(3.00\mu\text{F})\,(12.0\text{V})}{1.00\mu\text{F}+3.00\mu\text{F}} = 9.00\mu\text{C}.$$

(b) Capacitors 2 and 4 are also in series:

$$q_2 = q_4 = \frac{C_2C_4V}{C_2+C_4} = \frac{(2.00\mu\text{F})\,(4.00\mu\text{F})\,(12.0\text{V})}{2.00\mu\text{F}+4.00\mu\text{F}} = 16.0\mu\text{C}.$$

(c) $q_3 = q_1 = 9.00\mu\text{C}.$

(d) $q_4 = q_2 = 16.0\mu\text{C}.$

(e) With switch 2 also closed, the potential difference V_1 across C_1 must equal the potential difference across C_2 and is

$$V_1 = \frac{C_3+C_4}{C_1+C_2+C_3+C_4}\,V = \frac{(3.00\mu\text{F}+4.00\mu\text{F})(12.0\text{V})}{1.00\mu\text{F}+2.00\mu\text{F}+3.00\mu\text{F}+4.00\mu\text{F}} = 8.40\ \text{V}.$$

Thus, $q_1 = C_1V_1$ = (1.00 μF)(8.40 V) = 8.40 μC.

(f) Similarly, $q_2 = C_2V_1$ = (2.00 μF)(8.40 V) = 16.8 μC.

(g) $q_3 = C_3(V - V_1)$ = (3.00 μF)(12.0 V – 8.40 V) = 10.8 μC.

(h) $q_4 = C_4(V - V_1)$ = (4.00 μF)(12.0 V – 8.40 V) = 14.4 μC.

28. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_2}+\frac{1}{C_3}=\frac{C_2+C_3}{C_2C_3}.$$

Thus, $C_{\text{eq}} = C_2C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination, and the potential difference across the equivalent capacitor is given by q_2/C_{eq}. The potential difference across capacitor 1 is q_1/C_1, where q_1 is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{\text{eq}}$.

Now, some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If q_0 is the original charge, conservation of charge yields $q_1 + q_2 = q_0 = C_1 V_0$, where V_0 is the original potential difference across capacitor 1.

(a) Solving the two equations

$$\frac{q_1}{C_1}=\frac{q_2}{C_{\text{eq}}}$$

$$q_1+q_2=C_1V_0$$

for q_1 and q_2, we obtain

$$q_1=\frac{C_1^2V_0}{C_{\text{eq}}+C_1}=\frac{C_1^2V_0}{\dfrac{C_2C_3}{C_2+C_3}+C_1}=\frac{C_1^2\left(C_2+C_3\right)V_0}{C_1C_2+C_1C_3+C_2C_3}.$$

With $V_0 = 12.0$ V, $C_1 = 4.00\ \mu$F, $C_2 = 6.00\ \mu$F and $C_3 = 3.00\ \mu$F, we find $C_{\text{eq}} = 2.00\ \mu$F and $q_1 = 32.0\ \mu$C.

(b) The charge on capacitors 2 is

$$q_2=C_1V_0-q_1=(4.00\mu\text{F})(12.0\text{ V})-32.0\ \mu\text{C}=16.0\ \mu\text{C}.$$

(c) The charge on capacitor 3 is the same as that on capacitor 2:

$$q_3=C_1V_0-q_1=(4.00\ \mu\text{F})(12.0\text{ V})-32.0\ \mu\text{C}=16.0\ \mu\text{C}.$$

29. The energy stored by a capacitor is given by $U=\frac{1}{2}CV^2$, where V is the potential difference across its plates. We convert the given value of the energy to Joules. Since $1\text{ J}=1\text{ W}\cdot\text{s}$, we multiply by ($10^3$ W/kW)(3600 s/h) to obtain $10\text{ kW}\cdot\text{h}=3.6\times10^7\text{ J}$. Thus,

$$C=\frac{2U}{V^2}=\frac{2\left(3.6\times10^7\text{ J}\right)}{\left(1000\text{ V}\right)^2}=72\text{ F}.$$

30. Let $\mathcal{V}$ = 1.00 m^3. Using Eq. 25-25, the energy stored is

$$U = u\mathcal{V} = \frac{1}{2}\varepsilon_0 E^2 \mathcal{V} = \frac{1}{2}\left(8.85\times10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(150\ \mathrm{V/m})^2\left(1.00\ \mathrm{m}^3\right) = 9.96\times10^{-8}\ \mathrm{J}.$$

31. **THINK** The total electrical energy is the sum of the energies stored in the individual capacitors.

EXPRESS The energy stored in a charged capacitor is

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2.$$

Since we have two capacitors that are connected in parallel, the potential difference V across the capacitors is the same and the total energy is

$$U_{\text{tot}} = U_1 + U_2 = \frac{1}{2}(C_1 + C_2)V^2.$$

ANALYZE Substituting the values given, we have

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\ \mathrm{F} + 4.0\times10^{-6}\ \mathrm{F}\right)(300\ \mathrm{V})^2 = 0.27\ \mathrm{J}.$$

LEARN The energy stored in a capacitor is equal to the amount of work required to charge the capacitor.

32. (a) The capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(40\times10^{-4}\ \mathrm{m}^2\right)}{1.0\times10^{-3}\mathrm{m}} = 3.5\times10^{-11}\ \mathrm{F} = 35\ \mathrm{pF}.$$

(b) $q = CV$ = (35 pF)(600 V) = 2.1 × 10^{-8} C = 21 nC.

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(35\,\mathrm{pF})(21\,\mathrm{nC})^2 = 6.3\times10^{-6}\,\mathrm{J} = 6.3\,\mu\mathrm{J}.$

(d) $E = V/d$ = 600 V/1.0 × 10^{-3} m = 6.0 × 10^5 V/m.

(e) The energy density (energy per unit volume) is

$$u = \frac{U}{Ad} = \frac{6.3\times10^{-6}\,\text{J}}{\left(40\times10^{-4}\text{m}^2\right)\left(1.0\times10^{-3}\text{m}\right)} = 1.6\ \text{J/m}^3.$$

33. We use $E = q/4\pi\varepsilon_0 R^2 = V/R$. Thus

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{8000\,\text{V}}{0.050\text{ m}}\right)^2 = 0.11\text{ J/m}^3.$$

34. (a) The charge q_3 in the figure is $q_3 = C_3V = (4.00\,\mu\text{F})(100\text{ V}) = 4.00\times10^{-4}\text{ C}$.

(b) $V_3 = V = 100$ V.

(c) Using $U_i = \frac{1}{2}C_iV_i^2$, we have $U_3 = \frac{1}{2}C_3V_3^2 = 2.00\times10^{-2}\,\text{J}$.

(d) From the figure,

$$q_1 = q_2 = \frac{C_1C_2V}{C_1+C_2} = \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})(100\text{ V})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}} = 3.33\times10^{-4}\text{C}.$$

(e) $V_1 = q_1/C_1 = 3.33\times10^{-4}$ C/10.0 μF = 33.3 V.

(f) $U_1 = \frac{1}{2}C_1V_1^2 = 5.55\times10^{-3}\,\text{J}$.

(g) From part (d), we have $q_2 = q_1 = 3.33\times10^{-4}\text{C}$.

(h) $V_2 = V - V_1 = 100\text{ V} - 33.3\text{ V} = 66.7$ V.

(i) $U_2 = \frac{1}{2}C_2V_2^2 = 1.11\times10^{-2}\,\text{J}$.

35. The energy per unit volume is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 = \frac{e^2}{32\pi^2\varepsilon_0 r^4}.$$

(a) At $r = 1.00\times10^{-3}\text{m}$, with $e = 1.60\times10^{-19}\text{C}$ and $\varepsilon_0 = 8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$, we have $u = 9.16\times10^{-18}\text{ J/m}^3$.

(b) Similarly, at $r = 1.00\times10^{-6}\text{m}$, $u = 9.16\times10^{-6}\text{ J/m}^3$.

(c) At $r = 1.00\times10^{-9}\text{m}$, $u = 9.16\times10^{6}\text{ J/m}^3$.

(d) At $r = 1.00\times10^{-12}\,\text{m}$, $u = 9.16\times10^{18}\ \text{J/m}^3$.

(e) From the expression above, $u \propto r^{-4}$. Thus, for $r \to 0$, the energy density $u \to \infty$.

36. (a) We calculate the charged surface area of the cylindrical volume as follows:

$$A = 2\pi rh + \pi r^2 = 2\pi(0.20\ \text{m})(0.10\ \text{m}) + \pi(0.20\ \text{m})^2 = 0.25\ \text{m}^2$$

where we note from the figure that although the bottom is charged, the top is not. Therefore, the charge is $q = \sigma A = -0.50\ \mu\text{C}$ on the exterior surface, and consequently (according to the assumptions in the problem) that same charge q is induced in the interior of the fluid.

(b) By Eq. 25-21, the energy stored is

$$U = \frac{q^2}{2C} = \frac{(5.0\times10^{-7}\ \text{C})^2}{2(35\times10^{-12}\ \text{F})} = 3.6\times10^{-3}\ \text{J}.$$

(c) Our result is within a factor of three of that needed to cause a spark. Our conclusion is that it will probably not cause a spark; however, there is not enough of a safety factor to be sure.

37. **THINK** The potential difference between the plates of a parallel-plate capacitor depends on their distance of separation.

EPXRESS Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $C_i = \varepsilon_0 A/d_i$, the charge is $q_i = C_i V_i = \varepsilon_0 AV_i/d_i$. After the plates are pulled apart, their separation is d_f and the final potential difference is V_f. Thus, the final charge is $q_f = \varepsilon_0 AV_f/2d_f$. Since charge remains unchanged, $q_i = q_f$, we have

$$V_f = \frac{q_f}{C_f} = \frac{d_f}{\varepsilon_0 A}q_f = \frac{d_f}{\varepsilon_0 A}\frac{\varepsilon_0 A}{d_i}V_i = \frac{d_f}{d_i}V_i.$$

ANALYZE (a) With $d_i = 3.00\times10^{-3}\text{m}$, $V_i = 6.00\ \text{V}$ and $d_f = 8.00\times10^{-3}\text{m}$, the final potential difference is $V_f = 16.0\ \text{V}$.

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2}CV_i^2 = \frac{\varepsilon_0 AV_i^2}{2d_i} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.50\times10^{-4}\ \text{m}^2)(6.00\ \text{V})^2}{2(3.00\times10^{-3}\ \text{m})}$$
$$= 4.51\times10^{-11}\ \text{J}.$$

(c) The final energy stored is

$$U_f = \frac{1}{2}C_f V_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}V_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}\left(\frac{d_f}{d_i}V_i\right)^2 = \frac{d_f}{d_i}\left(\frac{\varepsilon_0 A V_i^2}{d_i}\right) = \frac{d_f}{d_i}U_i.$$

With $d_f / d_i = 8.00/3.00$, we have $U_f = 1.20\times10^{-10}$ J.

(d) The work done to pull the plates apart is the difference in the energy:

$$W = U_f - U_i = 7.52\times10^{-11} \text{ J}.$$

LEARN In a parallel-plate capacitor, the energy density (energy per unit volume) is given by $u = \varepsilon_0 E^2/2$ (see Eq. 25-25), where E is constant at all points between the plates. Thus, increasing the plate separation increases the volume (= Ad), and hence the total energy of the system.

38. (a) The potential difference across C_1 (the same as across C_2) is given by

$$V_1 = V_2 = \frac{C_3 V}{C_1 + C_2 + C_3} = \frac{(15.0\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F} + 15.0\,\mu\text{F}} = 50.0\,\text{V}.$$

Also, $V_3 = V - V_1 = V - V_2 = 100$ V – 50.0 V = 50.0 V. Thus,

$$q_1 = C_1 V_1 = (10.0\,\mu\text{F})(50.0\,\text{V}) = 5.00\times10^{-4}\,\text{C}$$
$$q_2 = C_2 V_2 = (5.00\,\mu\text{F})(50.0\,\text{V}) = 2.50\times10^{-4}\,\text{C}$$
$$q_3 = q_1 + q_2 = 5.00\times10^{-4}\,\text{C} + 2.50\times10^{-4}\,\text{C} = 7.50\times10^{-4}\,\text{C}.$$

(b) The potential difference V_3 was found in the course of solving for the charges in part (a). Its value is $V_3 = 50.0$ V.

(c) The energy stored in C_3 is $U_3 = C_3 V_3^2/2 = (15.0\,\mu\text{F})(50.0\,\text{V})^2/2 = 1.88\times10^{-2}\,\text{J}$.

(d) From part (a), we have $q_1 = 5.00\times10^{-4}\,\text{C}$, and

(e) $V_1 = 50.0$ V, as shown in (a).

(f) The energy stored in C_1 is $U_1 = \frac{1}{2}C_1 V_1^2 = \frac{1}{2}(10.0\,\mu\text{F})(50.0\,\text{V})^2 = 1.25\times10^{-2}\,\text{J}$.

(g) Again, from part (a), $q_2 = 2.50\times10^{-4}\,\text{C}$.

(h) V_2 = 50.0 V, as shown in (a).

(i) The energy stored in C_2 is $U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(5.00\,\mu\text{F})(50.0\,\text{V})^2 = 6.25\times10^{-3}\,\text{J}$.

39. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across 10 μF, then the voltage across the 20 μF capacitor is 50 V and the voltage across the 25 μF capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 25-21 or Eq. 25-22, we sum the energies on the capacitors and obtain U_{total} = 0.095 J.

40. If the original capacitance is given by $C = \varepsilon_0 A/d$, then the new capacitance is $C' = \varepsilon_0 \kappa A/2d$. Thus $C'/C = \kappa/2$ or

$$\kappa = 2C'/C = 2(2.6\text{ pF}/1.3\text{ pF}) = 4.0.$$

41. **THINK** Our system, a coaxial cable, is a cylindrical capacitor filled with polystyrene, a dielectric.

EXPRESS Using Eqs. 25-17 and 25-27, the capacitance of a cylindrical capacitor can be written as

$$C = \kappa C_0 = \frac{2\pi\kappa\varepsilon_0 L}{\ln(b/a)},$$

where C_0 is the capacitance without the dielectric, κ is the dielectric constant, L is the length, a is the inner radius, and b is the outer radius.

ANALYZE With $\kappa = 2.6$ for polystyrene, the capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\varepsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85\times10^{-12}\text{ F/m})}{\ln[(0.60\text{ mm})/(0.10\text{ mm})]} = 8.1\times10^{-11}\,\text{F/m} = 81\text{ pF/m}.$$

LEARN When the space between the plates of a capacitor is completely filled with a dielectric material, the capacitor increases by a factor κ, the dielectric constant characteristic of the material.

42. (a) We use $C = \varepsilon_0 A/d$ to solve for d:

$$d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(0.35\text{ m}^2)}{50\times10^{-12}\,\text{F}} = 6.2\times10^{-2}\text{ m}.$$

(b) We use $C \propto \kappa$. The new capacitance is

$$C' = C(\kappa/\kappa_{air}) = (50\text{ pf})(5.6/1.0) = 2.8\times10^2\text{ pF}.$$

43. The capacitance with the dielectric in place is given by $C = \kappa C_0$, where C_0 is the capacitance before the dielectric is inserted. The energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2$, so

$$\kappa = \frac{2U}{C_0V^2} = \frac{2(7.4\times10^{-6}\text{ J})}{(7.4\times10^{-12}\text{ F})(652\text{ V})^2} = 4.7.$$

According to Table 25-1, you should use Pyrex.

44. (a) We use Eq. 25-14:

$$C = 2\pi\varepsilon_0\kappa\frac{L}{\ln(b/a)} = \frac{(4.7)(0.15\text{ m})}{2\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\ln(3.8\text{ cm}/3.6\text{ cm})} = 0.73\text{ nF}.$$

(b) The breakdown potential is (14 kV/mm) (3.8 cm – 3.6 cm) = 28 kV.

45. Using Eq. 25-29, with $\sigma = q/A$, we have

$$\left|\vec{E}\right| = \frac{q}{\kappa\varepsilon_0 A} = 200\times10^3\text{ N/C}$$

which yields $q = 3.3 \times 10^{-7}$ C. Eq. 25-21 and Eq. 25-27 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\varepsilon_0 A} = 6.6\times10^{-5}\text{ J}\ .$$

46. Each capacitor has 12.0 V across it, so Eq. 25-1 yields the charge values once we know C_1 and C_2. From Eq. 25-9,

$$C_2 = \frac{\varepsilon_0 A}{d} = 2.21 \times 10^{-11}\text{ F}\ ,$$

and from Eq. 25-27,

$$C_1 = \frac{\kappa\varepsilon_0 A}{d} = 6.64 \times 10^{-11}\text{ F}\ .$$

This leads to

$$q_1 = C_1V_1 = 8.00 \times 10^{-10}\text{ C},\ \ q_2 = C_2V_2 = 2.66 \times 10^{-10}\text{ C}.$$

The addition of these gives the desired result: $q_{tot} = 1.06 \times 10^{-9}$ C. Alternatively, the circuit could be reduced to find the q_{tot}.

47. **THINK** Dielectric strength is the maximum value of the electric field a dielectric material can tolerate without breakdown.

EXPRESS The capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation. The electric field between the plates is given by $E = V/d$, where V is the potential difference between the plates. Thus, $d = V/E$ and $C = \kappa\varepsilon_0 AE/V$. Therefore, we find the plate area to be

$$A = \frac{CV}{\kappa\varepsilon_0 E}.$$

ANALYZE For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0\times10^{-8}\,\text{F})(4.0\times10^{3}\,\text{V})}{2.8(8.85\times10^{-12}\,\text{F/m})(18\times10^{6}\ \text{V/m})} = 0.63\ \text{m}^2.$$

LEARN If the area is smaller than the minimum value found above, then electric breakdown occurs and the dielectric is no longer insulating and will start to conduct.

48. The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with dielectric constants κ_1 and κ_2, respectively. Thus, (in SI units),

$$C = C_1 + C_2 = \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1+\kappa_2}{2}\right)$$
$$= \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.56\times10^{-4}\ \text{m}^2)}{5.56\times10^{-3}\ \text{m}}\left(\frac{7.00+12.00}{2}\right) = 8.41\times10^{-12}\ \text{F}.$$

49. We assume there is charge q on one plate and charge $-q$ on the other. The electric field in the lower half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1\varepsilon_0 A},$$

where A is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2\varepsilon_0 A}.$$

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V=\frac{E_1 d}{2}+\frac{E_2 d}{2}=\frac{qd}{2\varepsilon_0 A}\left[\frac{1}{\kappa_1}+\frac{1}{\kappa_2}\right]=\frac{qd}{2\varepsilon_0 A}\frac{\kappa_1+\kappa_2}{\kappa_1\kappa_2},$$

so

$$C=\frac{q}{V}=\frac{2\varepsilon_0 A}{d}\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}.$$

This expression is exactly the same as that for C_{eq} of two capacitors in series, one with dielectric constant κ_1 and the other with dielectric constant κ_2. Each has plate area A and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1\varepsilon_0 A/d$, the correct result for a parallel-plate capacitor with plate area A, plate separation d, and dielectric constant κ_1.

With $A=7.89\times10^{-4}\,\text{m}^2$, $d=4.62\times10^{-3}\,\text{m}$, $\kappa_1=11.0,$ and $\kappa_2=12.0,$ the capacitance is

$$C=\frac{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(7.89\times10^{-4}\ \text{m}^2)}{4.62\times10^{-3}\ \text{m}}\frac{(11.0)(12.0)}{11.0+12.0}=1.73\times10^{-11}\,\text{F}.$$

50. Let

$$C_1 = \varepsilon_0(A/2)\kappa_1/2d = \varepsilon_0 A\kappa_1/4d,$$
$$C_2 = \varepsilon_0(A/2)\kappa_2/d = \varepsilon_0 A\kappa_2/2d,$$
$$C_3 = \varepsilon_0 A\kappa_3/2d.$$

Note that C_2 and C_3 are effectively connected in series, while C_1 is effectively connected in parallel with the C_2-C_3 combination. Thus,

$$C=C_1+\frac{C_2C_3}{C_2+C_3}=\frac{\varepsilon_0 A\kappa_1}{4d}+\frac{(\varepsilon_0\,A/d)\,(\kappa_2/2)\,(\kappa_3/2)}{\kappa_2/2+\kappa_3/2}=\frac{\varepsilon_0 A}{4d}\left(\kappa_1+\frac{2\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right).$$

With $A=1.05\times10^{-3}\text{m}^2$, $d=3.56\times10^{-3}\text{m}$, $\kappa_1=21.0$, $\kappa_2=42.0$ and $\kappa_3=58.0$, we find the capacitance to be

$$C=\frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.05\times10^{-3}\ \text{m}^2)}{4(3.56\times10^{-3}\ \text{m})}\left(21.0+\frac{2(42.0)(58.0)}{42.0+58.0}\right)=4.55\times10^{-11}\text{F}.$$

51. **THINK** We have a parallel-plate capacitor, so the capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation.

EXPRESS The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. Since the separation can be written as $d = \kappa\varepsilon_0 A/C$, we have $E = VC/\kappa\varepsilon_0 A$. The free charge on the plates is $q_f = CV$.

ANALYZE (a) Substituting the values given, we find the magnitude of the field strength to be

$$E = \frac{VC}{\kappa\varepsilon_0 A} = \frac{(50\ \text{V})(100\times10^{-12}\ \text{F})}{5.4(8.85\times10^{-12}\ \text{F/m})(100\times10^{-4}\ \text{m}^2)} = 1.0\times10^{4}\ \text{V/m}.$$

(b) Similarly, we have $q_f = CV = (100 \times 10^{-12}\ \text{F})(50\ \text{V}) = 5.0 \times 10^{-9}\ \text{C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A},$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \varepsilon_0 AE = 5.0\times10^{-9}\ \text{C} - (8.85\times10^{-12}\ \text{F/m})(100\times10^{-4}\ \text{m}^2)(1.0\times10^{4}\ \text{V/m}) \\ &= 4.1\times10^{-9}\ \text{C} = 4.1\,\text{nC}. \end{aligned}$$

LEARN An alternative way to calculate the induced charge is to apply Eq. 25-35:

$$q_i = q_f\left(1-\frac{1}{\kappa}\right) = (5.0\,\text{nC})\left(1-\frac{1}{5.4}\right) = 4.1\,\text{nC}.$$

Note that there's no induced charge ($q_i = 0$) in the absence of dielectric ($\kappa = 1$).

52. (a) The electric field E_1 in the free space between the two plates is $E_1 = q/\varepsilon_0 A$ while that inside the slab is $E_2 = E_1/\kappa = q/\kappa\varepsilon_0 A$. Thus,

$$V_0 = E_1(d-b) + E_2 b = \left(\frac{q}{\varepsilon_0 A}\right)\left(d - b + \frac{b}{\kappa}\right),$$

and the capacitance is

$$C = \frac{q}{V_0} = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(115\times10^{-4}\ \text{m}^2)(2.61)}{(2.61)(0.0124\,\text{m}-0.00780\,\text{m})+(0.00780\,\text{m})} = 13.4\,\text{pF}.$$

(b) $q = CV = (13.4 \times 10^{-12}\text{ F})(85.5\text{ V}) = 1.15\text{ nC}$.

(c) The magnitude of the electric field in the gap is

$$E_1 = \frac{q}{\varepsilon_0 A} = \frac{1.15\times10^{-9}\ \text{C}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(115\times10^{-4}\ \text{m}^2\right)} = 1.13\times10^4\ \text{N/C}.$$

(d) Using Eq. 25-34, we obtain

$$E_2 = \frac{E_1}{\kappa} = \frac{1.13\times10^4\ \text{N/C}}{2.61} = 4.33\times10^3\ \text{N/C}.$$

53. (a) Initially, the capacitance is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.12\ \text{m}^2)}{1.2\times10^{-2}\ \text{m}} = 89\ \text{pF}.$$

(b) Working through Sample Problem 25.06 — "Dielectric partially filling the gap in a capacitor" algebraically, we find:

$$C = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.12\,\text{m}^2)(4.8)}{(4.8)(1.2-0.40)(10^{-2}\,\text{m})+(4.0\times10^{-3}\,\text{m})} = 1.2\times10^2\ \text{pF}.$$

(c) Before the insertion, $q = C_0V$ (89 pF)(120 V) = 11 nC.

(d) Since the battery is disconnected, q will remain the same after the insertion of the slab, with q = 11 nC.

(e) $E = q/\varepsilon_0 A = 11\times10^{-9}\ \text{C}/(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.12\ \text{m}^2) = 10\ \text{kV/m}.$

(f) $E' = E/\kappa$ = (10 kV/m)/4.8 = 2.1 kV/m.

(g) The potential difference across the plates is

$V = E(d - b) + E'b$ = (10 kV/m)(0.012 m – 0.0040 m)+ (2.1 kV/m)(0.40 × 10^{-3} m) = 88 V.

(h) The work done is

$$W_{\text{ext}} = \Delta U = \frac{q^2}{2}\left(\frac{1}{C}-\frac{1}{C_0}\right) = \frac{(11\times10^{-9}\,\text{C})^2}{2}\left(\frac{1}{89\times10^{-12}\,\text{F}}-\frac{1}{120\times10^{-12}\,\text{F}}\right) = -1.7\times10^{-7}\ \text{J}.$$

54. (a) We apply Gauss's law with dielectric: $q/\varepsilon_0 = \kappa EA$, and solve for κ:

$$\kappa = \frac{q}{\varepsilon_0 EA} = \frac{8.9\times10^{-7}\ \text{C}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.4\times10^{-6}\ \text{V/m}\right)\left(100\times10^{-4}\text{m}^2\right)} = 7.2.$$

(b) The charge induced is $q' = q\left(1-\frac{1}{\kappa}\right) = \left(8.9\times10^{-7}\ \text{C}\right)\left(1-\frac{1}{7.2}\right) = 7.7\times10^{-7}\ \text{C}.$

55. (a) According to Eq. 25-17 the capacitance of an air-filled spherical capacitor is given by

$$C_0 = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right).$$

When the dielectric is inserted between the plates the capacitance is greater by a factor of the dielectric constant κ. Consequently, the new capacitance is

$$C = 4\pi\kappa\varepsilon_0\left(\frac{ab}{b-a}\right) = \frac{23.5}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\cdot\frac{(0.0120\ \text{m})(0.0170\ \text{m})}{0.0170\ \text{m}-0.0120\ \text{m}} = 0.107\ \text{nF}.$$

(b) The charge on the positive plate is $q = CV = (0.107\ \text{nF})(73.0\ \text{V}) = 7.79\ \text{nC}.$

(c) Let the charge on the inner conductor be $-q$. Immediately adjacent to it is the induced charge q'. Since the electric field is less by a factor $1/\kappa$ than the field when no dielectric is present, then $-q + q' = -q/\kappa$. Thus,

$$q' = \frac{\kappa-1}{\kappa}q = 4\pi\left(\kappa-1\right)\varepsilon_0\frac{ab}{b-a}V = \left(\frac{23.5-1.00}{23.5}\right)(7.79\ \text{nC}) = 7.45\ \text{nC}.$$

56. (a) The potential across C_1 is 10 V, so the charge on it is

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\ \text{V}) = 100\ \mu\text{C}.$$

(b) Reducing the right portion of the circuit produces an equivalence equal to 6.00 μF, with 10.0 V across it. Thus, a charge of 60.0 μC is on it, and consequently also on the bottom right capacitor. The bottom right capacitor has, as a result, a potential across it equal to

$$V = \frac{q}{C} = \frac{60\ \mu\text{C}}{10\ \mu\text{F}} = 6.00\ \text{V}$$

which leaves 10.0 V – 6.00 V = 4.00 V across the group of capacitors in the upper right portion of the circuit. Inspection of the arrangement (and capacitance values) of that group reveals that this 4.00 V must be equally divided by C_2 and the capacitor directly below it (in series with it). Therefore, with 2.00 V across C_2 we find

$$q_2 = C_2V_2 = (10.0\ \mu\text{F})(2.00\ \text{V}) = 20.0\ \mu\text{C}.$$

57. **THINK** Figure 25-51 depicts a system of capacitors. The pair C_3 and C_4 are in parallel.

EXPRESS Since C_3 and C_4 are in parallel, we replace them with an equivalent capacitance $C_{34} = C_3 + C_4 = 30\ \mu\text{F}$. Now, C_1, C_2, and C_{34} are in series, and all are numerically 30 μF, we observe that each has one-third the battery voltage across it. Hence, 3.0 V is across C_4.

ANALYZE The charge on capacitor 4 is $q_4 = C_4V_4 = (15\ \mu\text{F})(3.0\ \text{V}) = 45\ \mu\text{C}$.

LEARN Alternatively, one may show that the equivalent capacitance of the arrangement is given by

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_{34}} = \frac{1}{30\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} = \frac{1}{10\ \mu\text{F}}$$

or $C_{1234} = 10\ \mu\text{F}$. Thus, the charge across C_1, C_2, and C_{34} are

$$q_1 = q_2 = q_{34} = q_{1234} = C_{1234}V = (10\ \mu\text{F})(9.0\ \text{V}) = 90\ \text{nC}.$$

Now, since C_3 and C_4 are in parallel, and $C_3 = C_4$, the charge on C_4 (as well as on C_3) is $q_3 = q_4 = q_{34}/2 = (90\ \mu\text{F})/2 = 45\ \mu\text{F}$.

58. (a) Here D is not attached to anything, so that the $6C$ and $4C$ capacitors are in series (equivalent to $2.4C$). This is then in parallel with the $2C$ capacitor, which produces an equivalence of $4.4C$. Finally the $4.4C$ is in series with C and we obtain

$$C_{\text{eq}} = \frac{(C)(4.4C)}{C + 4.4C} = 0.82C = 0.82(50\ \mu\text{F}) = 41\ \mu\text{F}$$

where we have used the fact that $C = 50\ \mu\text{F}$.

(b) Now, B is the point that is not attached to anything, so that the $6C$ and $2C$ capacitors are now in series (equivalent to $1.5C$), which is then in parallel with the $4C$ capacitor (and thus equivalent to $5.5C$). The $5.5C$ is then in series with the C capacitor; consequently,

$$C_{\text{eq}} = \frac{(C)(5.5C)}{C + 5.5C} = 0.85C = 42\ \mu\text{F}\ .$$

59. The pair C_1 and C_2 are in parallel, as are the pair C_3 and C_4; they reduce to equivalent values 6.0 μF and 3.0 μF, respectively. These are now in series and reduce to 2.0 μF,

across which we have the battery voltage. Consequently, the charge on the 2.0 μF equivalence is (2.0 μF)(12 V) = 24 μC. This charge on the 3.0 μF equivalence (of C_3 and C_4) has a voltage of

$$V = \frac{q}{C} = \frac{24\ \mu\text{C}}{3\ \mu\text{F}} = 8.0\ \text{V}.$$

Finally, this voltage on capacitor C_4 produces a charge (2.0 μF)(8.0 V) = 16 μC.

60. (a) Equation 25-22 yields

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(200\times10^{-12}\ \text{F}\right)\left(7.0\times10^{3}\ \text{V}\right)^2 = 4.9\times10^{-3}\ \text{J}.$$

(b) Our result from part (a) is much less than the required 150 mJ, so such a spark should not have set off an explosion.

61. Initially the capacitors C_1, C_2, and C_3 form a series combination equivalent to a single capacitor, which we denote C_{123}. Solving the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_1C_2 + C_2C_3 + C_1C_3}{C_1C_2C_3},$$

we obtain C_{123} = 2.40 μF. With V = 12.0 V, we then obtain $q = C_{123}V$ = 28.8 μC. In the final situation, C_2 and C_4 are in parallel and are thus effectively equivalent to $C_{24} = 12.0\ \mu\text{F}$. Similar to the previous computation, we use

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_{24}} + \frac{1}{C_3} = \frac{C_1C_{24} + C_{24}C_3 + C_1C_3}{C_1C_{24}C_3}$$

and find C_{1234} = 3.00 μF. Therefore, the final charge is $q = C_{1234}V$ = 36.0 μC.

(a) This represents a change (relative to the initial charge) of Δq = 7.20 μC.

(b) The capacitor C_{24} which we imagined to replace the parallel pair C_2 and C_4, is in series with C_1 and C_3 and thus also has the final charge q =36.0 μC found above. The voltage across C_{24} would be

$$V_{24} = \frac{q}{C_{24}} = \frac{36.0\ \mu\text{C}}{12.0\ \mu\text{F}} = 3.00\ \text{V}.$$

This is the same voltage across each of the parallel pairs. In particular, V_4 = 3.00 V implies that $q_4 = C_4V_4$ = 18.0 μC.

(c) The battery supplies charges only to the plates where it is connected. The charges on the rest of the plates are due to electron transfers between them, in accord with the new distribution of voltages across the capacitors. So, the battery does not directly supply the charge on capacitor 4.

62. In series, their equivalent capacitance (and thus their total energy stored) is smaller than either one individually (by Eq. 25-20). In parallel, their equivalent capacitance (and thus their total energy stored) is larger than either one individually (by Eq. 25-19). Thus, the middle two values quoted in the problem must correspond to the individual capacitors. We use Eq. 25-22 and find

(a) $100\ \mu\text{J} = \frac{1}{2}C_1(10\text{ V})^2 \Rightarrow C_1 = 2.0\ \mu\text{F}$;

(b) $300\ \mu\text{J} = \frac{1}{2}C_2(10\text{ V})^2 \Rightarrow C_2 = 6.0\ \mu\text{F}$.

63. Initially, the total equivalent capacitance is $C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 3.0\ \mu\text{F}$, and the charge on the positive plate of each one is $(3.0\ \mu\text{F})(10\text{ V}) = 30\ \mu\text{C}$. Next, the capacitor (call is C_1) is squeezed as described in the problem, with the effect that the new value of C_1 is 12 μF (see Eq. 25-9). The new total equivalent capacitance then becomes

$$C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 4.0\ \mu\text{F},$$

and the new charge on the positive plate of each one is $(4.0\ \mu\text{F})(10\text{ V}) = 40\ \mu\text{C}$.

(a) Thus we see that the charge transferred from the battery as a result of the squeezing is $40\ \mu\text{C} - 30\ \mu\text{C} = 10\ \mu\text{C}$.

(b) The total increase in positive charge (on the respective positive plates) stored on the capacitors is twice the value found in part (a) (since we are dealing with two capacitors in series): 20 μC.

64. (a) We reduce the parallel group C_2, C_3 and C_4, and the parallel pair C_5 and C_6, obtaining equivalent values $C' = 12\ \mu\text{F}$ and $C'' = 12\ \mu\text{F}$, respectively. We then reduce the series group C_1, C' and C'' to obtain an equivalent capacitance of $C_{eq} = 3\ \mu\text{F}$ hooked to the battery. Thus, the charge stored in the system is $q_{sys} = C_{eq}V_{bat} = 36\ \mu\text{C}$.

(b) Since $q_{sys} = q_1$, then the voltage across C_1 is

$$V_1 = \frac{q_1}{C_1} = \frac{36\ \mu\text{C}}{6.0\ \mu\text{F}} = 6.0\text{ V}.$$

The voltage across the series-pair C' and C'' is consequently $V_{bat} - V_1 = 6.0$ V. Since $C' = C''$, we infer $V' = V'' = 6.0/2 = 3.0$ V, which, in turn, is equal to V_4, the potential across C_4. Therefore,

$$q_4 = C_4V_4 = (4.0\ \mu\text{F})(3.0\text{ V}) = 12\ \mu\text{C}.$$

65. **THINK** We may think of the arrangement as two capacitors connected in series.

EXPRESS Let the capacitances be C_1 and C_2, with the former filled with the $\kappa_1 = 3.00$ material and the latter with the κ_2 = 4.00 material. Upon using Eq. 25-9, Eq. 25-27, and reducing C_1 and C_2 to an equivalent capacitance, we have

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\kappa_1 \varepsilon_0 A/d} + \frac{1}{\kappa_2 \varepsilon_0 A/d} = \left(\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2}\right)\frac{d}{\varepsilon_0 A}$$

or $C_{\text{eq}} = \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)\frac{\varepsilon_0 A}{d}$. The charge stored on the capacitor is $q = C_{\text{eq}}V$.

ANALYZE Substituting the values given, we find

$$C_{\text{eq}} = \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)\frac{\varepsilon_0 A}{d} = 1.52 \times 10^{-10} \text{ F},$$

Therefore, $q = C_{\text{eq}}V = 1.06 \times 10^{-9}$ C.

LEARN In the limit where $\kappa_1 = \kappa_2 = \kappa$, our expression for C_{eq} becomes $C_{\text{eq}} = \frac{\kappa \varepsilon_0 A}{2d}$, where $2d$ is the plate separation.

66. We first need to find an expression for the energy stored in a cylinder of radius R and length L, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\varepsilon_0 E^2$, where E is the magnitude of the electric field at that point. If q is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance r from the cylinder axis is given by (see Eq. 25-12)

$$E = \frac{q}{2\pi\varepsilon_0 L r},$$

and the energy density at that point is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi^2 \varepsilon_0 L^2 r^2}.$$

The corresponding energy in the cylinder is the volume integral $U_R = \int u\, d\mathcal{V}$. Now, $d\mathcal{V} = 2\pi r L\, dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2 \varepsilon_0 L^2 r^2} 2\pi r L\, dr = \frac{q^2}{4\pi\varepsilon_0 L}\int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\varepsilon_0 L}\ln\left(\frac{R}{a}\right).$$

To find an expression for the total energy stored in the capacitor, we replace R with b:

$$U_b = \frac{q^2}{4\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right).$$

We want the ratio U_R/U_b to be 1/2, so

$$\ln\frac{R}{a} = \frac{1}{2}\ln\frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln\left(\sqrt{b/a}\right)$, $\ln(R/a) = \ln\left(\sqrt{b/a}\right)$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.

67. (a) The equivalent capacitance is $C_{\text{eq}} = \dfrac{C_1C_2}{C_1+C_2} = \dfrac{(6.00\,\mu\text{F})(4.00\,\mu\text{F})}{6.00\,\mu\text{F}+4.00\,\mu\text{F}} = 2.40\,\mu\text{F}$.

(b) $q_1 = C_{\text{eq}}V = (2.40\ \mu\text{F})(200\text{ V}) = 4.80 \times 10^{-4}$ C.

(c) $V_1 = q_1/C_1 = 4.80 \times 10^{-4}\text{ C}/6.00\ \mu\text{F} = 80.0$ V.

(d) $q_2 = q_1 = 4.80 \times 10^{-4}$ C.

(e) $V_2 = V - V_1 = 200\text{ V} - 80.0\text{ V} = 120$ V.

68. (a) Now $C_{\text{eq}} = C_1 + C_2 = 6.00\ \mu\text{F} + 4.00\ \mu\text{F} = 10.0\ \mu\text{F}$.

(b) $q_1 = C_1V = (6.00\ \mu\text{F})(200\text{ V}) = 1.20 \times 10^{-3}$ C.

(c) $V_1 = 200$ V.

(d) $q_2 = C_2V = (4.00\ \mu\text{F})(200\text{ V}) = 8.00 \times 10^{-4}$ C.

(e) $V_2 = V_1 = 200$ V.

69. We use $U = \frac{1}{2}CV^2$. As V is increased by ΔV, the energy stored in the capacitor increases correspondingly from U to $U + \Delta U$: $U + \Delta U = \frac{1}{2}C(V+\Delta V)^2$. Thus,
$(1 + \Delta V/V)^2 = 1 + \Delta U/U$, or

$$\frac{\Delta V}{V} = \sqrt{1+\frac{\Delta U}{U}} - 1 = \sqrt{1+10\%} - 1 = 4.9\%\ .$$

70. (a) The length d is effectively shortened by b so $C' = \varepsilon_0 A/(d - b) = 0.708$ pF.

(b) The energy before, divided by the energy after inserting the slab is

$$\frac{U}{U'}=\frac{q^2/2C}{q^2/2C'}=\frac{C'}{C}=\frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d}=\frac{d}{d-b}=\frac{5.00}{5.00-2.00}=1.67.$$

(c) The work done is

$$W=\Delta U=U'-U=\frac{q^2}{2}\left(\frac{1}{C'}-\frac{1}{C}\right)=\frac{q^2}{2\varepsilon_0 A}(d-b-d)=-\frac{q^2 b}{2\varepsilon_0 A}=-5.44\text{ J.}$$

(d) Since $W < 0$, the slab is sucked in.

71. (a) $C' = \varepsilon_0 A/(d - b) = 0.708$ pF, the same as part (a) in Problem 25-70.

(b) The ratio of the stored energy is now

$$\frac{U}{U'}=\frac{\frac{1}{2}CV^2}{\frac{1}{2}C'V^2}=\frac{C}{C'}=\frac{\varepsilon_0 A/d}{\varepsilon_0 A/(d-b)}=\frac{d-b}{d}=\frac{5.00-2.00}{5.00}=0.600.$$

(c) The work done is

$$W=\Delta U=U'-U=\frac{1}{2}(C'-C)V^2=\frac{\varepsilon_0 A}{2}\left(\frac{1}{d-b}-\frac{1}{d}\right)V^2=\frac{\varepsilon_0 AbV^2}{2d(d-b)}=1.02\times10^{-9}\text{ J.}$$

(d) In Problem 25-70 where the capacitor is disconnected from the battery and the slab is sucked in, F is certainly given by $-dU/dx$. However, that relation does not hold when the battery is left attached because the force on the slab is not conservative. The charge distribution in the slab causes the slab to be sucked into the gap by the charge distribution on the plates. This action causes an increase in the potential energy stored by the battery in the capacitor.

72. (a) The equivalent capacitance is $C_{eq} = C_1C_2/(C_1 + C_2)$. Thus the charge q on each capacitor is

$$q=q_1=q_2=C_{eq}V=\frac{C_1C_2V}{C_1+C_2}=\frac{(2.00\mu\text{F})(8.00\mu\text{F})(300\text{V})}{2.00\mu\text{F}+8.00\mu\text{F}}=4.80\times10^{-4}\text{C.}$$

(b) The potential difference is $V_1 = q/C_1 = 4.80 \times 10^{-4}$ C/2.0 μF = 240 V.

(c) As noted in part (a), $q_2=q_1=4.80\times10^{-4}\text{C}$.

(d) $V_2 = V - V_1 = 300$ V – 240 V = 60.0 V.

Now we have $q'_1/C_1 = q'_2/C_2 = V'$ (V' being the new potential difference across each capacitor) and $q'_1 + q'_2 = 2q$. We solve for q'_1, q'_2 and V:

(e) $q_1' = \dfrac{2C_1q}{C_1+C_2} = \dfrac{2(2.00\,\mu\text{F})(4.80\times10^{-4}C)}{2.00\ \mu\text{F}+8.00\ \mu\text{F}} = 1.92\times10^{-4}\,\text{C}.$

(f) $V_1' = \dfrac{q_1'}{C_1} = \dfrac{1.92\times10^{-4}C}{2.00\,\mu\text{F}} = 96.0\,\text{V}.$

(g) $q_2' = 2q - q_1 = 7.68\times10^{-4}C.$

(h) $V_2' = V_1' = 96.0\,\text{V}.$

(i) In this circumstance, the capacitors will simply discharge themselves, leaving $q_1 = 0$,

(j) $V_1 = 0$,

(k) $q_2 = 0$,

(l) and $V_2 = V_1 = 0$.

73. The voltage across capacitor 1 is

$$V_1 = \frac{q_1}{C_1} = \frac{30\,\mu\text{C}}{10\,\mu\text{F}} = 3.0\ \text{V}\ .$$

Since $V_1 = V_2$, the total charge on capacitor 2 is

$$q_2 = C_2V_2 = (20\,\mu\text{F})(2\ \text{V}) = 60\,\mu\text{C}\ ,$$

which means a total of 90 μC of charge is on the pair of capacitors C_1 and C_2. This implies there is a total of 90 μC of charge also on the C_3 and C_4 pair. Since $C_3 = C_4$, the charge divides equally between them, so $q_3 = q_4 = 45\ \mu$C. Thus, the voltage across capacitor 3 is

$$V_3 = \frac{q_3}{C_3} = \frac{45\,\mu\text{C}}{20\,\mu\text{F}} = 2.3\,\text{V}\ .$$

Therefore, $|V_A - V_B| = V_1 + V_3 = 5.3$ V.

74. We use $C = \varepsilon_0\kappa A/d \propto \kappa/d$. To maximize C we need to choose the material with the greatest value of κ/d. It follows that the mica sheet should be chosen.

75. We cannot expect simple energy conservation to hold since energy is presumably dissipated either as heat in the hookup wires or as radio waves while the charge oscillates in the course of the system "settling down" to its final state (of having 40 V across the parallel pair of capacitors C and 60 μF). We do expect charge to be conserved. Thus, if Q is the charge originally stored on C and q_1, q_2 are the charges on the parallel pair after "settling down," then

$$Q = q_1 + q_2 \quad \Rightarrow \quad C(100\,\text{V}) = C(40\,\text{V}) + (60\,\mu\text{F})(40\,\text{V})$$

which leads to the solution $C = 40\ \mu$F.

76. One way to approach this is to note that since they are identical, the voltage is evenly divided between them. That is, the voltage across each capacitor is $V = (10/n)$ volt. With $C = 2.0 \times 10^{-6}$ F, the electric energy stored by each capacitor is $\frac{1}{2}CV^2$. The total energy stored by the capacitors is n times that value, and the problem requires the total be equal to 25×10^{-6} J. Thus,

$$\frac{n}{2}(2.0 \times 10^{-6}) \left(\frac{10}{n}\right)^2 = 25 \times 10^{-6},$$

which leads to $n = 4$.

77. **THINK** We have two parallel-plate capacitors that are connected in parallel. They both have the same plate separation and same potential difference across their plates.

EXPRESS The magnitude of the electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The surface charge density on the plate is $\sigma = q/A$.

ANALYZE (a) With $d = 0.00300$ m and $V = 600$ V, we have

$$E_A = \frac{V}{d} = \frac{600\ \text{V}}{3.00 \times 10^{-3}\ \text{m}} = 2.00 \times 10^5\ \text{V/m}.$$

(b) Since $d = 0.00300$ m and $V = 600$ V in capacitor B as well, $E_B = 2.00 \times 10^5$ V/m.

(c) For the air-filled capacitor, Eq. 25-4 leads to

$$\sigma_A = \frac{q_A}{A} = \frac{C_A V}{A} = \frac{(\varepsilon_0 A/d)V}{A} = \frac{\varepsilon_0 V}{d} = \varepsilon_0 E_A = (8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^5\ \text{V/m})$$
$$= 1.77 \times 10^{-6}\ \text{C/m}^2.$$

(d) For the dielectric-filled capacitor, we use Eq. 25-29:

$$\sigma_B = \kappa\varepsilon_0 E_B = (2.60)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^5\ \text{V/m}) = 4.60 \times 10^{-6}\ \text{C/m}^2.$$

(e) Although the discussion in Section 25-8 of the textbook is in terms of the charge being held fixed (while a dielectric is inserted), it is readily adapted to this situation (where comparison is made of two capacitors that have the same *voltage* and are identical except for the fact that one has a dielectric). The fact that capacitor B has a relatively large charge but only produces the field that A produces (with its smaller charge) is in line with the point being made (in the text) with Eq. 25-34 and in the material that follows. Adapting Eq. 25-35 to this problem, we see that the difference in charge densities between parts (c) and (d) is due, in part, to the (negative) layer of charge at the top surface of the dielectric; consequently,

$$\sigma_{\text{ind}} = \sigma_A - \sigma_B = (1.77\times10^{-6}\ \text{C/m}^2) - (4.60\times10^{-6}\ \text{C/m}^2) = -2.83\times10^{-6}\ \text{C/m}^2\ .$$

LEARN We note that the electric field in capacitor B is produced by both the charge on the plates ($\sigma_B A$) and the induced charges ($\sigma_{\text{ind}} A$), while the field in capacitor A is produced by the charge on the plates alone ($\sigma_A A$). Since $E_A = E_B$, we have $\sigma_A = \sigma_B + \sigma_{\text{ind}}$, or $\sigma_{\text{ind}} = \sigma_A - \sigma_B$.

78. (a) Put five such capacitors in series. Then, the equivalent capacitance is 2.0 μF/5 = 0.40 μF. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

(b) As one possibility, you can take three identical arrays of capacitors, each array being a five-capacitor combination described in part (a) above, and hook up the arrays in parallel. The equivalent capacitance is now C_{eq} = 3(0.40 μF) = 1.2 μF. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

79. (a) For a capacitor with surface area A and plate separation x its capacitance is given by $C_0 = \varepsilon_0 A/x$. The energy stored in the capacitor can be written as

$$U = \frac{q^2}{2C} = \frac{q^2}{2(\varepsilon_0 A/x)} = \frac{q^2 x}{2\varepsilon_0 A}.$$

The change in energy if the separation between plates increases to $x + dx$ is

$$dU = \frac{q^2}{2\varepsilon_0 A}dx.$$

Thus, the force between the plates is

$$F = -\frac{dU}{dx} = -\frac{q^2}{2\varepsilon_0 A}.$$

The negative sign means that the force between the plates is attractive.

(b) The magnitude of the electrostatic stress is

$$\frac{|F|}{A} = \frac{q^2}{2\varepsilon_0 A^2} = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

where $E = \sigma/\varepsilon_0$ is the magnitude of the electric field in the region between the plates.

80. The energy initially stored in one capacitor is $U_0 = q_0^2/2C = 4.00$ J. After a second capacitor is connected to it in parallel, with $q_1 = q_2 = q_0/2,$ the energy stored in the first capacitor becomes

$$U_1 = \frac{q_1^2}{2C} = \frac{(q_0/2)^2}{2C} = \frac{U_0}{4} = 1.00 \text{ J}$$

which is the same as that stored in the second capacitor. Thus, the total energy is

$$U = U_1 + U_2 = \frac{U_0}{2} = 2.00 \text{ J.}$$

(b) The wires connecting the capacitors have resistance, so some energy is converted to thermal energy in the wires, as well as electromagnetic radiation.